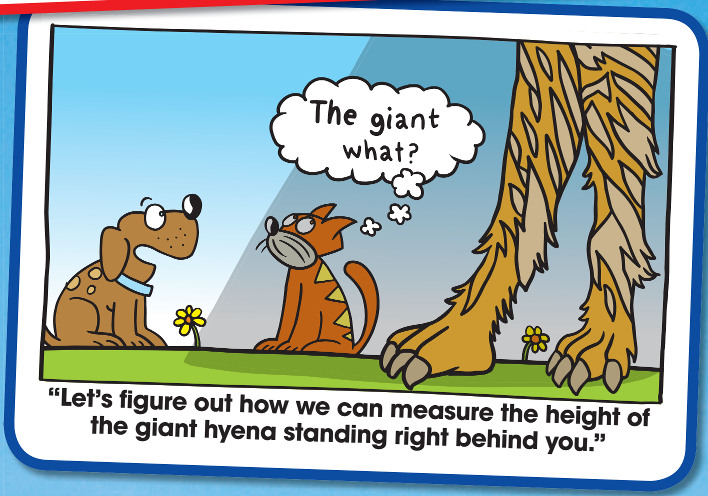
10 Square Root Functions and Geometry

"I'm pretty sure that Pythagoras was a Greek."

"I said 'Greek', not 'Geek'."



"Let's figure out how we can measure the height of the giant hyena standing right behind you."

Connections to Previous Learning

- Write, analyze, and solve one-variable linear equations.
- Find the areas of right triangles.

- Understand the connections between proportional relationships, lines, and linear equations.
- Draw and construct triangles and describe the relationships between them.
- Evaluate square roots of small perfect squares.

- Understand and apply the Pythagorean Theorem.
- Solve square root equations.

Chapter Summary

Section	Common Core State Standard	
10.1	Learning	F. IF.4 ★, F.IF.7b ★
10.2	Applying	N.RN.2 ★
10.3	Learning	8.G.6, 8.G.7
10.4	Learning	8.G.6 ★, 8.G.7 ★, 8.G.8 ★
★ Teaching is complete. Standard can be assessed.		

Pacing Guide for Chapter 10

Chapter Opener	1 Day
Section 1	2 Days
Section 2	2 Days
Study Help/Quiz	1 Day
Section 3	2 Days
Section 4	1 Day
Chapter Review / Chapter Tests	2 Days
Total Chapter 10	11 Days
Year-to-Date	137 Days

Technology for the *Teacher*

BigIdeasMath.com
Chapter at a Glance
Complete Materials List
Parent Letters: English and Spanish

Common Core State Standards

8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.

Additional Topics for Review

- Graphing functions
- Domain and range of a function
- Solving inequalities
- Solving equations
- Right triangles

Try It Yourself

1. 45 **2.** -10

3. -38

4. $(y + 3)(y + 9)$

5. $(n - 10)(n - 1)$

6. $(w + 6)(w - 8)$

7. $(z + 5)(z + 20)$

Record and Practice Journal Fair Game Review

1. -2 **2.** 29

3. 8 **4.** -13

5. 20 **6.** -48

7. 14 **8.** 3

9. 16 days since launch

10. $(v - 4)(v - 8)$

11. $(d + 3)(d + 6)$

12. $(k - 7)(k + 9)$

13. $(m + 2)(m - 12)$

14. $(t - 10)(t + 9)$

15. $(f + 9)(f - 3)$

16. $(a + 5)(a + 11)$

17. $(q - 4)(q - 17)$

18. $(x + 2)$

Math Background Notes

Vocabulary Review

- Square Root
- Perfect Square
- Polynomial
- Factoring Polynomials

Evaluating an Expression Involving a Square Root

- Students should know how to apply the order of operations to evaluate an expression.
- In applying the order of operations, finding the square root of a number is equivalent to raising a number to an exponent.

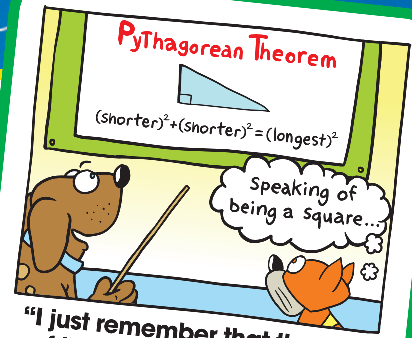
Factoring $x^2 + bx + c$

- Students should know how to factor a trinomial.
- Remind students that $x^2 + bx + c = (x + p)(x + q)$ when $p + q = b$ and $pq = c$. Also remind them that when c is positive, p and q have the same sign as b, and when c is negative, p and q have different signs.
- **Common Error:** Students may forget to check that the factors of c add up to b. Remind them that there are two steps to factoring polynomials, factoring c and checking that those factors add up to b. Encourage students to use tables as shown in the example.
- Remind students to check their answers using the FOIL Method.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Record and Practice Journal • Fair Game Review Skills Review Handbook Lesson Tutorials	Game Closet at *BigIdeasMath.com* Start the next section

What You Learned Before



"I just remember that the sum of the squares of the shorter sides is equal to the square of the longest side."

Evaluating an Expression Involving a Square Root (8.EE.2)

Example 1 **Evaluate $-4(\sqrt{121} - 16)$.**

$$\begin{aligned} -4(\sqrt{121} - 16) &= -4(11 - 16) && \text{Evaluate the square root.} \\ &= -4(-5) && \text{Subtract.} \\ &= 20 && \text{Multiply.} \end{aligned}$$

Try It Yourself

Evaluate the expression.

1. $7\sqrt{25} + 10$

2. $-8 - \sqrt{\frac{64}{16}}$

3. $-2(3\sqrt{4} + 13)$

Factoring $x^2 + bx + c$ (A.SSE.3a)

Example 2 **Factor $x^2 - 3x - 28$.**

Notice that $b = -3$ and $c = -28$. Because c is negative, the factors p and q must have different signs so that pq is negative.

Find two integer factors of -28 whose sum is -3.

Factors of −28	−28, 1	−1, 28	−14, 2	−2, 14	−7, 4	−4, 7
Sum of Factors	−27	27	−12	12	−3	3

The values of p and q are -7 and 4.

So, $x^2 - 3x - 28 = (x - 7)(x + 4)$.

Try It Yourself

Factor the polynomial.

4. $y^2 + 12y + 27$

5. $n^2 - 11n + 10$

6. $w^2 - 2w - 48$

7. $z^2 + 25z + 100$

10.1 Graphing Square Root Functions

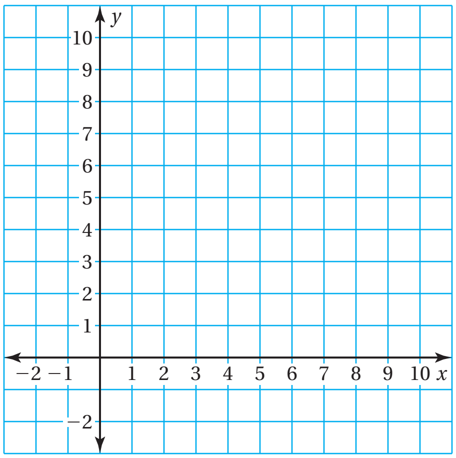
Essential Question How can you sketch the graph of a square root function?

1 ACTIVITY: Graphing Square Root Functions

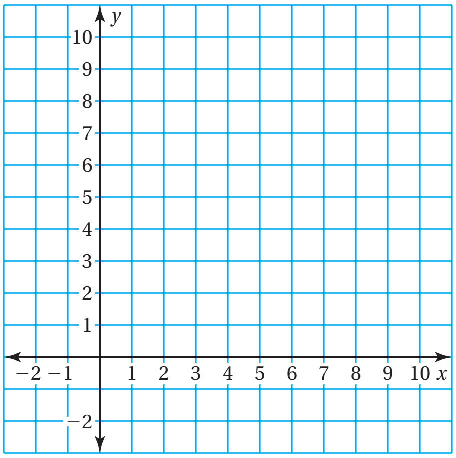
Work with a partner.

- Make a table of values for the function.
- Use the table to sketch the graph of the function.
- Describe the domain of the function.
- Describe the range of the function.

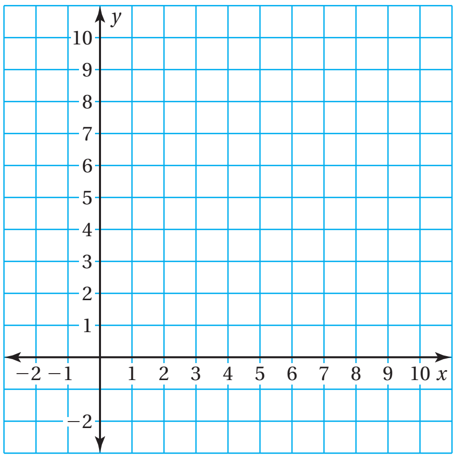
a. $y = \sqrt{x}$



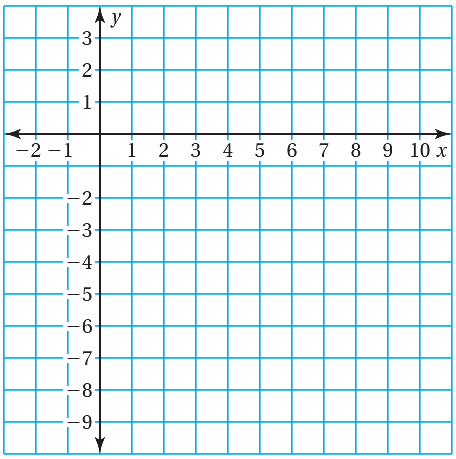
b. $y = \sqrt{x} + 2$



c. $y = \sqrt{x + 1}$



d. $y = -\sqrt{x}$



COMMON CORE

Square Root Functions

In this lesson, you will
- graph square root functions.
- compare graphs of square root functions.

Learning Standards
F.IF.4
F.IF.7b

Laurie's Notes

Introduction

Standards for Mathematical Practice

- **MP5 Use Appropriate Tools Strategically:** Students will graph square root functions using a table of values. They can use a calculator to approximate square roots.

Motivate

- Play a quick round of "30-second Un-Do." In this game, students list as many actions and undoing actions as they can in 30 seconds. For example: filling a sink and draining a sink.
- Have students share a few examples after their time is up.
- Play another round, but this time the actions must involve math operations.
- Have students share a few examples. Hopefully at least one student will have an example that does not involve addition, subtraction, multiplication, or division.
- Explain today's activities using more precise language. They involve taking square roots, the inverse operation of squaring.

Activity Notes

Activity 1

- Students have graphed a variety of functions, so the directions should need no additional explanation.
- **MP6 Attend to Precision:** When using a calculator, students need to be aware of the syntax for square roots. Some calculators require the radicand to be enclosed in parentheses. Students need to recognize the difference between $\sqrt{(x)} + 2$ and $\sqrt{(x+2)}$.

