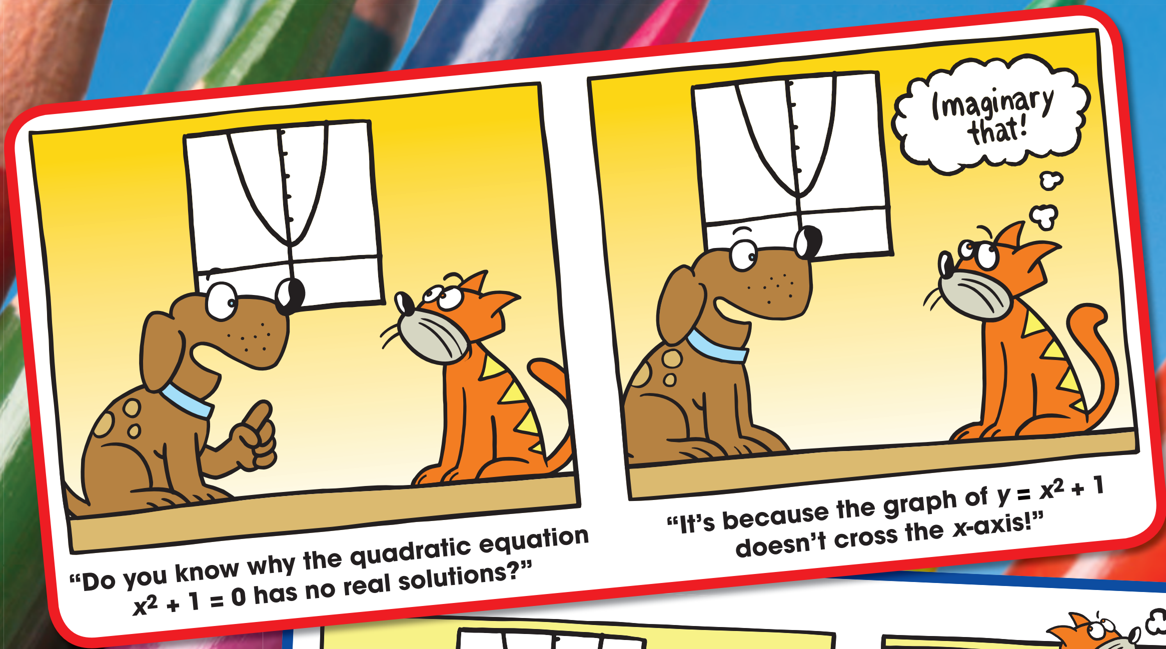
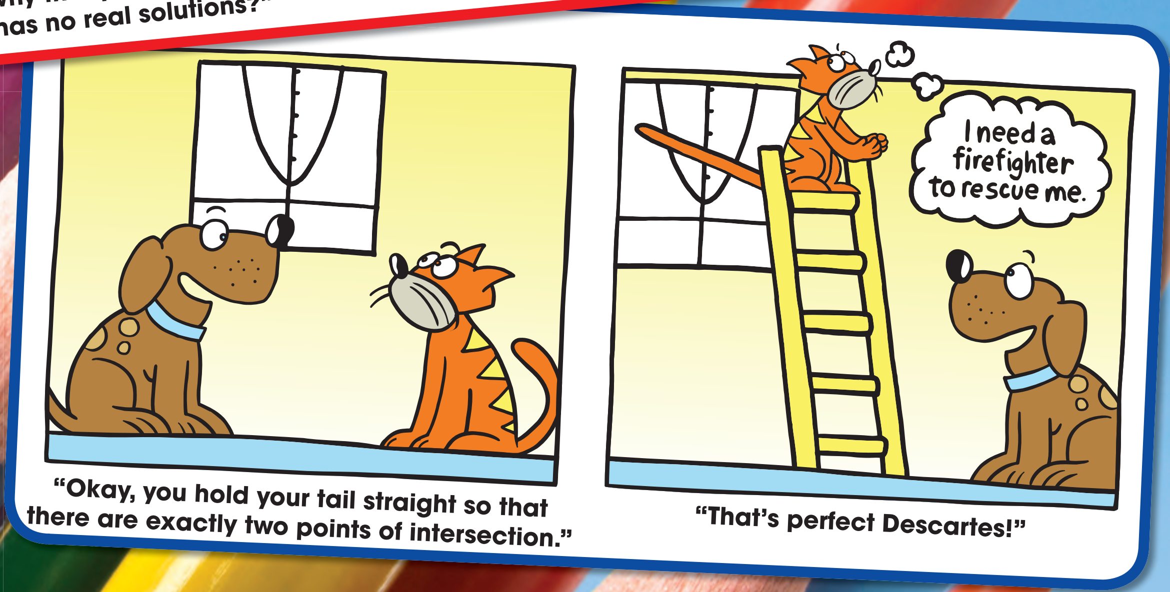
9 Solving Quadratic Equations





Connections to Previous Learning

• Write and solve one-step linear equations in one variable. • Evaluate expressions at specific values of their variables.
• Write and solve multi-step linear equations in one variable with one solution, no solution, or infinitely many solutions. • Evaluate square roots of perfect squares.
• Solve quadratic equations in one variable by graphing, using square roots, completing the square, and using the quadratic formula. • Derive the quadratic formula by completing the square. • Solve a system of equations consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

Chapter Summary

Section	Common Core State Standard	
9.1	Learning	A.REI.4b, A.REI.11 ★
9.2	Learning	A.REI.4b
9.3	Learning	A.REI.4a, A.REI.4b, A.SSE.3b ★, F.IF.8a ★
9.4	Learning	A.REI.4a ★, A.REI.4b ★
9.5	Learning	A.REI.7 ★
★ Teaching is complete. Standard can be assessed.		

Pacing Guide for Chapter 9

Chapter Opener	1 Day
Section 1	2 Days
Section 2	1 Day
Section 3	1 Day
Study Help / Quiz	1 Day
Section 4	3 Days
Section 5	2 Days
Chapter Review / Chapter Tests	2 Days
Total Chapter 9	13 Days
Year-to-Date	126 Days

Technology for the Teacher

BigIdeasMath.com
Chapter at a Glance
Complete Materials List
Parent Letters: English and Spanish

Common Core State Standards

8.EE.2 . . . Evaluate square roots of small perfect squares . . ..

N.RN.2 Rewrite expressions involving radicals . . . using the properties of exponents.

A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Additional Topics for Review

- Graphing quadratic functions
- Solving polynomial equations in factored form
- Factoring polynomials
- Special products of polynomials
- Solving simple equations

Try It Yourself

1. 9 **2.** -13

3. $\pm\frac{3}{5}$ **4.** -2.5

5. $3\sqrt{6}$ **6.** $4\sqrt{5}$

7. $10\sqrt{2}$ **8.** $(x+5)^2$

9. $(m-10)^2$ **10.** $(p+6)^2$

Record and Practice Journal Fair Game Review

1. -6 **2.** 11

3. $\frac{2}{7}$ **4.** ± 1.5

5. 3 ft **6.** 0.5 m

7. $2\sqrt{5}$ **8.** $3\sqrt{7}$

9. $6\sqrt{3}$ **10.** $12\sqrt{2}$

11. $5\sqrt{5}$ ft **12.** $8\sqrt{3}$ m

13. $(y-3)^2$ **14.** $(b+9)^2$

15. $(n+14)^2$ **16.** $(h-8)^2$

17. **a.** $(x-25)$ in.

b. $4(x-25)$ in.

Math Background Notes

Vocabulary Review

- Perfect square
- Radical sign
- Radicand
- Perfect Square Trinomial

Finding Square Roots

- Students should know how to find the square root of a perfect square.
- To find a square root of a given number, find a number that you can square to get the given number.
- **Teaching Tip:** Remind students that $\sqrt{n}$ represents the positive square root of n. Every number n has a positive square root $\sqrt{n}$ and a negative square root $-\sqrt{n}$.

Simplifying Square Roots

- Students should know how to simplify the square root of a number that is not a perfect square.
- **Teaching Tip:** The key to simplifying a square root is to factor using the greatest perfect square factor of the radicand.
- **Common Error:** Students may omit the step of using the Product Property of Square Roots and take the wrong quantity outside the radical sign. Emphasize the second and third steps in Example 3.

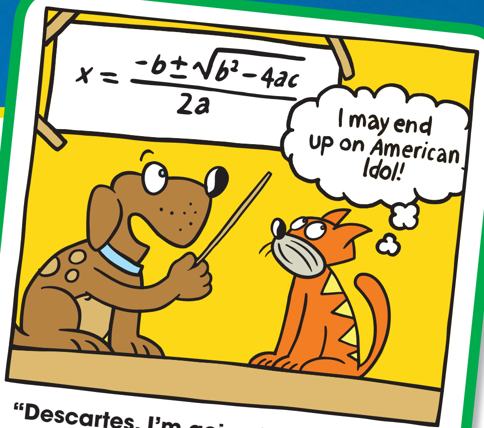
Factoring Perfect Square Trinomials

- Students should know how to factor a perfect square trinomial.
- Remind students of the Perfect Square Trinomial Patterns.
  $a^2 + 2ab + b^2 = (a+b)^2$
  $a^2 - 2ab + b^2 = (a-b)^2$

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Record and Practice Journal • Fair Game Review Skills Review Handbook Lesson Tutorials	Game Closet at *BigIdeasMath.com* Start the next section

What You Learned Before



"Descartes, I'm going to teach you to sing the Quadratic Formula."

Finding Square Roots (8.EE.2)

Example 1 Find $\sqrt{144}$.

Because $12^2 = 144$, $\sqrt{144} = \sqrt{12^2} = 12$.

Positive square root

Example 2 Find $-\sqrt{225}$.

Because $15^2 = 225$, $-\sqrt{225} = -\sqrt{15^2} = -15$.

Negative square root

Try It Yourself

Find the square root(s).

1. $\sqrt{81}$
2. $-\sqrt{169}$
3. $\pm\sqrt{\frac{9}{25}}$
4. $-\sqrt{6.25}$

Simplifying Square Roots (N.RN.2)

Example 3 Simplify $\sqrt{75}$.

$$\begin{aligned} \sqrt{75} &= \sqrt{25 \cdot 3} && \text{Factor using the greatest perfect square factor.} \\ &= \sqrt{25} \cdot \sqrt{3} && \text{Use the Product Property of Square Roots.} \\ &= 5\sqrt{3} && \text{Simplify.} \end{aligned}$$

Try It Yourself

5. Simplify $\sqrt{54}$.
6. Simplify $\sqrt{80}$.
7. Simplify $\sqrt{200}$.

Factoring Perfect Square Trinomials (A.SSE.2)

Example 4 Factor $x^2 + 14x + 49$.

$$\begin{aligned} x^2 + 14x + 49 &= x^2 + 2(x)(7) + 7^2 && \text{Write as } a^2 + 2ab + b^2. \\ &= (x + 7)^2 && \text{Perfect Square Trinomial Pattern} \end{aligned}$$

Example 5 Factor $y^2 - 10y + 25$.

$$\begin{aligned} y^2 - 10y + 25 &= y^2 - 2(y)(5) + 5^2 && \text{Write as } a^2 - 2ab + b^2. \\ &= (y - 5)^2 && \text{Perfect Square Trinomial Pattern} \end{aligned}$$

Try It Yourself

Factor the trinomial.

8. $x^2 + 10x + 25$
9. $m^2 - 20m + 100$
10. $p^2 + 12p + 36$

9.1 Solving Quadratic Equations by Graphing

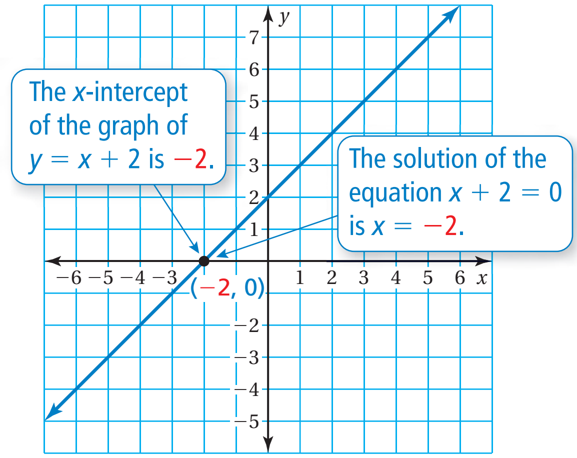
Essential Question How can you use a graph to solve a quadratic equation in one variable?

Earlier in the book, you learned that the x-intercept of the graph of

$$y = ax + b \qquad \text{2 variables}$$

is the same as the solution of

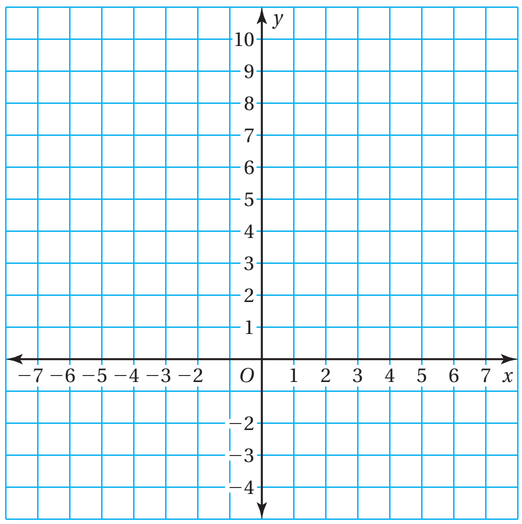
$$ax + b = 0. \qquad \text{1 variable}$$



1 ACTIVITY: Solving a Quadratic Equation by Graphing

Work with a partner.

a. Sketch the graph of $y = x^2 - 2x$.



b. What is the definition of an x-intercept of a graph? How many x-intercepts does this graph have? What are they?

c. What is the definition of a solution of an equation in x? How many solutions does the equation $x^2 - 2x = 0$ have? What are they?

d. Explain how you can verify that the x-values found in part (c) are solutions of $x^2 - 2x = 0$.

Common Core

Solving Quadratic Equations

In this lesson, you will
- solve quadratic equations by graphing.

Learning Standards
A.REI.4b
A.REI.11

Laurie's Notes

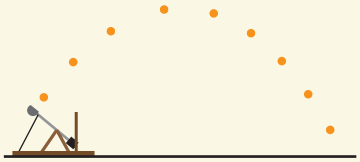
Introduction

Standards for Mathematical Practice

- **MP1a Make Sense of Problems:** When students solved quadratic equations in factored form, they used the Zero-Product Property and reasoned, "What value of x makes each factor zero?" Now they are reasoning "What values of x make the equation zero?" The values of the x-intercepts make the equation zero.

Motivate

- Show a picture of a projectile being launched from a trebuchet or a catapult.
- Ask students what they would like to know about the projectile. Hopefully a student will ask how long it takes for the projectile to land.
- Explain that in this section they will answer that type of question.



Activity Notes

Discuss

- Connect the projectile motion to the x-intercept dialogue.
- Also, connect the previous two chapters to this section by reminding students that they solved quadratic equations by factoring in Chapter 7, and they graphed quadratic functions in Chapter 8.

Activity 1

❓ "What do you know about the graph of $y = x^2 - 2x$?" opens up; y-intercept is 0; x-coordinate of the vertex is 1; The axis of symmetry is $x = 1$.

- Students should make an input-output table that includes x-values that show the key features of the graph.
- While students work, question different pairs of students about the vertex and the minimum value.
- **Big Idea:** In part (d), students should discuss evaluating the left side of the equation for the x-values found in part (c) and they should also discuss the x-intercept. It is not enough to solve the equation graphically because computational errors can influence the graph. An algebraic check is also important.
- **MP1a:** This approach to solving a quadratic equation should make sense to students if they think back to solving systems of equations. They can think of solving $x^2 - 2x = 0$ as solving the system $y = x^2 - 2x$ and $y = 0$. The graph of $y = 0$ is the x-axis, which intersects the graph of $y = x^2 - 2x$ at its x-intercepts.

Common Core State Standards

A.REI.4 Solve quadratic equations in one variable.

A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately . . ..

Previous Learning

Students should know how to graph a quadratic function and find the x-intercepts.

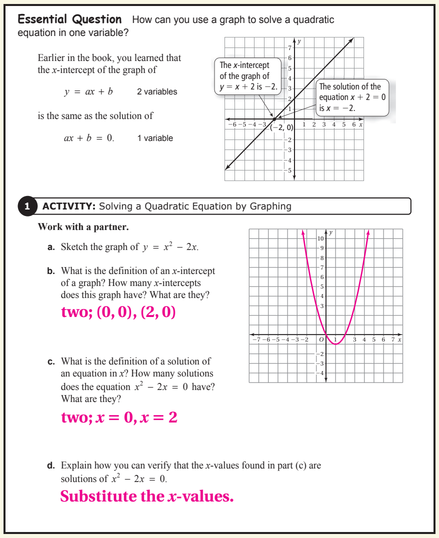
Technology for the Teacher

Dynamic Classroom

Lesson Plans

Complete Materials List

9.1 Record and Practice Journal



Essential Question How can you use a graph to solve a quadratic equation in one variable?

Earlier in the book, you learned that the x-intercept of the graph of

$y = ax + b$ 2 variables

is the same as the solution of

$ax + b = 0.$ 1 variable

1 ACTIVITY: Solving a Quadratic Equation by Graphing

Work with a partner.

a. Sketch the graph of $y = x^2 - 2x$.

b. What is the definition of an x-intercept of a graph? How many x-intercepts does this graph have? What are they?

two; (0, 0), (2, 0)

c. What is the definition of a solution of an equation in x? How many solutions does the equation $x^2 - 2x = 0$ have? What are they?

two; $x = 0$, $x = 2$

d. Explain how you can verify that the x-values found in part (c) are solutions of $x^2 - 2x = 0$.

Substitute the x-values.

Differentiated Instruction

Auditory

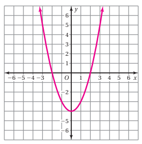
After Activity 1, ask students to discuss the differences between an equation of the form $ax^2 + bx + c = 0$ and its related function $y = ax^2 + bx + c$. Talk about how the equation can be solved by graphing its related function and discuss the relationship between solutions and x-intercepts.

9.1 Record and Practice Journal

2 ACTIVITY: Solving Quadratic Equations by Graphing

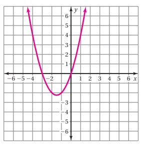
Work with a partner. Solve each equation by graphing.

a. $x^2 - 4 = 0$



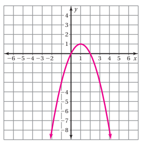
$x = -2, 2$

b. $x^2 + 3x = 0$



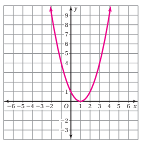
$x = -3, 0$

c. $-x^2 + 2x = 0$



$x = 0, 2$

d. $x^2 - 2x + 1 = 0$



$x = 1$

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you use a graph to solve a quadratic equation in one variable?

Get the equation equal to zero, set it equal to y, graph the resulting equation, and find its x-intercepts.

4. After you find a solution graphically, how can you check your result algebraically? Use your solutions in Activity 2 as examples.

Check that the x-values of the x-intercepts satisfy the original equation.

Laurie's Notes

Activity 2

- Explain to students that in Activity 2 they will solve four additional quadratic equations in one variable.
- As they are solving each equation by graphing, students should be checking the reasonableness of their graphs. For instance, the graph in part (c) is a parabola that opens down.
- While students work, probe different pairs of students about the vertex, the minimum or maximum value, the y-intercept, and the general shape of the graph.

? "In Activity 1, the equation had two solutions. Does each equation in Activity 2 have two solutions?" no; The equation in part (d) has only 1 solution.

? "Do you think it is possible to predict how many solutions a quadratic equation in one variable will have?" Answers will vary.

What Is Your Answer?

- **MP1a:** In Question 4, students are making sense of a problem in more than one way. It is important for students to realize that the x-intercepts correspond to the solutions of the equation, so substituting these values for x should result in a true equation.

Closure

- **Writing Prompt:** To solve a quadratic equation by graphing you . . .

2 ACTIVITY: Solving Quadratic Equations by Graphing

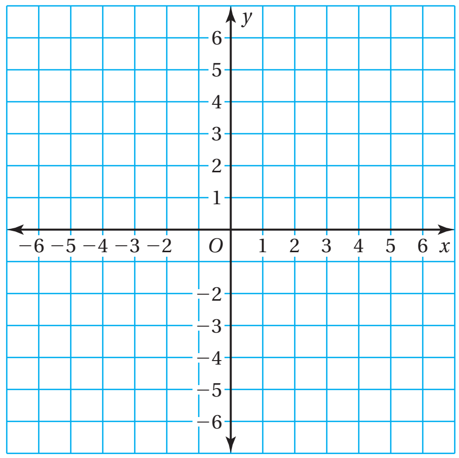
Math Practice 6

Use Clear Definitions

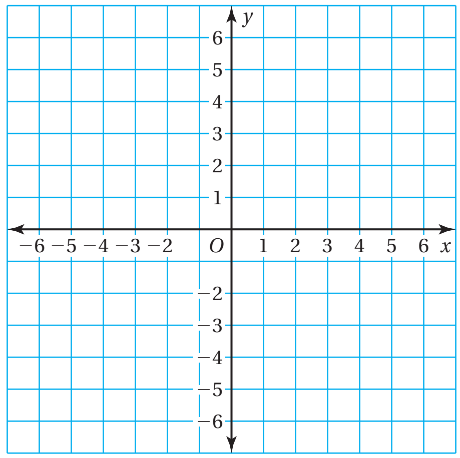
How is the solution of the equation represented by the graph of the equation?

Work with a partner. Solve each equation by graphing.

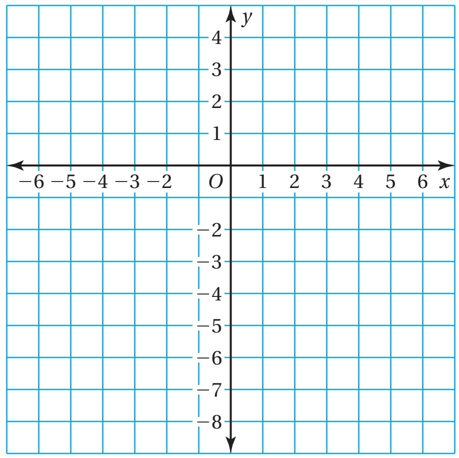
a. $x^2 - 4 = 0$



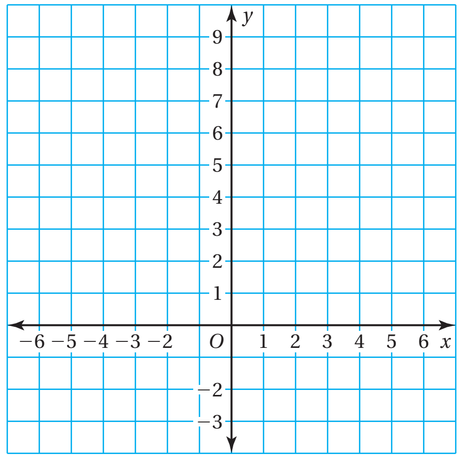
b. $x^2 + 3x = 0$



c. $-x^2 + 2x = 0$



d. $x^2 - 2x + 1 = 0$



What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you use a graph to solve a quadratic equation in one variable?

4. After you find a solution graphically, how can you check your result algebraically? Use your solutions in Activity 2 as examples.

Practice Use what you learned about solving quadratic equations to complete Exercises 5–7 on page 459.

9.1 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
quadratic equation, *p. 456*

A **quadratic equation** is a nonlinear equation that can be written in the standard form $ax^2 + bx + c = 0$, where $a \neq 0$.

In Chapter 7, you solved quadratic equations by factoring. You can also solve quadratic equations in standard form by finding the x-intercept(s) of the graph of the related function $y = ax^2 + bx + c$.

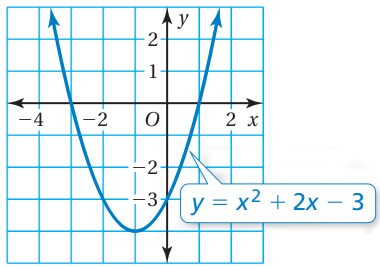
EXAMPLE 1 Solving a Quadratic Equation: Two Real Solutions

Solve $x^2 + 2x - 3 = 0$ by graphing.

Step 1: Graph the related function $y = x^2 + 2x - 3$.

Step 2: Find the x-intercepts. They are -3 and 1.

So, the solutions are $x = -3$ and $x = 1$.



Remember
The solutions of a quadratic equation are also called roots.

Check Check each solution in the original equation.

$x^2 + 2x - 3 = 0$	Original equation	$x^2 + 2x - 3 = 0$
$(-3)^2 + 2(-3) - 3 \stackrel{?}{=} 0$	Substitute.	$1^2 + 2(1) - 3 \stackrel{?}{=} 0$
$0 = 0$ ✓	Simplify.	$0 = 0$ ✓

EXAMPLE 2 Solving a Quadratic Equation: One Real Solution

Solve $x^2 - 8x = -16$ by graphing.

Step 1: Rewrite the equation in standard form.

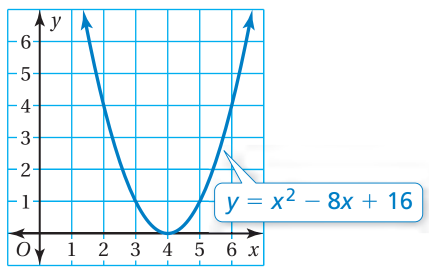
$$x^2 - 8x = -16 \qquad \text{Write the equation.}$$

$$x^2 - 8x + 16 = 0 \qquad \text{Add 16 to each side.}$$

Step 2: Graph the related function $y = x^2 - 8x + 16$.

Step 3: Find the x-intercept. The only x-intercept is at the vertex $(4, 0)$.

So, the solution is $x = 4$.



Study Tip
You can also solve the equation in Example 2 by factoring.

$x^2 - 8x + 16 = 0$

$(x - 4)(x - 4) = 0$

So, $x = 4$.

On Your Own

Now You're Ready
Exercises 8–10

Solve the equation by graphing. Check your solution(s).

1. $x^2 - x - 2 = 0$
2. $x^2 + 7x + 10 = 0$
3. $x^2 + x = 12$
4. $x^2 + 1 = 2x$
5. $x^2 + 4x = 0$
6. $x^2 + 10x = -25$

Laurie's Notes

Introduction

Connect

- **Yesterday:** Students developed an understanding of solving quadratic equations by graphing. (MP1a)
- **Today:** Students will solve quadratic equations by graphing.

Motivate

? "What do water fountains and a kicked football have in common?" Answers will vary.

- This may prompt a lot of different responses. Listen for responses such as their paths have the same shape or they have a maximum height.

? "What questions can you answer using the equation that relates the time the football has been in the air and its height?" Answers will vary.

- Mention that today students will use a quadratic equation to determine when the football is at a specific height.

Lesson Notes

Discuss

- **MP1a Make Sense of Problems:** Throughout this lesson, discuss with students the different graphing methods they can use to solve these types of equations as well as the different checks they can use. They can check by factoring, using a graphing utility, or substituting solutions back into the original equation.
- Write the definition of the standard form of a quadratic equation.

? "How can you solve a quadratic equation such as $x^2 - 5x + 4 = 0$?" Factor the left side as $(x - 4)(x - 1) = 0$. Then solve for x.

? "Do you think all quadratic equations can be factored?" no

- Explain that today students will solve quadratic equations by writing the equation in standard form and graphing the related function $y = ax^2 + bx + c$.

Example 1

- Use an input-output table to graph the related function. You might suggest a domain from -3 to 3.

? "What values of x give an output of 0?" -3 and 1

? **MP6 Attend to Precision:** "Why is it necessary to check the solutions algebraically?" You could have made an error in graphing the equation.

- If time permits, asks students about the factored form of $x^2 + 2x - 3$.
- Remind students that the solutions of a quadratic equation are also called roots.

Example 2

? "How does $x^2 - 8x = -16$ differ from the equation in Example 1?" It is not written in standard form.

- **MP6:** "Add 16 to each side" is another way of saying "use the Addition Property of Equality."

Goal Today's lesson is solving **quadratic equations** by graphing.

Technology for the Teacher

Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Extra Example 1

Solve $x^2 - x - 6 = 0$ by graphing.
$x = -2, x = 3$

Extra Example 2

Solve $x^2 + 9x = -20$ by graphing.
$x = -4, x = -5$

On Your Own

1. $x = -1, x = 2$
2. $x = -5, x = -2$
3. $x = -4, x = 3$
4. $x = 1$
5. $x = -4, x = 0$
6. $x = -5$

Extra Example 3

Solve $x^2 = -3x - 4$ by graphing.
no real solutions

On Your Own

7. no real solutions

8. $x = -6, x = -1$

9. no real solutions

Laurie's Notes

Example 2 (continued)

- In graphing the related function, note that a domain of -3 to 3 (like in Example 1) will not include the x-intercept. Students should understand that when $x = 0$, $y = 16$ and when $x = 1$, $y = 9$. The positive leading coefficient means that the graph opens up, so the vertex must be at an x-value greater than 0. There is no need to evaluate the equation for negative values of x.
- The solution can be checked by factoring as noted in the Study Tip.

On Your Own

- If time is a concern, have students do only the even or odd exercises.

Example 3

- Take time to work through both methods. Showing two pathways for approaching the problem helps deepen students' understanding of the problem.
- Because there are no x-intercepts when there are no solutions, it is not possible to perform a check by substituting x-values in the original equation.
- **Connection:** The second method connects to solving a system of equations (Chapter 4). In this method, each side of the equation is treated as a function and graphed.

On Your Own

- Remind students to check the reasonableness of their solution(s).