? "What is the domain of each function?" All have a domain of $x \geq 0$ except part (c), which has a domain of $x \geq -1$.

? "What is the range of each function?" Listen for correct answers.

- After students have finished all four graphs, you could have them check their work using a graphing calculator.
- You could also ask volunteers to describe the differences in the graphs. Students should be able to describe the transformations in parts (b)–(d).

Common Core State Standards

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

Previous Learning

Students should know how to graph a function using a table of values.

Technology for the Teacher

Dynamic Classroom

Lesson Plans
Complete Materials List

10.1 Record and Practice Journal

Essential Question How can you sketch the graph of a square root function?

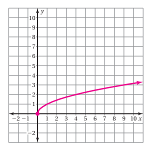
1 ACTIVITY: Graphing Square Root Functions

Work with a partner.

- Make a table of values for the function.
- Use the table to sketch the graph of the function.
- Describe the domain of the function.
- Describe the range of the function.

a. $y = \sqrt{x}$

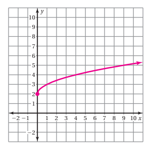
x	-1	0	1	4	9
y	und.	0	1	2	3



Domain: $x \geq 0$

b. $y = \sqrt{x} + 2$

x	-1	0	1	4	9
y	und.	2	3	4	5



Domain: $x \geq 0$

English Language Learners

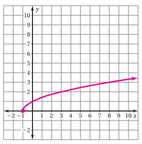
Vocabulary

Students may think of *domain* as the set of all real numbers. Review the definition. The domain is the set of all possible input values. For a square root function, a number that makes the radicand negative is *not* a possible input value.

10.1 Record and Practice Journal

c. $y = \sqrt{x+1}$

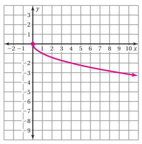
x	-2	-1	0	3	8
y	und.	0	1	2	3



Domain: $x \geq -1$

d. $y = -\sqrt{x}$

x	-1	0	1	4	9
y	und.	0	-1	-2	-3



Domain: $x \geq 0$

2 ACTIVITY: Writing Square Root Functions

Work with a partner. Write a square root function, $y = f(x)$, that has the given values. Then use the function to complete the table.

a.

x	$f(x)$
-4	0
-3	1
-2	1.41
-1	1.73
0	2
1	2.24

$f(x) = \sqrt{x+4}$

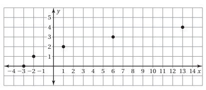
b.

x	$f(x)$
-4	1
-3	2
-2	2.41
-1	2.73
0	3
1	3.24

$f(x) = \sqrt{x+4} + 1$

3 ACTIVITY: Writing a Square Root Function

Work with a partner. Write a square root function, $y = f(x)$, that has the given points on its graph. Explain how you found your function.



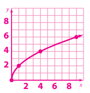
$f(x) = \sqrt{x+3}$

What Is Your Answer?

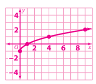
4. **IN YOUR OWN WORDS** How can you sketch the graph of a square root function? Summarize a procedure for sketching the graph. Then use your procedure to sketch the graph of each function.

Make a table, plot the ordered pairs, and draw a smooth curve through the points.

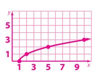
a. $y = 2\sqrt{x}$



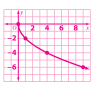
b. $y = \sqrt{x} - 1$



c. $y = \sqrt{x-1}$



d. $y = -2\sqrt{x}$



Laurie's Notes

Activity 2

- Ordered pairs have been given so that students can determine the equation of the square root function.
- Students may need to use trial and error. For instance, students may believe part (a) is $y = \sqrt{x} + 2$ because of the y-intercept. They need to test all three ordered pairs in the equation.
- Probe students as you walk around to see if they recognize any patterns between the two parts of the activity. The y-values differ by 1. Does that influence their thinking as they start the second part?
- It is good practice to have students identify the domain and range of each function.

Activity 3

? "How is this activity like the last activity?" Ordered pairs are given, but in a graph instead of a table.

- Students have graphed several square root functions. If they have not made an observation about the general shape of the graph, ask about it now. They may describe it as half of a parabola that has been rotated.

What Is Your Answer?

- **MP7 Look for and Make Use of Structure:** Students have not graphed $y = a\sqrt{x}$ or $y = -a\sqrt{x}$. Ask them how a affects the graph of $y = \sqrt{x}$.

Closure

- Compare the graph of $y = \sqrt{x} + 2$ to the graph of $y = \sqrt{x}$. translation 2 units up

2 ACTIVITY: Writing Square Root Functions

Math Practice 4

Use a Table

How can you use values of the domain and range to help write a function?

Work with a partner. Write a square root function, $y = f(x)$, that has the given values. Then use the function to complete the table.

a.

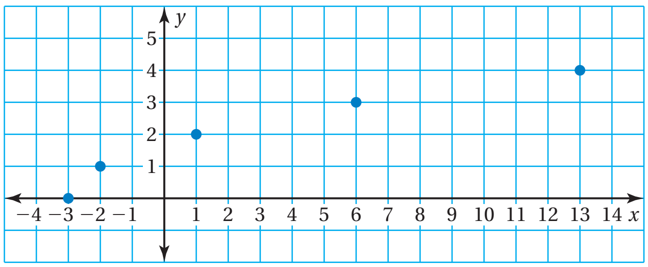
x	$f(x)$
-4	0
-3	1
-2	
-1	
0	2
1	

b.

x	$f(x)$
-4	1
-3	2
-2	
-1	
0	3
1	

3 ACTIVITY: Writing a Square Root Function

Work with a partner. Write a square root function, $y = f(x)$, that has the given points on its graph. Explain how you found your function.



What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you sketch the graph of a square root function? Summarize a procedure for sketching the graph. Then use your procedure to sketch the graph of each function.

a. $y = 2\sqrt{x}$

b. $y = \sqrt{x} - 1$

c. $y = \sqrt{x - 1}$

d. $y = -2\sqrt{x}$

Practice Use what you learned about the graphs of square root functions to complete Exercises 3–8 on page 506.

10.1 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

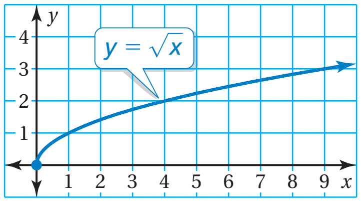
Key Vocabulary

square root function, *p. 504*

Key Idea

Square Root Function

A **square root function** is a function that contains a square root with the independent variable in the radicand. The most basic square root function is $y = \sqrt{x}$.



The value of the radicand in the square root function cannot be negative. So, the domain of a square root function includes x-values for which the radicand is greater than or equal to 0.

EXAMPLE 1 Finding the Domain of a Square Root Function

Find the domain of $y = 3\sqrt{x - 5}$.

The radicand cannot be negative. So, $x - 5$ is greater than or equal to 0.

$x - 5 \geq 0$ — Write an inequality for the domain.

$x \geq 5$ — Add 5 to each side.

∴ The domain is the set of real numbers greater than or equal to 5.

On Your Own

Now You're Ready
Exercises 9–14

Find the domain of the function.

1. $y = 10\sqrt{x}$
2. $y = \sqrt{x} + 7$
3. $y = \sqrt{-x + 1}$

EXAMPLE 2 Comparing Graphs of Square Root Functions

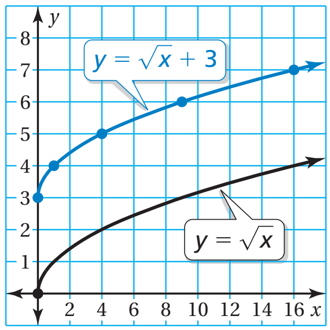
Graph $y = \sqrt{x} + 3$. Describe the domain and range. Compare the graph to the graph of $y = \sqrt{x}$.

Step 1: Make a table of values.

x	0	1	4	9	16
y	3	4	5	6	7

Step 2: Plot the ordered pairs.

Step 3: Draw a smooth curve through the points.



∴ From the graph, you can see that the domain is $x \geq 0$ and the range is $y \geq 3$. The graph of $y = \sqrt{x} + 3$ is a translation 3 units up of the graph of $y = \sqrt{x}$.

Remember

When graphing, remember $f(x) + k$ is a vertical translation of $f(x)$.

Laurie's Notes

Introduction

Connect

- **Yesterday:** Students explored graphs of square root functions. (MP5, MP6, MP7)
- **Today:** Students will graph square root functions.

Motivate

- Use a graphing calculator to graph $y = \sqrt{x}$, $y = \sqrt{x} - 3$, and $y = \sqrt{x - 3}$. Then ask students how the 3 affects the last two graphs.
- In today's lesson, students will graph square root functions.

Lesson Notes

Discuss

- **Big Idea:** A *square root function* is an example of a *radical function.* There are other radical functions, such as $y = \sqrt[3]{x}$. This should make sense from when students learned about rational exponents and *n*th roots.
- Point out to students that the list of functions they have studied is growing: linear, absolute value, exponential, quadratic, and radical.

Key Idea

- Write the Key Idea, which defines a square root function. The parent function is $y = \sqrt{x}$.
- Say, "You cannot take the square root of a negative number. So, the domain of a square root function includes inputs for which the radicand is nonnegative." Give examples to support this statement.

Example 1

? "Could $x = 8$?" yes "Could $x = 6$?" yes "Could $x = 4$?" no

- Continue to solve the problem as shown.

On Your Own

- **Neighbor Check:** Have students work independently and then have their neighbors check their work. Have students discuss any discrepancies.

Example 2

? "Can you use $x = 1, 2, 3, 4$, and 5 to graph the function?" Yes, but it is easier to use perfect squares so that the outputs are whole numbers.

- **MP6 Attend to Precision:** Stress that you take the square root of the input and then add three, not add three and then take the square root.
- The functions have the same domain. The range of $y = \sqrt{x} + 3$ is $y \geq 3$.

Goal Today's lesson is graphing **square root functions**.

Technology for the Teacher

Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Extra Example 1

Find the domain of $y = \sqrt{-x + 5}$.

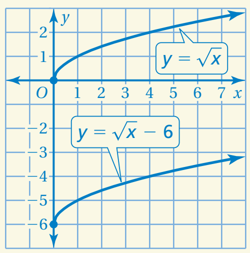
$x \leq 5$

On Your Own

1. $x \geq 0$
2. $x \geq 0$
3. $x \leq 1$

Extra Example 2

Graph $y = \sqrt{x} - 6$. Describe the domain and range. Compare the graph to the graph of $y = \sqrt{x}$.



The domain is $x \geq 0$ and the range is $y \geq -6$. The graph of $y = \sqrt{x} - 6$ is a translation 6 units down of the graph of $y = \sqrt{x}$.

Extra Example 3

Graph $y = -\sqrt{x+1} - 2$. Describe the domain and range. Compare the graph to the graph of $y = \sqrt{x}$.
See Additional Answers.

On Your Own

4–6. See Additional Answers.