Discuss

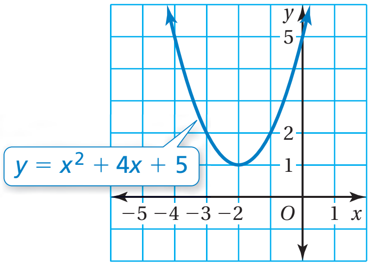
? "What are the possible numbers of points of intersection for the graph of a linear function and the graph of a quadratic function?" 0, 1, or 2
- **Connection:** Be sure to point out the connection between this question and the Summary box.
- Note how the number of solutions is related to the position of the vertex relative to the x-axis and the direction in which the parabola opens.

? "What can you say about the vertex of a parabola that opens down when the corresponding equation has two solutions?" The vertex must be above the x-axis.

EXAMPLE 3 Solving a Quadratic Equation: No Real Solutions

Solve $-x^2 = 4x + 5$ by graphing.

Method 1: Rewrite the equation in standard form and graph the related function $y = x^2 + 4x + 5$.

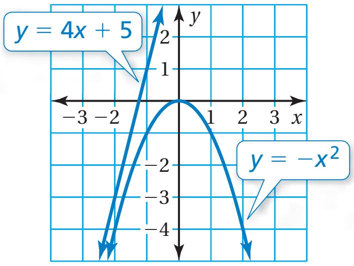


There are no x-intercepts. So, $-x^2 = 4x + 5$ has no real solutions.

Method 2: Graph each side of the equation.

$$y = -x^2 \qquad \text{Left side}$$

$$y = 4x + 5 \qquad \text{Right side}$$



The graphs do not intersect. So, $-x^2 = 4x + 5$ has no real solutions.

On Your Own

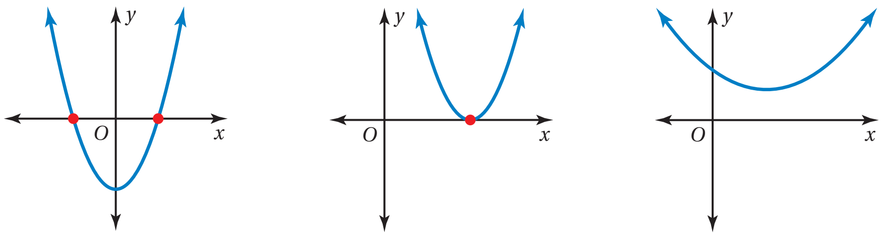
Now You're Ready
Exercises 11–16

Solve the equation by graphing. Check your solution(s).

7. $x^2 = 3x - 3$
8. $x^2 + 7x = -6$
9. $2x + 5 = -x^2$

Summary

Quadratic equations may have two real solutions, one real solution, or no real solutions.



• two real solutions • two x-intercepts	• one real solution • one x-intercept	• no real solutions • no x-intercepts

EXAMPLE 4 Real-Life Application

A football player kicks a football 2 feet above the ground with an upward velocity of 75 feet per second. The function $h = -16t^2 + 75t + 2$ gives the height h (in feet) of the football after t seconds. After how many seconds is the football 50 feet above the ground?

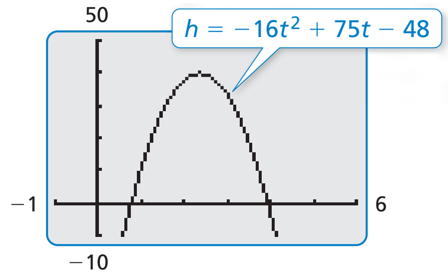
To determine when the football is 50 feet above the ground, find the t-values for which $h = 50$. So, solve the equation $-16t^2 + 75t + 2 = 50$.

Step 1: Rewrite the equation in standard form.

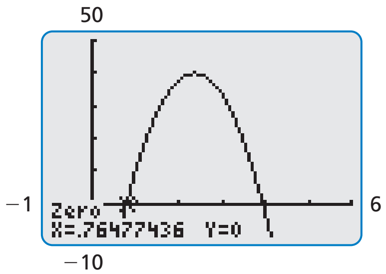
$-16t^2 + 75t + 2 = 50$ — Write the equation.

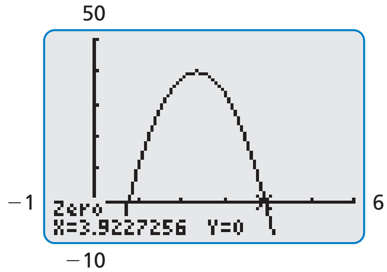
$-16t^2 + 75t - 48 = 0$ — Subtract 50 from each side.

Step 2: Use a graphing calculator to graph the related function $h = -16t^2 + 75t - 48$.



Step 3: Use the *zero* feature to find the zeros of the function.





The football is 50 feet above the ground after about 0.8 second and about 3.9 seconds.

Remember

A zero of a function $y = f(x)$ is an x-value for which the value of the function is zero.

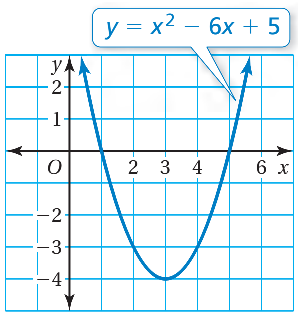
On Your Own

Now You're Ready
Exercise 18

10. **WHAT IF?** After how many seconds is the football 65 feet above the ground?

Summary

- The *solutions*, or *roots*, of $x^2 - 6x + 5 = 0$ are $x = 1$ and $x = 5$.
- The *x-intercepts* of the graph of $y = x^2 - 6x + 5$ are 1 and 5.
- The *zeros* of the function $f(x) = x^2 - 6x + 5$ are 1 and 5.



Laurie's Notes

Example 4

- **MP4 Model with Mathematics:** The position function $h = -16t^2 + v_0 t + s_0$ represents the height h (in feet) of an object where v_0 is the initial upward velocity (in feet per second), s_0 is the initial height (in feet) of the object, and t is the time (in seconds). In this example, the initial upward velocity is 75 feet per second and the initial height is 2 feet.
- **MP2 Reason Abstractly and Quantitatively:** Students must reason abstractly and quantitatively to relate the model and its graph to the height of the football.
- Students are accustomed to seeing functions with ordered pairs (x, y). Remind students that each ordered pair (t, h) represents the height h of the object at time t.
- **Misconception:** The graph shows the height of the object over time, not the path of the object.
- Discuss with students what the two zeros actually represent and why the football is at the same height at two different times.
- **Connection:** Some students may ask if this problem can be solved using Method 2 in Example 3. The answer is yes. Graph each side of the equation where $y = -16t^2 + 75t + 2$ represents the left side and $y = 50$ represents the right side. The solutions are the x-values of the points of intersection of the graphs.
- This is a significant problem where many connections can be made. Do not overwhelm students with all of them until they are ready.
- **MP6:** Note that the actual zeros have been rounded to the nearest tenth of a second. This degree of precision is sufficient in this context.
- Take time to discuss the language being highlighted in the last Summary box.

Closure

- When a quadratic equation has two solutions, what do you know about the graph of its related function? It has two x-intercepts.

Extra Example 4

The function $h = -16t^2 + 32t$ gives the height h of a soccer ball t seconds after it is kicked. After how many seconds is the soccer ball 15 feet above the ground?
0.75 second and 1.25 seconds

On Your Own

10. about 1.1 seconds and about 3.6 seconds

English Language Learners

Vocabulary
To help your students understand the terminology used in this section, have them write the quadratic equation
$(x + 20)(x - 10) = 0$
in standard form and then write the related function of the form
$f(x) = ax^2 + bx + c$.
Then write the following on the board. After a volunteer reads each statement, have students write it in their notebooks.

- solutions, or roots, of $(x + 20)(x - 10) = 0$: $x = -20$, $x = 10$
- x-intercepts of the graph of f: -20 and 10
- zeros of f: -20 and 10

Vocabulary and Concept Check

1. It is a nonlinear equation that can be written in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
2. $x^2 + x - 4 = 0$; It is the only equation in standard form.
3. Use the graph to find the x-intercepts.
4. The roots, or solutions, of an equation are the same as the zeros of the related function or the x-intercepts of its graph.

Practice and Problem Solving

5. $x = 4, x = 6$
6. no real solutions
7. $x = -6$
8. $x = 0, x = 4$
9. $x = 3$
10. $x = -1, x = 7$
11. no real solutions
12. $x = -2, x = 1$
13. $x = -2$
14. $x = -5, x = 3$
15. $x = 7$
16. no real solutions
17. **a.** The x-intercepts give the horizontal positions of the ball where it is struck and where it lands.

    b. 5 yards
18. about 0.6 second and about 1.3 seconds

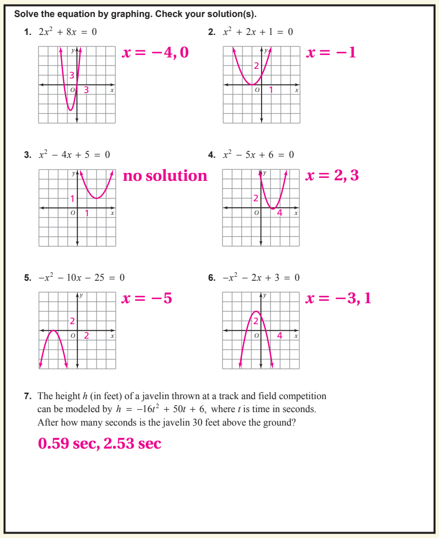
Assignment Guide and Homework Check

Level	Assignment	Homework Check
Average	1–4, 5–29 odd, 32, 33, 46–50	5, 13, 21, 25, 33
Advanced	1–4, 14–34 even, 41–50	16, 26, 34, 41, 43

Common Errors

- **Exercise 4** Students may think these are all identical terms. Remind them of the Summary box that addresses these terms.
- **Exercise 6** Students may try using the y-intercept as a solution. Remind students that the solutions are given by the x-intercepts. When there are no x-intercepts, there are no solutions.
- **Exercises 8–16** Solutions obtained graphically may be incorrect or not exact. Make sure students use a sound graphical approach. Also, remind them to check their answers.
- **Exercise 18** Students may give only one solution. There are two solutions representing the time when the ball is 16 feet above the ground.

9.1 Record and Practice Journal



Solve the equation by graphing. Check your solution(s).

1. $2x^2 + 8x = 0$ — $x = -4, 0$
2. $x^2 + 2x + 1 = 0$ — $x = -1$
3. $x^2 - 4x + 5 = 0$ — no solution
4. $x^2 - 5x + 6 = 0$ — $x = 2, 3$
5. $-x^2 - 10x - 25 = 0$ — $x = -5$
6. $-x^2 - 2x + 3 = 0$ — $x = -3, 1$
7. The height h (in feet) of a javelin thrown at a track and field competition can be modeled by $h = -16t^2 + 50t + 6$, where t is time in seconds. After how many seconds is the javelin 30 feet above the ground?

   0.59 sec, 2.53 sec

9.1 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **VOCABULARY** What is a quadratic equation?

2. **WHICH ONE DOESN'T BELONG?** Which equation does *not* belong with the other three? Explain your reasoning.

$x^2 + 5x = 20$	$x^2 + x - 4 = 0$	$x^2 - 6 = 4x$	$7x + 12 = x^2$

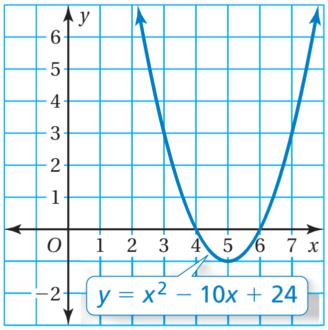
3. **WRITING** How can you use a graph to find the number of solutions of a quadratic equation?

4. **WRITING** How are solutions, roots, x-intercepts, and zeros related?

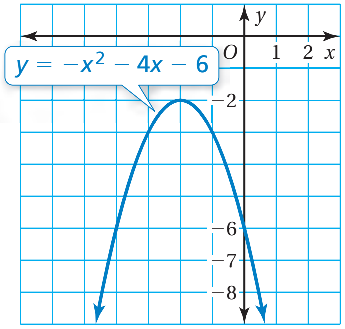
Practice and Problem Solving

Determine the solution(s) of the equation. Check your solution(s).

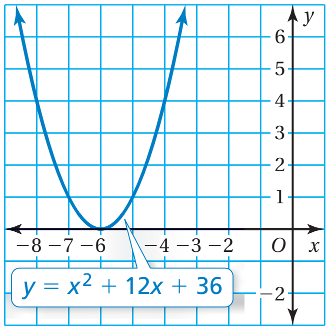
5. $x^2 - 10x + 24 = 0$



6. $-x^2 - 4x - 6 = 0$



7. $x^2 + 12x + 36 = 0$



Solve the equation by graphing. Check your solution(s).

1 2 8. $x^2 - 4x = 0$

9. $x^2 - 6x + 9 = 0$

10. $x^2 - 6x - 7 = 0$

3 11. $x^2 - 2x + 5 = 0$

12. $x^2 + x - 2 = 0$

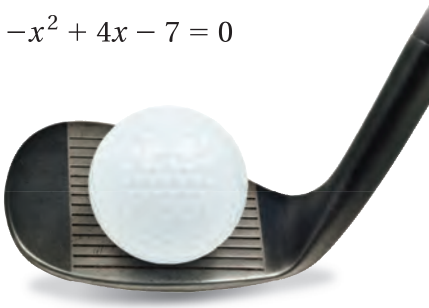
13. $x^2 + 4x + 4 = 0$

14. $-x^2 - 2x + 15 = 0$

15. $-x^2 + 14x - 49 = 0$

16. $-x^2 + 4x - 7 = 0$

17. **FLOP SHOT** The height y (in yards) of a flop shot in golf can be modeled by $y = -x^2 + 5x$, where x is the horizontal distance (in yards).



a. Interpret the x-intercepts of the graph of the equation.

b. How far away does the golf ball land?

4 18. **VOLLEYBALL** The height h (in feet) of an underhand volleyball serve can be modeled by $h = -16t^2 + 30t + 4$, where t is the time in seconds. After how many seconds is the ball 16 feet above the ground?

Rewrite the equation in standard form. Then solve the equation by graphing. Check your solution(s) with a graphing calculator.

19. $x^2 = 6x - 8$

20. $x^2 = -1 - 2x$

21. $x^2 = -x - 3$

22. $x^2 = 2x - 4$

23. $5x - 6 = x^2$

24. $3x - 18 = -x^2$

Solve the equation by using Method 2 from Example 3. Check your solution(s).

25. $x^2 = 10 - 3x$

26. $4 - 4x = -x^2$

27. $5x - 7 = x^2$

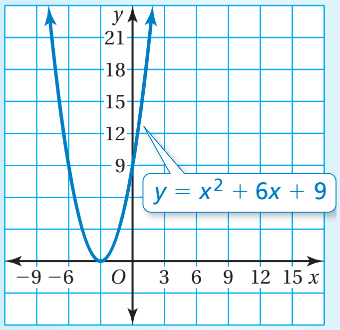
28. $x^2 = 6x - 10$

29. $x^2 = -2x - 1$

30. $x^2 - 8x = 9$

31. **REASONING** Example 3 shows two methods for solving a quadratic equation. Which method do you prefer? Explain your reasoning.

32. **ERROR ANALYSIS** Describe and correct the error in solving the equation.



The only solution of the equation $x^2 + 6x + 9 = 0$ is $x = 9$.

33. **BASEBALL** A baseball player throws a baseball with an upward velocity of 24 feet per second. The release point is 6 feet above the ground. The function $h = -16t^2 + 24t + 6$ gives the height h (in feet) of the baseball after t seconds.

a. How long is the ball in the air if no one catches it?

b. How long does the ball remain above 6 feet?

34. **SOFTBALL** You throw a softball straight up into the air with an upward velocity of 40 feet per second. The release point is 5 feet above the ground. The function $h = -16t^2 + 40t + 5$ gives the height h (in feet) of the softball after t seconds.

a. How long is the ball in the air if you miss it?

b. How long is the ball in the air if you catch it at a height of 5 feet?

Common Errors

- **Exercises 19–24** Students may make sign errors when rewriting the equation in standard form. Remind them that they need to use the Addition or Subtraction Property of Equality to add or subtract the same quantity on each side.
- **Exercises 25–30** Students may give solutions in terms of *x*- and *y*-coordinates. Tell them that the original equation was only in *x*. When Method 2 is used, the solutions are the *x*-values of the points of intersection.
- **Exercises 35–40** Students may round their answers incorrectly. Make sure they are using the features of the graphing utility correctly and that they understand how to find solutions rounded to the nearest tenth.
- **Exercise 41** Students may have trouble setting up the equation in part (a). Explain the coefficients of the function for the height of a projectile.
- **Exercise 42** Students may not know where to start. Remind them to first write a function for the height of the keg using the initial velocity and initial height. Then write an equation to find when the keg reaches a height of 16.5 feet. When they find that there is no solution, they should realize that the keg does not clear the wall.

Practice and Problem Solving

19. $x^2 - 6x + 8 = 0$; $x = 2, x = 4$

20. $x^2 + 2x + 1 = 0$; $x = -1$

21. $x^2 + x + 3 = 0$; no real solutions

22. $x^2 - 2x + 4 = 0$; no real solutions

23. $x^2 - 5x + 6 = 0$; $x = 2, x = 3$

24. $x^2 + 3x - 18 = 0$; $x = -6, x = 3$

25. $x = -5, x = 2$

26. $x = 2$

27. no real solutions

28. no real solutions

29. $x = -1$

30. $x = -1, x = 9$

31. *Sample answer:* Method 2; You do not have to rewrite the equation.

32. The *y*-intercept was used instead of the *x*-intercept. The correct answer is $x = -3$.

33. **a.** about 1.7 seconds

b. 1.5 seconds

34. **a.** about 2.6 seconds

b. 2.5 seconds

English Language Learners

Class Activity

Form groups of 2 to 4 students with at least one English language learner and one English speaker. Select an exercise for each group and have them work together to create a poster for the exercise to present to the class. This will allow students to discuss concepts in small groups and create a visual display to aid their understanding.

Practice and Problem Solving

35. $x \approx -5.8, x \approx -0.2$

36. $x \approx -0.6, x \approx 3.6$

37. $x \approx -5.7, x \approx 0.7$

38. $x \approx -3.4, x \approx 1.4$

39. $x \approx 0.6, x \approx 3.4$

40. $x \approx 0.7, x \approx 8.3$

41. **a.** $h = -16t^2 + 20t + 8$

b. about 1.6 seconds

42. See *Taking Math Deeper*.

43. sometimes; There are 2 x-intercepts when c is positive.

44. always; In each case, the parabola opens away from the x-axis.

45. never; A quadratic equation can never have more than 2 solutions.

Fair Game Review

46. 24 **47.** 81

48. $6\sqrt{3}$ **49.** $25\sqrt{2}$

50. D

Mini-Assessment

Solve the equation by graphing. Check your solution(s).

1. $x^2 + 4x + 5 = 0$ no real solutions
2. $x^2 + 7x - 8 = 0$ $x = -8, x = 1$
3. $x^2 = 10x - 25$ $x = 5$
4. $x^2 + 3x = 18$ $x = -6, x = 3$

Taking Math Deeper

Exercise 42

This exercise is good practice for using different methods to solve a problem.

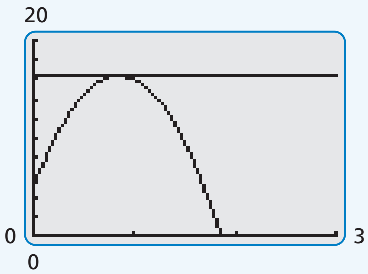
1 Write the function that gives the height of the keg. Then write a system of equations that can be used to solve the problem.

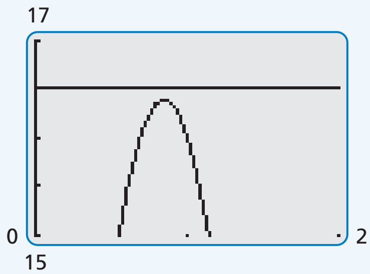
The keg is released at a height of 5 feet with an upward velocity of 27 feet per second. So, the function is $y = -16t^2 + 27t + 5$. To determine if the keg reaches a height of 16.5 feet, use the following system.

$y = 16.5$
$y = -16t^2 + 27t + 5$

Remember 6 inches is one-half of a foot.

2 Graph the system. Depending on the viewing window, the graphs may look like they intersect. When a proper viewing window is used, you can see that they do not intersect.





3 Answer the questions.

a. Because the graphs do not intersect, the throw is not high enough to clear the wall.

b. Yes, a taller competitor may release the keg at a greater height. This changes the value for the initial height of the function. Students can verify this by graphing $y = -16t^2 + 27t + 5.5$ instead of $y = -16t^2 + 27t + 5$.

Note: You may want to have students solve this problem using the *maximum* feature of their graphing calculators.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension Start the next section

Use a graphing calculator to approximate the zeros of the function to the nearest tenth.

35. $f(x) = x^2 + 6x + 1$ **36.** $f(x) = x^2 - 3x - 2$ **37.** $f(x) = x^2 + 5x - 4$

38. $f(x) = -x^2 - 2x + 5$ **39.** $f(x) = -x^2 + 4x - 2$ **40.** $f(x) = -x^2 + 9x - 6$

41. MODELING A dirt bike launches off a ramp that is 8 feet tall. The upward velocity of the dirt bike is 20 feet per second.

a. Write a function that models the height h (in feet) of the dirt bike after t seconds.

b. After how many seconds does the dirt bike land?

42. WORLD'S STRONGEST MAN One of the events in the World's Strongest Man competition is the keg toss. In this event, competitors try to throw kegs of various weights over a wall that is 16 feet 6 inches high.

a. A competitor releases a keg 5 feet above the ground with an upward velocity of 27 feet per second. Is this throw high enough to clear the wall? Explain your reasoning.

b. Do the heights of the competitors factor into their success at this event? Explain your reasoning.

Reasoning **Determine whether the statement is *sometimes*, *always*, or *never* true. Justify your answer.**

43. The graph of $y = ax^2 + c$ has two x-intercepts when $a = -2$.

44. The graph of $y = ax^2 + c$ has no x-intercepts when a and c have the same sign.

45. The graph of $y = ax^2 + bx + c$ has more than two zeros when $a \neq 0$.

Fair Game Review What you learned in previous grades & lessons

Simplify the expression. *(Section 6.1)*

46. $4\sqrt{36}$ **47.** $9\sqrt{81}$ **48.** $2\sqrt{27}$ **49.** $5\sqrt{50}$

50. MULTIPLE CHOICE Which expression is equivalent to $\left(\frac{2x^3}{3m^5}\right)^2$? *(Section 6.2)*

Ⓐ $\frac{2x^5}{3m^7}$ Ⓑ $\frac{2x^6}{3m^{10}}$ Ⓒ $\frac{4x^5}{9m^7}$ Ⓓ $\frac{4x^6}{9m^{10}}$

9.2 Solving Quadratic Equations Using Square Roots

Essential Question How can you determine the number of solutions of a quadratic equation of the form $ax^2 + c = 0$?

1 ACTIVITY: The Number of Solutions of $ax^2 + c = 0$

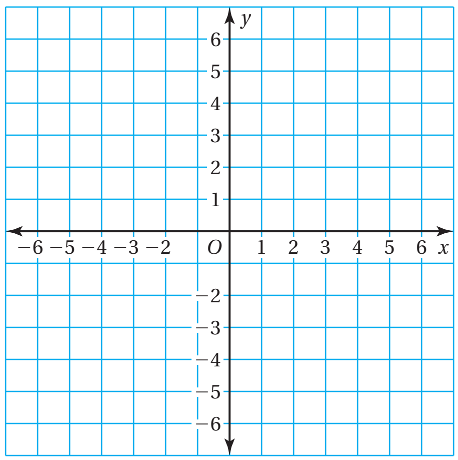
Work with a partner. Solve each equation by graphing. Explain how the number of solutions of

$$ax^2 + c = 0 \quad \text{Quadratic equation}$$

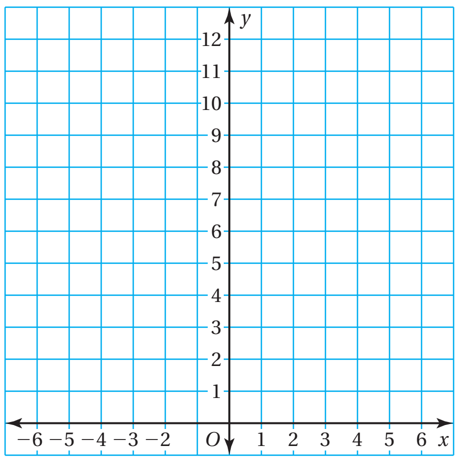
relates to the graph of

$$y = ax^2 + c. \quad \text{Quadratic function}$$

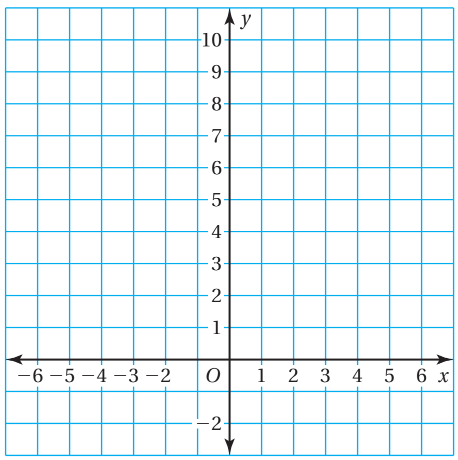
a. $x^2 - 4 = 0$



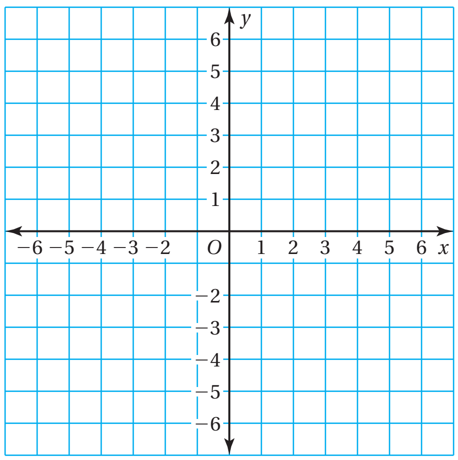
b. $2x^2 + 5 = 0$



c. $x^2 = 0$



d. $x^2 - 5 = 0$



Common Core

Solving Quadratic Equations

In this lesson, you will

- solve quadratic equations by taking square roots.

Learning Standard
A.REI.4b

Laurie's Notes

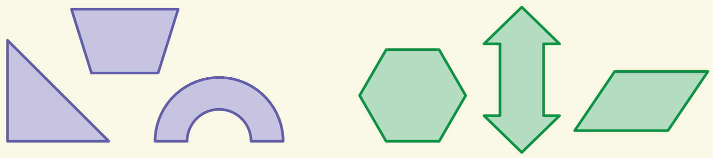
Introduction

Standards for Mathematical Practice

- **MP3a Construct Viable Arguments** and **MP8 Look for and Express Regularity in Repeated Reasoning:** Students are asked to make a conjecture about the number of solutions of quadratic equations after graphing the related functions.

Motivate

- All of today's graphs will be symmetric about the y-axis.
- Draw two collections of shapes. In the first, each shape has one line of symmetry. In the second, each shape has no symmetry or more than one line of symmetry.



- Ask students to figure out how the groups have been sorted.
- ? "In which group would you place a parabola?" group on left

Activity Notes

Discuss

- Today you want students to think about the function $y = x^2$ (1 x-intercept) and the effect of adding a constant c that shifts the graph up (no x-intercepts) or down (2 x-intercepts).

Activity 1

- ? "What do you know about the graph of $y = x^2$?" It is a parabola with vertex (0, 0) that opens up.
- Students can make a table to graph the function. Make sure they choose a domain that displays the key features of the graph.
- Refer to the related function in part (c), $y = x^2$, as the *parent function* or *basic quadratic function*.
- **MP3a:** Conjectures will likely relate the number of solutions to the number of x-intercepts of the graph.
- Students might go further and say that when c is positive, there are no solutions, and when c is negative, there are two solutions. Note however that this is not true when $a < 0$.

Common Core State Standards

A.REI.4b Solve quadratic equations . . . by taking square roots . . ..

Previous Learning

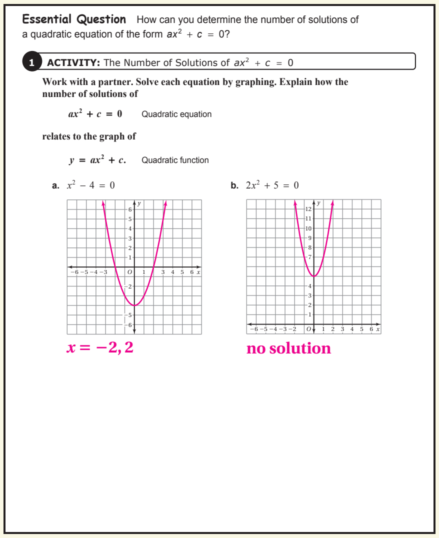
Students should know how to solve quadratic equations by graphing. They should also know how to find the square roots of a number.

Technology for the Teacher

Dynamic Classroom

Lesson Plans
Complete Materials List

9.2 Record and Practice Journal



Essential Question How can you determine the number of solutions of a quadratic equation of the form $ax^2 + c = 0$?

1 ACTIVITY: The Number of Solutions of $ax^2 + c = 0$

Work with a partner. Solve each equation by graphing. Explain how the number of solutions of

$ax^2 + c = 0$ Quadratic equation

relates to the graph of

$y = ax^2 + c.$ Quadratic function

a. $x^2 - 4 = 0$

$x = -2, 2$

b. $2x^2 + 5 = 0$

no solution

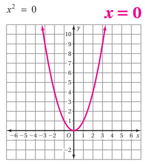
English Language Learners

Vocabulary

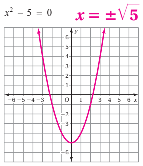
Reinforce the terms "exact solution" and "estimated solution" by asking students which term describes their solutions to $x^2 - 5 = 0$ in each activity.