Extra Example 4

In Example 4, at what depth does the velocity of the tsunami exceed 150 meters per second? about 2300 meters

On Your Own

7. domain: $x \geq 0$;
range: $y \geq 0$

8. about 1000 meters

Differentiated Instruction

Kinesthetic

Use masking tape to create a large coordinate plane on the floor of the classroom. Divide the class into groups of four. Assign each group a square root function and have them create a table of values to graph the function. Have each group model their function in the coordinate plane with string or yarn. Start with the function $y = \sqrt{x}$. Have students describe the transformation(s) as they graph the remaining functions.

Laurie's Notes

Example 3

? "How do you think this graph will compare to the graph of $y = \sqrt{x}$?"
Students may have an idea about the graph being reflected about the x-axis and translated 2 units, but they may be unsure of the direction.

? "What is the domain of this function? Explain." $x \geq 2$; because the radicand is negative when $x < 2$

- Make a table of values. Note that perfect squares were not selected. Two more than a perfect square would allow for easy computations.
- Work through the rest of the problem as shown.

On Your Own

- If time is short, make sure students try Question 6. Students may try to graph the function without making a table of values. Knowing where (0, 0) is transformed helps to "anchor" the graph.

Example 4

- Ask a student to read the problem.
- **MP4 Model with Mathematics:** The velocity of the tsunami is a function of the depth of the water.
- **MP5 Use Appropriate Tools Strategically:** Explain how to enter the function into a graphing calculator and set the viewing window as shown.
- Use the *trace* feature to answer the question. The units are meters per second.
- **Connection:** Have students identify a familiar distance that is about 200 meters. The tsunami travels that distance in one second.
- **Extension:** Graph $y = 200$ and use the *intersect* feature to find the solution.

On Your Own

- **Think-Pair-Share:** Students should read each question independently and then work in pairs to answer the questions. When they have answered the questions, the pair should compare their answers with another group and discuss any discrepancies.

Closure

- **Exit Ticket:** Compare the graph of $y = \sqrt{x+1} - 3$ to the graph of $y = \sqrt{x}$.
translation 1 unit to the left and 3 units down

EXAMPLE 3 Comparing Graphs of Square Root Functions

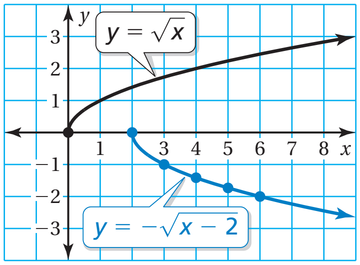
Graph $y = -\sqrt{x - 2}$. Describe the domain and range. Compare the graph to the graph of $y = \sqrt{x}$.

Study Tip

The graph of $f(x - h)$ is a horizontal translation of $f(x)$.

Step 1: Make a table of values.

x	2	3	4	5	6
y	0	-1	-1.4	-1.7	-2



Step 2: Plot the ordered pairs.

Step 3: Draw a smooth curve through the points.

From the graph, you can see that the domain is $x \geq 2$ and the range is $y \leq 0$. The graph of $y = -\sqrt{x - 2}$ is a reflection of the graph of $y = \sqrt{x}$ in the x-axis and then a translation 2 units to the right.

On Your Own

Now You're Ready
Exercises 16–21

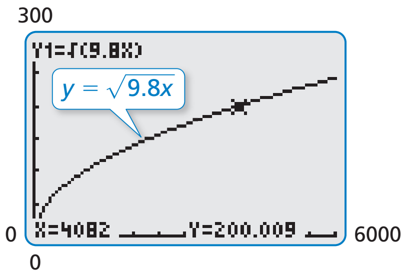
Graph the function. Describe the domain and range. Compare the graph to the graph of $y = \sqrt{x}$.

4. $y = \sqrt{x} - 4$ **5.** $y = \sqrt{x + 5}$ **6.** $y = -\sqrt{x + 1} + 2$

EXAMPLE 4 Real-Life Application

The velocity y (in meters per second) of a tsunami can be modeled by the function $y = \sqrt{9.8x}$, where x is the water depth (in meters). Use a graphing calculator to graph the function. At what depth does the velocity of the tsunami exceed 200 meters per second?

Step 1: Enter the function $y = \sqrt{9.8x}$ into your calculator and graph it. Because the radicand cannot be negative, use only nonnegative values of x.



Step 2: Use the *trace* feature to find where the value of y is about 200.

The velocity exceeds 200 meters per second at a depth of about 4100 meters.

On Your Own

7. Find the domain and range of the function in Example 4.

8. **WHAT IF?** In Example 4, at what depth does the velocity of the tsunami exceed 100 meters per second?

10.1 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

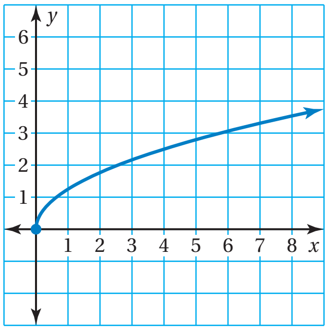
1. **VOCABULARY** Is $y = 2x\sqrt{5}$ a square root function? Explain.
2. **REASONING** How do you find the domain of a square root function?

Practice and Problem Solving

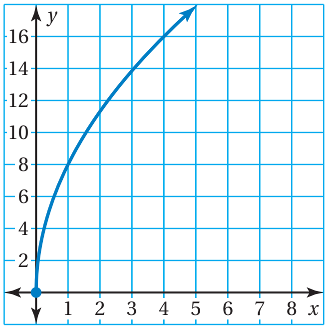
Match the function with its graph.

3. $y = 8\sqrt{x}$
4. $y = \frac{5}{4}\sqrt{x}$
5. $y = -4\sqrt{x}$

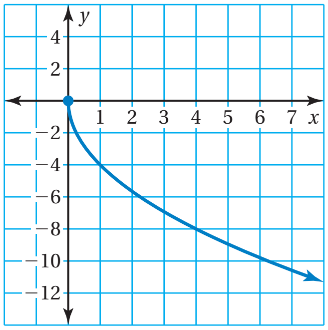
A.



B.



C.



Graph the function. Describe the domain.

6. $y = 3\sqrt{x}$
7. $y = 7\sqrt{x}$
8. $y = -0.5\sqrt{x}$

Find the domain of the function.

1

9. $y = 5\sqrt{x}$
10. $y = \sqrt{x} + 1$
11. $y = \sqrt{x - 2}$
12. $y = \sqrt{-x - 1}$
13. $y = 2\sqrt{x + 4}$
14. $y = \frac{1}{2}\sqrt{-x + 2}$

15. **FIRE** The nozzle pressure of a fire hose allows firefighters to control the amount of water they spray on a fire. The flow rate f (in gallons per minute) can be modeled by the function $f = 120\sqrt{p}$, where p is the nozzle pressure (in pounds per square inch).

    a. Use a graphing calculator to graph the function.

    b. Use the *trace* feature to approximate the nozzle pressure that results in a flow rate of 300 gallons per minute.

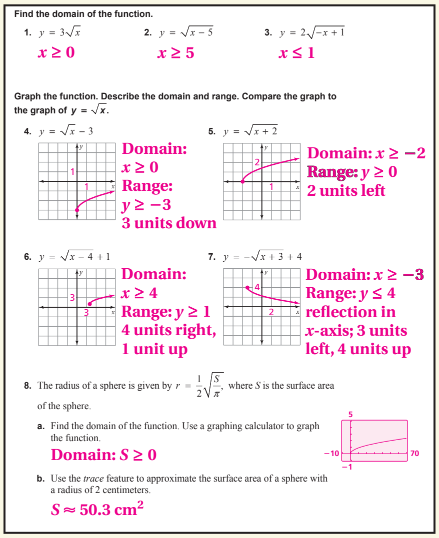
Assignment Guide and Homework Check

Level	Assignment	Homework Check
Average	1, 2, 7–21 odd, 22, 26, 28 – 31	7, 13, 15, 19, 22
Advanced	1, 2, 6 –20 even, 15, 23–27, 28 – 31	14, 15, 20, 26, 27

Common Errors

- **Exercises 12 and 14** Students may forget to reverse the direction of the inequality symbol when dividing each side by a negative number. Remind them of the Division Property of Inequality.
- **Exercises 10 – 14 and 16 – 21** Students may treat constants added to a radical expression as part of the radicand, or vice versa. Encourage them to identify the radicand before solving the exercise.

10.1 Record and Practice Journal



Find the domain of the function.

1. $y = 3\sqrt{x}$ **$x \geq 0$**
2. $y = \sqrt{x - 5}$ **$x \geq 5$**
3. $y = 2\sqrt{-x + 1}$ **$x \leq 1$**

Graph the function. Describe the domain and range. Compare the graph to the graph of $y = \sqrt{x}$.

4. $y = \sqrt{x} - 3$ **Domain: $x \geq 0$ Range: $y \geq -3$ 3 units down**
5. $y = \sqrt{x + 2}$ **Domain: $x \geq -2$ Range: $y \geq 0$ 2 units left**
6. $y = \sqrt{x - 4} + 1$ **Domain: $x \geq 4$ Range: $y \geq 1$ 4 units right, 1 unit up**
7. $y = -\sqrt{x + 3} + 4$ **Domain: $x \geq -3$ Range: $y \leq 4$ reflection in x-axis; 3 units left, 4 units up**

8. The radius of a sphere is given by $r = \frac{1}{2}\sqrt{\frac{S}{\pi}}$, where S is the surface area of the sphere.

   a. Find the domain of the function. Use a graphing calculator to graph the function.

   Domain: $S \geq 0$

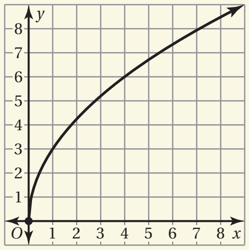
   b. Use the *trace* feature to approximate the surface area of a sphere with a radius of 2 centimeters.

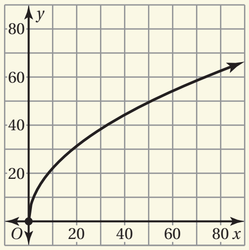
   $S \approx 50.3\ \text{cm}^2$

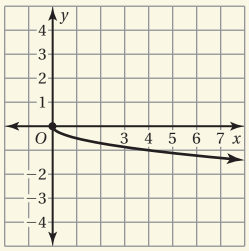
Vocabulary and Concept Check

1. no; It is a linear function. A square root function contains a square root with the independent variable in the radicand.
2. The radicand cannot be negative. Write and solve an inequality with the radicand greater than or equal to zero.

Practice and Problem Solving

3. B
4. A
5. C
6. 

   domain: $x \geq 0$
7. 

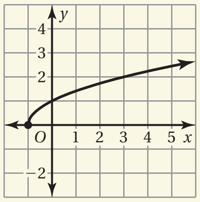
   domain: $x \geq 0$
8. 

   domain: $x \geq 0$
9. $x \geq 0$
10. $x \geq 0$
11. $x \geq 2$
12. $x \leq -1$
13. $x \geq -4$
14. $x \leq 2$
15. See Additional Answers.