9.2 Record and Practice Journal

c. $x^2 = 0$ $x = 0$



d. $x^2 - 5 = 0$ $x = \pm\sqrt{5}$



The number of solutions of $ax^2 + c = 0$ equals the number of x-intercepts of $y = ax^2 + c$.

2 ACTIVITY: Estimating Solutions

Work with a partner. Complete each table. Use the completed tables to estimate the solutions of $x^2 - 5 = 0$. Explain your reasoning. $x \approx \pm 2.235$

a.

x	$x^2 - 5$
2.21	−0.1159
2.22	−0.0716
2.23	−0.0271
2.24	0.0176
2.25	0.0625
2.26	0.1076

b.

x	$x^2 - 5$
−2.21	−0.1159
−2.22	−0.0716
−2.23	−0.0271
−2.24	0.0176
−2.25	0.0625
−2.26	0.1076

3 ACTIVITY: Using Technology to Estimate Solutions

Work with a partner. Two equations are equivalent when they have the same solutions.

a. Are the equations $x^2 - 5 = 0$ and $x^2 = 5$ equivalent? Explain your reasoning.

yes; They have the same solutions.

b. Use the square root key on a calculator to estimate the solutions of $x^2 - 5 = 0$. Describe the accuracy of your estimates.

$x \approx \pm 2.236$; estimates are off by about 0.001.

c. Write the *exact* solutions of $x^2 - 5 = 0$.

$x \approx \pm\sqrt{5}$

What Is Your Answer?

4. IN YOUR OWN WORDS How can you determine the number of solutions of a quadratic equation of the form $ax^2 + c = 0$?

Graph the equation and count the number of x-intercepts.

5. Write the exact solutions of each equation. Then use a calculator to estimate the solutions.

a. $x^2 - 2 = 0$ $x = \pm\sqrt{2}$ $x \approx \pm 1.414$

b. $3x^2 - 15 = 0$ $x = \pm\sqrt{5}$ $x \approx \pm 2.236$

c. $x^2 = 8$ $x = \pm\sqrt{8}$ $x \approx \pm 2.828$

Laurie's Notes

Activity 2

- A calculator is a helpful tool for this activity.

? "In part (a), what is happening to the x-values?" increasing by 0.01

? "How do the tables help you estimate the solutions of $x^2 - 5 = 0$?" The consecutive x-values at which the expression values change in sign from negative to positive indicate an approximate solution.

- **MP2 Reason Abstractly and Quantitatively:** In exploring the table of values, you are asking your students to reason quantitatively.

Activity 3

- Students are using a calculator to confirm the estimates in Activity 2.
- **MP6 Attend to Precision:** In part (b), students should have obtained approximate solutions such as ± 2.236067977. In part (c), the *exact* solutions are $\pm\sqrt{5}$. Discuss the use of exact and approximate solutions. This draws attention to the concept of precision.

What Is Your Answer?

- **MP3a** and **MP8:** In Question 4, expect student conjectures to relate the number of solutions to the number of x-intercepts of the graph. Students may also try to distinguish between whether c is a positive or negative number. If so, make sure the conjecture is correct for both $a > 0$ and $a < 0$.
- **MP6:** In Question 5, discuss whether the solutions obtained on a calculator for a quadratic equation of the form $ax^2 + c = 0$ will always be estimates.

Closure

- Besides graphing, describe how to find the solutions of $x^2 - 9 = 0$. Add 9 to each side and then take the square root of each side.

2 ACTIVITY: Estimating Solutions

Work with a partner. Complete each table. Use the completed tables to estimate the solutions of $x^2 - 5 = 0$. Explain your reasoning.

a.

x	$x^2 - 5$
2.21	
2.22	
2.23	
2.24	
2.25	
2.26	

b.

x	$x^2 - 5$
−2.21	
−2.22	
−2.23	
−2.24	
−2.25	
−2.26	

3 ACTIVITY: Using Technology to Estimate Solutions

Math Practice 5

Choose Appropriate Tools

What different types of technology can be used to answer the questions? Which tool would be the most appropriate and why?

Work with a partner. Two equations are equivalent when they have the same solutions.

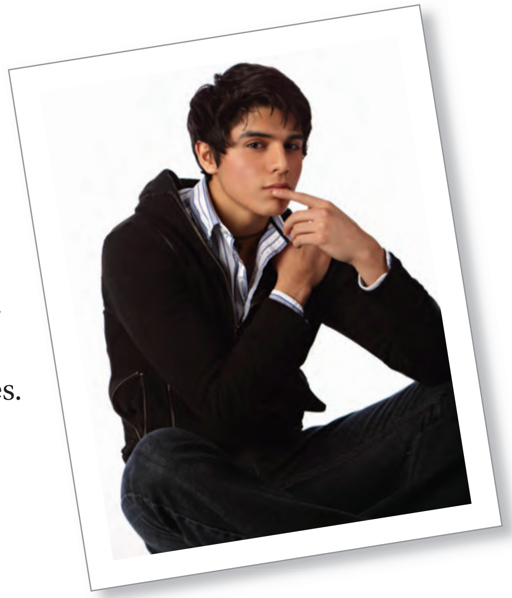
a. Are the equations

$$x^2 - 5 = 0 \quad \text{and} \quad x^2 = 5$$

equivalent? Explain your reasoning.

b. Use the square root key on a calculator to estimate the solutions of $x^2 - 5 = 0$. Describe the accuracy of your estimates.

c. Write the *exact* solutions of $x^2 - 5 = 0$.



What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you determine the number of solutions of a quadratic equation of the form $ax^2 + c = 0$?

5. Write the exact solutions of each equation. Then use a calculator to estimate the solutions.

a. $x^2 - 2 = 0$ **b.** $3x^2 - 15 = 0$ **c.** $x^2 = 8$

Practice

Use what you learned about quadratic equations to complete Exercises 3–5 on page 466.

9.2 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

In Section 6.1, you studied properties of square roots. Here you will use square roots to solve quadratic equations of the form $ax^2 + c = 0$.

Key Idea

Solving Quadratic Equations Using Square Roots

You can solve $x^2 = d$ by taking the square root of each side.

- When $d > 0$, $x^2 = d$ has two real solutions, $x = \pm\sqrt{d}$.
- When $d = 0$, $x^2 = d$ has one real solution, $x = 0$.
- When $d < 0$, $x^2 = d$ has no real solutions.

EXAMPLE 1 Solving Quadratic Equations Using Square Roots

a. Solve $3x^2 - 27 = 0$ using square roots.

$3x^2 - 27 = 0$	Write the equation.
$3x^2 = 27$	Add 27 to each side.
$x^2 = 9$	Divide each side by 3.
$x = \pm\sqrt{9}$	Take the square root of each side.
$x = \pm 3$	Simplify.

∴ The solutions are $x = 3$ and $x = -3$.

b. Solve $x^2 - 10 = -10$ using square roots.

$x^2 - 10 = -10$	Write the equation.
$x^2 = 0$	Add 10 to each side.
$x = 0$	Take the square root of each side.

∴ The only solution is $x = 0$.

Remember

The square of a real number cannot be negative. That is why the equation in part (c) has no real solutions.

c. Solve $-5x^2 + 11 = 16$ using square roots.

$-5x^2 + 11 = 16$	Write the equation.
$-5x^2 = 5$	Subtract 11 from each side.
$x^2 = -1$	Divide each side by -5.

∴ The equation has no real solutions.

On Your Own

Now You're Ready
Exercises 12–20

Solve the equation using square roots.

1. $-3x^2 = -75$ **2.** $x^2 + 12 = 10$ **3.** $4x^2 - 15 = -15$

Laurie's Notes

Introduction

Connect

- **Yesterday:** Students developed an understanding of solving quadratic equations of the form $ax^2 + c = 0$. (MP2, MP3a, MP6, MP8)
- **Today:** Students will solve quadratic equations of the form $ax^2 + c = 0$ using square roots.

Motivate

- Tell students that today they will find the dimensions of a touch tank. Ask if any of them have visited an aquarium that has a touch tank.

Lesson Notes

Discuss

- **MP7 Look for and Make Use of Structure:** Help students see the similarities in solving $3x - 27 = 0$ and $3x^2 - 27 = 0$. In this lesson, students will solve the latter by performing one last step—taking the square root of each side.
- **FYI:** You can rewrite $ax^2 + c = 0$ as $x^2 = -\frac{c}{a}$. This is simplified as $x^2 = d$ in the Key Idea.

Key Idea

- Write the Key Idea on the board.
- ? "How does this Key Idea connect to the graphing you did in the activity?" The number of solutions was shown to be 2, 1, or 0.
- Explain that today students will solve quadratic equations algebraically. Solving $ax^2 + c = 0$ is similar to solving the linear equation $ax + c = 0$.

Example 1

- Work through the three parts as shown.
- Take time to discuss with students how to solve $27 = 3x^2$. For some students, having the x^2-term on the right side of the equation makes the equation appear quite different.
- **Common Error:** Students often forget the negative square root when taking the square root of each side of an equation.

On Your Own

- **Think-Pair-Share:** Students should read each question independently and then work in pairs to answer the questions. When they have answered the questions, the pair should compare their answers with another group and discuss any discrepancies.

Goal Today's lesson is solving quadratic equations using square roots.

Technology for the Teacher

Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Extra Example 1

Solve each equation using square roots.

a. $2x^2 - 32 = 0$ $x = 4, x = -4$

b. $5x^2 + 8 = 8$ $x = 0$

c. $x^2 + 8 = 5$ no real solutions

On Your Own

1. $x = 5, x = -5$
2. no real solutions
3. $x = 0$

Extra Example 2

Solve $(x - 3)^2 = 49$ using square roots. $x = -4$, $x = 10$

Extra Example 3

In Example 3, the volume of the tank is 450 cubic feet. Find the length and width of the tank.
width: about 7.1 ft; length: about 21.2 ft

On Your Own

4. $x = -7$
5. $x = 1.5$, $x = 4.5$
6. $x = \dfrac{-1 + \sqrt{35}}{2}$, $x = \dfrac{-1 - \sqrt{35}}{2}$
7. width: about 5.9 ft; length: about 17.7 ft

Differentiated Instruction

Kinesthetic
In Example 3, have students check the answer for reasonableness by using the length and width to sketch the base of the tank on graph paper (using feet as units). The drawing represents one of the three layers of unit cubes needed to fill the tank. Are the length and width reasonable?

Laurie's Notes

Example 2

? "How do you read $(x - 1)^2 = 25$?" Listen for "the quantity x minus 1 squared equals 25" or "x minus 1 quantity squared equals 25."

- Make sure students understand that this is about a quantity that is squared, and it is equal to 25, which is also a quantity squared.
- One technique is to place your hand or a few fingers over the expression in the parentheses and say, "Something squared is 25. Taking the square root of each side of the equation makes sense."
- Read the solution, "1 plus or minus 5." There are two solutions: 1 plus 5 and 1 minus 5.
- **Connection:** Check the solutions using a graphing calculator. The x-intercepts are -4 and 6. The vertex of the parabola occurs halfway between these numbers at $x = 1$.
- **Extension:** Discuss whether it is helpful to start the problem by first expanding $(x - 1)^2$. This expansion results in $x^2 - 2x + 1 = 25$. Have students solve this equation by factoring.

Example 3

- Discuss what the length and width represent in terms of the diagram.
- Work through the problem as shown.

? "What is a reasonable estimate of $\sqrt{30}$? Explain." About 5.5 because $\sqrt{25} = 5$, $\sqrt{36} = 6$, and 30 is about halfway between 25 and 36.

- Discuss the Study Tip.
- **MP6 Attend to Precision:** Point out to students that they should wait to round their answers until after they have substituted to find the length. Otherwise, they would calculate the length as $3(5.5) = 16.5$ feet.

On Your Own

- In Question 7, ask students how the new volume will affect the original dimensions in Example 3. They will increase.

Closure

- **Exit Ticket:** Match each equation with its number of solutions.

1. $2x^2 + 8 = 40$ C	**A.** 0 solutions
2. $2x^2 - 8 = -40$ A	**B.** 1 solution
3. $2x^2 = 0$ B	**C.** 2 solutions

EXAMPLE 2 Solving a Quadratic Equation Using Square Roots

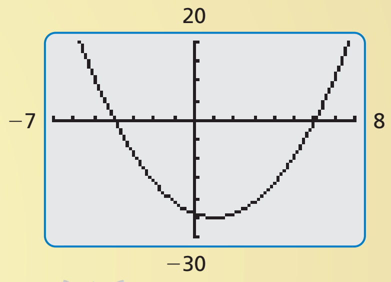
Solve $(x-1)^2 = 25$ using square roots.

$(x-1)^2 = 25$	Write the equation.
$x - 1 = \pm 5$	Take the square root of each side.
$x = 1 \pm 5$	Add 1 to each side.

So, the solutions are $x = 1 + 5 = 6$ and $x = 1 - 5 = -4$.

Check

Use a graphing calculator to check your answer. Rewrite the equation as $(x-1)^2 - 25 = 0$. Graph the related function $y = (x-1)^2 - 25$ and find the x-intercepts, or zeros. The zeros are -4 and 6, so the solution checks.



EXAMPLE 3 Real-Life Application

A touch tank has a height of 3 feet. Its length is 3 times its width. The volume of the tank is 270 cubic feet. Find the length and width of the tank.

The length ℓ is 3 times the width w, so $\ell = 3w$. Write an equation using the formula for the volume of a rectangular prism.

$V = \ell wh$	Write the formula.
$270 = 3w(w)(3)$	Substitute 270 for V, $3w$ for ℓ, and 3 for h.
$270 = 9w^2$	Multiply.
$30 = w^2$	Divide each side by 9.
$\pm\sqrt{30} = w$	Take the square root of each side.

The solutions are $\sqrt{30}$ and $-\sqrt{30}$. Use the positive solution.

So, the width is $\sqrt{30} \approx 5.5$ feet and the length is $3\sqrt{30} \approx 16.4$ feet.

Study Tip

Use the positive square root because negative solutions do not make sense in this context. Length and width cannot be negative.

On Your Own

Now You're Ready
Exercises 23–28 and 32

Solve the equation using square roots.

4. $(x+7)^2 = 0$
5. $4(x-3)^2 = 9$
6. $(2x+1)^2 = 35$
7. **WHAT IF?** In Example 3, the volume of the tank is 315 cubic feet. Find the length and width of the tank.

9.2 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **REASONING** How many real solutions does the equation $x^2 = d$ have when d is positive? 0? negative?

2. **WHICH ONE DOESN'T BELONG?** Which equation does *not* belong with the other three? Explain your reasoning.

$x^2 = 9$ $\qquad$ $x^2 = 2$ $\qquad$ $x^2 = -7$ $\qquad$ $x^2 = 21$

Practice and Problem Solving

Determine the number of solutions of the equation. Then use a calculator to estimate the solutions.

3. $x^2 - 11 = 0$
4. $x^2 + 10 = 0$
5. $2x^2 - 3 = 0$

Determine the number of solutions of the equation. Then solve the equation using square roots.

6. $x^2 = 25$
7. $x^2 = -36$
8. $x^2 = 8$
9. $x^2 = 21$
10. $x^2 = 0$
11. $x^2 = 169$

Solve the equation using square roots.

1 12. $x^2 - 16 = 0$
13. $x^2 + 12 = 0$
14. $x^2 + 6 = 0$
15. $x^2 - 61 = 0$
16. $2x^2 - 98 = 0$
17. $-x^2 + 9 = 9$
18. $x^2 + 13 = 7$
19. $-4x^2 - 5 = -5$
20. $-3x^2 + 8 = 8$

21. **ERROR ANALYSIS** Describe and correct the error in solving the equation.

Solve $2x^2 - 33 = 39$.

$2x^2 = 72$	Add 33 to each side.
$x^2 = 36$	Divide each side by 2.
$x = 6$	Take the square root of each side.

The solution is $x = 6$.

22. **WAREHOUSE** A box falls off a warehouse shelf from a height of 16 feet. The function $h = -16x^2 + 16$ gives the height h (in feet) of the box after x seconds. When does it hit the floor?

Assignment Guide and Homework Check

Level	Assignment	Homework Check
Average	1, 2, 3–25 odd, 29, 32, 37–40	11, 13, 21, 23, 32
Advanced	1, 2, 12–28 even, 30–40	14, 20, 22, 24, 32

Common Errors

- **Exercises 3–20** Students may forget the negative square root when taking the square root of each side of the equation. Remind them to account for the negative square root when appropriate.
- **Exercises 3–20** Students may try to take the square root of a negative number. Remind them that the square of a real number cannot be negative.
- **Exercises 23–28** Students may use the Addition Property of Equality incorrectly. For example, a student may rewrite $(x - 1)^2 = 35$ as $x^2 = 36$ Remind students that the Addition Property of Equality cannot be applied to a term that is within grouping symbols.

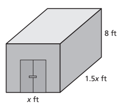
9.2 Record and Practice Journal

Solve the equation using square roots.

1. $x^2 - 64 = 0$
   $x = \pm 8$
2. $x^2 + 18 = 0$
   no solution
3. $x^2 - 11 = -11$
   $x = 0$
4. $3x^2 - 75 = 0$
   $x = \pm 5$
5. $2x^2 + 12 = 9$
   no solution
6. $-6x^2 + 4 = 4$
   $x = 0$

Solve the equation using square roots. Use a graphing calculator to check your solution(s).

7. $(x - 4)^2 = 49$
   $x = 11$ and $x = -3$
8. $16(x + 3)^2 = 36$
   $x = -\frac{3}{2}$ and $x = -\frac{9}{2}$
9. The volume of a shed is 972 cubic feet. Find the width x of the shed.
   9 ft



Vocabulary and Concept Check

1. 2; 1; 0
2. $x^2 = -7$; It is the only equation with no real solutions.

Practice and Problem Solving

3. 2; $x \approx 3.317$, $x \approx -3.317$
4. 0; no real solutions
5. 2; $x \approx 1.225$, $x \approx -1.225$
6. 2; $x = 5$, $x = -5$
7. 0; no real solutions
8. 2; $x = 2\sqrt{2}$, $x = -2\sqrt{2}$
9. 2; $x = \sqrt{21}$, $x = -\sqrt{21}$
10. 1; $x = 0$
11. 2; $x = 13$, $x = -13$
12. $x = 4$, $x = -4$
13. no real solutions
14. no real solutions
15. $x = \sqrt{61}$, $x = -\sqrt{61}$
16. $x = 7$, $x = -7$
17. $x = 0$
18. no real solutions
19. $x = 0$
20. $x = 0$
21. When taking the square root of each side, the student forgot the negative root. $x = 6$, $x = -6$
22. after 1 second
23. $x = -3$
24. $x = -1$, $x = 3$
25. $x = -4$, $x = 5$

Practice and Problem Solving

26. $x = \frac{1}{2}, x = 2$

27. $x = -\frac{7}{3}, x = \frac{1}{3}$

28. $x = -\frac{1}{2}, x = \frac{9}{2}$

29. 8 in. by 8 in.

30. $3\sqrt{13}$ cm by $2\sqrt{13}$ cm

31. 12 ft

32. length = 52.4 in., width = 26.2 in.

33. See *Taking Math Deeper.*

34. Find two integers or decimals that you know the root is between and then use a table of values.

35. **a.** two solutions: a is positive and c is negative, or c is positive and a is negative.

b. one solution: $c = 0$

c. no solutions: a and c are both positive or both negative.

36. $(-3, 9)$, $(3, 9)$; They intersect at $\left(\sqrt{9}, 9\right)$ and $\left(-\sqrt{9}, 9\right)$.

Fair Game Review

37. $x^2 + 10x + 25$

38. $w^2 - 14w + 49$

39. $4y^2 - 12y + 9$

40. B

Mini-Assessment

Solve the equation using square roots.

1. $x^2 = 100$ $x = 10, x = -10$

2. $x^2 + 10 = 0$ no real solutions

3. $5x^2 - 7 = -7$ $x = 0$

4. $2(x + 2)^2 = 72$ $x = 4, x = -8$

Taking Math Deeper

Exercise 33

You can solve this problem by recalling a property of similar figures.

1 The ratio of the areas of two similar figures is equal to the square of the ratio of their corresponding side lengths. Because squares are similar, we can use this property to find x.

2 Use the property to write an equation. Because the area of the inner square is 25% of the area of the rug, the ratio of the area of the inner square to the area of the rug is $\frac{1}{4}$.

$$\frac{\text{Area of inner square}}{\text{Area of rug}} = \left(\frac{\text{side length of inner square}}{\text{side length of rug}}\right)^2$$

$$\frac{1}{4} = \left(\frac{x}{6}\right)^2$$

3 Solve the equation.

$$\frac{1}{4} = \left(\frac{x}{6}\right)^2$$

$$\frac{1}{4} = \frac{x^2}{36}$$

$$9 = x^2$$

$$3 = x$$

So, the side length of the inner square is 3 feet.

Project

Research Tibetan rugs and Persian carpets. Which type would you buy?

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension Start the next section

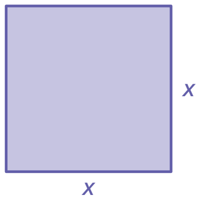
Solve the equation using square roots. Use a graphing calculator to check your solution(s).

2 **23.** $(x + 3)^2 = 0$ **24.** $(x - 1)^2 = 4$ **25.** $(2x - 1)^2 = 81$

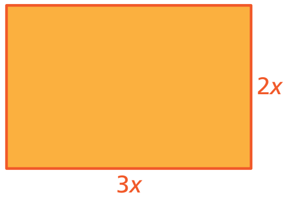
26. $(4x - 5)^2 = 9$ **27.** $9(x + 1)^2 = 16$ **28.** $4(x - 2)^2 = 25$

Use the given area A to find the dimensions of the figure.

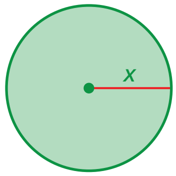
29. $A = 64$ in.^{2}



30. $A = 78$ cm^2



31. $A = 144\pi$ ft^2



3 **32.** **POND** An in-ground pond has the shape of a rectangular prism. The pond has a height of 24 inches and a volume of 33,000 cubic inches. The pond's length is 2 times its width. Find the length and width of the pond.

33. **AREA RUG** The design of a square area rug for your living room is shown. You want the area of the inner square to be 25% of the total area of the rug. Find the side length x of the inner square.

34. **WRITING** How can you approximate the roots of a quadratic equation when the roots are not integers?

35. **LOGIC** Given the equation $ax^2 + c = 0$, describe the values of a and c so the equation has the following number of solutions.

a. two solutions **b.** one solution **c.** no solutions

36. **Reasoning** Without graphing, where do the graphs of $y = x^2$ and $y = 9$ intersect? Explain.

Fair Game Review What you learned in previous grades & lessons

Find the product. *(Section 7.4)*

37. $(x + 5)^2$ **38.** $(w - 7)^2$ **39.** $(2y - 3)^2$

40. **MULTIPLE CHOICE** What is an explicit equation for $a_1 = -3$, $a_n = a_{n-1} + 2$? *(Section 6.7)*

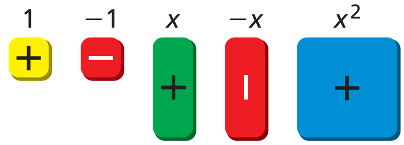
Ⓐ $a_n = 2n - 3$ Ⓑ $a_n = 2n - 5$ Ⓒ $a_n = n + 2$ Ⓓ $a_n = -3n + 2$

9.3 Solving Quadratic Equations by Completing the Square

Essential Question How can you use "completing the square" to solve a quadratic equation?

1 EXAMPLE: Solving by Completing the Square

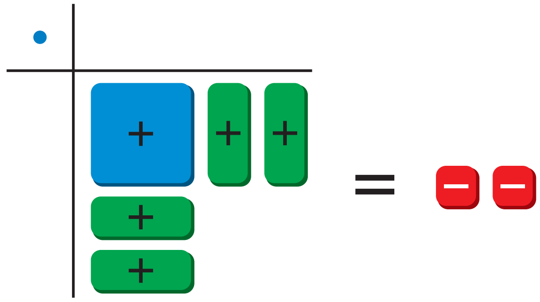
Work with a partner. Five different algebra tiles are shown at the right.



Solve $x^2 + 4x = -2$ by completing the square.

Step 1: Use algebra tiles to model the equation

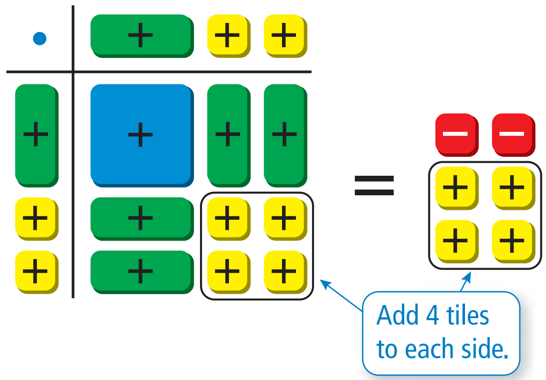
$$x^2 + 4x = -2.$$



Step 2: Add four yellow tiles to the left side of the equation so that it is a perfect square. Balance the equation by also adding four yellow tiles to the right side.

$$x^2 + 4x + 4 = -2 + 4$$

$$(x + 2)^2 = 2$$



Step 3: Take the square root of each side of the equation and simplify.

$$x + 2 = \pm\sqrt{2}$$

$$x = -2 \pm \sqrt{2}$$

Check Check each solution in the original equation.

$$x^2 + 4x = -2$$

$$(-2 + \sqrt{2})^2 + 4(-2 + \sqrt{2}) \stackrel{?}{=} -2$$

$$4 - 4\sqrt{2} + 2 - 8 + 4\sqrt{2} \stackrel{?}{=} -2$$

$$4 + 2 - 8 \stackrel{?}{=} -2$$

$$-2 = -2 \checkmark$$

Now you check the other solution.

Common Core

Solving Quadratic Equations

In this lesson, you will

- solve quadratic equations by completing the square.
- solve real-life problems.

Learning Standards
A.REI.4a
A.REI.4b
A.SSE.3b
F.IF.8a

Laurie's Notes

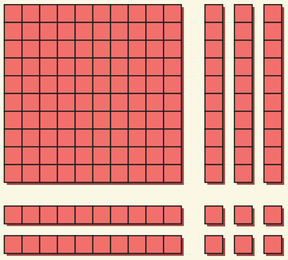
Introduction

Standards for Mathematical Practice

- **MP5 Use Appropriate Tools Strategically:** Students are using algebra tiles to model the technique of *completing the square*.

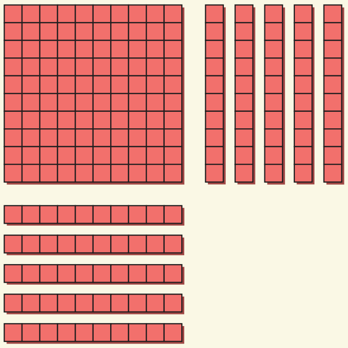
Motivate

- Students should recall using base 10 pieces in an array to represent multiplication.
- Display the model.



- ❓ "What multiplication problem does it represent?"
  $12 \times 13 = 100 + 50 + 6 = 156$
- Discuss the value of each piece (1), the dimensions of the array ($10 + 2$ by $10 + 3$), and the answer.
- Display the model.

$15^2 =$



- ❓ "What is missing from the model?" 25 pieces in the bottom right corner

Activity Notes

Discuss

- **Connection:** Squaring a number (like 15^2) connects to today's work with completing the square, another technique for solving quadratic equations.

Activity 1

- Review the names and dimensions of the algebra tiles.
- Students should model the equation with the goal of adding 1-tiles to form a square array on the left side. To make this possible, start by arranging half of the x-tiles vertically and half horizontally.
- Add the number of 1-tiles needed to form a square on the left side. Add the same number of 1-tiles to the right side.
- Remind students how multiplication of binomials was modeled (rectangular array).
- Explain how to use the dimensions of the square formed on the left side of the equation to write it as the square of a binomial. The vertical and horizontal dimensions are each $x + 2$, so the left side of the equation represents $(x + 2)^2$.
- **MP2 Reason Abstractly and Quantitatively:** Students have obtained a concrete model of $(x + 2)^2 = 2$. They will now reason abstractly, using algebra to manipulate the equation.
- It is important to work through the checking process.

Common Core State Standards

A.REI.4a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions . . ..
A.REI.4b Solve quadratic equations by . . . completing the square . . ..
A.SSE.3b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
F.IF.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Previous Learning

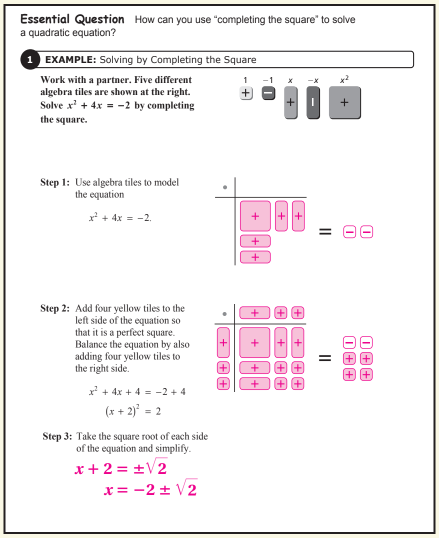
Students should know how to solve quadratic equations using square roots.

Technology for the Teacher

Dynamic Classroom

Lesson Plans
Complete Materials List

9.3 Record and Practice Journal



Essential Question How can you use "completing the square" to solve a quadratic equation?

1 EXAMPLE: Solving by Completing the Square

Work with a partner. Five different algebra tiles are shown at the right. Solve $x^2 + 4x = -2$ by completing the square.

Step 1: Use algebra tiles to model the equation

$x^2 + 4x = -2.$

Step 2: Add four yellow tiles to the left side of the equation so that it is a perfect square. Balance the equation by also adding four yellow tiles to the right side.

$x^2 + 4x + 4 = -2 + 4$
$(x + 2)^2 = 2$

Step 3: Take the square root of each side of the equation and simplify.

$x + 2 = \pm\sqrt{2}$
$x = -2 \pm \sqrt{2}$

English Language Learners

Vocabulary

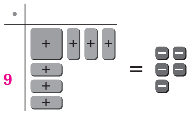
Discuss the use of the word *square* as a noun and as a verb. For example, to square (verb) the number 15, you use a model in the shape of a square (noun). The result is the square (noun) of 15. In the phrase *completing the square,* the word square is used as a noun.

9.3 Record and Practice Journal

2 ACTIVITY: Solving by Completing the Square

Work with a partner.

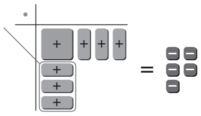
- Write the equation modeled by the algebra tiles.
  $x^2 + 6x = -5$
- Use algebra tiles to complete the square.
  $x^2 + 6x + 9 = -5 + 9$
- Write the solutions of the equation.
  $x = -5, -1$
- Check each solution.
  $(-5)^2 + 6(-5) = -5$
  $(-1)^2 + 6(-1) = -5$



3 ACTIVITY: Writing a Rule

Work with a partner.

- What does this group of tiles represent?
  $3x$
- How is the coefficient of x for this group of tiles related to the coefficient of x in the equation from Activity 2?
  It is half.
- How is it related to the number of tiles you add to each side when completing the square?
  The number of tiles you add to each side when completing the square is the square of this coefficient.



- **WRITE A RULE** Fill in the blanks.

  To complete the square, take ___half___ of the coefficient of the x-term and ___square___ it. ___Add___ this number to each side of the equation.

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use "completing the square" to solve a quadratic equation?
   Factor the left-hand side as the square of a binomial and evaluate the addition on the right-hand side of the equation. Then take the square root of each side and solve for x.

5. Solve each quadratic equation by completing the square.

   a. $x^2 - 2x = 1$ $\quad x = 1 \pm \sqrt{2}$
   b. $x^2 - 4x = -1$ $\quad x = 2 \pm \sqrt{3}$
   c. $x^2 + 4x = -3$ $\quad x = -3, -1$

Laurie's Notes

Activity 2

- Having guided students through the first activity, students should be able to work with a partner on this activity.

? "What equation is modeled by the algebra tiles?" $x^2 + 6x = -5$

? "What equation is represented by the tiles after completing the square?" $x^2 + 6x + 9 = 4$

- **MP3a Construct Viable Arguments:** Asking students to explain the thinking behind their solutions is helpful for them and for their peers.

Activity 3

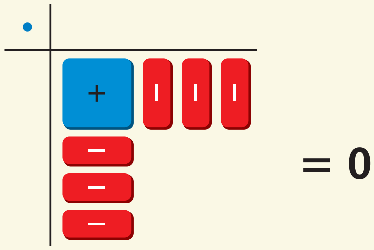
- Encourage students to read through the whole activity before attempting to answer the questions. This will help them understand the focus of the activity.
- **MP2** and **MP7 Look for and Make Use of Structure:** In this activity, students compare the structure of the concrete model to the structure of the abstract equation it represents. They use these comparisons to deduce an algebraic rule for completing the square.
- The activity is intended for students to answer the first question as $3x$, realize that the coefficient 3 is one-half of the coefficient in the equation, and then realize that you square it and add that number of 1-tiles to each side. The model helps visualize this.

What Is Your Answer?

- **MP3a:** In Question 4, answers should include an understanding of writing the quadratic equation in a form that allows you to solve it using square roots.

Closure

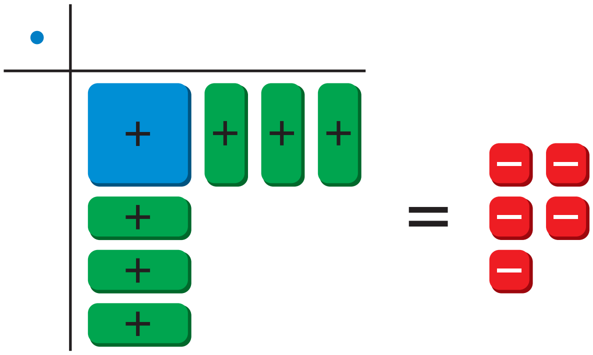
- Ask students to repeat Activity 2 for the model shown and then compare the two problems. Procedurally it is the same (add nine 1-tiles to each side, write the equation modeled, and solve).



2 ACTIVITY: Solving by Completing the Square

Work with a partner.

- Write the equation modeled by the algebra tiles.
- Use algebra tiles to complete the square.
- Write the solutions of the equation.
- Check each solution.



3 ACTIVITY: Writing a Rule

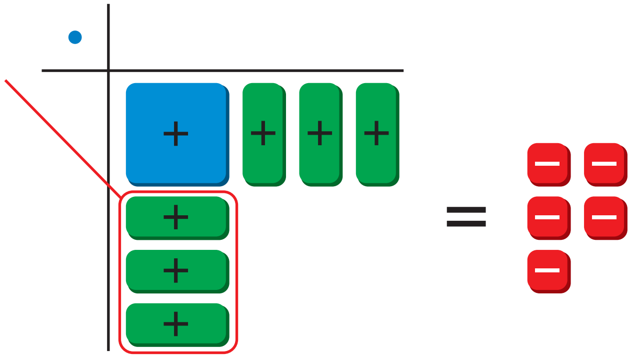
Math Practice 6

State the Meaning of Symbols

Which algebra tiles do you need to add to complete the square? How can you represent the tiles in the equation?

Work with a partner.

- What does this group of tiles represent?
- How is the coefficient of x for this group of tiles related to the coefficient of x in the equation from Activity 2? How is it related to the number of tiles you add to each side when completing the square?
- **WRITE A RULE** Fill in the blanks.



To complete the square, take __________ of the coefficient of the x-term and __________ it. __________ this number to each side of the equation.

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use "completing the square" to solve a quadratic equation?

5. Solve each quadratic equation by completing the square.

   a. $x^2 - 2x = 1$ **b.** $x^2 - 4x = -1$ **c.** $x^2 + 4x = -3$

Practice Use what you learned about quadratic equations to complete Exercises 3–5 on page 472.

9.3 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
completing the square, *p. 470*

Another method for solving quadratic equations is **completing the square**. In this method, a constant c is added to the expression $x^2 + bx$ so that $x^2 + bx + c$ is a perfect square trinomial.

Key Idea

Completing the Square

Words To complete the square for an expression of the form $x^2 + bx$, follow these steps.

Step 1: Find one-half of b, the coefficient of x.
Step 2: Square the result from Step 1.
Step 3: Add the result from Step 2 to $x^2 + bx$.

Factor the resulting expression as the square of a binomial.

Algebra $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

EXAMPLE 1 Completing the Square

Complete the square for each expression. Then factor the trinomial.

a. $x^2 + 6x$

Step 1: Find one-half of b. $\frac{b}{2} = \frac{6}{2} = 3$

Step 2: Square the result from Step 1. $3^2 = 9$

Step 3: Add the result from Step 2 to $x^2 + bx$. $x^2 + 6x + 9$

$x^2 + 6x + 9 = (x + 3)^2$

b. $x^2 - 9x$

Step 1: Find one-half of b. $\frac{b}{2} = \frac{-9}{2}$

Step 2: Square the result from Step 1. $\left(\frac{-9}{2}\right)^2 = \frac{81}{4}$

Step 3: Add the result from Step 2 to $x^2 + bx$. $x^2 - 9x + \frac{81}{4}$

$x^2 - 9x + \frac{81}{4} = \left(x - \frac{9}{2}\right)^2$

On Your Own

Now You're Ready
Exercises 12–17

Complete the square for each expression. Then factor the trinomial.

1. $x^2 + 10x$ **2.** $x^2 - 4x$ **3.** $x^2 + 7x$

Laurie's Notes

Introduction

Connect

- **Yesterday:** Students solved quadratic equations of the form $x^2 + bx = d$ using algebra tiles. (MP2, MP3a, MP5, MP7)
- **Today:** Students will solve quadratic equations by *completing the square.*

Motivate

- Write the sequence on the board: 1, 4, ____, 16, ____, 36, . . .
- ? "What numbers are missing and what is the pattern? Explain." 9 and 25; The terms of the sequence are perfect squares.
- ? "What is a perfect square trinomial?" a trinomial that can be factored as $(a \pm b)^2$

Lesson Notes

Discuss

- **MP7 Look for and Make Use of Structure:** In expressions of the form $x^2 + bx$, students will use the coefficient of the x-term to determine the constant that must be added to form a perfect square trinomial.
- In the previous section, students solved quadratic equations of the form $x^2 = d$. Now they will solve quadratic equations of the form $x^2 + bx = d$.

Key Idea

- Write the Key Idea on the board.
- ? "How does one-half of b in Step 1 connect to the algebra tiles activity?" Half of the x-tiles were placed on each dimension of the square.
- ? "How does Step 2 connect to the algebra tiles activity?" This gives the number of 1-tiles that were added to each side of the equation.

Example 1

- This example helps students become familiar with the technique of completing the square before they use it to solve an equation.
- It may be helpful to ask students what is missing that would make it a perfect square trinomial.

$$x^2 + 6x + \underline{\ ?\ } = (x + \underline{\ ?\ })^2$$

- Work through both parts as shown.

On Your Own

- **Neighbor Check:** Have students work independently and then have their neighbors check their work. Have students discuss any discrepancies.
- Ask a volunteer to discuss the solution of Question 3. The odd coefficient may challenge some students.

Goal Today's lesson is solving quadratic equations by **completing the square**.

Technology for the Teacher

Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Extra Example 1

Complete the square for each expression. Then factor the trinomial.

a. $x^2 + 8x$ $x^2 + 8x + 16 = (x + 4)^2$

b. $x^2 - x$ $x^2 - x + \frac{1}{4} = \left(x - \frac{1}{2}\right)^2$

On Your Own

1. $x^2 + 10x + 25 = (x + 5)^2$
2. $x^2 - 4x + 4 = (x - 2)^2$
3. $x^2 + 7x + \frac{49}{4} = \left(x + \frac{7}{2}\right)^2$

Extra Example 2

Solve $x^2 + 4x - 5 = 7$ by completing the square. $x = -6, x = 2$

Extra Example 3

In Example 3, you throw the stone with an upward velocity of 48 feet per second. The function $h = -16t^2 + 48t + 16$ gives the height h of the stone after t seconds. When does the stone land in the water? after about 3.3 seconds

On Your Own

4. $x = -3, x = 9$
5. $x = -6 + \sqrt{34}$,
   $x = -6 - \sqrt{34}$
6. $x = -6, x = 4$
7. after about 4.2 seconds

Differentiated Instruction

Visual

In each step of Example 1(a), look back at the model in Activity 2 and discuss how the step is connected to the model.

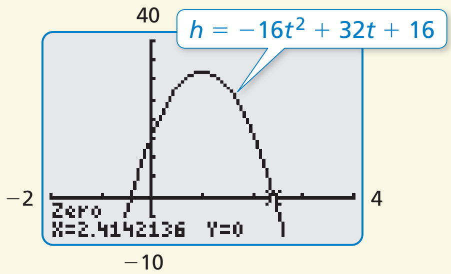
Laurie's Notes

Example 2

- Write on the board: $x^2 + 8x + \underline{\ ?\ } = \underline{\ ?\ }$.
- ? "If each blank is replaced by 0, how do you complete the square?" Add 16 to each side.
- ? "If the first blank is replaced by 0 and the second blank is replaced by a nonzero number, how do you complete the square?" Add 16 to each side.
- ? "If each blank is replaced by a nonzero number, how do you complete the square?" Isolate $x^2 + 8x$ on the left side and then add 16 to each side.
- **Common Error:** Students often forget the negative square root when taking the square root of each side of the equation.
- Note that when 4 is subtracted from each side in the last step, the next step is written as $x = -4 \pm 6$ This form is preferred to $x = \pm 6 - 4$.

Example 3

- **MP4 Modeling with Mathematics:** Read through the problem. Remind students that they have used the vertical motion model previously.
- ? "What do the coefficients and the constant term have in common?" They are all divisible by 16.
- Discuss the Study Tip about having a leading coefficient of 1.
- Work through the problem as shown.
- ? "Why is −0.4 discounted as a solution?" Time cannot be negative.
- **MP5 Use Appropriate Tools Strategically:** Graph the function. The graph shows the two solutions, but only the positive solution makes sense.



On Your Own

- In Question 7, what has changed from Example 3? the upward velocity

Closure

You have studied the methods of factoring, graphing, using square roots and completing the square to solve quadratic equations. State which method you would use to solve each equation below. (Sample answers provided.)

a. $4x^2 - 12 = 4$ square roots
b. $x^2 - 8x = 0$ completing the square
c. $x^2 + 4x + 3 = 0$ graphing or factoring

To solve a quadratic equation by completing the square, write the equation in the form $x^2 + bx = d$.

EXAMPLE 2 Solving a Quadratic Equation by Completing the Square

Solve $x^2 + 8x - 3 = 17$ by completing the square.

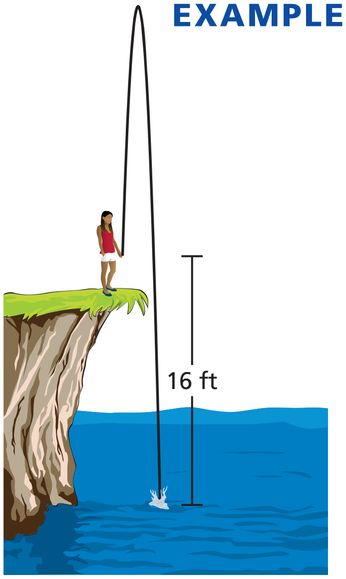
$x^2 + 8x - 3 = 17$	Write the equation.
$x^2 + 8x = 20$	Add 3 to each side.
Complete the square. → $x^2 + 8x + 16 = 20 + 16$	Add $\left(\frac{8}{2}\right)^2$, or 16, to each side.
$(x + 4)^2 = 36$	Factor $x^2 + 8x + 16$.
$x + 4 = \pm 6$	Take the square root of each side.
$x = -4 \pm 6$	Subtract 4 from each side.

The solutions are $x = -4 + 6 = 2$ and $x = -4 - 6 = -10$.

Common Error

When completing the square, be sure to add to *both* sides of the equation.

EXAMPLE 3 Real-Life Application



You throw a stone from a height of 16 feet with an upward velocity of 32 feet per second. The function $h = -16t^2 + 32t + 16$ gives the height h of the stone after t seconds. When does the stone land in the water?

Find the t-values for which $h = 0$. So, solve $-16t^2 + 32t + 16 = 0$.

$-16t^2 + 32t + 16 = 0$	Write the equation.
$t^2 - 2t - 1 = 0$	Divide each side by -16.
$t^2 - 2t = 1$	Add 1 to each side.
Complete the square. → $t^2 - 2t + 1 = 1 + 1$	Add $\left(\frac{-2}{2}\right)^2$, or 1, to each side.
$(t - 1)^2 = 2$	Factor $t^2 - 2t + 1$.
$t - 1 = \pm\sqrt{2}$	Take the square root of each side.
$t = 1 \pm \sqrt{2}$	Add 1 to each side.

The solutions are $x = 1 + \sqrt{2} \approx 2.4$ and $x = 1 - \sqrt{2} \approx -0.4$. Use the positive solution.

The stone lands in the water after about 2.4 seconds.

Study Tip

Before completing the square, make sure the leading coefficient is 1.

On Your Own

Now You're Ready
Exercises 18–23

Solve the equation by completing the square.

4. $x^2 - 6x = 27$
5. $x^2 + 12x + 3 = 1$
6. $2x^2 + 4x + 10 = 58$
7. **WHAT IF?** In Example 3, the function $h = -16t^2 + 64t + 16$ gives the height h (in feet) of the stone after t seconds. When does the stone land in the water?

9.3 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

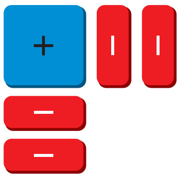
Vocabulary and Concept Check

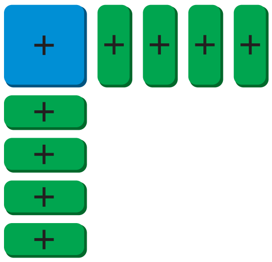
1. **VOCABULARY** Explain how to complete the square for an expression of the form $x^2 + bx$.

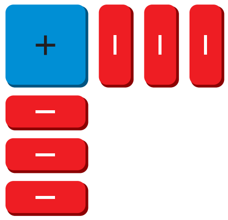
2. **WRITING** For what values of b is it easier to complete the square for $x^2 + bx$? Explain.

Practice and Problem Solving

Use algebra tiles to complete the square. Then write the perfect square trinomial.

3. 

4. 

5. 

Find the value of c that completes the square.

6. $x^2 - 8x + c$
7. $x^2 + 4x + c$
8. $x^2 - 2x + c$
9. $x^2 - 14x + c$
10. $x^2 + 12x + c$
11. $x^2 + 18x + c$

Complete the square for the expression. Then factor the trinomial.

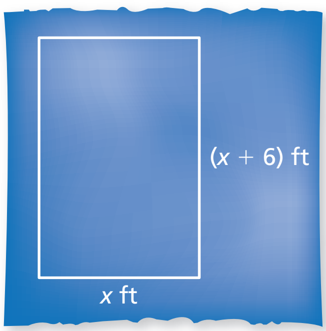
1 12. $x^2 - 10x$
13. $x^2 + 16x$
14. $x^2 + 22x$
15. $x^2 - 40x$
16. $x^2 - 3x$
17. $x^2 + 5x$

Solve the equation by completing the square.

2 18. $x^2 + 2x = 3$
19. $x^2 - 6x = 16$
20. $x^2 + 4x + 7 = -6$
21. $x^2 + 5x - 7 = -14$
22. $2x^2 - 8x = 10$
23. $2x^2 - 3x + 1 = 0$

24. **ERROR ANALYSIS** Describe and correct the error in solving the equation.

$$x^2 + 8x = 10$$
$$x^2 + 8x + 16 = 10$$
$$(x + 4)^2 = 10$$
$$x + 4 = \pm\sqrt{10}$$
$$x = -4 \pm \sqrt{10}$$



25. **PATIO** The area of the new patio is 216 square feet.

a. Write an equation for the area of the patio.

b. Find the dimensions of the patio by completing the square.

Assignment Guide and Homework Check

Level	Assignment	Homework Check
Average	1–4, 7–23 odd, 24, 26, 32–36	7, 15, 19, 24
Advanced	1, 2, 12–24 even, 26–36	16, 20, 24, 28

Common Errors

- **Exercises 6–23** Students may forget to divide the x-coefficient by 2 before squaring. Remind them of this process.
- **Exercises 12–23** Students may factor the trinomial as $\left[x + \left(\frac{b}{2}\right)^2\right]^2$ instead of $\left(x + \frac{b}{2}\right)^2$. Remind them that in the factored form of the trinomial, the term $\frac{b}{2}$ should not be squared.
- **Exercises 18–23** Students may not add the same value to each side of the equation when completing the square. Remind students that to form an equivalent equation, they must add the same quantity to each side.

9.3 Record and Practice Journal

Complete the square for the expression. Then factor the trinomial.

1. $x^2 + 8x$
   $x^2 + 8x + 16 = (x + 4)^2$
2. $x^2 - 6x$
   $x^2 - 6x + 9 = (x - 3)^2$
3. $x^2 - 20x$
   $x^2 - 20x + 100 = (x - 10)^2$
4. $x^2 + 7x$
   $x^2 + 7x + \frac{49}{4} = \left(x + \frac{7}{2}\right)^2$

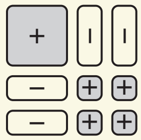
Solve the equation by completing the square.

5. $x^2 - 2x = 8$
   $x = -2, 4$
6. $x^2 + 12x + 9 = -5$
   $x = -6 \pm \sqrt{22}$
7. $x^2 + 5x + 13 = 1$
   no solution
8. $2x^2 - 6x + 3 = 11$
   $x = -1, 4$
9. Your backyard is rectangular and has an area of 760 square meters. The length of the yard is 18 meters more than the width. Find the length and width of the backyard.
   length: 38 m; width: 20 m

Vocabulary and Concept Check

1. Add $\left(\frac{b}{2}\right)^2$.
2. A perfect square trinomial is a trinomial that can be factored as the square of a binomial.
   Example:
   $(x + 2)^2 = x^2 + 4x + 4$

Practice and Problem Solving

3. 
   $x^2 - 4x + 4$

4–5. See Additional Answers.

6. 16 **7.** 4

8. 1 **9.** 49

10. 36 **11.** 81

12. $x^2 - 10x + 25 = (x - 5)^2$

13. $x^2 + 16x + 64 = (x + 8)^2$

14. $x^2 + 22x + 121 = (x + 11)^2$

15. $x^2 - 40x + 400 = (x - 20)^2$

16. $x^2 - 3x + \frac{9}{4} = \left(x - \frac{3}{2}\right)^2$

17. $x^2 + 5x + \frac{25}{4} = \left(x + \frac{5}{2}\right)^2$

18. $x = -3, x = 1$

19. $x = -2, x = 8$

20. no real solutions

21. no real solutions

22. $x = -1, x = 5$

23. $x = \frac{1}{2}, x = 1$

24. 16 was not added to both sides. $x = -4 \pm \sqrt{26}$

25. **a.** $216 = x(x + 6)$

b. 12 ft by 18 ft

Practice and Problem Solving

26. ± 10

27. Divide each side by 3.

28. **a.** $x = -6, x = 2$

b. Evaluate $y = x^2 + 4x - 12$ when x is the mean of the solutions.

29. **a.** after about 4.4 seconds

b. 96 ft; The vertex is (2, 96). The maximum is the y-coordinate of the vertex.

30. See *Taking Math Deeper*.

31. $x(x + 1) = 42$; 6, 7

32. **a.** $y = (x + 2)^2 - 1$; The minimum value is -1.

b. $c - \dfrac{b^2}{4}$

Fair Game Review

33. $2\sqrt{3}$ **34.** $6\sqrt{2}$

35. $2\sqrt{5}$ **36.** B

Mini-Assessment

Solve the equation by completing the square.

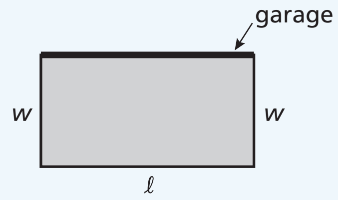
1. $x^2 + 2x = 15$ $x = -5, x = 3$
2. $x^2 - 12x = -32$ $x = 4, x = 8$
3. $x^2 + 6x + 3 = -7$ no real solutions
4. $2x^2 - 11x + 4 = 10$ $x = -\frac{1}{2}, x = 6$
5. A toy rocket is launched from the top of a building. The function $h = -16t^2 + 64t + 64$ gives the height h of the rocket after t seconds. When does the rocket hit the ground? (Round your answer to the nearest tenth of a second.) after about 4.8 seconds

Taking Math Deeper

Exercise 30

The key to this exercise is that the garage forms one side of the garden. So, fencing is needed for only three of the sides.

1 Draw a diagram of the situation. Let ℓ represent the length of the garden and let w represent the width.



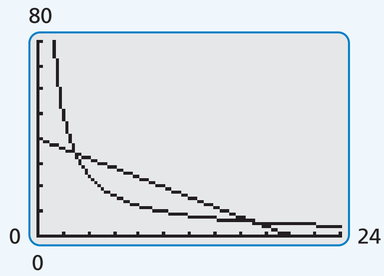
2 Use the information about the perimeter to write an equation.

$$\ell + 2w = 40, \text{ or } \ell = 40 - 2w$$

Use the information about the area to write an equation.

$$\ell w = 100, \text{ or } \ell = \frac{100}{w}$$

3 Graph the two equations and find the points of intersection. Let w be the independent variable.



The graphs intersect at about (2.9, 34.1) and (17.1, 5.9). So, the possible dimensions of the garden are 2.9 feet by 34.1 feet and 17.1 feet by 5.9 feet.

Many students will choose the garden that is 17.1 feet by 5.9 feet because it is less narrow.

Challenge students to explain why these dimensions do not add up to a perimeter of 40 feet.

Project

Research standard garage sizes. Does this play a roll in your answer?

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension • School-to-Work Start the next section

26. **NUMBER SENSE** Find the value of b that makes $x^2 + bx + 25$ a perfect square trinomial.

27. **REASONING** You are completing the square to solve $3x^2 + 6x = 12$. What is the first step?

28. **REASONING** Consider the equation $x^2 + 4x - 12 = 0$.

    a. Solve the equation by completing the square.

    b. Explain how to use the solutions to find the minimum value of $y = x^2 + 4x - 12$.

29. **TOY ROCKET** The function $h = -16t^2 + 64t + 32$ gives the height h (in feet) of a toy rocket after t seconds.

    a. When does the rocket hit the ground?

    b. What is the maximum height of the rocket? Justify your answer.

30. **ROSE GARDEN** You plant a rectangular rose garden along the side of your garage. You enclose 3 sides of the garden with 40 feet of fencing. The total area of the garden is 100 square feet. Find the possible dimensions of the garden. Round to the nearest tenth. Which size garden would you choose?

31. **PRECISION** The product of two consecutive positive integers is 42. Write and solve an equation to find the integers.

32. **Structure** Begin solving $x^2 + 4x + 3 = 0$ by completing the square. Stop when you obtain an equation of the form $(x + p)^2 = q$.

    a. Write the related function in vertex form. Without graphing, determine the maximum or minimum value of the function.

    b. Find the minimum value of $y = x^2 + bx + c$.

Fair Game Review What you learned in previous grades & lessons

Simplify $\sqrt{b^2 - 4ac}$ for the given values. *(Section 6.1)*

33. $a = 3, b = -6, c = 2$

34. $a = -2, b = 4, c = 7$

35. $a = 1, b = 6, c = 4$

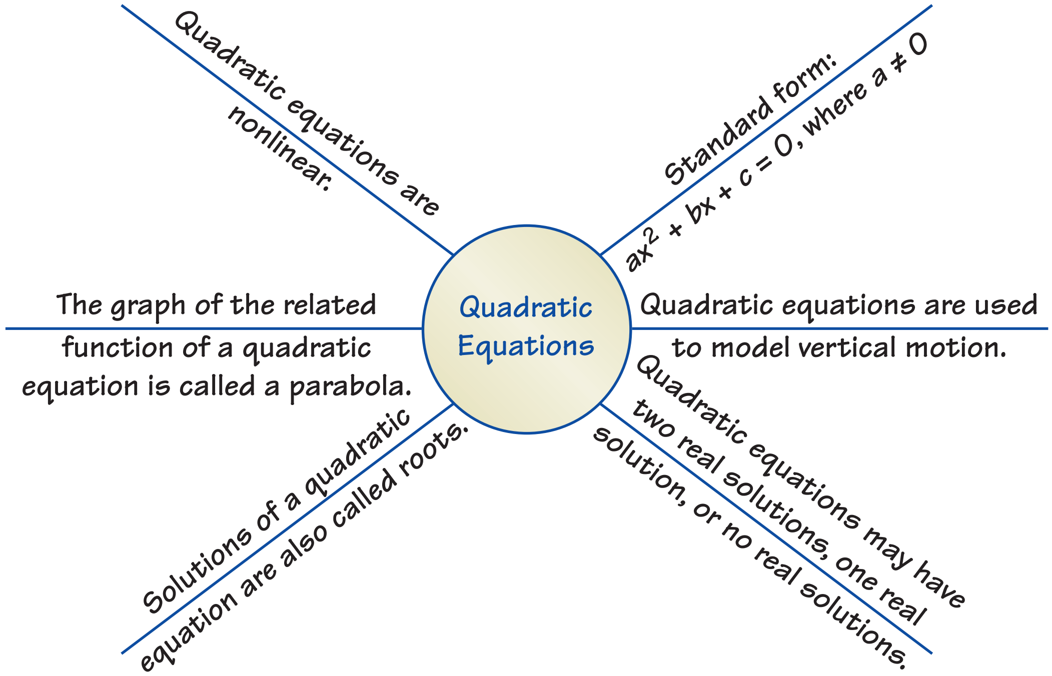
36. **MULTIPLE CHOICE** What are the solutions of $x^2 - 49 = 0$? *(Section 9.2)*

    Ⓐ $x = 7$ Ⓑ $x = -7, x = 7$ Ⓒ $x = 0, x = 7$ Ⓓ no solution

9 Study Help

Check It Out
Graphic Organizer
BigIdeasMath.com

You can use an **information wheel** to organize information about a topic. Here is an example of an information wheel for quadratic equations.



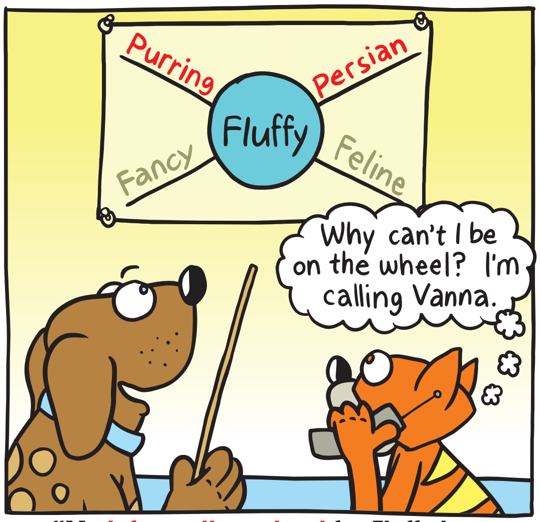
On Your Own

Make information wheels to help you study these topics.