Practice and Problem Solving

16–21. See Additional Answers.

22. The student graphed $y = \sqrt{x} + 1$ instead of $y = \sqrt{x + 1}$.



23. a. *Sample answer:* $y = \sqrt{x} + 1$

b. *Sample answer:* $y = -\sqrt{x}$

24. domain: yes, as long as the radicand is not negative; range: yes, the function could be a reflection or a vertical translation

25. See Additional Answers.

26. yes; Solve $40 = \sqrt{30d(0.75)}$ for d to find that the skid marks are about 71 feet long.

27. See *Taking Math Deeper.*

Fair Game Review

28. $x = 0, x = 8$

29. $x = -3$

30. $x = -2, x = 3$

31. B

Mini-Assessment

Graph the function. Describe the domain and range. Compare the graph to the graph of $y = \sqrt{x}$.

1. $y = \sqrt{x} + 5$

2. $y = \sqrt{x - 7}$

3. $y = \sqrt{x + 3} - 4$

4. $y = -\sqrt{x - 4} + 6$

1–4. See Additional Answers.

Taking Math Deeper

Exercise 27

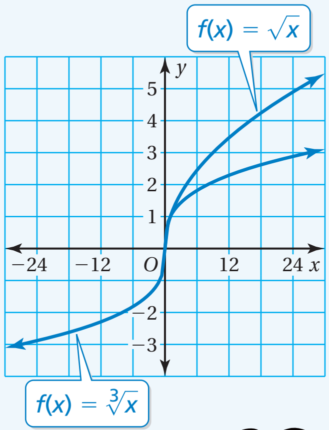
This exercise previews the most basic cube root function, $y = \sqrt[3]{x}$, which students will study in a future course.

1 Make a table of values for $g(x) = \sqrt[3]{x}$.

x	−27	−8	−1	0	1	8	27
$g(x)$	−3	−2	−1	0	1	2	3

Notice that *perfect cubes* were chosen to make evaluating easier.

2 Graph the functions in the same coordinate plane.



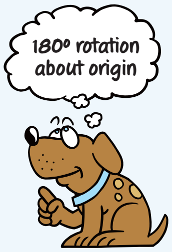
3 Compare the graphs.

Similarities:

- nonlinear functions
- pass through the origin
- y increases as x increases

Differences:

	Domain	Range	Symmetry
$f(x) = \sqrt{x}$	$x \geq 0$	$y \geq 0$	none
$g(x) = \sqrt[3]{x}$	all real numbers	all real numbers	about the origin



Project

Research *even and odd functions*. Of the functions you have studied, what types can be even functions? odd functions? Is $y = \sqrt[3]{x}$ even, odd, or neither?

Reteaching and Enrichment Strategies

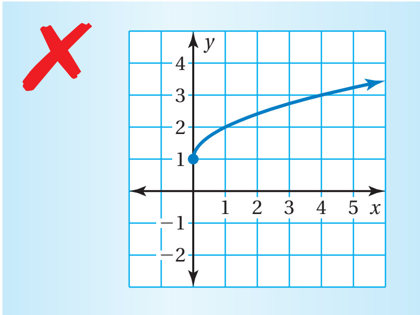
If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension Start the next section

Graph the function. Describe the domain and range. Compare the graph to the graph of $y = \sqrt{x}$.

2 3 **16.** $y = \sqrt{x} - 2$ **17.** $y = \sqrt{x} + 4$ **18.** $y = \sqrt{x + 4}$

19. $y = \sqrt{x + 2} - 2$ **20.** $y = -\sqrt{x - 3}$ **21.** $y = -\sqrt{x - 1} + 3$

22. ERROR ANALYSIS Describe and correct the error in graphing the function $y = \sqrt{x + 1}$.



23. OPEN-ENDED Consider the graph of $y = \sqrt{x}$.

a. Write a function that is a vertical translation of the graph of $y = \sqrt{x}$.

b. Write a function that is a reflection of the graph of $y = \sqrt{x}$.

24. REASONING Can the domain of a square root function include negative numbers? Can the range include negative numbers? Explain your reasoning.

25. GEOMETRY The radius of a circle is given by $r = \sqrt{\frac{A}{\pi}}$, where A is the area of the circle.

a. Find the domain of the function. Use a graphing calculator to graph the function.

b. Use the *trace* feature to approximate the area of a circle with a radius of 3 inches.

26. PROBLEM SOLVING The speed S (in miles per hour) of a van before it skids to a stop can be modeled by the equation $S = \sqrt{30df}$, where d is the length (in feet) of the skid marks and f is the drag factor of the road surface. Suppose the drag factor is 0.75 and the speed of the van was 40 miles per hour. Is the length of the skid marks more than 65 feet long? Explain your reasoning.

27. Precision Compare the graphs of the functions $f(x) = \sqrt{x}$ and $g(x) = \sqrt[3]{x}$.

Fair Game Review What you learned in previous grades & lessons

Solve the equation. *(Section 7.5)*

28. $x(x - 8) = 0$ **29.** $(x + 3)^2 = 0$ **30.** $(x + 2)(x - 3) = 0$

31. MULTIPLE CHOICE What are the next three terms of the geometric sequence 240, 120, 60, 30, . . .? *(Section 6.7)*

Ⓐ 20, 10, 5 Ⓑ 15, 7.5, 3.75 Ⓒ 20, 10, 0 Ⓓ 15, 10, 5

Extension 10.1 Rationalizing the Denominator

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
simplest form of a radical expression, *p. 508*
rationalizing the denominator, *p. 508*
conjugates, *p. 509*

In Section 6.1, you used properties to simplify radical expressions. A radical expression is in **simplest form** when the following are true.

- No radicands have perfect square factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.

When a radicand in the denominator of a fraction is not a perfect square, multiply the fraction by an appropriate form of 1 to eliminate the radical from the denominator. This process is called **rationalizing the denominator**.

EXAMPLE 1 Simplifying a Radical Expression

Simplify $\sqrt{\frac{1}{3}}$.

$\sqrt{\frac{1}{3}} = \frac{\sqrt{1}}{\sqrt{3}}$ — Quotient Property of Square Roots

$= \frac{\sqrt{1}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ — Multiply by $\frac{\sqrt{3}}{\sqrt{3}}$.

$= \frac{\sqrt{1 \cdot 3}}{\sqrt{3 \cdot 3}}$ — Product Property of Square Roots

$= \frac{\sqrt{3}}{\sqrt{9}}$ — Simplify.

$= \frac{\sqrt{3}}{3}$ — Evaluate the square root.

Study Tip
Rationalizing the denominator works because you multiply the numerator and denominator by the same nonzero number a, which is the same as multiplying by $\frac{a}{a}$, or 1.

Practice

Simplify the expression.

1. $\frac{1}{\sqrt{10}}$ **2.** $\frac{\sqrt{2}}{\sqrt{7}}$ **3.** $\sqrt{\frac{9}{2}}$

4. $\sqrt{\frac{10}{21}}$ **5.** $\sqrt{\frac{5}{18}}$ **6.** $\sqrt{\frac{40}{48}}$

7. $\frac{4}{\sqrt{5}} + \frac{1}{\sqrt{5}}$ **8.** $\sqrt{3} - \frac{2}{\sqrt{12}}$ **9.** $\sqrt{\frac{16}{15}} - \frac{1}{3}$

10. REASONING Explain why for any number a, $\sqrt{a^2} = |a|$. Use this rule to simplify the expression $\sqrt{\frac{x^2}{2}}$.

Multi-Language Glossary at BigIdeasMath.com

Laurie's Notes

Introduction

Connect

- **Yesterday:** Students graphed square root functions. (MP4, MP5, MP6)
- **Today:** Students will simplify square root expressions by rationalizing the denominator.

Motivate

- **Story time:** Tell a story about seeing a large ship on the horizon while at the beach. A friend says, "How far away is it?" You say, "About 3 miles. I can give you a more exact answer with a calculator."
- Tell students that by the end of the lesson, they will be able to make a similar approximation.

Lesson Notes

Discuss

- Remind students that they simplified radical expressions in Chapter 6.
- Discuss what must be true for a radical expression to be in simplest form.
- Explain the procedure called *rationalizing the denominator*.
- Point out that the name of this procedure makes sense because you rewrite the expression with a *rational* number in the *denominator*.
- Connect these previously avoided expressions with the continuous domains of square root functions. For example, when you graph $y = \sqrt{x}$, you draw the smooth curve through $\left(\frac{1}{2}, \sqrt{\frac{1}{2}}\right)$.

Example 1

- Explain that to simplify, you must "remove" the radical in the denominator.
- Remind students that whatever is done to the denominator must be done to the numerator and vice versa. This results in multiplication by 1, so it does not change the value of the expression.

? "What form of 1 is used in this example? Why is it a wise choice?" $\frac{\sqrt{3}}{\sqrt{3}}$; It results in a perfect square radicand in the denominator.

- Finish the problem. Take a moment to use a calculator to show that the original and final expressions are equivalent.

Practice

- In Exercise 5, students should recognize that multiplying by $\frac{\sqrt{2}}{\sqrt{2}}$ results in a perfect square radicand in the denominator.
- Exercises 7–9 require students to perform operations with fractions.

Common Core State Standards

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities . . ..
F.IF.7b Graph square root . . . functions . . ..

Goal Today's lesson is simplifying square root expressions.

Technology for the Teacher

Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Extra Example 1

Simplify $\sqrt{\frac{1}{6}}$. $\frac{\sqrt{6}}{6}$

Practice

1. $\frac{\sqrt{10}}{10}$
2. $\frac{\sqrt{14}}{7}$
3. $\frac{3\sqrt{2}}{2}$
4. $\frac{\sqrt{210}}{21}$
5. $\frac{\sqrt{10}}{6}$
6. $\frac{\sqrt{30}}{6}$
7. $\sqrt{5}$
8. $\frac{2\sqrt{3}}{3}$
9. $\frac{4\sqrt{15} - 5}{15}$
10. The square root of a number cannot be negative. You must include absolute value symbols for when $a < 0$; $\frac{|x|\sqrt{2}}{2}$

Record and Practice Journal Extension 10.1 Practice

See Additional Answers.

Extra Example 2

Simplify $\dfrac{2}{5+\sqrt{2}}$. $\dfrac{10-2\sqrt{2}}{23}$

Extra Example 3

In Example 3, how far can you see when your eye level is 6 feet above the water? 3 miles

Practice

11. $-3+3\sqrt{3}$

12. $-5\sqrt{3}-10$

13. $-2\sqrt{2}+2\sqrt{7}$

14. $\dfrac{\sqrt{210}}{2} \approx 7.25$ mi

Mini-Assessment

Simplify the expression.