1. solving quadratic equations by graphing
2. solving quadratic equations using square roots
3. solving quadratic equations by completing the square

After you complete this chapter, make information wheels for the following topics.

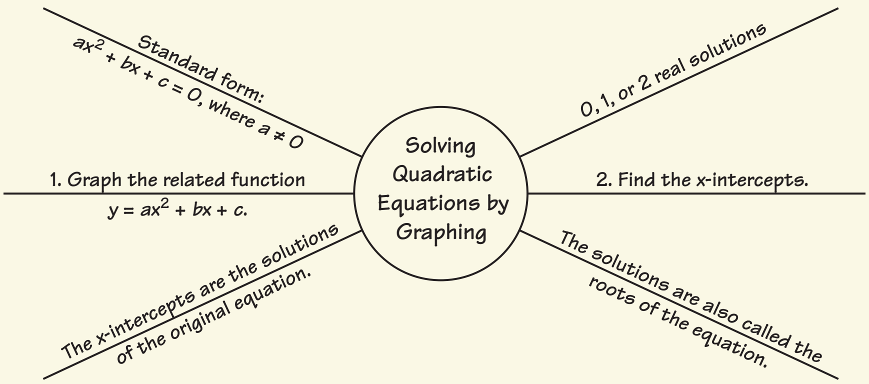
4. solving quadratic equations using the quadratic formula
5. choosing a solution method for solving quadratic equations
6. solving systems of linear and quadratic equations



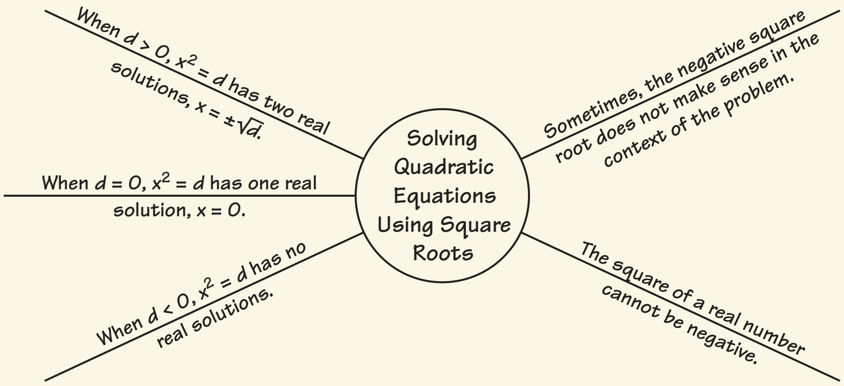
"My **information wheel** for Fluffy has matching adjectives and nouns."

Sample Answers

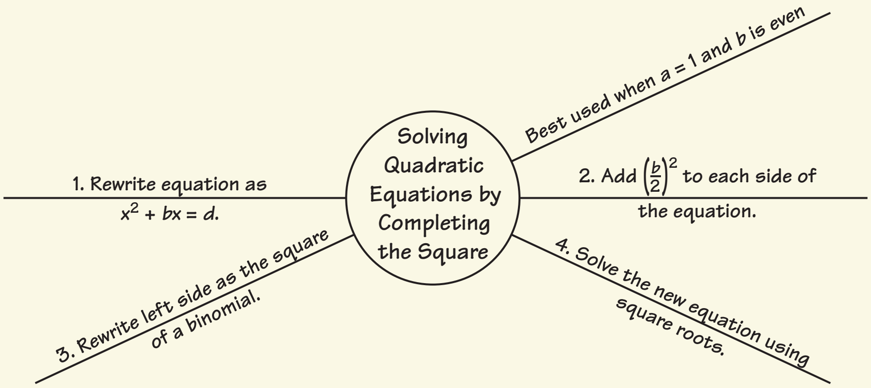
1.



2.



3.



List of Organizers

Available at *BigIdeasMath.com*

Comparison Chart
Concept Circle
Definition (Idea) and Example Chart
Example and Non-Example Chart
Formula Triangle
Four Square
Information Frame
Information Wheel
Notetaking Organizer
Process Diagram
Summary Triangle
Word Magnet
Y Chart

About this Organizer

A **Information Wheel** can be used to organize information about a concept. Students write the concept in the middle of the “wheel.” Then students write information related to the concept on the “spokes” of the wheel. Related information can include, but is not limited to: vocabulary words or terms, definitions, formulas, procedures, examples, and visuals. This type of organizer serves as a good summary tool because any information related to a concept can be included.

Technology for the Teacher

Editable Graphic Organizer

Answers

1. $x = -1, x = 3$
2. no real solutions
3. $x = -5$
4. $x = -7, x = -2$
5. $x = -1, x = 8$
6. no real solutions
7. $x = 4, x = -4$
8. no real solutions
9. $x = 7, x = 9$
10. $x = -9, x = 5$
11. $x = 1 + \sqrt{10}, x = 1 - \sqrt{10}$
12. $x = -7, x = 1$
13. $x = 1 + 2\sqrt{2}, x = 1 - 2\sqrt{2}$
14. Because $x^2 = 100$ has the form $x^2 = d$ with $d > 0$, there are 2 real solutions.
15. length: $4\sqrt{19} \approx 17.44$ m

    width: $\sqrt{19} \approx 4.36$ m
16. **a.** about 0.19 second and about 2.31 seconds

    b. 28 ft

Technology for the Teacher

Online Assessment
Assessment Book
ExamView® Assessment Suite

Alternative Quiz Ideas

100% Quiz	Math Log
Error Notebook	Notebook Quiz
Group Quiz	Partner Quiz
Homework Quiz	Pass the Paper

100% Quiz

This is a quiz where students are given the answers and then they have to explain and justify each answer.

Reteaching and Enrichment Strategies

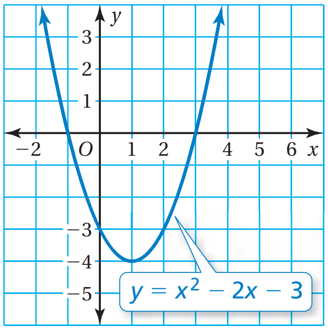
If students need help. . .	If students got it. . .
Resources by Chapter • Study Help • Practice A and Practice B • Puzzle Time Lesson Tutorials *BigIdeasMath.com*	Resources by Chapter • Enrichment and Extension • School-to-Work Game Closet at *BigIdeasMath.com* Start the next section

9.1–9.3 Quiz

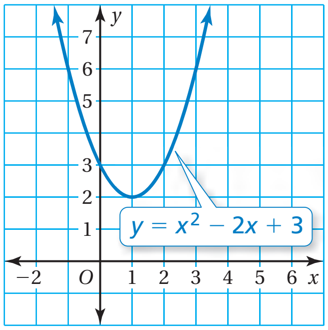
Check It Out
Progress Check
BigIdeasMath.com

Determine the solution(s) of the equation. Check your solution(s). *(Section 9.1)*

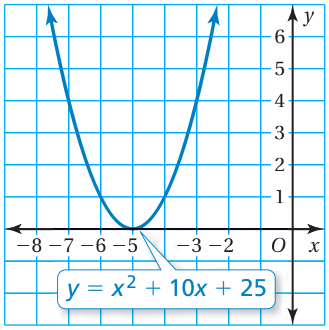
1. $x^2 - 2x - 3 = 0$



2. $x^2 - 2x + 3 = 0$



3. $x^2 + 10x + 25 = 0$



Solve the equation by graphing. Check your solution(s). *(Section 9.1)*

4. $x^2 + 9x + 14 = 0$

5. $x^2 - 7x = 8$

6. $x + 1 = -x^2$

Solve the equation using square roots. *(Section 9.2)*

7. $4x^2 = 64$

8. $-3x^2 + 6 = 10$

9. $(x - 8)^2 = 1$

Solve the equation by completing the square. *(Section 9.3)*

10. $x^2 + 4x = 45$

11. $x^2 - 2x - 1 = 8$

12. $2x^2 + 12x + 20 = 34$

13. $-4x^2 + 8x + 44 = 16$

14. REASONING Explain how to determine the number of real solutions of $x^2 = 100$ without solving. *(Section 9.2)*

15. VOLUME The length of a rectangular prism is 4 times its width. The volume of the prism is 380 cubic meters. Find the length and width of the prism. *(Section 9.2)*

16. PROBLEM SOLVING A cannon launches a cannonball from a height of 3 feet with an upward velocity of 40 feet per second. The function $h = -16t^2 + 40t + 3$ gives the height h (in feet) of the cannonball after t seconds. *(Section 9.1 and Section 9.3)*

a. After how many seconds is the cannonball 10 feet above the ground?

b. What is the maximum height of the cannonball?

9.4 Solving Quadratic Equations Using the Quadratic Formula

Essential Question How can you use the discriminant to determine the number of solutions of a quadratic equation?

1 ACTIVITY: Deriving the Quadratic Formula

Work with a partner. The following steps show one method of solving $ax^2 + bx + c = 0$. Explain what was done in each step.

$$ax^2 + bx + c = 0$$ ← 1. Write the equation.

$$4a^2x^2 + 4abx + 4ac = 0$$ ← 2. What was done?

$$4a^2x^2 + 4abx + 4ac + b^2 = b^2$$ ← 3. What was done?

$$4a^2x^2 + 4abx + b^2 = b^2 - 4ac$$ ← 4. What was done?

$$(2ax + b)^2 = b^2 - 4ac$$ ← 5. What was done?

$$2ax + b = \pm\sqrt{b^2 - 4ac}$$ ← 6. What was done?

$$2ax = -b \pm \sqrt{b^2 - 4ac}$$ ← 7. What was done?

Quadratic Formula: $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ ← 8. What was done?

Common Core

Solving Quadratic Equations

In this lesson, you will

- solve quadratic equations by the quadratic formula.
- use discriminants to determine the number of real solutions of quadratic equations.

Learning Standards
A.REI.4a
A.REI.4b

2 ACTIVITY: Deriving the Quadratic Formula by Completing the Square

- Solve $ax^2 + bx + c = 0$ by completing the square. (*Hint:* Subtract c from each side, divide each side by a, and then proceed by completing the square.)
- Compare this method with the method in Activity 1. Explain why you think $4a$ and b^2 were chosen in Steps 2 and 3 of Activity 1.

Laurie's Notes

Introduction

Standards for Mathematical Practice

- **MP1a Make Sense of Problems:** Students read and explain the steps provided for deriving the quadratic formula. This helps deepen their understanding of the quadratic formula and the process for deriving a formula.
- **MP3b Critique the Reasoning of Others:** Students derive the quadratic formula on their own in Activity 2 by completing the square. Then they compare the two methods.

Motivate

- **Story Time:** Share with students some history about the quadratic formula.
  - Around 400 B.C., the Babylonians and Chinese use a method called "completing the square" to solve problems involving areas.
  - Around 300 B.C., the Greek mathematicians Pythagoras and Euclid use geometry to find a general procedure for solving a quadratic equation, but their methods are not considered useful.
  - Around 700 A.D., the Hindu mathematician named Brahmagupta finds the general solution for the quadratic equation. He uses irrational numbers and also recognizes the two roots in the solution.
  - Around 1100 A.D., another Hindu mathematician named Baskhara finds the complete solution. He recognizes that any positive number has two square roots.
  - In 1637, the French mathematician René Descartes publishes *La Géométrie* which presents the quadratic formula in its present form.
- Look back at Examples 2 and 3 in Section 6.1. Students will be evaluating and simplifying these types of expressions in this section.

Activity Notes

Discuss

- Tell students that today they will see one way of deriving the quadratic formula algebraically. Then they will derive the quadratic formula on their own by completing the square. They will also learn about the discriminant and discover how it relates to the number of solutions.

Activity 1

- A derivation of the quadratic formula is shown in this activity. Students justify each step.
- If students get stuck on a step, ask them what is different from the last step. Mathematically, what has changed?
- When students are finished, discuss the justifications.
- Students sometimes lose sight of what they have accomplished. Starting with the quadratic equation in general form, they solve for x algebraically to produce a formula for finding the solutions of any quadratic equation.

Common Core State Standards

A.REI.4a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

A.REI.4b Solve quadratic equations by . . . the quadratic formula . . ..

Previous Learning

Students should know how to solve quadratic equations by completing the square. Students should know how to find the square roots of positive numbers.

Technology for the Teacher

Dynamic Classroom

Lesson Plans

Complete Materials List

9.4 Record and Practice Journal

Essential Question How can you use the discriminant to determine the number of solutions of a quadratic equation?

1 ACTIVITY: Deriving the Quadratic Formula

Work with a partner. The following steps show one method of solving $ax^2 + bx + c = 0$. Explain what was done in each step.

$ax^2 + bx + c = 0$	1. Write the equation.
$4a^2x^2 + 4abx + 4ac = 0$	2. Multiply the equation by $4a$.
$4a^2x^2 + 4abx + 4ac + b^2 = b^2$	3. Add b^2 to each side.
$4a^2x^2 + 4abx + b^2 = b^2 - 4ac$	4. Subtract $4ac$ from each side.
$(2ax + b)^2 = b^2 - 4ac$	5. Factor.
$2ax + b = \pm\sqrt{b^2 - 4ac}$	6. Take the square root of each side.
$2ax = -b \pm \sqrt{b^2 - 4ac}$	7. Subtract b from each side.
Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	8. Divide each side by $2a$.

Differentiated Instruction

Visual, Auditory

In Activity 3, explain that each graph represents just one example of each solution type. Show other graphs for each solution type and discuss the common characteristics of these graphs.

9.4 Record and Practice Journal

2 ACTIVITY: Deriving the Quadratic Formula by Completing the Square

- Solve $ax^2 + bx + c = 0$ by completing the square. (*Hint:* Subtract c from each side, divide each side by a, and then proceed by completing the square.)

$x = -\frac{b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{c}{a}}$

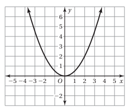
- Compare this method with the method in Activity 1. Explain why you think $4a$ and b^2 were chosen in Steps 2 and 3 of Activity 1.

Check students' work.

3 ACTIVITY: Writing a Rule

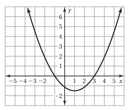
Work with a partner. In the quadratic formula in Activity 1, the expression under the radical sign, $b^2 - 4ac$, is called the discriminant. For each graph, decide whether the corresponding discriminant is equal to 0, is greater than 0, or is less than 0. Explain your reasoning.

a. 1 rational solution



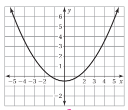
equals 0

b. 2 rational solutions



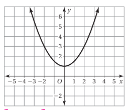
greater than 0

c. 2 irrational solutions



greater than 0

d. no real solutions



less than 0

What Is Your Answer?

4. IN YOUR OWN WORDS How can you use the discriminant to determine the number of solutions of a quadratic equation?

positive discriminant: two solutions
zero discriminant: one solution
negative discriminant: no real solutions

5. Use the quadratic formula to solve each quadratic equation.

a. $x^2 + 2x - 3 = 0$ **$x = -3, 1$**

b. $x^2 - 4x + 4 = 0$ **$x = 2$**

c. $x^2 + 4x + 5 = 0$ **no real solution**

6. Use the Internet to research *imaginary numbers*. How are they related to quadratic equations?

An imaginary number is a number whose square is negative.
When the discriminant is negative.

Laurie's Notes

Activity 2

- In Section 9.3, students learned how to solve a quadratic equation by completing the square. They are using that method to derive the quadratic formula and compare it to the method used in Activity 1.
- **MP1b Persevere in Solving Problems:** Do not be too quick to rescue your students. Give them time to wrestle with the derivation. If they get stuck, have them refer back to their notes from the last section. Believe that your students can persevere in deriving the formula. Unlike many problems, they know what the formula should look like when they finish because of Activity 1.

Activity 3

- **Teaching Tip:** Begin by having students identify a, b, and c for a quadratic equation such as $3x^2 - 4x + 8 = 0$. Then have them substitute the values for a, b, and c into the quadratic formula.
- **Common Error:** Students may make mistakes when using negative signs. For $3x^2 - 4x + 8 = 0$, the value of b is -4. So, in the quadratic formula, $-b = -(4) = 4$
- ? "How many solutions does a quadratic equation have when the value of the discriminant is 0?" 1

What Is Your Answer?

- **MP7 Look for and Make Use of Structure:** In Question 4, students should try to list rules for the number of solutions based on the value of the discriminant.
- To solve each quadratic equation in Question 5, tell students they need to evaluate the quadratic formula for the values of a, b, and c from the equation.

Closure

- Without referring to your notes, write the quadratic formula and explain how to find a, b, and c. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; The values of a, b, and c come from the standard form of the quadratic equation $ax^2 + bx + c = 0$.

3 ACTIVITY: Analyzing the Solutions of an Equation

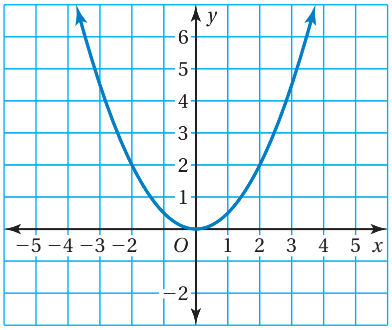
Math Practice 1

Explain the Meaning

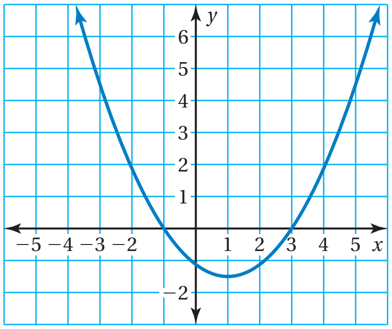
What does it mean for an equation to have a solution? How does this compare to the graph of the equation?

Work with a partner. In the quadratic formula in Activity 1, the expression under the radical sign, $b^2 - 4ac$, is called the discriminant. For each graph, decide whether the corresponding discriminant is equal to 0, is greater than 0, or is less than 0. Explain your reasoning.

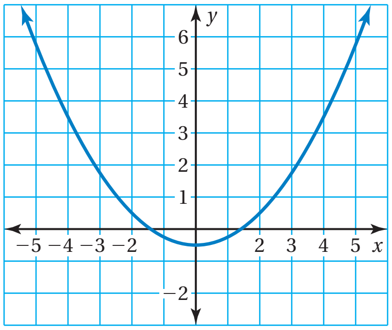
a. 1 rational solution



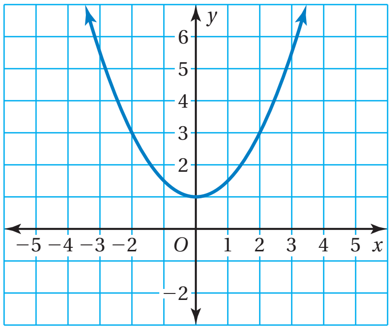
b. 2 rational solutions



c. 2 irrational solutions



d. no real solutions



What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use the discriminant to determine the number of solutions of a quadratic equation?

5. Use the quadratic formula to solve each quadratic equation.

   a. $x^2 + 2x - 3 = 0$ **b.** $x^2 - 4x + 4 = 0$ **c.** $x^2 + 4x + 5 = 0$

6. Use the Internet to research *imaginary numbers*. How are they related to quadratic equations?

Practice Use what you learned about quadratic equations to complete Exercises 9–11 on page 481.

9.4 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
quadratic formula, *p. 478*
discriminant, *p. 480*

Another way to solve quadratic equations is to use the *quadratic formula.*

Key Idea

Quadratic Formula

The real solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $a \neq 0$ and $b^2 - 4ac \geq 0$. This is called the **quadratic formula.**

EXAMPLE 1 Solving a Quadratic Equation Using the Quadratic Formula

Solve $2x^2 - 5x + 3 = 0$ using the quadratic formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — Quadratic Formula

$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(3)}}{2(2)}$ — Substitute 2 for *a*, −5 for *b*, and 3 for *c*.

$= \frac{5 \pm \sqrt{1}}{4}$ — Simplify.

$= \frac{5 \pm 1}{4}$ — Evaluate the square root.

So, the solutions are $x = \frac{5+1}{4} = \frac{3}{2}$ and $x = \frac{5-1}{4} = 1$.

Study Tip

You can use the roots of a quadratic equation to factor the related expression. In Example 1, you can use 1 and $\frac{3}{2}$ to factor $2x^2 - 5x + 3$ as $(x - 1)(2x - 3)$.

Check Check each solution in the original equation.

$2x^2 - 5x + 3 = 0$	Original equation	$2x^2 - 5x + 3 = 0$
$2\left(\frac{3}{2}\right)^2 - 5\left(\frac{3}{2}\right) + 3 \stackrel{?}{=} 0$	Substitute.	$2(1)^2 - 5(1) + 3 \stackrel{?}{=} 0$
$\frac{9}{2} - \frac{15}{2} + 3 \stackrel{?}{=} 0$	Simplify.	$2 - 5 + 3 \stackrel{?}{=} 0$
$0 = 0$ ✓	Simplify.	$0 = 0$ ✓

On Your Own

Now You're Ready
Exercises 12–14

Solve the equation using the quadratic formula.

1. $x^2 - 6x + 5 = 0$
2. $4x^2 + x - 3 = 0$
3. $-6x^2 + 7x - 2 = 0$

Laurie's Notes

Introduction

Connect

- **Yesterday:** Students derived the quadratic formula. (MP1, MP3b, MP7)
- **Today:** Students will solve quadratic equations using the quadratic formula.

Motivate

- There are many online videos of students singing the quadratic formula to the tune "Pop Goes the Weasel." Share one with students. You could return to the video at the end of the period when the students may be ready to sing along.

Lesson Notes

Discuss

- **MP2 Reason Abstractly and Quantitatively:** Students will decontextualize a quadratic model to solve for the roots and then interpret the roots in the context of the problem.
- Discuss with students the methods they have learned to solve quadratic equations. The form of the equation and the tools available often dictate the best method for solving. The quadratic formula is another method that can be used for solving quadratic equations.

Key Idea

- Write the Key Idea on the board.
- ? "Why must one side of the quadratic equation be equal to 0?" So you can determine the values of *a, b,* and *c*.
- ? "Why is there a restriction that $a \neq 0$?" When $a = 0$, the equation is not a quadratic equation. Also, the denominator in the quadratic formula would be 0, and you cannot divide by 0.
- ? "Why is there a restriction that $b^2 - 4ac \geq 0$?" The expression $b^2 - 4ac$ is under a radical sign in the formula. You cannot take the square root of a negative number.

Example 1

- Write the equation on the board and ask students to identify *a, b,* and *c*. Be sure students include the sign of each number, such as $b = -5$.
- As you substitute each value into the formula, point at and read aloud the corresponding term in the quadratic equation.
- Review the order of operations as you simplify.
- **Representation:** Students may still have trouble working with the plus/minus symbol $\pm$. Read the expression slowly and translate: "5 plus or minus the square root of 1 represents the two values: 5 plus the square root of 1, and 5 minus the square root of 1."
- ? **Connection:** "What type of numbers are $\frac{3}{2}$ and 1?" rational

Goal Today's lesson is solving quadratic equations using the **quadratic formula**.

Technology for the Teacher

Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Extra Example 1

Solve $2x^2 + 7x + 3 = 0$ using the quadratic formula.

$x = -3, x = -\frac{1}{2}$

On Your Own

1. $x = 1, x = 5$
2. $x = -1, x = \frac{3}{4}$
3. $x = \frac{1}{2}, x = \frac{2}{3}$

Laurie's Notes

Example 1 (continued)

- "Recall in Chapter 7 you factored quadratic polynomials. Can $2x^2 - 5x + 3$ be factored?" yes; It factors as $(x - 1)(2x - 3)$.
- When the factored form is set equal to 0 the roots are 1 and $\frac{3}{2}$. Point out the Study Tip.
- **MP2:** By considering how the solutions of the quadratic equation are related to the factors of the related quadratic expression, students attend to the meaning of the quantities, using important reasoning that leads to a deeper understanding.

On Your Own

- Students should check their work with a neighbor after completing each question. Ask students who finish quickly to write the quadratic expressions in factored form.
- Discuss the solutions for Question 3. Rational roots are often more difficult for students to connect to the factors.

Example 2

- "Based on the discriminant, why is there only one solution?" The discriminant is 0, so adding or subtracting 0 gives the same number.
- Because there is one rational solution $x = -\frac{5}{2}$, students should understand that the expression can be factored as $(2x + 5)^2$.

Example 3

- Read through the problem statement. Ask questions to make sure that students understand the model. Interpret the y-intercept of the graph.
- "What is the first step in solving this equation?" Subtract 30 from each side.
- **MP2:** Students decontextualize the model to solve algebraically. Once the solutions are found, the context is considered again.
- Consider using technology to graph the function $y = 0.34x^2 + 3.0x - 21$. Use the graph to approximate the zeros. Then solve the corresponding equation by using the quadratic formula.
- "Do both solutions make sense in the context of the problem? Explain." no; Only the positive solution makes sense in the context of the problem.
- "Can the related expression be factored using only integers? Explain." no; Both solutions are irrational.
- "Do you think the trend shown by the graph will continue?" Students should recognize that while the graph increases, the number of breeding pairs cannot increase indefinitely due to the limits of nature.

On Your Own

- If time is a concern, have students do only the odd exercises.

Extra Example 2

Solve $4x^2 - 12x + 9 = 0$ using the quadratic formula.

$x = -\frac{3}{2}$

Extra Example 3

In Example 3, when were there about 55 breeding pairs?

2003

On Your Own

4. $x = \frac{1}{2}$
5. $x = -\frac{4}{5}, x = 1$
6. $x = -\frac{5}{3}, x = 1$
7. 2006

EXAMPLE 2 Solving a Quadratic Equation Using the Quadratic Formula

Solve $4x^2 + 20x + 25 = 0$ using the quadratic formula.

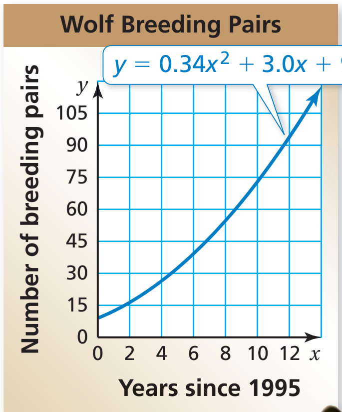
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

$$= \frac{-20 \pm \sqrt{20^2 - 4(4)(25)}}{2(4)} \quad \text{Substitute 4 for } a\text{, 20 for } b\text{, and 25 for } c.$$

$$= \frac{-20 \pm \sqrt{0}}{8} = -\frac{5}{2} \quad \text{Simplify.}$$

The solution is $x = -\frac{5}{2}$.

EXAMPLE 3 Real-Life Application



The number y of Northern Rocky Mountain wolf breeding pairs x years since 1995 can be modeled by $y = 0.34x^2 + 3.0x + 9$. When were there about 30 breeding pairs?

To determine when there were 30 breeding pairs, find the x-values for which $y = 30$. So, solve the equation $30 = 0.34x^2 + 3.0x + 9$.

$$30 = 0.34x^2 + 3.0x + 9 \quad \text{Write the equation.}$$

$$0 = 0.34x^2 + 3.0x - 21 \quad \text{Write in standard form.}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

$$= \frac{-3.0 \pm \sqrt{3.0^2 - 4(0.34)(-21)}}{2(0.34)} \quad \text{Substitute 0.34 for } a\text{, 3.0 for } b\text{, and } -21 \text{ for } c.$$

$$= \frac{-3.0 \pm \sqrt{37.56}}{0.68} \quad \text{Simplify.}$$

The solutions are $x = \frac{-3.0 + \sqrt{37.56}}{0.68} \approx 5$ and $x = \frac{-3.0 - \sqrt{37.56}}{0.68} \approx -13$.

Because x represents the number of years since 1995, x is greater than or equal to zero. So, there were about 30 breeding pairs 5 years after 1995, in 2000.

On Your Own

Now You're Ready
Exercises 15–23

Solve the equation using the quadratic formula.

4. $4x^2 - 4x + 1 = 0$ **5.** $-5x^2 + x = -4$ **6.** $3x^2 + 2x = 5$

7. WHAT IF? In Example 3, when were there about 85 breeding pairs?

The expression $b^2 - 4ac$ in the quadratic formula is the **discriminant**.

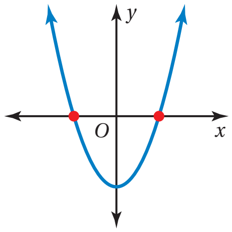
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \leftarrow \text{discriminant}$$

You can use the discriminant to determine the number of real solutions of a quadratic equation.

Key Idea

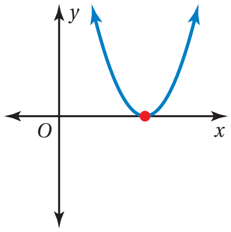
Interpreting the Discriminant

$b^2 - 4ac > 0$



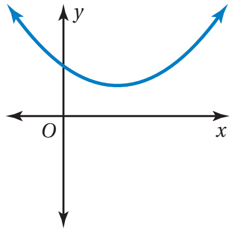
- two real solutions
- two x-intercepts

$b^2 - 4ac = 0$



- one real solution
- one x-intercept

$b^2 - 4ac < 0$



- no real solutions
- no x-intercepts

Study Tip

The solutions of a quadratic equation may be real numbers or *imaginary numbers*. You will study imaginary numbers in a future course.

EXAMPLE 4 Determining the Number of Real Solutions

a. Determine the number of real solutions of $x^2 + 8x - 3 = 0$.

$$\begin{aligned} b^2 - 4ac &= 8^2 - 4(1)(-3) && \text{Substitute 1 for } a\text{, 8 for } b\text{, and } -3 \text{ for } c. \\ &= 64 + 12 && \text{Simplify.} \\ &= 76 && \text{Add.} \end{aligned}$$

The discriminant is greater than 0, so the equation has two real solutions.

b. Determine the number of real solutions of $2x^2 + 7 = 6x$.