1. $\sqrt{\dfrac{8}{3}}$ $\dfrac{2\sqrt{6}}{3}$

2. $\sqrt{2}-\dfrac{3}{\sqrt{5}}$ $\dfrac{5\sqrt{2}-3\sqrt{5}}{5}$

3. $\dfrac{3}{2+\sqrt{7}}$ $-2+\sqrt{7}$

Laurie's Notes

Discuss

- ? "What is $(x-4)(x+4)$?" x^2-16
- Write $4+\sqrt{2}$ and $4-\sqrt{2}$ and say that these are called *conjugates*.
- ? "What is $(4+\sqrt{2})(4-\sqrt{2})$?" $16-\sqrt{4}=16-2=14$

Example 2

- Write the expression and say, "We need to use conjugates to simplify this expression."
- ? "How can you simplify this expression?" Multiply the numerator and denominator by the conjugate of $3+\sqrt{5}$.
- Finish the problem. Take a moment to use a calculator to show that the original and final expressions are equivalent.

Example 3

- **MP4 Model with Mathematics:** The given model for the distance you can see to the horizon is a square root function. Mathematically proficient students are able to use many function types to model real-world phenomena.
- Ask a volunteer to read the problem.
- Sketch a diagram that shows *h* and *d*. Note that the units for *h* are feet and the units for *d* are miles.
- Explain that this is how you approximated the distance to the ship in the Motivate.
- Substitute 5 for *h* and continue to solve the problem as shown.
- **MP5 Use Appropriate Tools Strategically:** Graph the function using a graphing calculator to check your answer.

Practice

- Ask volunteers to show their work at the board.

Closure

- **Exit Ticket:** Simplify $\dfrac{8}{\sqrt{12}}$. $\dfrac{4\sqrt{3}}{3}$

Common Core

Square Root Functions

In this extension, you will

- simplify radical expressions.

Learning Standards
F.IF.4
F.IF.7b

The binomials $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$ are called **conjugates.** You can use conjugates to simplify radical expressions that involve a sum or difference of radicals in the denominator.

EXAMPLE 2 Simplifying a Radical Expression

Simplify $\dfrac{1}{3 + \sqrt{5}}$.

$$\frac{1}{3+\sqrt{5}} = \frac{1}{3+\sqrt{5}} \cdot \frac{3-\sqrt{5}}{3-\sqrt{5}} \qquad \text{The conjugate of } 3+\sqrt{5} \text{ is } 3-\sqrt{5}.$$

$$= \frac{1(3-\sqrt{5})}{3^2-(\sqrt{5})^2} \qquad \text{Sum and Difference Pattern}$$

$$= \frac{3-\sqrt{5}}{4} \qquad \text{Simplify.}$$

Study Tip

Notice that the product of conjugates is a rational number.

EXAMPLE 3 Real-Life Application

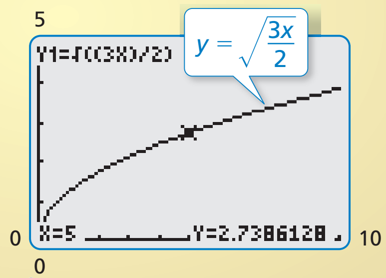
The distance d (in miles) that you can see to the horizon with your eye level h feet above the water is given by $d = \sqrt{\dfrac{3h}{2}}$. How far can you see when your eye level is 5 feet above the water?

$$d = \sqrt{\frac{3(5)}{2}} \qquad \text{Substitute 5 for } h.$$

$$= \frac{\sqrt{15}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \qquad \text{Multiply by } \frac{\sqrt{2}}{\sqrt{2}}.$$

$$= \frac{\sqrt{30}}{2} \qquad \text{Simplify.}$$

Check



You can see $\dfrac{\sqrt{30}}{2}$, or about 2.74 miles.

Practice

Simplify the expression.

11. $\dfrac{6}{1+\sqrt{3}}$ **12.** $\dfrac{5}{\sqrt{3}-2}$ **13.** $\dfrac{10}{\sqrt{2}+\sqrt{7}}$

14. **WHAT IF?** In Example 3, how far can you see when your eye level is 35 feet above the water?

10.2 Solving Square Root Equations

Essential Question How can you solve an equation that contains square roots?

1 ACTIVITY: Analyzing a Free-Falling Object

Work with a partner. The table shows the time *t* (in seconds) that it takes a free-falling object (with no air resistance) to fall *d* feet.

a. Sketch the graph of t as a function of d.

b. Use your graph to estimate the time it takes for a free-falling object to fall 240 feet.

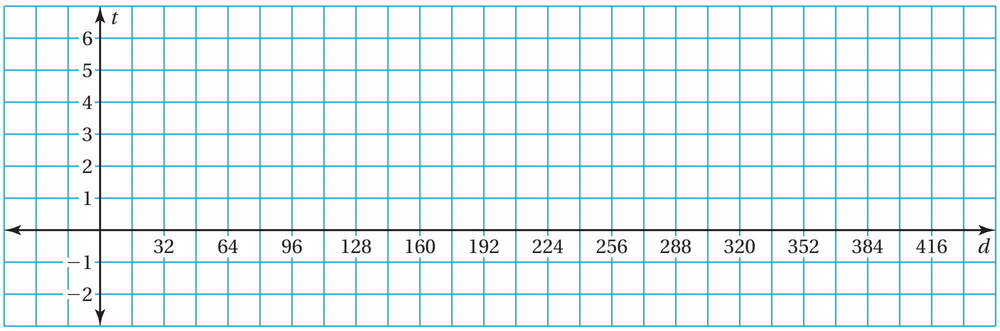
c. The relationship between d and t is given by the function

$$t = \sqrt{\frac{d}{16}}.$$

Use this function to check the estimate you obtained from the graph.

d. Consider a free-falling object that takes 5 seconds to hit the ground. How far did it fall? Explain your reasoning.

d feet	*t* seconds
0	0.00
32	1.41
64	2.00
96	2.45
128	2.83
160	3.16
192	3.46
224	3.74
256	4.00
288	4.24
320	4.47



COMMON CORE

Radical Functions

In this lesson, you will
- solve square root equations, including those with square roots on both sides.
- identify extraneous solutions.

Applying Standard N.RN.2

Laurie's Notes

Introduction

Standards for Mathematical Practice

- **MP6 Attend to Precision:** When working with a square root function, students recognize that small changes in the domain ($x > 1$) result in even smaller changes in the range. It is important to include a few decimal places in the result. To the nearest whole number, $y = \sqrt{x}$ rounds to 20 for values of x between 381 and 420. The context of the problem will help students determine the degree of precision needed.

Motivate

- Slide a math book off the edge of your desk or a table and let it hit the floor. This sound should get your students' attention.
- ? "How long do you think it took for the book to hit the floor from the moment it started falling?" Answers will vary.
- Drop the book again. This time, do it from your shoulder height.
- ? "How long do you think it took for the book to hit the floor from the moment it started falling?" Answers will vary. However, answers should be different from the first height.
- ? "Suppose I drop the book from the height of the ceiling. How long do you think it would take the book to hit the floor?" It would take longer than the first two heights.
- Explain that today they will investigate the time it takes for an object to fall from different heights.

Activity Notes

Activity 1

- As students begin to work on this activity make sure that they are plotting (d, t) and not (t, d).
- ? "Should the plotted points be connected with a smooth curve?" yes
- ? "What is the independent variable?" d; the distance d (in feet) that the free-falling object falls
- ? "What is the dependent variable?" t; the time (in seconds) that the free-falling object falls
- When answering part (b), students will most likely interpolate and guess a time about halfway between 3.74 and 4.00 seconds.
- Students use substitution to answer part (c).
- When answering part (d), students should substitute 5 for t and solve. To solve for d, students must square each side of the equation. Then multiply each side by 16.
- If time permits, investigate the equation on a graphing calculator. Adjusting the viewing window to see the key features of the graph is a good exercise for students. In this problem, the scale for the x-axis is much different than the scale for the y-axis.

Common Core State Standards

N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Previous Learning

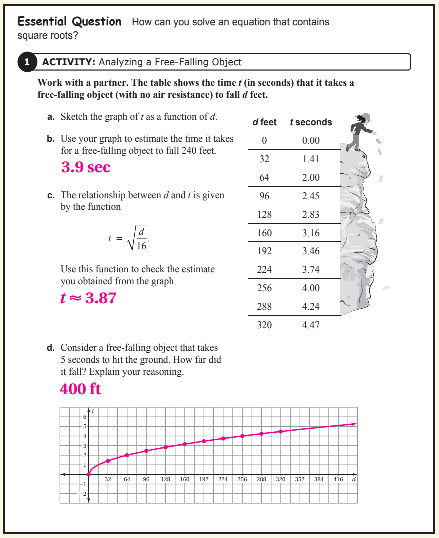
Students should know how to solve linear and quadratic equations.

Technology for the Teacher

Dynamic Classroom

Lesson Plans
Complete Materials List

10.2 Record and Practice Journal



Essential Question How can you solve an equation that contains square roots?

1 ACTIVITY: Analyzing a Free-Falling Object

Work with a partner. The table shows the time t (in seconds) that it takes a free-falling object (with no air resistance) to fall d feet.

a. Sketch the graph of t as a function of d.

b. Use your graph to estimate the time it takes for a free-falling object to fall 240 feet.
3.9 sec

c. The relationship between d and t is given by the function

$$t = \sqrt{\frac{d}{16}}.$$

Use this function to check the estimate you obtained from the graph.
$t \approx 3.87$

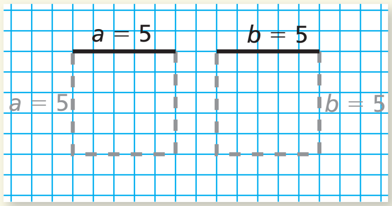
d feet	t seconds
0	0.00
32	1.41
64	2.00
96	2.45
128	2.83
160	3.16
192	3.46
224	3.74
256	4.00
288	4.24
320	4.47

d. Consider a free-falling object that takes 5 seconds to hit the ground. How far did it fall? Explain your reasoning.
400 ft

Differentiated Instruction

Visual

Help students visualize that if two expressions are equal, then the squares of the expressions are equal. On graph paper, have students draw two horizontal line segments, 5 units in length.



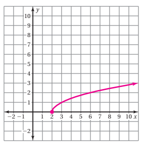
Tell them to draw two squares using the line segments and find the area of each. Point out that $a = b$ and $a^2 = b^2$.

10.2 Record and Practice Journal

2 ACTIVITY: Solving a Square Root Equation

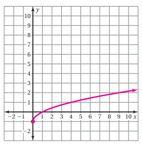
Work with a partner. Sketch the graph of each function. Then find the value of x such that $f(x) = 2$. Explain your reasoning.

a. $f(x) = \sqrt{x - 2}$



$x = 6; f(6) = 2$

b. $f(x) = \sqrt{x} - 1$



$x = 9; f(9) = 2$

3 ACTIVITY: Solving a Square Root Function

Work with a partner. The speed s (in feet per second) of the free-falling object in Activity 1 is given by the function

$$s = \sqrt{64d}.$$

Find the distance traveled for each speed.

a. $s = 8$ ft/sec

$d = 1$ ft

b. $s = 16$ ft/sec

$d = 4$ ft

c. $s = 24$ ft/sec

$d = 9$ ft

What Is Your Answer?

4. IN YOUR OWN WORDS How can you solve an equation that contains square roots? Summarize a procedure for solving a square root equation. Then use your procedure to solve each equation.

Solve for the square root, then square both sides of the equation, then solve for the variable.

a. $\sqrt{x} + 2 = 3$

$x = 1$

b. $4 - \sqrt{x} = 1$

$x = 9$

c. $5 = \sqrt{x + 20}$

$x = 5$

d. $-3 = -2\sqrt{x}$

$x = \frac{9}{4}$

Laurie's Notes

Activity 2

- Students will need to make a table of values to graph the functions.

? "In part (a), what x-values allow for easy computation?" x-values that are 2 more than a perfect square

x	2	3	6	11	18	27
$f(x)$	0	1	2	3	4	5

- Students may make a comment about the scale of the axes at this point. This is very different from graphing linear, quadratic, and exponential functions. When plotting by hand, you do not want to use consecutive integer values for the domain.

? "How did you solve for the value of x that gives a function value of 2?" Listen for students describing the undoing process.

- **Extension:** Be sure to discuss the transformation of the graph $y = \sqrt{x}$ in each part. Discuss horizontal and vertical translations.

Activity 3

- Students have already solved a few square root problems. Check to be sure that students are squaring each side of the equation, then dividing by 64.

? "When the speed doubled from 8 ft/sec to 16 ft/sec, did the distance traveled double?" no; It quadrupled. "When the speed tripled from 8 ft/sec to 24 ft/sec, did the distance traveled triple?" no; It increased by a factor of 9.

What Is Your Answer?

- At this point, students have solved enough equations that they should know to isolate the expression (with the square root symbol) that contains the variable.

Closure

- Suppose the object in Activity 1 takes 3 seconds to hit the ground. How far did it fall? 144 ft

Math Practice 1

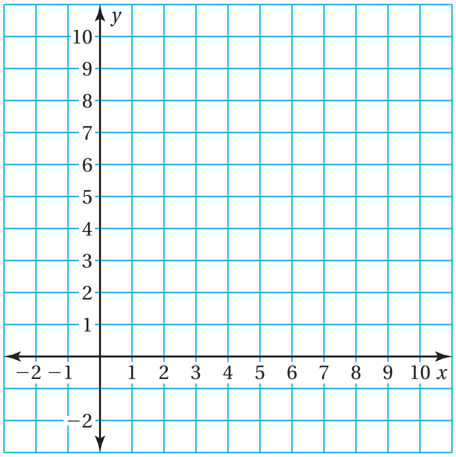
Interpret a Solution

How can you use a graph to find the value of *x* given the value of the function?

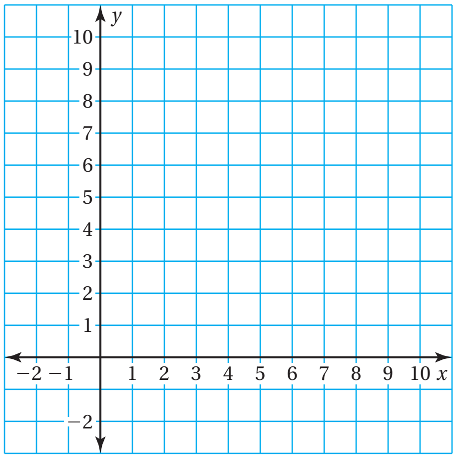
2 ACTIVITY: Solving a Square Root Equation

Work with a partner. Sketch the graph of each function. Then find the value of x such that $f(x) = 2$. Explain your reasoning.

a. $f(x) = \sqrt{x - 2}$



b. $f(x) = \sqrt{x} - 1$



3 ACTIVITY: Solving a Square Root Equation

Work with a partner. The speed s (in feet per second) of the free-falling object in Activity 1 is given by the function

$$s = \sqrt{64d}.$$

Find the distance traveled for each speed.

a. $s = 8$ ft/sec **b.** $s = 16$ ft/sec **c.** $s = 24$ ft/sec

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you solve an equation that contains square roots? Summarize a procedure for solving a square root equation. Then use your procedure to solve each equation.

a. $\sqrt{x} + 2 = 3$ **b.** $4 - \sqrt{x} = 1$

c. $5 = \sqrt{x + 20}$ **d.** $-3 = -2\sqrt{x}$

Practice Use what you learned about solving square root equations to complete Exercises 3–5 on page 515.

10.2 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
square root equation, p. 512
extraneous solution, p. 513

A **square root equation** is an equation that contains a square root with a variable in the radicand. To solve a square root equation, use properties of equality to isolate the square root by itself on one side of the equation, then use the following property.

Key Idea

Squaring Each Side of an Equation

Words If two expressions are equal, then their squares are also equal.

Algebra If $a = b$, then $a^2 = b^2$.

EXAMPLE 1 Solving Square Root Equations

a. Solve $\sqrt{x} + 5 = 13$.

$\sqrt{x} + 5 = 13$	Write the equation.
$\sqrt{x} = 8$	Subtract 5 from each side.
$(\sqrt{x})^2 = 8^2$	Square each side of the equation.
$x = 64$	Simplify.

The solution is $x = 64$.

Check

$\sqrt{x} + 5 = 13$

$\sqrt{64} + 5 \stackrel{?}{=} 13$

$8 + 5 \stackrel{?}{=} 13$

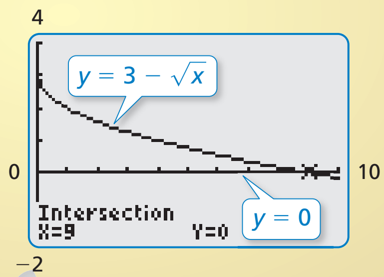
$13 = 13$ ✓

b. Solve $3 - \sqrt{x} = 0$.

$3 - \sqrt{x} = 0$	Write the equation.
$3 = \sqrt{x}$	Add $\sqrt{x}$ to each side.
$3^2 = (\sqrt{x})^2$	Square each side of the equation.
$9 = x$	Simplify.

The solution is $x = 9$.

Check



On Your Own

Now You're Ready
Exercises 6–11

Solve the equation. Check your solution.

1. $\sqrt{x} = 6$
2. $\sqrt{x} - 7 = 3$
3. $\sqrt{x} + 15 = 22$
4. $1 - \sqrt{x} = -2$

Multi-Language Glossary at BigIdeasMath.com

Laurie's Notes

Introduction

Connect

- **Yesterday:** Students used graphs to help them solve equations involving square roots. (MP6)
- **Today:** Students will solve square root equations.

Motivate

- Read an excerpt, or find a brief online video from "The Pit and the Pendulum" by Edgar Allan Poe. Your students may have some familiarity with this classic literary work.
- This introduction provides an interesting transition to the work students will be engaged in today. Share with them that they will indeed work on a pendulum problem.

Lesson Notes

Discuss

- Discuss the square root equations students solved in the activity. They used properties of equality to isolate the square root before squaring each side of the equation.
- **MP7 Look for and Make Use of Structure:** Solving square root equations requires that students recognize the structure of the equation. The equations below all involve subtracting 5 as the first step.
  $2x + 5 = 13$ $\quad$ $x^2 + 5 = 13$ $\quad$ $\sqrt{x} + 5 = 13$
  In doing so, the term involving the variable x is isolated.

Key Idea

- Write the Key Idea which simply says that if you have an equation, squaring each side preserves the equality.

Example 1

- ? "To isolate the term involving x, what is the first step?" Subtract 5 from each side.
- ? "How do you undo the square root of x?" Square each side of the equation.
- In part (b), students may ask if the result is the same when you subtract 3 from each side first, then square each side. Students can verify this on their own.
- Remind students to check their solutions.

On Your Own

- **Think-Pair-Share**: Students should read each question independently and then work in pairs to answer the questions. When they have answered the questions, the pair should compare their answers with another group and discuss any discrepancies.

Goal Today's lesson is solving **square root equations**.

Technology for the Teacher

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Answer Presentation Tool

Extra Example 1

Solve each equation. Check your solution.

a. $\sqrt{x} + 1 = 4$ $\quad x = 9$

b. $7 - \sqrt{x} = -5$ $\quad x = 144$

On Your Own

1. $x = 36$ $\quad$ **2.** $x = 100$

3. $x = 49$ $\quad$ **4.** $x = 9$

Extra Example 2

Solve $3\sqrt{x+1} - 6 = 9$. Check your solution. $x = 24$

On Your Own

5. $x = 12$
6. $x = 10$
7. $x = 30$

Extra Example 3

Solve $\sqrt{2x+2} = \sqrt{3x-7}$. Check your solution. $x = 9$

On Your Own

8. $x = 8$
9. $x = \frac{1}{4}$

English Language Learners

Vocabulary

English language learners may mistake *extraneous solution* for *extra solution*. The prefix *extra-* means more than is usual or necessary. The definition of *extraneous* is not forming a necessary part. So, an extraneous solution is not a necessary part of the solution.

Laurie's Notes

Example 2

- Write the following equations on the board.

  $4(x + 2) + 3 = 19$

  $4(x + 2)^2 + 3 = 19$

  $4\sqrt{x+2} + 3 = 19$

- Discuss strategies for solving the first two equations before solving the third equation. You want students to see the underlying structure of the equation and recognize that they know the steps necessary to solve.
- **Common Error:** When squaring $\left(\sqrt{x+2}\right)^2$ students may square the x and square the 2.
- **Teaching Tip:** When squaring each side of the equation, it is a good idea to enclose the square root expression in parentheses.
- Remind students to check the solution.

On Your Own

- Ask for volunteers to share their work at the board for each question.
- If time permits, have students use graphing calculators to check their solutions. Treat each side of the equation as a function and graph each side. Then find the intersection point.

Example 3

- Students generally do not have a problem solving this type of example. Before they begin, have them consider what each side of the equation looks like graphically.
- ? "What is the first step?" Square each side of the equation.
- ? "What is the result after squaring each side?" $2x - 1 = x + 4$
- Remind students that this is an equation with variables on both sides.
- Use a graphing calculator to check the solution.
- ? "Will two square root functions always intersect?" no "Can they intersect at more than one point?" yes; for example, $y = \sqrt{2x}$ and $y = \sqrt{x-2} + 2$; Listen for student understanding of the shape of the square root function.

On Your Own

- **Neighbor Check:** Have students work independently and then have their neighbors check their work. Have students discuss any discrepancies.

EXAMPLE 2 Solving a Square Root Equation

Solve $4\sqrt{x+2}+3=19$.

$4\sqrt{x+2}+3=19$	Write the equation.
$4\sqrt{x+2}=16$	Subtract 3 from each side.
$\sqrt{x+2}=4$	Divide each side by 4.
$(\sqrt{x+2})^2=4^2$	Square each side of the equation.
$x+2=16$	Simplify.
$x=14$	Subtract 2 from each side.