Write the equation in standard form: $2x^2 - 6x + 7 = 0$.

$$\begin{aligned} b^2 - 4ac &= (-6)^2 - 4(2)(7) && \text{Substitute 2 for } a\text{, } -6 \text{ for } b\text{, and 7 for } c. \\ &= 36 - 56 && \text{Simplify.} \\ &= -20 && \text{Subtract.} \end{aligned}$$

The discriminant is less than 0, so the equation has no real solutions.

On Your Own

Now You're Ready
Exercises 27–32

Determine the number of real solutions of the equation.

8. $-x^2 + 4x - 4 = 0$ **9.** $6x^2 + 2x = -1$ **10.** $\frac{1}{2}x^2 = 7x - 1$

Laurie's Notes

Key Idea

- Discuss the discriminant and ask students to find the discriminant in each of the previous examples.
- Write the Key Idea on the board.
- **MP1a Make Sense of Problems:** This key idea helps connect previous lessons in this chapter. Ask a volunteer to discuss the ways to recognize each number of solutions when solving by graphing, factoring, and the quadratic formula.
- Discuss the Study Tip.

Example 4

- Work through each part as shown.
- ? After writing the equation in part (b) ask, "What are the values of a, b, and c?" Check for understanding—students need to first subtract $6x$ from each side, and then recognize that the value of b is -6, not 6.

On Your Own

- **Think-Pair-Share:** Students should read each question independently and then work in pairs to answer the questions. When they have answered the questions, the pair should compare their answers with another group and discuss any discrepancies.
- Students are often surprised that a small change in any of the three parameters (a, b, and c) can produce a big change in the graph and subsequent x-intercepts. If students finish early, have them use a graphing calculator to explore small changes in one of the parameters.

Closure

- For quadratic equations of the form $x^2 + 4x + c = 0$ determine the values of c that yield a quadratic equation with 2 roots, 1 root, and no roots. when $c < 4$, two roots; when $c = 4$, one root; when $c > 4$, no roots

Extra Example 4

a. Determine the number of real solutions of $x^2 + 2x + 5 = 0$. 0

b. Determine the number of real solutions of $4x^2 + 25 = 20x$. 1

On Your Own

8. 1 **9.** 0

10. 2

English Language Learners

Vocabulary

To check that your students can distinguish between the expressions *quadratic polynomial, quadratic equation,* and *quadratic formula*, have them state an example or explanation of each.

Discuss how the word *discriminant* relates to the word *discriminate*, which means to distinguish between things.

Vocabulary and Concept Check

1. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
2. discriminant > 0: 2 real solutions; discriminant $= 0$: 1 real solution; discriminant < 0: no real solutions

Practice and Problem Solving

3. $x^2 - 7x = 0$; $a = 1, b = -7, c = 0$
4. $x^2 - 4x + 12 = 0$; $a = 1, b = -4, c = 12$
5. $-2x^2 - 5x + 1 = 0$; $a = -2, b = -5, c = 1$
6. $-4x^2 + 3x + 2 = 0$; $a = -4, b = 3, c = 2$
7. $x^2 - 6x + 4 = 0$; $a = 1, b = -6, c = 4$
8. $3x^2 + 8x + 3 = 0$; $a = 3, b = 8, c = 3$
9. $x = 6$
10. no real solutions
11. $x = -1, x = 11$
12. $x = -\frac{1}{2}, x = 1$
13. no real solutions
14. $x = \frac{1}{3}$
15. $x = \frac{2}{3}, x = \frac{3}{2}$
16. no real solutions
17. $x = \frac{1}{4}$
18. $x \approx 0.7, x \approx 4.3$
19. $x \approx -4.2, x \approx 2.2$
20. $x \approx -0.3, x \approx 1.1$

Assignment Guide and Homework Check

Level	Assignment	Homework Check
Average	1, 2, 9–21 odd, 27–29, 34, 35, 47–50	13, 15, 21, 27, 34
Advanced	1, 2, 16–22 even, 31–35, 37, 40, 43–50	16, 20, 31, 34, 44

For Your Information

- **Exercises 3–20** Note that there are two possible ways to write standard form. For example, the standard form in Exercise 6 could be written as $4x^2 - 3x - 2 = 0$ or $-4x^2 + 3x + 2 = 0$. The values of a, b, and c should come from one equation or the other.

Common Errors

- **Exercises 3–20** Students may make sign mistakes when identifying the values of a, b, and c. Emphasize how the signs are determined.
- **Exercise 23** Students may solve for h when $d = 0$. Explain that the distance from the pier is given by d, so they need to solve for d when $h = 0$.

9.4 Record and Practice Journal

Solve the equation using the quadratic formula. Round to the nearest tenth, if necessary.

1. $x^2 + 3x - 18 = 0$
   $x = -6, 3$
2. $x^2 + 8x + 16 = 0$
   $x = -4$
3. $x^2 - 5x + 7 = 0$
   no solution
4. $3x^2 - 10x - 8 = 0$
   $x = -\frac{2}{3}, 4$
5. $4x^2 - 12x = -9$
   $x = \frac{3}{2}$
6. $4x - 3 = 2x^2$
   no solution
7. $x^2 + 2x - 6 = 0$
   $x = -3.6, 1.6$
8. $-2x^2 - 11x = -5$
   $x = -5.9, 0.4$
9. The deer population in a forest from 2000 to 2010 can be modeled by $y = -0.1x^2 + 1.1x + 3$, where y is hundreds of deer and x is the number of years since 2000.
   a. When was the deer population about 500?
   2002 and 2008 ($x \approx 2.3, 8.7$)
   b. Do you think this model can be used for future years? Explain your reasoning. no; It will eventually predict negative population.

Use the discriminant to determine the number of real solutions of the equation.

10. $x^2 + 8x + 13 = 0$
    2
11. $2x^2 - 6x = -9$
    0
12. $9x^2 + 4 = 12x$
    1

9.4 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **VOCABULARY** Write the formula that can be used to solve any quadratic equation.
2. **VOCABULARY** What does the discriminant tell you about the number of solutions of a quadratic equation?

Practice and Problem Solving

Write the equation in standard form. Then identify the values of *a*, *b*, and *c* that you would use to solve the equation using the quadratic formula.

3. $x^2 = 7x$
4. $x^2 - 4x = -12$
5. $-2x^2 + 1 = 5x$
6. $3x + 2 = 4x^2$
7. $4 - 6x = -x^2$
8. $-8x = 3x^2 + 3$

Solve the equation using the quadratic formula. Round to the nearest tenth, if necessary.

9. $x^2 - 12x + 36 = 0$
10. $x^2 + 7x + 16 = 0$
11. $x^2 - 10x - 11 = 0$

1 12. $2x^2 - x - 1 = 0$
13. $2x^2 - 6x + 5 = 0$
14. $9x^2 - 6x + 1 = 0$

2 15. $6x^2 - 13x = -6$
16. $-3x^2 + 6x = 4$
17. $1 - 8x = -16x^2$

18. $x^2 - 5x + 3 = 0$
19. $x^2 + 2x = 9$
20. $5x^2 - 2 = 4x$

ERROR ANALYSIS **Describe and correct the error in solving the equation.**

21. $3x^2 - 7x - 6 = 0$

$$x = \frac{-7 \pm \sqrt{(-7)^2 - 4(3)(-6)}}{2(3)}$$
$$= \frac{-7 \pm \sqrt{121}}{6}$$
$$x = \frac{2}{3} \text{ and } x = -3$$

22. $-2x^2 + 9x = 4$

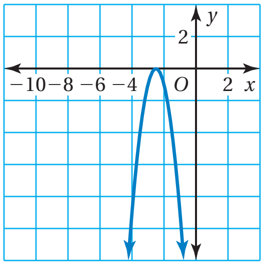
$$x = \frac{-9 \pm \sqrt{9^2 - 4(-2)(4)}}{2(-2)}$$
$$= \frac{-9 \pm \sqrt{113}}{-4}$$
$$x \approx -0.41 \text{ and } x \approx 4.91$$

3 23. **PIER** A swimmer takes a running jump off a pier. The path of the swimmer can be modeled by the equation $h = -0.1d^2 + 0.1d + 3$, where h is the height (in feet) and d is the horizontal distance (in feet). How far from the pier does the swimmer enter the water?

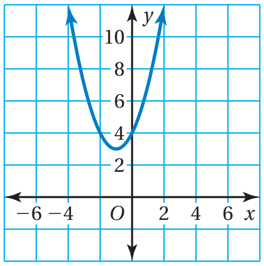
Match the discriminant with the corresponding graph.

24. $b^2 - 4ac > 0$ **25.** $b^2 - 4ac = 0$ **26.** $b^2 - 4ac < 0$

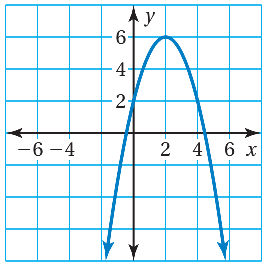
A.



B.



C.



Use the discriminant to determine the number of real solutions of the equation.

4 **27.** $x^2 - 6x + 10 = 0$ **28.** $x^2 - 5x - 3 = 0$ **29.** $2x^2 - 12x = -18$

30. $4x^2 = 4x - 1$ **31.** $-\frac{1}{4}x^2 + 4x = -2$ **32.** $-5x^2 + 8x = 9$

33. REPEATED REASONING You use the quadratic formula to solve an equation.

a. You obtain solutions that are integers. Could you have used factoring to solve the equation? Explain your reasoning.

b. You obtain solutions that are fractions. Could you have used factoring to solve the equation? Explain your reasoning.

c. Make a generalization about quadratic equations with rational solutions.

34. STOPPING A CAR The distance d (in feet) it takes to stop a car traveling v miles per hour can be modeled by $d = 0.05v^2 + 2.2v$. It takes a car 235 feet to stop. How fast was the car going when the brakes were applied?

35. FISHING The amount y of trout (in tons) caught in a lake from 1990 to 2009 can be modeled by $y = -0.08x^2 + 1.6x + 10$, where x is the number of years since 1990.

a. When were about 15 tons of trout caught in the lake?

b. Do you think this model can be used for future years? Explain your reasoning.

36. ERROR ANALYSIS Describe and correct the error in finding the number of solutions of the equation $2x^2 - 5x - 2 = -11$.

✗

$$b^2 - 4ac = (-5)^2 - 4(2)(-2)$$
$$= 25 - (-16)$$
$$= 41$$

The equation has two solutions.

Common Errors

- **Exercises 27–32** If students use a calculator to evaluate the discriminant, they may make keystroke errors. For example, they might enter -4^2 instead of (-4^2).
- **Exercise 34** Students may solve for d when $v = 0$. Explain that 235 is the distance it takes the car to stop. So they need substitute 235 for d and solve for v.
- **Exercise 35** Students may neglect to give two answers. Tell them to be sure to decide whether both answers make sense in the context of the situation. In this case, they do.
- **Exercises 37–39** Students may fail to write the equation in standard form before finding the discriminant.
- **Exercise 46** Students may not know how to solve this problem. The students need to use the projectile motion model to write an expression involving v and set the expression equal to the height of the branch. The key is realizing that the minimum velocity is given by the value of v for which this equation has one solution.

Practice and Problem Solving

21. used -7 for $-b$ instead of $-(-7) = 7$; $x = -\frac{2}{3}$, $x = 3$

22. used $c = 4$ instead of $c = -4$; $x = \frac{1}{2}$, $x = 4$

23. 6 ft **24.** C

25. A **26.** B

27. 0 **28.** 2

29. 1 **30.** 1

31. 2 **32.** 0

33. **a.** yes; When the solutions m and n are integers, the standard form can be factored as $(x - m)(x - n) = 0$.

b. yes; When the solutions $\frac{m}{n}$ and $\frac{h}{k}$ are fractions, the standard form can be factored as $(nx - m)(kx - h) = 0$.

c. Any quadratic equation with rational solutions can be solved by factoring.

34. 50 miles per hour

35. **a.** 1994 and 2006

b. no; The model predicts negative numbers of fish caught after 2015.

36. Standard form was not used; $2x^2 - 5x + 9 = 0$; no real solutions

Differentiated Instruction

Kinesthetic

In their notebooks, have students create a table listing the methods they have learned for solving quadratic equations: *by graphing, using square roots, by completing the square,* and *using the quadratic formula*. A description of the method and an example with its solution should be written for each method.

Practice and Problem Solving

37. 2

38. 1

39. 0

40–42. Sample answers are given.

40. **a.** $c = 2$ **b.** $c = -5$

41. **a.** $c = 8$ **b.** $c = 2$

42. **a.** $c = -20$ **b.** $c = 4$

43. 2; When a and c have different signs, $b^2 - 4ac$ is positive.

44. rational; When the discriminant is a perfect square, the quadratic formula will have integers in the numerator which give rational solutions.

45. See *Taking Math Deeper*.

46. about 24.7 feet per second

Fair Game Review

47. $(1, -1)$

48. infinitely many solutions

49. no solution

50. A

Mini-Assessment

Solve the equation using the quadratic formula. Round to the nearest tenth, if necessary.

1. $x^2 - 2x - 99 = 0$
$x = -9, x = 11$

2. $3x^2 + 16x - 35 = 0$
$x = -7, x \approx 1.7$

3. $4x^2 - 6x = -7$
no real solutions

4. $-3x^2 + 12x = 8$
$x \approx 0.8, x \approx 3.2$

Taking Math Deeper

Exercise 45

For this problem, it is important for students to read the problem carefully and list the given information before looking for an entry point to the solution.

1 Write an equation that represents the amount of fencing needed for both pastures and one that represents the area of each pasture.

Amount of fencing needed for both pastures:

$x + x + x + x + y + y + y = 1050$ There is 1050 feet of fencing.

$4x + 3y = 1050$ Combine like terms.

Area of each pasture:

$xy = 15{,}000$ Area = length × width

2 **a.** Solving $4x + 3y = 1050$ for y produces $y = 350 - \frac{4}{3}x$.

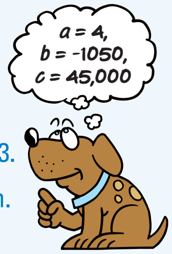
3 **b.** Substitute for y in the equation $xy = 15{,}000$ and solve for x.

$x\left(350 - \frac{4}{3}x\right) = 15{,}000$ Substitute for y.

$350x - \frac{4}{3}x^2 = 15{,}000$ Distributive Property

$1050x - 4x^2 = 45{,}000$ Multiply each side by 3.

$4x^2 - 1050x + 45{,}000 = 0$ Write in standard form.



Using the quadratic formula, the solutions are

$$x = \frac{1050 + \sqrt{382{,}500}}{8} \approx 208.6 \text{ and } x = \frac{1050 - \sqrt{382{,}500}}{8} \approx 53.9.$$

So, the possible lengths and widths of each section are:
$x = 208.6$ feet and $y = 71.9$ feet or $x = 53.9$ feet and $y = 278.1$ feet.

Note: You may want to challenge students by having them solve this problem using different methods.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension • School-to-Work Start the next section

Use the discriminant to determine how many times the graph of the related function intersects the *x*-axis.

37. $x^2 + 5x - 1 = 0$ **38.** $4x^2 + 4x = -1$ **39.** $4 - 3x = -6x^2$

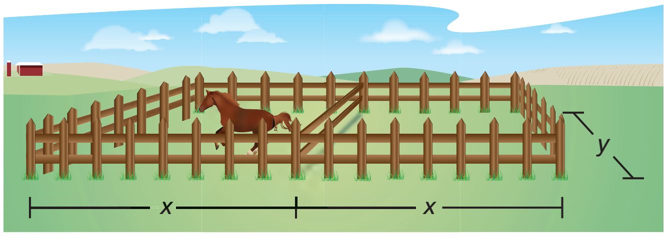
Give a value for *c* where (a) you can factor to solve the equation and (b) you must use the quadratic formula to solve the equation.

40. $x^2 + 3x + c = 0$ **41.** $x^2 - 6x + c = 0$ **42.** $x^2 - 8x + c = 0$

43. **REASONING** How many solutions does $ax^2 + bx + c = 0$ have when a and c have different signs? Explain your reasoning.

44. **REASONING** When the discriminant is a perfect square, are the solutions of $ax^2 + bx + c = 0$ rational or irrational? Assume a, b, and c are integers. Explain your reasoning.

45. **PROBLEM SOLVING** A rancher constructs two rectangular horse pastures that share a side, as shown. The pastures are enclosed by 1050 feet of fencing. Each pasture has an area of 15,000 square feet.



a. Show that $y = 350 - \frac{4}{3}x$.

b. Find the possible lengths and widths of each pasture.

46. Critical Thinking You are trying to hang a tire swing. To get the rope over a tree branch that is 15 feet high, you tie the rope to a weight and throw it over the branch. You release the weight at a height of 5.5 feet. What is the minimum upward velocity needed to reach the branch?

Fair Game Review What you learned in previous grades & lessons

Solve the system of linear equations. *(Section 4.4)*

47. $x + y = 0$, $3x + 2y = 1$

48. $2x - 2y = 4$, $-x + y = -2$

49. $2x - 4y = -1$, $-3x + 6y = -5$

50. **MULTIPLE CHOICE** What is the solution of the equation $7x + 3x = 5x - 10$? *(Section 1.3)*

Ⓐ $x = -2$ Ⓑ $x = -\frac{2}{3}$ Ⓒ $x = 2$ Ⓓ $x = 4$

Extension 9.4 Choosing a Solution Method

Check It Out
Lesson Tutorials
BigIdeasMath.com

The table shows five methods for solving quadratic equations. While there is no one correct method, some methods may be easier to use than others. Some advantages and disadvantages of each method are shown.

COMMON CORE

Solving Quadratic Equations

In this extension, you will
- choose a method to solve quadratic equations.

Learning Standards
A.REI.4a
A.REI.4b

Key Ideas

Methods for Solving Quadratic Equations

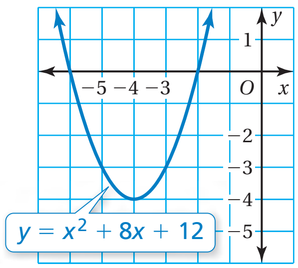
Method	Advantages	Disadvantages
Factoring *(Lessons 7.6–7.9)*	• Straightforward when equation can be factored easily	• Some equations are not factorable.
Graphing *(Lesson 9.1)*	• Can easily see the number of solutions • Use when approximate solutions are sufficient. • Can use a graphing calculator	• May not give exact solutions
Using Square Roots *(Lesson 9.2)*	• Use to solve equations of the form $x^2 = d$.	• Can only be used for certain equations
Completing the Square *(Lesson 9.3)*	• Best used when $a = 1$ and b is even	• May involve difficult calculations
Quadratic Formula *(Lesson 9.4)*	• Can be used for *any* quadratic equation • Gives exact solutions	• Takes time to do calculations

EXAMPLE 1 Solving a Quadratic Equation Using Different Methods

Solve $x^2 + 8x + 12 = 0$ using two different methods.

Method 1: Solve by graphing. Graph the related function $y = x^2 + 8x + 12$.

The x-intercepts are -6 and -2.



So, the solutions are $x = -6$ and $x = -2$.

Method 2: Solve by factoring.

$x^2 + 8x + 12 = 0$ Write the equation.

$(x + 2)(x + 6) = 0$ Factor left side.

$x + 2 = 0$ *or* $x + 6 = 0$ Use Zero-Product Property.

$x = -2$ *or* $x = -6$ Solve for x.

The solutions are $x = -2$ and $x = -6$.

Study Tip

Notice that each method produces the same solutions, $x = -6$ and $x = -2$.

Laurie's Notes

Introduction

Connect

- **Yesterday:** Students solved quadratic equations using the quadratic formula. (MP1a, MP2)
- **Today:** Students will choose methods to solve quadratic equations.

Motivate

- Write three multiplication problems and three solution methods on the board.
- Tell students to choose a different solution method for each problem.

Problems	Solution methods
13×20	calculator
13×24	paper and pencil
1.3×2.42	mental math

- Explain that in today's lesson, students will choose methods to solve quadratic equations.

Lesson Notes

Key Ideas

- Write the chart on the board. Discuss the advantages and disadvantages of each method as you fill in the chart.
- ? "What is an example of a quadratic equation that factors easily?"
  Sample answers: $x^2 - 16 = 0$, $x^2 + 2x + 1 = 0$
- ? "What is an example of a quadratic equation that does not factor?"
  Sample answers: $x^2 + 5 = 0$, $x^2 - 4x - 2 = 0$
- Discuss with students that solutions found using a graphing utility are often approximated.
- ? "In a quadratic equation of the form $x^2 = d$, which term is missing?"
  the x-term
- Tell students that it is possible to write a program in their calculators or in a spreadsheet that computes solutions using the quadratic formula.

Example 1

- **MP5 Use Appropriate Tools Strategically:** Write the equation on the board and ask students to strategically choose and support two methods for solving this equation.
- **MP3b Critique the Reasoning of Others:** Have students critique each method. Allow personal preference. Look for comments that are thoughtful and reasonable.
- Work through both methods as shown in the text.

Common Core State Standards

A.REI.4a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
A.REI.4b Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation . . ..

Goal Today's lesson is choosing a method for solving a quadratic equation.

Technology for the Teacher

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Extra Example 1

Solve $x^2 + x - 20 = 0$ using two different methods.
$x = -5$, $x = 4$

Practice

1. $x = -7 + \sqrt{41}$, $x = -7 - \sqrt{41}$
2. $x = 1$, $x = 9$
3. $x = 6$, $x = -6$

Record and Practice Journal Extension 9.4 Practice

See Additional Answers.

Extra Example 2

Solve $2x^2 - 98 = 0$ using any method. Explain your choice of method.
$x = 7, x = -7$; square roots, because no x-term

Extra Example 3

Solve $x^2 - 4x = 6$ using any method. Explain your choice of method.
$x = 2 + \sqrt{10}, x = 2 - \sqrt{10}$; The quadratic formula is most convenient.

Practice

4–12. Sample explanations are given.

4. $x = -12, x = 1$; factors easily

5. $x = 1, x = -1$; square roots, because no x-term

6. $x = \frac{1 + \sqrt{21}}{10}, x = \frac{1 - \sqrt{21}}{10}$; The quadratic formula is most convenient.

7. $x = -5, x = 8$; factors easily

8. $x = -2, x = -10$; factors easily

9. no real solutions; completing the square, because $a = 1$ and b is even

10. no real solutions; square roots, because no x-term

11. $x = -4, x = 3$; factors easily

12. $x = -7, x = 1$; factors easily

Mini-Assessment

Solve the equation using any method. Explain your choice of method.

1. $x^2 + 4x = -1$ $x = -2 + \sqrt{3}, x = -2 - \sqrt{3}$; The quadratic formula is most convenient.
2. $3x^2 = 12$ $x = 2, x = -2$; square roots, because no x-term
3. $x^2 - 5x + 6 = 0$ $x = 2, x = 3$; factors easily

Laurie's Notes

Example 2

- Write the equation as shown.
- "Can this equation be solved by factoring? Explain." no; $x^2 - 10x - 1$ does not factor.
- "Can this equation be solved using square roots? Explain." no; There is an x-term.
- "Can this equation be solved by completing the square? Explain." yes; $a = 1$ and b is even.
- Work through the problem as shown.
- If time permits, use the quadratic formula to solve and verify that the same solutions are found. Discuss which method the students prefer.

Example 3

- "Could you use either square roots or completing the square to solve this equation? Explain." Square roots will not work because there is an x-term. Completing the square is not convenient because $a \neq 1$ and b is odd.
- Consider factoring before using the quadratic formula. In this example, there are many possible products of polynomials for $a = 2$ and $c = -24$. Factoring is possible, but not easy.
- Work through the problem as shown.
- "How do you know this equation was factorable?" rational solutions
- "What are the factors?" $(x - 8)$ and $(2x + 3)$

Practice

- **MP5:** It is important that the students strategically choose the two methods they use in Exercises 1–3.
- **MP3a Construct Viable Arguments:** In explaining the methods they choose, students have to think and reason about the equations. Different students will make different choices, so it is important for students to share their thinking with the class.

Closure

- Write a quadratic equation that you would *not* solve using square roots. Look for equations that have an x-term.
- Write a quadratic equation that you would *not* solve by factoring. Look for equations that are not easily factorable.

EXAMPLE 2 Choosing a Method

Solve $x^2 - 10x = 1$ using any method. Explain your choice of method.

The coefficient of the x^2-term is 1 and the coefficient of the x-term is an even number. So, solve by completing the square.

$x^2 - 10x = 1$	Write the equation.
Complete the square. → $x^2 - 10x + 25 = 1 + 25$	Add $\left(\frac{-10}{2}\right)^2$, or 25, to each side.
$(x-5)^2 = 26$	Factor $x^2 - 10x + 25$.
$x - 5 = \pm\sqrt{26}$	Take the square root of each side.
$x = 5 \pm \sqrt{26}$	Add 5 to each side.

The solutions are $x = 5 + \sqrt{26} \approx 10.1$ and $x = 5 - \sqrt{26} \approx -0.1$.

EXAMPLE 3 Choosing a Method

Solve $2x^2 - 13x - 24 = 0$ using any method. Explain your choice of method.

The equation is not easily factorable and the numbers are somewhat large. So, solve using the quadratic formula.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic Formula
$= \dfrac{-(-13) \pm \sqrt{(-13)^2 - 4(2)(-24)}}{2(2)}$	Substitute 2 for a, -13 for b, and -24 for c.
$= \dfrac{13 \pm \sqrt{361}}{4}$	Simplify.
$= \dfrac{13 \pm 19}{4}$	Evaluate the square root.

The solutions are $x = \dfrac{13 + 19}{4} = 8$ and $x = \dfrac{13 - 19}{4} = -\dfrac{3}{2}$.

Practice

Solve the equation using two different methods.

1. $x^2 + 14x = -8$
2. $x^2 - 10x + 9 = 0$
3. $-4x^2 + 144 = 0$

Solve the equation using any method. Explain your choice of method.

4. $x^2 + 11x - 12 = 0$
5. $9x^2 - 5 = 4$
6. $5x^2 - x - 1 = 0$
7. $x^2 - 3x - 40 = 0$
8. $x^2 + 12x + 5 = -15$
9. $x^2 = 2x - 5$
10. $-8x^2 - 2 = 14$
11. $x^2 + x - 12 = 0$
12. $x^2 + 6x + 9 = 16$

9.5 Solving Systems of Linear and Quadratic Equations

Essential Question How can you solve a system of two equations when one is linear and the other is quadratic?

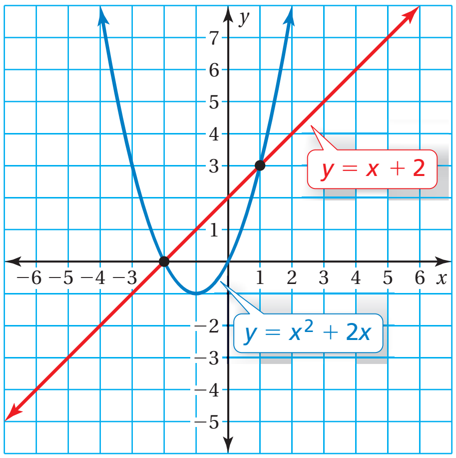
1 ACTIVITY: Solving a System of Equations

Work with a partner. Solve the system of equations using the given strategy. Which strategy do you prefer? Why?

System of Equations:

$y = x + 2$ Linear

$y = x^2 + 2x$ Quadratic



a. Solve by Graphing

Graph each equation and find the points of intersection of the line and the parabola.

b. Solve by Substitution

Substitute the expression for y from the quadratic equation into the linear equation to obtain

$$x^2 + 2x = x + 2.$$

Solve this equation and substitute each x-value into the linear equation $y = x + 2$ to find the corresponding y-value.

c. Solve by Elimination

Eliminate y by subtracting the linear equation from the quadratic equation to obtain

$$\begin{aligned} y &= x^2 + 2x \\ \underline{y} &= \underline{\quad\;\; x + 2} \\ 0 &= x^2 + \quad x - 2. \end{aligned}$$

Solve this equation and substitute each x-value into the linear equation $y = x + 2$ to find the corresponding y-value.

COMMON CORE

Solving Systems of Equations

In this lesson, you will
- solve systems of linear and quadratic equations.

Learning Standard
A.REI.7

Laurie's Notes

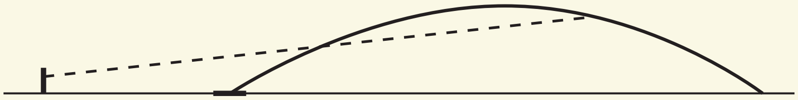
Introduction

Standards for Mathematical Practice

- **MP1a Make Sense of Problems** and **MP8 Look for and Express Regularity in Repeated Reasoning:** In this activity, students extend the methods they learned for solving a system of linear equations to solving a system with a nonlinear equation. Students make sense of the problem by connecting the new process to what they know about solving linear systems and solving quadratic equations.

Motivate

- **Story Time:** Share a story about shooting clay pigeons launched into the air by a machine. In the last round, you hit the target on the way down. (Sketch the diagram shown below.)



- Connect this to today's activity. Explain that you prefer to hit the target on the way up, because it is closer to you.

Activity Notes

Discuss

- ? "What is a system of linear equations?" a set of two or more linear equations in the same variables
- ? "How do you solve a system of linear equations?" graphing, substitution, or elimination
- Explain that today's activity is about systems of equations that include quadratic equations.

Activity 1

- As students graph the two equations, you should hear comments about slope, y-intercepts, the parabola opening up, and so on.
- **Big Idea:** The two equations are solved for y. So, substitution results in the expressions being set equal. The equation that results connects to solving quadratic equations by factoring, from a previous chapter.
- **Common Error:** When using elimination, students often subtract the left side of the equations but add on the right side.
- A preference for one method over another is often related to a student's comfort level with each method.
- **MP1a:** Summarize the multiple approaches used in the activity to make sense of the problem.