The solution is $x = 14$.

Check

$4\sqrt{x+2}+3=19$

$4\sqrt{14+2}+3\stackrel{?}{=}19$

$4\sqrt{16}+3\stackrel{?}{=}19$

$4(4)+3\stackrel{?}{=}19$

$16+3\stackrel{?}{=}19$

$19=19$ ✓

On Your Own

Now You're Ready Exercises 13–18

Solve the equation. Check your solution.

5. $\sqrt{x+4}+7=11$ **6.** $8\sqrt{x-1}=24$ **7.** $15=6+\sqrt{3x-9}$

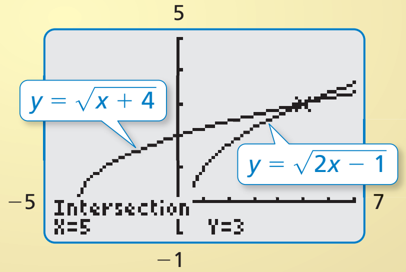
EXAMPLE 3 Solving an Equation with Square Roots on Both Sides

Solve $\sqrt{2x-1}=\sqrt{x+4}$.

$\sqrt{2x-1}=\sqrt{x+4}$	Write the equation.
$(\sqrt{2x-1})^2=(\sqrt{x+4})^2$	Square each side of the equation.
$2x-1=x+4$	Simplify.
$x-1=4$	Subtract x from each side.
$x=5$	Add 1 to each side.

The solution is $x = 5$.

Check



On Your Own

Now You're Ready Exercises 22–27

Solve the equation. Check your solution.

8. $\sqrt{3x+1}=\sqrt{4x-7}$ **9.** $\sqrt{x}=\sqrt{5x-1}$

Squaring each side of an equation can sometimes introduce a solution that is *not* a solution of the original equation. This solution is called an **extraneous solution.** Be sure to always substitute your solutions into the original equation to check for extraneous solutions.

EXAMPLE 4 Identifying an Extraneous Solution

$x = \sqrt{x + 6}$	Original equation
$x^2 = \left(\sqrt{x + 6}\,\right)^2$	Square each side of the equation.
$x^2 = x + 6$	Simplify.
$x^2 - x - 6 = 0$	Subtract x and 6 from each side.
$(x - 3)(x + 2) = 0$	Factor.
$(x - 3) = 0 \quad or \quad (x + 2) = 0$	Use Zero-Product Property.
$x = 3 \quad or \quad x = -2$	Solve for x.

Check

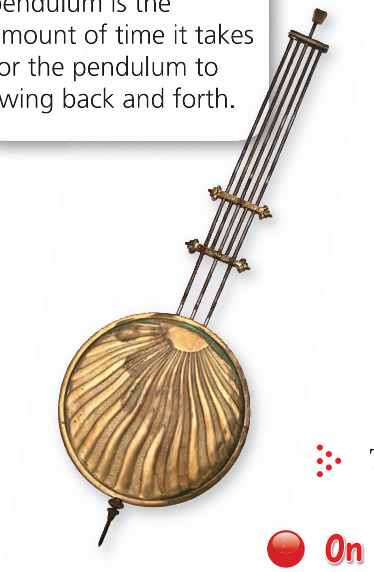
$3 \stackrel{?}{=} \sqrt{3 + 6}$	Substitute for x.	$-2 \stackrel{?}{=} \sqrt{-2 + 6}$
$3 \stackrel{?}{=} \sqrt{9}$	Simplify.	$-2 \stackrel{?}{=} \sqrt{4}$
$3 = 3$ ✓		$-2 \neq 2$ ✗

Because $x = -2$ does not check in the original equation, it is an extraneous solution. The only solution is $x = 3$.

EXAMPLE 5 Real-Life Application

Study Tip

The period of a pendulum is the amount of time it takes for the pendulum to swing back and forth.



The period P (in seconds) of a pendulum is given by the function $P = 2\pi\sqrt{\frac{L}{32}}$, where L is the pendulum length (in feet). What is the length of a pendulum that has a period of 2 seconds?

$2 = 2\pi\sqrt{\frac{L}{32}}$	Substitute 2 for P in the function.
$\frac{1}{\pi} = \sqrt{\frac{L}{32}}$	Divide each side by 2π and simplify.
$\frac{1}{\pi^2} = \frac{L}{32}$	Square each side and simplify.
$\frac{32}{\pi^2} = L$	Multiply both sides by 32.
$3.2 \approx L$	Use a calculator.

The length of the pendulum is about 3.2 feet.

On Your Own

Now You're Ready
Exercises 31–36

10. Solve $\sqrt{x - 1} = x - 3$. Check your solution.

11. **WHAT IF?** In Example 5, what is the length of a pendulum that has a period of 4 seconds? Is your result twice the length in Example 5? Explain.

Laurie's Notes

Example 4

- Instead of telling students about extraneous solutions, work through the example and when checking the solution, emphasize that something is wrong. Then explain the possibility of introducing extraneous roots when you square each side of an equation.
- Write the original equation. Take time to discuss the graph of each side.
- ? "What are the intersection possibilities for the graphs of a linear function and a square root function?" 0, 1, or 2 points of intersection; Show students generic graphs for each case.
- Work through the problem as shown. Review how to factor a trinomial.
- **Connection:** Tell students that -2 and 3 are solutions of the quadratic equation $x^2 - x - 6 = 0$. But they are not both solutions of the original square root equation.
- **Extension:** Graph each side of the original equation on a graphing calculator. Use the *intersect* feature to find the solution.

Example 5

- It is time to return to "The Pit and the Pendulum!" Ask a student to read the problem. The Study Tip describes the period of a pendulum. Use a string with a weight on it to model the pendulum motion and to identify the period.
- Take time to identify the independent and dependent variables. Be sure to include the units of measure.
- Set the function equal to 2, which is the period of the pendulum.
- Students may want to divide by 2 and then divide by π. This can be done in one step.
- Continue to work through the problem and use a calculator to approximate the answer to the nearest tenth.
- **Extension:** Have a piece of string available that is 3.2 feet long. Place a weight on the string and model a period of 2 seconds.

On Your Own

- Discuss Question 11. This is similar to Activity 3.

Closure

- Solve each equation.

a. $2x + 1 = 10$ $x = \frac{9}{2}$

b. $x^2 + 1 = 10$ $x = 3, x = -3$

c. $\sqrt{x} + 1 = 10$ $x = 81$

Extra Example 4

Solve $x = \sqrt{x + 20}$. Check your solution.
$x = 5$

Extra Example 5

In Example 5, what is the length of a pendulum that has a period of 3 seconds? about 7.3 feet

On Your Own

10. $x = 5$

11. about 13 feet; no; It is about 4 times the length of the pendulum in Example 5.

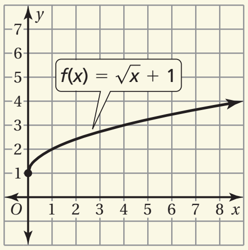
Vocabulary and Concept Check

1. no; The radicand does not contain a variable.

2. Squaring each side of a square root equation can introduce extraneous solutions, which are not solutions of the original equation.

Practice and Problem Solving

3.



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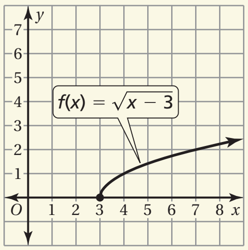
4.



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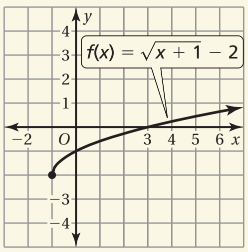
5.



24

6. $x = 81$ 7. $x = 144$

8. $x = 16$ 9. $x = 121$

10. $x = 0$ 11. $x = 225$

12. The student did not square the negative coefficient; $x = 4$

13. $x = 19$ 14. $x = 60$

15. $x = 45$ 16. $x = 34$

Assignment Guide and Homework Check

Level	Assignment	Homework Check
Average	1, 2, 6–38 even, 19, 43, 51–54	12, 16, 20, 24, 34, 38
Advanced	1, 2, 13–37 odd, 43–50, 51–54	19, 21, 35, 45, 50

For Your Information

- **Exercise 20** BASE stands for Buildings, Antennas, Spans (bridges), and Earth (cliffs).
- **Exercise 44** Students can use graphing calculators for this exercise.

Common Errors

- **Exercises 6–11, 13–18** Students may square each side of the equation before isolating the square root on one side of the equation.

10.2 Record and Practice Journal

Solve the equation. Check your solution.

1. $\sqrt{x} + 4 = 9$
$x = 25$

2. $-2 = 6 - \sqrt{x}$
$x = 64$

3. $7 = 1 + 2\sqrt{x + 4}$
$x = 5$

4. $\sqrt{5x - 11} - 3 = 5$
$x = 15$

5. $\sqrt{4x - 3} = \sqrt{x + 6}$
$x = 3$

6. $\sqrt{8x + 1} = \sqrt{7x + 7}$
$x = 6$

7. $x = \sqrt{12x - 32}$
$x = 4, 8$

8. $\sqrt{4x + 13} = x - 2$
$x = 9$

9. The formula $\frac{S}{8} = \sqrt{df}$ relates the speed S (in feet per second), drag factor f, and distance d (in feet) it takes for a car to come to a stop after the driver applies the brakes. A car travels at 80 feet per second and the drag factor is $\frac{2}{3}$. What distance does it take for the car to stop once the driver applies the brakes?
150 ft

10.2 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **VOCABULARY** Is $x\sqrt{3} = 4$ a square root equation? Explain your reasoning.
2. **WRITING** Why should you check every solution of a square root equation?

Practice and Problem Solving

Sketch the graph of the function. Then find the value of x such that $f(x) = 3$.

3. $f(x) = \sqrt{x} + 1$
4. $f(x) = \sqrt{x - 3}$
5. $f(x) = \sqrt{x + 1} - 2$

Solve the equation. Check your solution.

1
6. $\sqrt{x} = 9$
7. $7 = \sqrt{x} - 5$
8. $\sqrt{x} + 6 = 10$
9. $\sqrt{x} + 12 = 23$
10. $4 - \sqrt{x} = 4$
11. $-8 = 7 - \sqrt{x}$

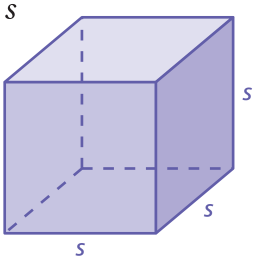
12. **ERROR ANALYSIS** Describe and correct the error in solving the equation.