Common Core State Standards

A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

Previous Learning

Students should know how to solve systems of linear equations. They should also know how to solve quadratic equations.

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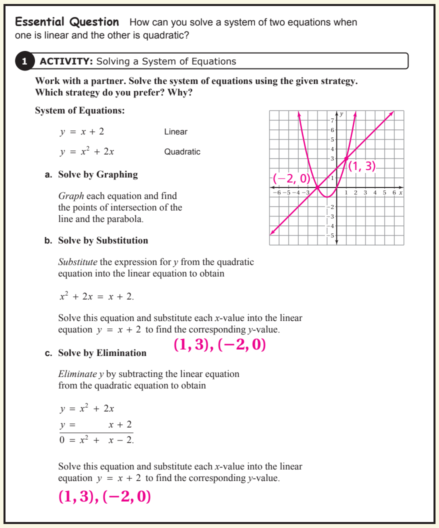
Lesson Plans
Complete Materials List

9.5 Record and Practice Journal



Essential Question How can you solve a system of two equations when one is linear and the other is quadratic?

1 ACTIVITY: Solving a System of Equations

Work with a partner. Solve the system of equations using the given strategy. Which strategy do you prefer? Why?

System of Equations:

$y = x + 2$ Linear

$y = x^2 + 2x$ Quadratic

a. Solve by Graphing

Graph each equation and find the points of intersection of the line and the parabola.

b. Solve by Substitution

Substitute the expression for y from the quadratic equation into the linear equation to obtain

$x^2 + 2x = x + 2.$

Solve this equation and substitute each x-value into the linear equation $y = x + 2$ to find the corresponding y-value.

$(1, 3), (-2, 0)$

c. Solve by Elimination

Eliminate y by subtracting the linear equation from the quadratic equation to obtain

$$\begin{aligned} y &= x^2 + 2x \\ y &= \phantom{x^2 + {}} x + 2 \\ \hline 0 &= x^2 + x - 2. \end{aligned}$$

Solve this equation and substitute each x-value into the linear equation $y = x + 2$ to find the corresponding y-value.

$(1, 3), (-2, 0)$

English Language Learners

Vocabulary

To help students recall the methods of substitution and elimination, write a system of linear equations.

$y = 3x - 1$
$y = 2x + 5$

Then ask which method uses each step shown below.

a. $3x - 1 = 2x + 5$ Substitution

b. $y = 3x - 1$
$\underline{y = 2x + 5}$
$0 = x - 6$ Elimination

9.5 Record and Practice Journal

2 ACTIVITY: Analyzing Systems of Equations

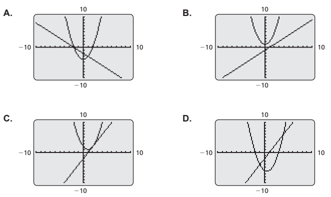
Work with a partner. Match each system of equations with its graph. Then solve the system of equations.

a. $y = x^2 - 4$ A
$y = -x - 2$
$(-2, 0), (1, -3)$

b. $y = x^2 - 2x + 2$ C
$y = 2x - 2$
$(2, 2)$

c. $y = x^2 + 1$ B
$y = x - 1$
no solution

d. $y = x^2 - x - 6$ D
$y = 2x - 2$
$(4, 6), (-1, -4),$



What Is Your Answer?

3. IN YOUR OWN WORDS How can you solve a system of two equations when one is linear and the other is quadratic?

Solve by graphing, by substitution, or by elimination.

4. Summarize your favorite strategy for solving a system of two equations when one is linear and the other is quadratic.

Check students' work.

5. Write a system of equations (one linear and one quadratic) that has the following number of solutions.

a. no solutions **b.** one solution **c.** two solutions

Your systems should be different from those in the activities.

Sample answers:

a. $y = x^2$, $y = -1$ **b.** $y = x^2$, $y = 0$ **c.** $y = x^2$, $y = 1$

Laurie's Notes

Activity 2

- In Activity 1, the focus was on the different solution methods. In Activity 2, the focus is on the number of solutions.
- The graphs visually suggest three different cases for the number of solutions. These are similar to the three cases for the number of x-intercepts of the graph of a quadratic function.
- **MP6 Attend to Precision:** Students could approximate the solution(s) for each system. Instead, make sure students find the exact solution(s).

? "What happened when you solved the system in part (c) algebraically?" found that the system has no real solutions

What Is Your Answer?

- As a follow-up to Question 4, discuss different ways to check solutions, such as solving in two different ways or substituting the solutions back into the original equations.
- Question 5 takes time and students are likely to begin by using trial and error.

? "In Question 5, which of the three cases was the most challenging and why?" Answers will vary. Many students will say that finding a system with one solution was the most challenging.

Closure

- **Writing Prompt:** Consider the graphs of $y = x^2$ and $y = -4$. As the constant function (horizontal line) is translated up, . . . the system of equations formed by the two equations goes from having no solutions, to one solution (when $y = 0$), and then to two solutions.

2 ACTIVITY: Analyzing Systems of Equations

Math Practice 8

Evaluate Results

How can you check the solution of the system of equations to verify that your answer is reasonable?

Work with a partner. Match each system of equations with its graph. Then solve the system of equations.

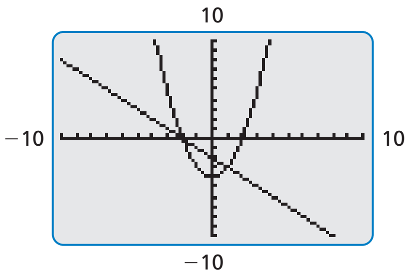
a. $y = x^2 - 4$
$y = -x - 2$

b. $y = x^2 - 2x + 2$
$y = 2x - 2$

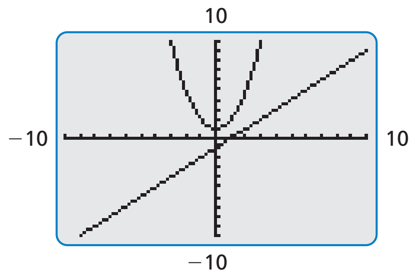
c. $y = x^2 + 1$
$y = x - 1$

d. $y = x^2 - x - 6$
$y = 2x - 2$

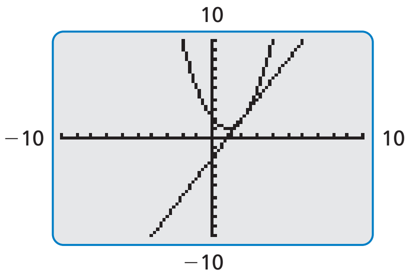
A.



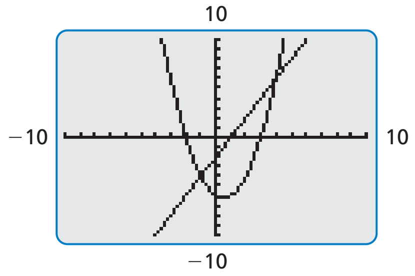
B.



C.



D.



What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you solve a system of two equations when one is linear and the other is quadratic?

4. Summarize your favorite strategy for solving a system of two equations when one is linear and the other is quadratic.

5. Write a system of equations (one linear and one quadratic) that has the following number of solutions.

   a. no solutions **b.** one solution **c.** two solutions

   Your systems should be different from those in the activities.

Practice

Use what you learned about systems of equations to complete Exercises 3–5 on page 490.

9.5 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

You learned methods for solving systems of linear equations in Chapter 4. You can use similar methods to solve systems of linear and quadratic equations.

- Solving by Graphing (Section 4.1 and Section 9.1)
- Solving by Substitution (Section 4.2)
- Solving by Elimination (Section 4.3)

EXAMPLE 1 Solving a System of Linear and Quadratic Equations

Solve the system by substitution.

$\boldsymbol{y = x^2 + x - 1}$ Equation 1

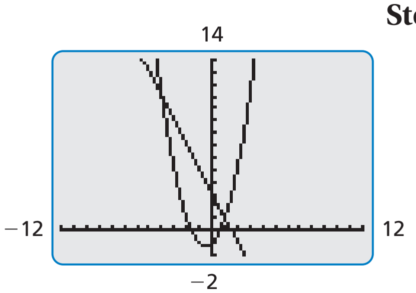
$\boldsymbol{y = -2x + 3}$ Equation 2

Step 1: The equations are already solved for y.

Step 2: Substitute $-2x + 3$ for y in Equation 1 and solve for x.

$y = x^2 + x - 1$	Equation 1
$-2x + 3 = x^2 + x - 1$	Substitute $-2x + 3$ for y.
$3 = x^2 + 3x - 1$	Add $2x$ to each side.
$0 = x^2 + 3x - 4$	Subtract 3 from each side.
$0 = (x + 4)(x - 1)$	Factor right side.
$x + 4 = 0 \quad or \quad x - 1 = 0$	Use Zero-Product Property.
$x = -4 \quad or \quad x = 1$	Solve for x.

Step 3: Substitute -4 and 1 for x in Equation 2 and solve for y.



$y = -2x + 3$	Equation 2	$y = -2x + 3$
$= -2(-4) + 3$	Substitute.	$= -2(1) + 3$
$= 8 + 3$	Multiply.	$= -2 + 3$
$= 11$	Add.	$= 1$

So, the solutions are $(-4, 11)$ and $(1, 1)$.

On Your Own

Now You're Ready
Exercises 6–11

Solve the system by substitution. Check your solution(s).

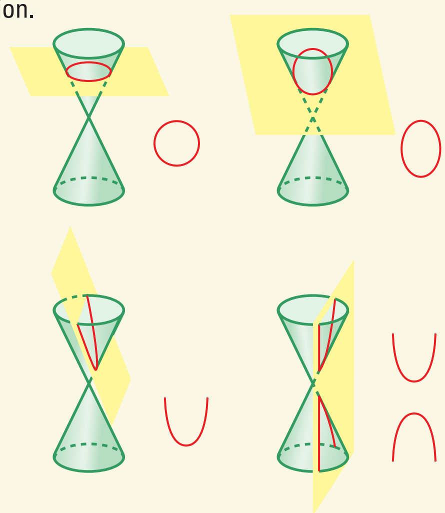
1. $y = x^2 + 9$
   $y = 9$
2. $y = -5x$
   $y = x^2 - 3x - 3$
3. $y = -3x^2 + 2x + 1$
   $y = 5 - 3x$

Laurie's Notes

Introduction

Connect
- **Yesterday:** Students developed a conceptual understanding of solving a system of linear and quadratic equations. (MP1a, MP6, MP8)
- **Today:** Students will solve systems of equations consisting of one linear equation and one quadratic equation.

Motivate
- Show an illustration of the four conic sections. Discuss how a plane can intersect a double-napped cone to form a circle, a parabola, an ellipse or a hyperbola.
- Today students will look at the intersection of a line and a parabola in a plane.



Lesson Notes

Discuss
- **MP7 Look for and Make Use of Structure:** There are no new skills in this lesson. Prior skills are applied to a different type of system. Discuss the strategies students have already learned for solving a linear system.

? "When you solved a system of linear equations, what were the possible numbers of solutions? Explain what your answers imply graphically." no solution: The lines are parallel and do not intersect; one solution: The lines intersect; infinitely many solutions: The lines are the same.

? "What are the possible numbers of solutions for a system with one linear equation and one quadratic equation? Explain." no solutions: The line and parabola do not intersect; one solution: The line intersects the parabola at one point; two solutions: The line intersects the parabola at two points.

- Some students may believe that there can be infinitely many solutions for a system of linear and quadratic equations. Explain that an entire parabola is a curve that can be intersected by a line in one or two points at most.
- If time permits, look back at Section 9.1, Example 3. Method 2 is actually solving this type of system by graphing.

Example 1
- Write the system on the board.

? "Because each equation is already solved for y, what method do you suggest using? Explain." substitution; No work is needed before setting the two expressions equal to each other.

- Collect like terms on one side, but do so in two steps, identifying the quantity added or subtracted to each side in each step.

? "So, are the solutions $x = -4$ and $x = 1$?" no; The solutions are ordered pairs, so we still need to solve for the corresponding y-values.

Goal Today's lesson is solving systems of linear and quadratic equations.

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Extra Example 1
Solve the system by substitution.

$y = x^2 - 4$ Equation 1

$y = -2x - 1$ Equation 2

$(-3, 5), (1, -3)$

On Your Own
1. $(0, 9)$
2. $(-3, 15), (1, -5)$
3. no real solutions

Extra Example 2

Solve the system by elimination.

$y = 2x^2 + 7x + 3$ Equation 1

$y = -x + 3$ Equation 2

(0, 3), (−4, 7)

Extra Example 3

In Example 3, how many solutions does the system have when Equation 2 is changed to $y = x - 4$? Explain.

0; The graph of $y = x - 4$ is a translation 1 unit down of the graph of $y = x - 3$, so it does not intersect the parabola.

On Your Own

4. $\left(-\sqrt{5}, 5 - \sqrt{5}\right)$, $\left(\sqrt{5}, 5 + \sqrt{5}\right)$
5. $\left(\frac{1}{3}, -2\frac{1}{3}\right)$, $\left(-\frac{2}{3}, -7\frac{1}{3}\right)$
6. no real solutions
7. no; The system has 2 solutions because the graph of $y = x - 2$ is a translation 1 unit up of the graph of $y = x - 3$.

Differentiated Instruction

Kinesthetic

In Example 3, it is difficult to tell from the graph that there is exactly one solution. Have your students use an algebraic method to verify the solution.

Laurie's Notes

Example 1 (continued)

? "How can we find the y-value that corresponds to each x-value?" Substitute the x-value into one of the original equations.

- Either equation can be used to determine the y-values, though the linear equation is easier. Discuss ways of checking solutions.

On Your Own

- Have students check with a neighbor after completing each question. Ask students who finish quickly to check by graphing.

Example 2

- When solving by elimination, it is helpful to line up like terms as in Step 1.
- In Step 1, be sure that students correctly subtract each term. They are subtracting $-3x$ and -8, which means they "add the opposite."

? "How can we check the solution?" Graph each equation in the system. The graphs do not intersect. So, there are no solutions.

Example 3

- Have students graph the system in a standard viewing window of a graphing calculator.

? "Can you determine the solution(s) in a standard viewing window?" Answers will vary. Students may say no and that they need to zoom in.

- Remind students that the solution is the point of intersection. Students should be familiar with the several ways to determine the point of intersection using a graphing calculator.
- Take time to discuss the three wrong answers. These three distractors point out some common misconceptions that your students may have.

On Your Own

? "In Question 6, how does using elimination to determine that this system has no solution differ from using elimination to determine that a system of two linear equations has no solution?" Using the quadratic formula leads to an expression that contains the square root of a negative number. A linear system that has no solution leads to an equation that is never true, such as $0 = 7$.

Closure

- **Exit Ticket:** Solve the system using any method. (−1, 0) and (4, 10)

  $y = 2x + 2$

  $y = x^2 - x - 2$

EXAMPLE 2 Solving a System of Linear and Quadratic Equations

Solve the system by elimination. $\boldsymbol{y = x^2 - 3x - 2}$ Equation 1

$\boldsymbol{y = -3x - 8}$ Equation 2

Step 1: Subtract.

$$\begin{array}{rcll} y &=& x^2 - 3x - 2 & \text{Equation 1} \\ \underline{y} &=& \underline{\quad -3x - 8} & \text{Equation 2} \\ 0 &=& x^2 \quad\quad + 6 & \text{Subtract the equations.} \end{array}$$

Step 2: Solve for x.

$$\begin{array}{rcll} 0 &=& x^2 + 6 & \text{Equation from Step 1} \\ -6 &=& x^2 & \text{Subtract 6 from each side.} \end{array}$$

The square of a real number cannot be negative. So, the system has no real solutions.

EXAMPLE 3 Analyze a System of Equations

Which statement about the system is valid?

$\boldsymbol{y = 2x^2 + 5x - 1}$ Equation 1

$\boldsymbol{y = x - 3}$ Equation 2

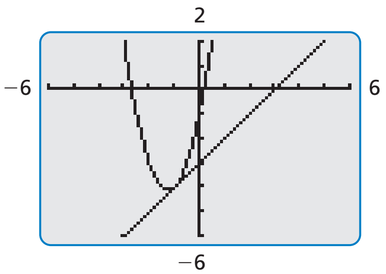
(A) There is one solution because the graph of $y = x - 3$ has one y-intercept.

(B) There is one solution because $y = x - 3$ has one zero.

(C) There is one solution because the graphs of $y = 2x^2 + 5x - 1$ and $y = x - 3$ intersect at one point.

(D) There are two solutions because the graph of $y = 2x^2 + 5x - 1$ has two x-intercepts.

Use a graphing calculator to graph the system. The graphs of $y = 2x^2 + 5x - 1$ and $y = x - 3$ intersect at only one point, $(-1, -4)$.



So, the correct answer is (C).

On Your Own

Now You're Ready
Exercises 12–17

Solve the system by elimination. Check your solution(s).

4. $y = x^2 + x$
   $y = x + 5$

5. $y = 9x^2 + 8x - 6$
   $y = 5x - 4$

6. $y = 2x + 5$
   $y = -3x^2 + x - 4$

7. **WHAT IF?** In Example 3, does the system still have one solution when Equation 2 is changed to $y = x - 2$? Explain.

9.5 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **VOCABULARY** What is a solution of a system of linear and quadratic equations?

2. **WRITING** How is solving a system of linear and quadratic equations similar to solving a system of linear equations? How is it different?

Practice and Problem Solving

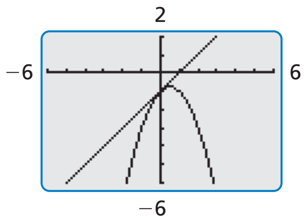
Match the system of equations with its graph. Then solve the system.

3. $y = x^2 - 2x + 1$
   $y = x + 1$

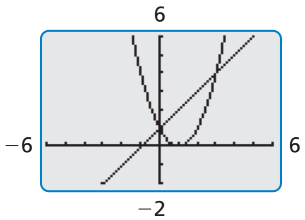
4. $y = x^2 + 3x + 2$
   $y = -x - 3$

5. $y = x - 1$
   $y = -x^2 + x - 1$

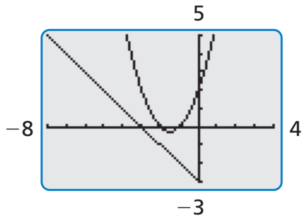
A.



B.



C.



Solve the system by substitution. Check your solution(s).

1 6. $y = x - 5$
   $y = x^2 + 4x - 5$

7. $y = -2x^2$
   $y = 4x + 2$

8. $y = -x + 7$
   $y = -x^2 - 2x - 1$

9. $y = -x^2 + 7$
   $y - 2x = 4$

10. $y - 5 = -x^2$
    $y = 5$

11. $y = 2x^2 + 3x - 4$
    $y - 4x = 2$

Solve the system by elimination. Check your solution(s).

2 12. $y = -x^2 - 2x + 2$
    $y = 4x + 2$

13. $y = -2x^2 + x - 3$
    $y = 2x - 2$

14. $y = 2x - 1$
    $y = x^2$

15. $y = -2x$
    $y - x^2 = 3x$

16. $y - 1 = x^2 + x$
    $y = -x - 2$

17. $y = \frac{1}{2}x - 7$
    $y + 4x = x^2 - 2$

18. **MOVIES** The attendances y for two movies can be modeled by the following equations, where x is the number of days since the movies opened.

$y = -x^2 + 35x + 100$ Movie A

$y = -5x + 275$ Movie B

When is the attendance for each movie the same?

Assignment Guide and Homework Check

Level	Assignment	Homework Check
Average	1–5, 7–17 odd, 18, 23, 27–30	7, 13, 18, 23
Advanced	1, 2, 6–18 even, 22–30	8, 16, 23, 25

Common Errors

- **Exercises 6–11** Students may give only the *x*-values instead of the coordinates of each solution. Remind them each solution is an ordered pair.
- **Exercises 15–17** Students may fail to solve for *y* before subtracting vertically. Remind students to solve both equations for *y* and to line up like terms before subtracting.
- **Exercises 19–21** Estimates obtained graphically may be incorrect or not exact. Make sure their graphical approach is correct. Also, remind them to check their answers.

9.5 Record and Practice Journal

Solve the system by substitution. Check your solution(s).

1. $y = x^2 + 5x - 4$
$y = 3x - 1$
$(-3, -10), (1, 2)$

2. $y = 4x + 2$
$y = x^2 + 6$
$(2, 10)$

3. $y = -3x^2$
$y - 1 = 2x$
no solution

4. $y - x = 2x^2 - 5$
$y = x + 3$
$(-2, 1), (2, 5)$

Solve the system by elimination. Check your solution(s).

5. $y = 4 - 2x$
$y = -x^2 + 2x$
$(2, 0)$

6. $y = x^2 + 5x + 8$
$y = -2x + 2$
$(-1, 4), (-6, 14)$

7. $y + 6x = 7$
$y = -2x^2 + 9x$
$\left(\frac{1}{2}, 4\right), (7, -35)$

8. $y - 4 = x^2 + 5x$
$y = 3x - 2$
no solution

9. The weekly profit *y* (in dollars) for two street vendors can be modeled by the following equations, where *x* is the number of weeks since the beginning of the year.

$$y = -x^2 + 9x + 100$$
$$y = 5x + 103$$

When is the weekly profit for each vendor the same?
weeks 1 and 3

Vocabulary and Concept Check

1. A solution of a system of linear and quadratic equations is an ordered pair that is a solution of each equation in the system.

2. Similarities: You can solve either type of system by elimination, substitution, or graphing.

Differences: Solving a linear system involves finding the intersection(s) of 2 lines or solving a linear equation. Solving a system of linear and quadratic equations involves finding the intersection(s) of a line and a parabola or solving a quadratic equation.

Practice and Problem Solving

3. B; (0, 1), (3, 4)

4. C; no real solutions

5. A; $(0, -1)$

6. $(0, -5), (-3, -8)$

7. $(-1, -2)$

8. no real solutions

9. $(-3, -2), (1, 6)$

10. $(0, 5)$

11. $\left(-\frac{3}{2}, -4\right), (2, 10)$

12. $(-6, -22), (0, 2)$

13. no real solutions

14. $(1, 1)$

15. $(0, 0), (-5, 10)$

16. no real solutions

17. $(2, -6), \left(\frac{5}{2}, -\frac{23}{4}\right)$

18. after 5 days and after 35 days

Practice and Problem Solving

19. $(-1, -6), (-2, -9)$

20. no real solutions

21. $(2, 2)$

22. *Sample answer:* graphing calculator; It is easier to use a graphing calculator, especially when fractions are involved.

23. *Sample answer:* The viewing window of the graphing calculator is too small. By increasing the window you can see that $(5, 14)$ is also a solution.

24. **a.** $y = 30x + 290$

b. $(1, 320), (34, 1310)$

25. **a.** 2 **b.** 0

26. See *Taking Math Deeper.*

Fair Game Review

27. $x = -1, x = 1$

28. $x = -1, x = 5$

29. no real solutions

30. C

Mini-Assessment

Solve the system.

1. $y = x^2 + 2$
$y = 6$ $(-2, 6), (2, 6)$

2. $y = x^2 - 7x + 12$
$y = x - 4$ $(4, 0)$

3. $y = x^2 - 3x + 5$
$y + 2x = 3$ no real solutions

4. The system of equations represents the annual revenues y (in thousands of dollars) of two companies t years after 2010.

$y = 3t^2 + 32t + 10$

$y = 45t + 40$

In what year are the revenues of the two companies equal? 2016

Taking Math Deeper

Exercise 26

You can solve this problem algebraically by setting up the system using the general form of each type of equation.

1 Write the system.

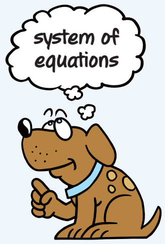
$y = mx + n$ General form of a linear equation

$y = ax^2 + bx + c$ General form of a quadratic equation

(*Note:* m and n are used in the general form of a linear equation because a and b are used in the general form of a quadratic equation.)

2 Begin solving the system by subtracting the equations.

$$\begin{aligned} y &= \phantom{ax^2 +{}} mx + n \\ y &= ax^2 + bx + c \\ \hline 0 &= -ax^2 + (m - b)x + (n - c) \end{aligned}$$



Notice that the resulting equation is a quadratic equation in one variable. The solutions of this equation represent the solutions of the system.

You learned previously that quadratic equations must have 0, 1, or 2 solutions. So, the system must have 0, 1, or 2 solutions.

3 Interpret the result.

This system represents ALL possible systems of linear and quadratic equations. So, a system of linear and quadratic equations cannot have an infinite number of solutions.

Project

Research cubic functions. Make a conjecture about the number of possible solutions of a system of linear and cubic equations.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension • School-to-Work • Financial Literacy Start the next section

Solve the system using a graphing calculator.

19. $y = x^2 + 6x - 1$
$y = 3x - 3$

20. $y = \frac{1}{4}x - 12$
$y = x^2 - 6x$

21. $y = \frac{1}{2}x^2$
$y = 2x - 2$

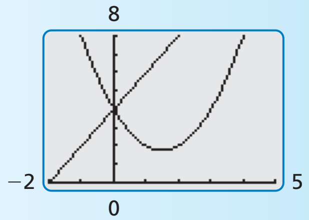
22. CHOOSE TOOLS Do you prefer to solve systems of equations by hand or using a graphing calculator? Explain your reasoning.

23. ERROR ANALYSIS Describe and correct the error in solving the system of equations.

✗

$y = x^2 - 3x + 4$
$y = 2x + 4$

The only solution of the system of equations is (0, 4).



24. WEBSITES The function $y = -x^2 + 65x + 256$ models the number y of subscribers to a website, where x is the number of days since the website was launched. The number of subscribers to a competitor's website can be modeled by a linear function. The websites have the same number of subscribers on days 1 and 34.

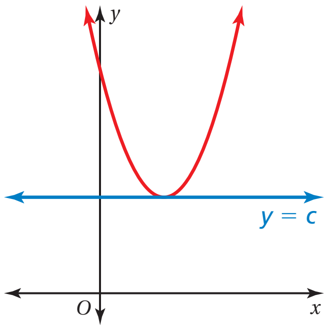
a. Write a linear function that models the number of subscribers to the competitor's website.

b. Solve the system to verify the function from part (a).

25. REASONING The graph shows a quadratic function and the linear function $y = c$.

a. How many solutions will the system have when you change the linear equation to $y = c + 2$?

b. How many solutions will the system have when you change the linear equation to $y = c - 2$?



26. **Writing** Can a system of linear and quadratic equations have an infinite number of solutions? Explain your reasoning.

Fair Game Review What you learned in previous grades & lessons

Solve the equation by graphing. Check your solution(s). *(Section 9.1)*

27. $x^2 = 1$

28. $x^2 - 4x - 5 = 0$

29. $-x^2 = 2x + 7$

30. MULTIPLE CHOICE What is the factored form of the polynomial $x^2 - 36$? *(Section 7.9)*

Ⓐ $(x + 6)^2$ Ⓑ $(x - 6)^2$ Ⓒ $(x + 6)(x - 6)$ Ⓓ $x + 6$

9.4–9.5 Quiz

Check It Out
Progress Check
BigIdeasMath.com

Solve the equation using the quadratic formula. *(Section 9.4)*

1. $x^2 + 8x - 20 = 0$

2. $13x = 2x^2 + 6$

3. $9 - 24x = -16x^2$

Use the discriminant to determine the number of real solutions of the equation. *(Section 9.4)*

4. $x^2 + 6x - 13 = 0$

5. $-8x^2 - x = 5$

6. $\frac{3}{4}x^2 = 3x - 3$

7. Solve $x^2 + 10x + 21 = 0$ using two different methods. *(Section 9.4)*

Solve the equation using any method. Explain your choice of method. *(Section 9.4)*

8. $x^2 + 4x - 11 = 0$

9. $-4x^2 + 1 = 0$

10. $52 = x^2 - 2x$

Solve the system. *(Section 9.5)*

11. $y = x^2 - 16$
$y = -7$

12. $y = x^2 + 2x + 1$
$y = 2x + 2$

13. $y = x^2 - 5x + 8$
$y = -3x - 4$

14. **BACTERIA** The numbers y of two types of bacteria after t hours are given by the models below. *(Section 9.5)*

$y = 3t^2 + 8t + 20$ Type 1

$y = 27t + 60$ Type 2

a. As t increases, which type grows more quickly? Explain.

b. When are the numbers of Type 1 and Type 2 bacteria the same?

c. When are there more Type 1 bacteria than Type 2? When are there more Type 2 bacteria than Type 1? Use a graph to support your answer.

15. **CELLULAR PHONE CALLS** The average monthly bill y (in dollars) for a customer's cell phone x years after 2000 can be modeled by $y = -0.2x^2 + 2x + 45$. When was the average monthly bill about \$50? *(Section 9.4)*

16. **REASONING** Do you think the model in Exercise 15 can be used for future years? Explain using a graphing calculator to support your answer. *(Section 9.4)*

Alternative Assessment Options

Math Chat	Student Reflective Focus Question
Structured Interview	Writing Prompt

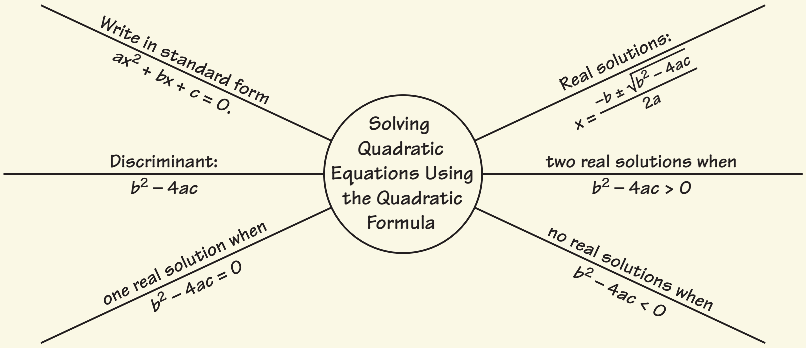
Math Chat

- Have individual students work problems from the quiz on the board. The student explains the process used and justifies each step. Students in the class ask questions of the student presenting.
- The teacher explores the thought process of the student presenting, but does not teach or ask leading questions.

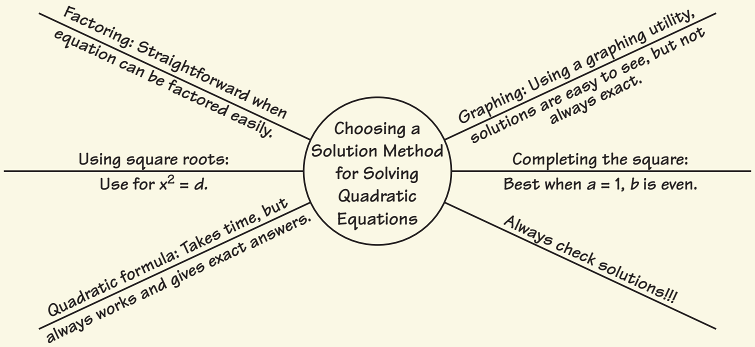
Study Help Sample Answers

Remind students to complete Graphic Organizers for the rest of the chapter.

4.



5.



6. Available at *BigIdeasMath.com*

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
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Answers

1. $x = -10, x = 2$
2. $x = \frac{1}{2}, x = 6$
3. $x = \frac{3}{4}$
4. 2
5. 0
6. 1
7. $x = -7, x = -3$
8. $x = -2 + \sqrt{15}, x = -2 - \sqrt{15}$; *Sample answer:* completing the square, because $a = 1$ and b is even
9. $x = \frac{1}{2}, x = -\frac{1}{2}$; *Sample answer:* square roots, because no x-term
10. $x = 1 + \sqrt{53}, x = 1 - \sqrt{53}$; *Sample answer:* completing the square, because $a = 1$ and b is even
11. $(-3, -7), (3, -7)$
12. $(-1, 0), (1, 4)$
13. no real solutions

14–16. See Additional Answers.

Technology for the Teacher

Online Assessment
Assessment Book
ExamView® Assessment Suite

For the Teacher
Additional Review Options

- *BigIdeasMath.com*
- Online Assessment
- Game Closet at *BigIdeasMath.com*
- Vocabulary Help
- Resources by Chapter

Answers

1. $x = 3, x = 6$
2. no real solutions
3. $x = -4$
4. $x = 0$
5. no real solutions
6. $x = -10, x = 6$

Review of Common Errors

- **Exercises 1–3** Solutions obtained graphically may be incorrect or not exact. Make sure students use a sound graphical approach. Also, remind them to check their answers.
- **Exercise 5** Students may try to take the square root of a negative number. Remind them that the square of a real number cannot be negative.
- **Exercise 6** Students may forget the negative square root when taking the square root of each side of the equation. Remind them to account for the negative square root when appropriate.
- **Exercises 7–9** Students may forget to divide the x-coefficient by 2 before squaring. Remind them of this process.
- **Exercises 7–9** Students may not add the same value to each side of the equation when completing the square. Remind students that to form an equivalent equation, they must add the same quantity to each side.
- **Exercise 10** Students may stop after finding the length ℓ. They need to find the perimeter.
- **Exercises 11–13** Students may make sign mistakes when identifying the values of *a, b,* and *c*. Emphasize how the signs are determined.
- **Exercises 14–16** Students may not explain their reasoning. Remind them to read the directions carefully.
- **Exercises 17–19** Students may give only the x-values instead of the coordinates of each solution. Remind them each solution is an ordered pair.

9 Chapter Review

Check It Out
Vocabulary Help
BigIdeasMath.com

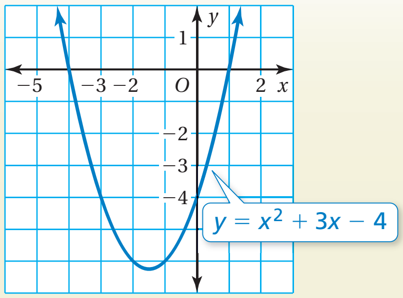
Review Key Vocabulary

quadratic equation, *p. 456*
completing the square, *p. 470*
quadratic formula, *p. 478*
discriminant, *p. 480*

Review Examples and Exercises

9.1 Solving Quadratic Equations by Graphing *(pp. 454–461)*

Solve $x^2 + 3x - 4 = 0$ by graphing.



Step 1: Graph the related function $y = x^2 + 3x - 4$.

Step 2: Find the x-intercepts. They are -4 and 1.

So, the solutions are $x = -4$ and $x = 1$.

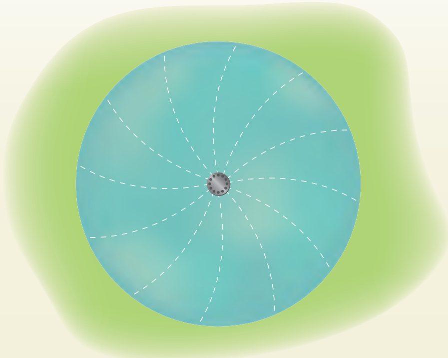
Exercises

Solve the equation by graphing. Check your solution(s).

1. $x^2 - 9x + 18 = 0$ **2.** $x^2 - 2x = -4$ **3.** $-8x - 16 = x^2$

9.2 Solving Quadratic Equations Using Square Roots *(pp. 462–467)*

A sprinkler sprays water that covers a circular region of 90π square feet. Find the diameter of the circle.



Write an equation using the formula for the area of a circle.

$A = \pi r^2$		Write the formula.
$90\pi = \pi r^2$		Substitute 90π for A.
$90 = r^2$		Divide each side by π.
$\pm\sqrt{90} = r$		Take the square root of each side.

A diameter cannot be negative, so use the positive square root. The diameter is twice the radius. So, the diameter is $2\sqrt{90}$.

The diameter of the circle is $2\sqrt{90} \approx 19$ feet.

Exercises

Solve the equation using square roots.

4. $x^2 - 10 = -10$ **5.** $4x^2 = -100$ **6.** $(x + 2)^2 = 64$

9.3 Solving Quadratic Equations by Completing the Square (pp. 468–473)

Solve $x^2 - 6x + 4 = 11$ by completing the square.

$x^2 - 6x + 4 = 11$	Write the equation.
$x^2 - 6x = 7$	Subtract 4 from each side.
Complete the square. → $x^2 - 6x + 9 = 7 + 9$	Add $\left(\frac{-6}{2}\right)^2$, or 9, to each side.
$(x - 3)^2 = 16$	Factor $x^2 - 6x + 9$.
$x - 3 = \pm 4$	Take the square root of each side.
$x = 3 \pm 4$	Add 3 to each side.

The solutions are $x = 3 + 4 = 7$ and $x = 3 - 4 = -1$.

Exercises

Solve the equation by completing the square.

7. $x^2 + x + 10 = 0$ **8.** $x^2 + 2x + 5 = 4$ **9.** $2x^2 - 4x = 10$

10. **CREDIT CARD** The width w of a credit card is 3 centimeters shorter than the length ℓ. The area is 46.75 square centimeters. Find the perimeter.

9.4 Solving Quadratic Equations Using the Quadratic Formula (pp. 476–485)

Solve $-3x^2 + x = -8$ using the quadratic formula.

$-3x^2 + x = -8$	Write original equation.
$-3x^2 + x + 8 = 0$	Write in standard form.
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic Formula
$= \dfrac{-1 \pm \sqrt{1^2 - 4(-3)(8)}}{2(-3)}$	Substitute -3 for a, 1 for b, and 8 for c.
$= \dfrac{-1 \pm \sqrt{97}}{-6}$	Simplify.

The solutions are $x = \dfrac{-1 + \sqrt{97}}{-6} \approx -1.5$ and $x = \dfrac{-1 - \sqrt{97}}{-6} \approx 1.8$.

Exercises

Solve the equation using the quadratic formula.

11. $x^2 + 2x - 15 = 0$ **12.** $2x^2 - x + 8 = 3$ **13.** $-5x^2 + 10x = 5$

Solve the equation using any method. Explain your choice of method.

14. $x^2 - 121 = 0$ **15.** $x^2 - 4x + 4 = 0$ **16.** $x^2 - 4x = -1$

Review Game

Choosing a Solution Method

Materials:

- flash cards
- paper
- pencil

Directions:

- Make flash cards ahead of time by writing quadratic equations large enough for your students to see.
- Split the class into two teams. Select a spokesperson for each team.

Playing a round:

- Lift a flash card to show a quadratic equation. The two teams race to determine:
  (1) a list of the methods that can be used to solve the equation
  (2) the solution of the equation
- The first spokesperson to raise his or her hand answers both parts.
- Next, the spokesperson from the other team either confirms or corrects the first team's answers.

Round scoring:

- The correct answer to Part (1) is a list of any of the methods (factoring, graphing, using square roots, completing the square, or quadratic formula) that can be used to solve the equation. The correct answer to Part (2) includes all solutions of the equation.
- The first team earns 2 points for each part they answer correctly. The other team earns 1 point for confirming a correct part and 3 points for correcting a wrong part.

Who wins?
The team with the greatest number of points after the last round wins.

For the Student Additional Practice

- Lesson Tutorials
- Multi-Language Glossary
- Self-Grading Progress Check
- *BigIdeasMath.com*
  Dynamic Student Edition
  Student Resources

Answers

7. no real solutions

8. $x = -1$

9. $x = 1 + \sqrt{6}, x = 1 - \sqrt{6}$

10. 28 cm

11. $x = -5, x = 3$

12. no real solutions

13. $x = 1$

14. $x = 11, x = -11$; *Sample answer:* square roots, because no x-term

15. $x = 2$; *Sample answer:* factoring, because the left side is a perfect square trinomial

16. $x = 2 + \sqrt{3}, x = 2 - \sqrt{3}$; *Sample answer:* completing the square, because $a = 1$ and b is even

17. $(1, -5)$

18. $\left(1 - \sqrt{15}, 7 - 2\sqrt{15}\right)$, $\left(1 + \sqrt{15}, 7 + 2\sqrt{15}\right)$

19. no real solutions

My Thoughts on the Chapter

What worked. . .

Teacher Tip

Not allowed to write in your teaching edition? Use sticky notes to record your thoughts.

What did not work. . .

What I would do differently. . .

9.5 Solving Systems of Linear and Quadratic Equations (pp. 486–491)

Solve the system by substitution.

$y = 2x^2 - 5$ Equation 1

$y = -x + 1$ Equation 2

Step 1: The equations are already solved for y.

Step 2: Substitute $-x + 1$ for y in Equation 1 and solve for x.

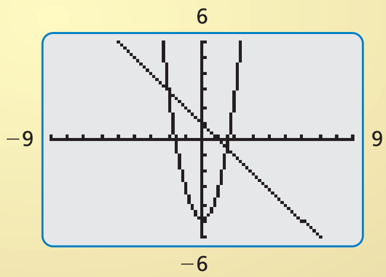
$$
\begin{aligned}
y &= 2x^2 - 5 && \text{Equation 1}\\
-x + 1 &= 2x^2 - 5 && \text{Substitute } -x + 1 \text{ for } y.\\
1 &= 2x^2 + x - 5 && \text{Add } x \text{ to each side.}\\
0 &= 2x^2 + x - 6 && \text{Subtract 1 from each side.}\\
0 &= (2x - 3)(x + 2) && \text{Factor right side.}\\
2x - 3 = 0 \quad &or \quad x + 2 = 0 && \text{Use Zero-Product Property.}\\
x = \frac{3}{2} \quad &or \quad x = -2 && \text{Solve for } x.
\end{aligned}
$$

Step 3: Substitute $\frac{3}{2}$ and -2 for x in Equation 2 and solve for y.

$$
\begin{aligned}
y &= -x + 1 && \text{Equation 2} && y = -x + 1\\
&= -\frac{3}{2} + 1 && \text{Substitute.} && = -(-2) + 1\\
&= -\frac{1}{2} && \text{Simplify.} && = 3
\end{aligned}
$$

So, the solutions are $\left(\frac{3}{2}, -\frac{1}{2}\right)$ and $(-2, 3)$.

Check



Exercises

Solve the system. Check your solution(s).

17. $y = x^2 - 2x - 4$
$y = -5$

18. $y = x^2 - 9$
$y = 2x + 5$

19. $y = 2 - 3x$
$y = -x^2 - 5x - 4$

9 Chapter Test

Check It Out
Test Practice
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Solve the equation by graphing.

1. $x^2 - 7x + 12 = 0$ **2.** $x^2 + 12x = -36$ **3.** $x + 1 = -x^2$

Solve the equation using square roots.

4. $14 = 2x^2$ **5.** $x^2 + 9 = 5$ **6.** $(4x + 3)^2 = 16$

Solve the equation by completing the square.

7. $x^2 - 8x + 15 = 0$ **8.** $x^2 - 6x = 10$

9. $x^2 - 8x = -9$ **10.** $16 = x^2 - 16x - 20$

Solve the equation using the quadratic formula.

11. $5x^2 + x - 4 = 0$ **12.** $9x^2 + 6x + 1 = 0$ **13.** $-2x^2 + 3x + 7 = 0$

14. **REASONING** Use the discriminant to determine how many times the graph of $y = 4x^2 - 4x + 1$ intersects the x-axis.

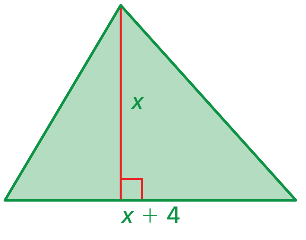
15. **CHOOSING A METHOD** Solve $x^2 - 9x - 10 = 0$ using any method. Explain your choice of method.

Solve the system.

16. $y = x^2 - 4x - 2$
$y = -4x + 2$

17. $y = -5x^2 + x - 1$
$y = -7$

18. **GEOMETRY** The area of the triangle is 35 square feet. Use a quadratic equation to find the length of the base. Round your answer to the nearest tenth.



19. **SNOWBOARDING** A snowboarder leaves an 8-foot-tall ramp with an upward velocity of 28 feet per second. The function $h = -16t^2 + 28t + 8$ gives the height h (in feet) of the snowboarder after t seconds. How many points does the snowboarder earn with a perfect landing?

Criteria	Scoring
Maximum height	1 point per foot
Time in air	5 points per second
Perfect landing	25 points

Test Item References

Chapter Test Questions	Section to Review	Common Core State Standards
1–3	9.1	A.REI.4, A.REI.11
4–6	9.2	A.REI.4b
7–10	9.3	A.REI.4a, A.REI.4b, A.SSE.3b, F.IF.8a
11–14, 15, 18, 19	9.4	A.REI.4a, A.REI.4b
16, 17	9.5	A.REI.7

Test-Taking Strategies

Remind students to quickly look over the entire test before they start so that they can budget their time. Have students use the **Stop** and **Think** strategy before they answer each question.

Common Errors

- **Exercises 1–3** Solutions obtained graphically may be incorrect or not exact. Make sure students use a sound graphical approach. Also, remind them to check their answers.
- **Exercises 4–6** Students may forget the negative square root when taking the square root of each side of the equation. Remind them to account for the negative square root when appropriate.
- **Exercises 7–10** Students may not add the same value to each side of the equation when completing the square. Remind students that to form an equivalent equation, they must add the same quantity to each side.
- **Exercises 11–13** Students may make sign mistakes when identifying the values of *a, b,* and *c*. Emphasize how the signs are determined.
- **Exercises 16 and 17** Students may give only the *x*-values instead of the coordinates of each solution. Remind them each solution is an ordered pair.
- **Exercise 19** Students may have a difficult time starting the problem. Explain how to approach this problem one part at a time: Find the maximum height, the time in the air, and the points awarded.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials *BigIdeasMath.com* Skills Review Handbook	Resources by Chapter • Enrichment and Extension • School-to-Work • Financial Literacy Game Closet at *BigIdeasMath.com* Start Standards Assessment

Answers

1. $x = 3, x = 4$
2. $x = -6$
3. no real solutions
4. $x = \sqrt{7}, x = -\sqrt{7}$
5. no real solutions
6. $x = -\frac{7}{4}, x = \frac{1}{4}$
7. $x = 3, x = 5$
8. $x = 3 + \sqrt{19}, x = 3 - \sqrt{19}$
9. $x = 4 + \sqrt{7}, x = 4 - \sqrt{7}$
10. $x = -2, x = 18$
11. $x = -1, x = \frac{4}{5}$
12. $x = -\frac{1}{3}$
13. $x = \frac{3 + \sqrt{65}}{4}, x = \frac{3 - \sqrt{65}}{4}$
14. 1
15. $x = -1, x = 10$, *Sample answer:* factors easily
16. $(-2, 10), (2, -6)$
17. $(-1, -7), \left(\frac{6}{5}, -7\right)$
18. 10.6 ft
19. $55\frac{1}{4}$ points

Technology for the Teacher

Online Assessment
Assessment Book
ExamView® Assessment Suite

Test Taking Strategies

Available at *BigIdeasMath.com*

After Answering Easy Questions, Relax
Answer Easy Questions First
Estimate the Answer
Read All Choices before Answering
Read Question before Answering
Solve Directly or Eliminate Choices
Solve Problem before Looking at Choices
Use Intelligent Guessing
Work Backwards

About this Strategy

When taking a multiple choice test, be sure to read each question carefully and thoroughly. When taking a timed test, it is often best to skim the test and answer the easy questions first. Be careful that you record your answer in the correct position on the answer sheet.

Answers

1. B

2. I

3. D

4. -4

5. H

Technology for the Teacher

Common Core State Standards Support
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Assessment Book
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Item Analysis

1. **A.** The student confuses the patterns for the square of a binomial.

B. Correct answer

C. The student represents the product as a sum of two squares.

D. The student confuses the patterns for the square of a binomial.

2. **F.** The student incorrectly rewrites the related equation as $y = (x - 25)^2 + 5$ and solves by graphing.

G. The student is confused by the negative sign.

H. The student incorrectly rewrites the related equation as $y = (3x + 1)^2 + 9$ and solves by graphing.

I. Correct answer

3. **A.** The student thinks the values represent solutions, *not* zeros.

B. The student incorrectly thinks the graph crosses the x-axis at (1, 0) and (3, 0).

C. The student does not know that the axis of symmetry is halfway between the x-intercepts.

D. Correct answer

4. **Gridded response:** Correct answer: -4

Common error: The student substitutes -12 for x and evaluates as -52.

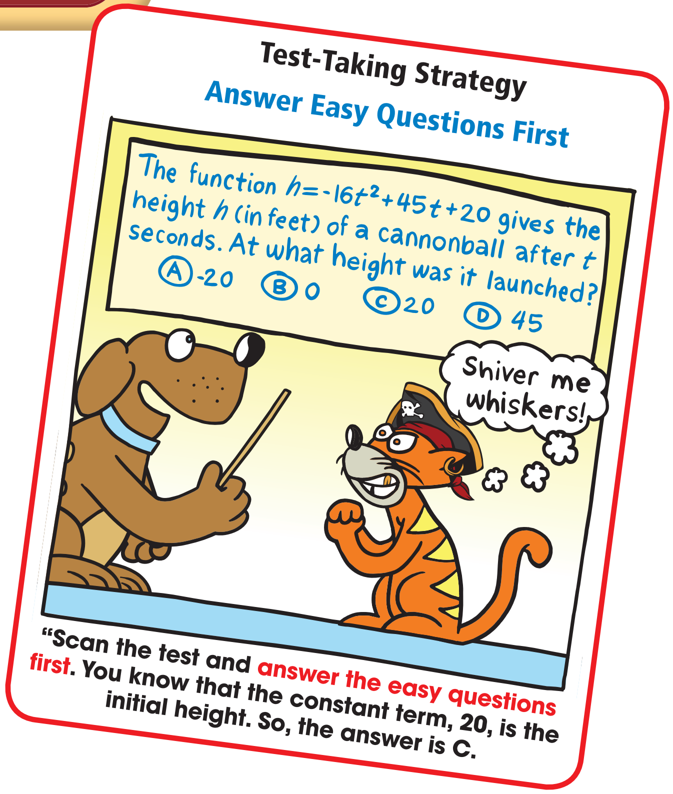
5. **F.** The student chooses an integer value of x close to where the maximum occurs.

G. The student incorrectly estimates the value from the graph.

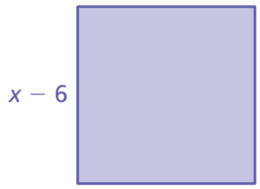
H. Correct answer

I. The student chooses an integer value of x close to where the maximum occurs.

9 Standards Assessment



1. Which expression represents the area of the square? *(A.APR.1)*



A. $x^2 + 12x + 36$ **C.** $x^2 + 36$

B. $x^2 - 12x + 36$ **D.** $x^2 - 12x - 36$

2. Which of the following equations has *no* real solutions? *(A.REI.4b)*

F. $(x - 25)^2 = 5$ **H.** $(3x + 1)^2 = 9$

G. $-4x^2 = 0$ **I.** $2x^2 + 1 = -1$

3. Use the solution below to determine which statement about the function $y = x^2 + 4x + 3$ is false. *(A.SSE.3a)*

$$x^2 + 4x + 3 = 0$$
$$(x+3)(x+1) = 0$$
$$x + 3 = 0 \quad or \quad x + 1 = 0$$
$$x = -3 \qquad\qquad x = -1$$

A. The zeros are -1 and -3.

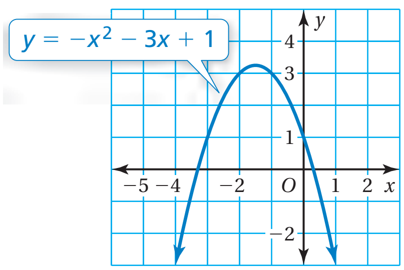
B. The graph crosses the x-axis at $(-1, 0)$ and $(-3, 0)$.

C. The axis of symmetry of the graph is $x = -2$.

D. The maximum value occurs when $x = -2$.

4. For $f(x) = 5x + 8$, what value of x makes $f(x) = -12$? *(F.IF.2)*

5. Which line represents the axis of symmetry of the graph of the quadratic function? *(F.IF.4)*



F. $x = -2$ **H.** $x = -\frac{3}{2}$

G. $x = -\frac{5}{3}$ **I.** $x = -1$

6. What are the *exact* roots of the quadratic equation $3x^2 + x - 1 = 0$? *(A.REI.4b)*

A. $-0.8, 0.4$

B. $-0.77, 0.43$

C. $\frac{-1 - \sqrt{13}}{6}, \frac{-1 + \sqrt{13}}{6}$

D. $-\frac{3}{4}, \frac{1}{2}$

7. The function $h = -16t^2 + 60t + 2$ gives the height h (in feet) of a soccer ball after t seconds. Which of the following statements is true? *(A.REI.4b)*

F. The soccer ball reaches a height of 60 feet.

G. It takes the soccer ball 2.5 seconds to reach its maximum height.

H. The soccer ball hits the ground after about 5 seconds.

I. The soccer ball is kicked from a height of 2 feet.

8. Which *best* describes the solutions of the system of equations below? *(A.REI.7)*

$y = x^2 + 2x - 8$ Equation 1

$y = 5x + 2$ Equation 2

A. Their graphs intersect at one point, $(-2, -8)$. So, there is one solution.

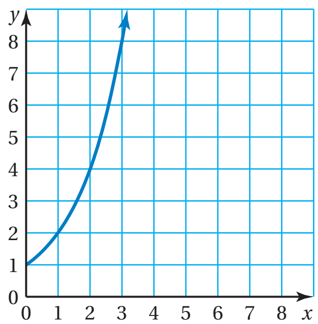
B. Their graphs intersect at two points, $(-2, -8)$ and $(5, 27)$. So, there are two solutions.

C. Their graphs do not intersect. So, there is no solution.

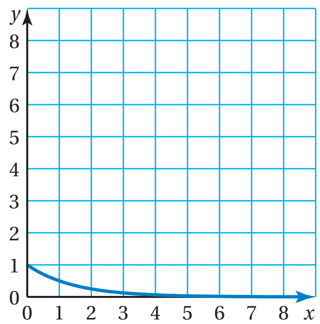
D. The graph of $y = x^2 + 2x - 8$ has two x-intercepts. So, there are two solutions.

9. Which graph shows exponential growth? *(F.LE.1c)*

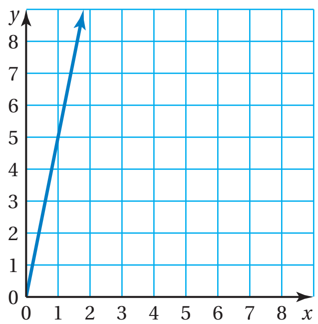
F.



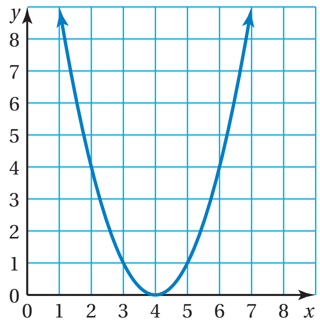
H.



G.



I.



Item Analysis (continued)

6. **A.** The student rounds the solutions.
   B. The student rounds the solutions.
   C. Correct answer
   D. The student incorrectly approximates the solutions by graphing.

7. **F.** The student graphs the function and estimates that the graph reaches a height of 60 feet.
   G. The student makes a calculation error.
   H. The student makes a calculation error.
   I. Correct answer

8. **A.** Using a graphing calculator, the student graphs the system in a standard viewing window and does not see the second point of intersection.
   B. Correct answer
   C. Using a graphing calculator, the student graphs the system in a window that does not show either point of intersection.
   D. The student confuses the solutions of a system of equations with the solutions of a quadratic equation.

9. **F.** Correct answer
   G. The student confuses the graphs of exponential growth and exponential decay models.
   H. The student randomly chooses a graph that rises from left to right.
   I. The student chooses the graph of a parabola because the chapter is about solving quadratic equations.

Answers

6. C
7. I
8. B
9. F

Answers

10. *Part A:* up

Part B: (0, 4)

Part C: $x = -\frac{1}{2}$

Part D: $\left(-\frac{1}{2}, \frac{13}{4}\right)$

11. A

12. 114

13. G

Answer for Extra Example

1. **A.** The student confuses the vertical line $x = d$ for a horizontal line that does not intersect the parabola.

B. Correct answer

C. The student confuses the vertical line $x = d$ for a horizontal line that intersects the parabola at two points.

D. The student confuses the vertical line $x = d$ for a horizontal line.

Item Analysis (continued)

10. **2 points** The student demonstrates a thorough understanding of how the graph of a quadratic function is related to its standard form $y = ax^2 + bx + c$. The student finds each part correctly, shows the work, and gives sound explanations.

1 point The student's work and explanation demonstrate a partial understanding. The student is unable to find one or two of the parts correctly and not all of the explanations are adequate.

0 points The student provides no response, a completely incorrect or incomprehensible response, or a response that demonstrates insufficient understanding of how the graph of a quadratic equation is related to the standard form of its equation.

11. **A.** Correct answer

B. The student incorrectly factors the perfect square trinomial.

C. The student uses $-\left(\frac{b^2}{2}\right)$ instead of $\left(\frac{b^2}{2}\right)$.

D. The student confuses this problem for the type of real-life problem in which you only use the positive root.

12. **Gridded response:** Correct answer: 114

Common error: The student forgets to write the equation in standard quadratic form and uses $c = 13$ instead of $c = -13$.

13. **F.** The student confuses the range and the domain.

G. Correct answer

H. The student thinks the zeros are the points where the graph crosses the x-axis.

I. The student confuses minimum and maximum.

Extra Example

1. Which statement best describes the number of solutions of the system, where *a, b, c,* and *d* are real numbers? *(A.REI.7)*

$y = ax^2 + bx + c$ Equation 1

$x = d$ Equation 2

A. There are no real solutions.

B. There is one solution.

C. There are two solutions.

D. There may be one, two, or no real solutions.

10. For Parts A–D, use the function $y = 3x^2 + 3x + 4$ to find each characteristic without using a graph. Show your work and explain your reasoning. *(F.IF.4)*

Think
Solve
Explain

Part A direction the graph of the function opens

Part B y-intercept of the graph of the function

Part C axis of symmetry of the graph of the function

Part D vertex of the graph of the function

11. Jamie is solving the equation $x^2 - 14x + 7 = 18$ by completing the square.

$$x^2 - 14x + 7 = 18$$
$$x^2 - 14x = 11$$
$$x^2 - 14x + 49 = 11$$
$$(x - 7)^2 = 11$$
$$x - 7 = \pm\sqrt{11}$$
$$x = 7 \pm \sqrt{11}$$

What should Jamie do to correct the error that he made? *(A.REI.4b)*

A. Add 49 to each side of the equation.

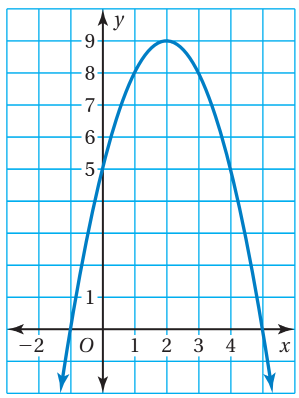
B. Factor $x^2 - 14x + 49$ as $(x + 7)^2$.

C. Subtract 49 from each side of the equation instead of adding 49.

D. Only use the positive square root of 11.

12. What is the value of the discriminant for the quadratic equation $1.5x^2 - 6x = 13$? *(A.REI.4b)*

13. Which of the following statements is true about the quadratic function shown in the graph? *(A.REI.4b)*



F. The range is all real numbers.

G. The domain is all real numbers.

H. The zeros are $(-1, 0)$ and $(5, 0)$.

I. A minimum occurs at the vertex.