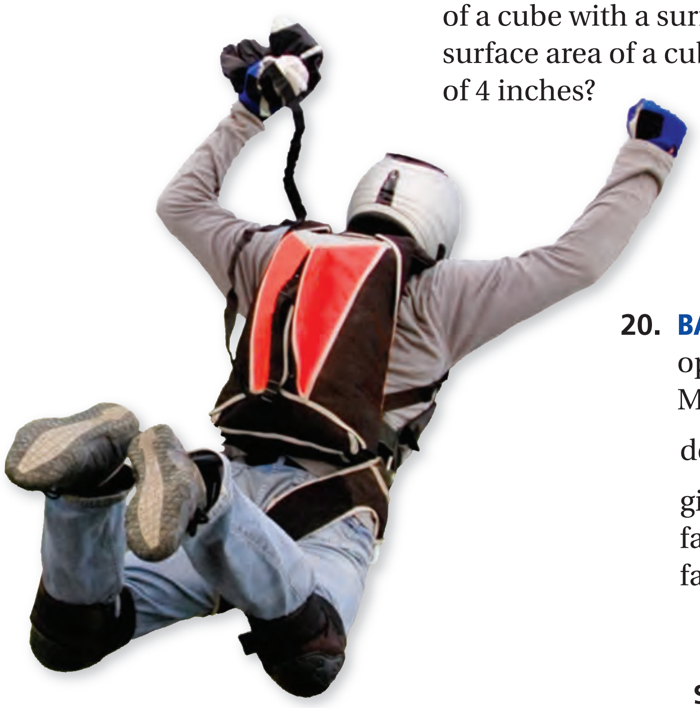
$$4 - \sqrt{x} = 2$$
$$-\sqrt{x} = -2$$
$$-x = 4$$
$$x = -4$$

Solve the equation. Check your solution.

2
13. $\sqrt{x - 3} + 5 = 9$
14. $2\sqrt{x + 4} = 16$
15. $25 = 7 + 3\sqrt{x - 9}$
16. $\sqrt{\frac{x}{2} - 1} + 14 = 18$
17. $-1 = \sqrt{5x + 1} - 7$
18. $12 = 19 - \sqrt{3x - 11}$

19. **CUBE** The formula $s = \sqrt{\frac{A}{6}}$ gives the edge length s of a cube with a surface area of A. What is the surface area of a cube with an edge length of 4 inches?





20. **BASE JUMPING** The Cave of Swallows is a natural open-air pit cave in the state of San Luis Potosi, Mexico. The 1220-foot deep cave is a popular destination for BASE jumpers. The formula $t = \sqrt{\frac{d}{16}}$ gives the distance d (in feet) a BASE jumper free falls in t seconds. How far does the BASE jumper fall in 3 seconds?

21. **WRITING** Explain how you would solve $\sqrt{m+4} - \sqrt{3m} = 0$.

Solve the equation. Check your solution.

3 **22.** $\sqrt{2x-9} = \sqrt{x}$

23. $\sqrt{x+1} = \sqrt{4x-8}$

24. $\sqrt{3x+1} = \sqrt{7x-19}$

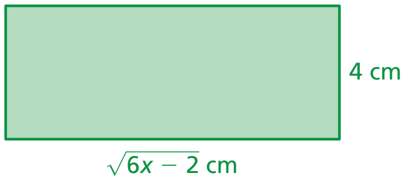
25. $\sqrt{8x-7} = \sqrt{6x+7}$

26. $\sqrt{2x+1} - \sqrt{4x} = 0$

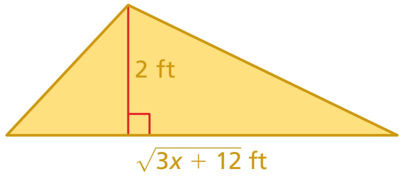
27. $\sqrt{5x} - \sqrt{8x-2} = 0$

Find the value of *x*.

28. Perimeter = 28 cm



29. Area = $\sqrt{5x-4}$ ft^2



30. **OPEN-ENDED** Write a square root equation of the form $\sqrt{ax} + b = c$ that has a solution of 9.

Solve the equation. Check your solution.

4 **31.** $x = \sqrt{5x-4}$

32. $\sqrt{9x-14} = x$

33. $\sqrt{3x+10} = x$

34. $2x = \sqrt{6-10x}$

35. $x - 1 = \sqrt{3-x}$

36. $\sqrt{-4x-19} = x + 4$

37. **ERROR ANALYSIS** Describe and correct the error in solving the equation.

$$x = \sqrt{12-4x}$$
$$x^2 = 12 - 4x$$
$$x^2 + 4x - 12 = 0$$
$$(x-2)(x+6) = 0$$
$$x = 2 \quad \text{or} \quad x = -6$$

38. **REASONING** Explain how to use mental math to find the solution of $\sqrt{2x} + 5 = 1$.

Determine whether the statement is *true* or *false*.

39. If $\sqrt{a} = b$, then $(\sqrt{a})^2 = b^2$.

40. If $\sqrt{a} = \sqrt{b}$, then $a = b$.

41. If $a^2 = b^2$, then $a = b$.

42. If $a^2 = \sqrt{b}$, then $a^4 = (\sqrt{b})^2$.

43. **ELECTRICITY** The formula $V = \sqrt{PR}$ relates the voltage V (in volts), power P (in watts), and resistance R (in ohms) of an electrical circuit. What is the resistance of a 1000-watt hair dryer on a 120-volt circuit?

Common Errors

- **Exercise 37** Students may have trouble determining what the error is because the equation is solved correctly. However, the extraneous solution is not eliminated. Remind students to check their solutions.
- **Exercises 48 and 49** Students may forget that when you square an expression with two terms, the result is an expression with three terms. Remind them of this process.
- **Exercise 50** Students may forget to convert 1.5 cubic feet to cubic inches before finishing the problem.

Practice and Problem Solving

17. $x = 7$ **18.** $x = 20$

19. 96 in.^{2} **20.** 144 ft

21. Add $\sqrt{3m}$ to each side. Square each side. Subtract m from each side. Divide each side by 2. Check your solution.

22. $x = 9$ **23.** $x = 3$

24. $x = 5$ **25.** $x = 7$

26. $x = \frac{1}{2}$ **27.** $x = \frac{2}{3}$

28. $x = 17$ **29.** $x = 8$

30. *Sample answer:* $\sqrt{4x} + 10 = 16$

31. $x = 1, x = 4$

32. $x = 2, x = 7$

33. $x = 5$ **34.** $x = \frac{1}{2}$

35. $x = 2$ **36.** no solution

37. $x = -6$ does not check in the original equation, so it is extraneous. $x = 2$ is the only solution.

38. See Additional Answers.

39. true **40.** true

41. false **42.** true

43. 14.4 ohms

Differentiated Instruction

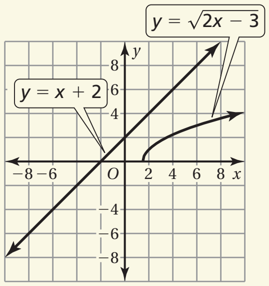
Organization

Students may have difficulty determining the order of the steps needed to solve a radical equation. Suggest they draw a box around the term containing the radical and isolate that term on one side. Then continue the process to solve for x.

$$\boxed{3\sqrt{x+1}} - 8 = 1$$
$$\boxed{3\sqrt{x+1}} = 9$$
$$\sqrt{x+1} = 3$$
$$x + 1 = 9$$
$$x = 8$$

Practice and Problem Solving

44. a.



The graphs do not intersect. The equation has no solutions.

b. no solution; Solving algebraically and graphically give the same result.

c. *Sample answer:* graphing; Using a graphing calculator is convenient and accurate.

45. about 29.2 ft

46. 2; 4

47. $\sqrt{x+2}$ has one term and $\sqrt{x} + 2$ has two terms. When you square $\sqrt{x+2}$, the result is the radicand, $x + 2$. When you square $\sqrt{x} + 2$, the result is an expression with 3 terms that includes a radical, $x + 4\sqrt{x} + 4$.

48. $x = 5$ **49.** $x = -\frac{7}{16}$

50. See *Taking Math Deeper*.

Fair Game Review

51. 92° **52.** 80°

53. 90° **54.** B

Mini-Assessment

Solve the equation. Check your solution.

1. $\sqrt{x} - 4 = 6$ $x = 100$
2. $\sqrt{x+7} = \sqrt{2x-4}$ $x = 11$
3. $x = \sqrt{5x+6}$ $x = 6$

Taking Math Deeper

Exercise 50

This problem reviews many previous skills such as converting units for volume and finding the percent of a number.

1 **a.** Substitute for A and b_2 in the formula and simplify to get $h = \sqrt{336 - 16h}$. Then solve for h.

$$h^2 = 336 - 16h \qquad \text{Square each side.}$$

$$h^2 + 16h - 336 = 0 \qquad \text{Write in standard form.}$$

$$h = 12 \text{ or } h = -28 \qquad \text{Factor and solve for } h.$$

The height of the speaker box is 12 inches.

2 **b.** Use the formula for the area of a trapezoid to find b_1.

$$A = \frac{1}{2}h(b_1 + b_2) \qquad \text{Area of a trapezoid}$$

$$168 = \frac{1}{2}(12)(b_1 + 16) \qquad \text{Substitute.}$$

$$12 = b_1 \qquad \text{Solve for } b_1.$$

The length of b_1 is 12 inches.

3 **c.** Convert 1.5 cubic feet to cubic inches.

$$1.5 \text{ ft}^3 \cdot \left(\frac{12 \text{ in.}}{1 \text{ ft}}\right)^3 = 1.5 \cancel{\text{ft}^3} \cdot \frac{1728 \text{ in.}^3}{1 \cancel{\text{ft}^3}} = 2592 \text{ in.}^3$$

So, the minimum volume is $0.9(2592) = 2332.8$ in.^{3} and the maximum volume is $1.1(2592) = 2851.2$ in.^{3}.

The volume of a prism is the area of the base times the height of the prism. In this problem, the height of the prism is the width w of the speaker box. Use the maximum and minimum volumes to find the range of the width.

$$2332.8 = 168w \qquad 2851.2 = 168w$$

$$13.9 \approx w \qquad 17.0 \approx w$$

To the nearest tenth, the range of the width is $13.9 \le w \le 17.0$.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension Start the next section

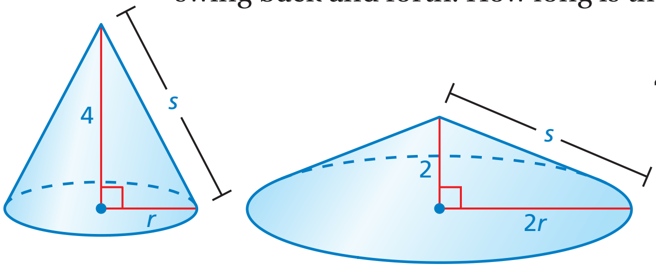
44. **CHOOSE TOOLS** Consider the equation $x + 2 = \sqrt{2x - 3}$.

a. Graph each side of the equation in the same coordinate plane. Solve the equation by finding points of intersection.

b. Solve the equation algebraically. How does your solution compare to the solution in part (a)?

c. Which method do you prefer? Explain your reasoning.

45. **TRAPEZE** The time t (in seconds) it takes a trapeze artist to swing back and forth is given by the function $t = 2\pi\sqrt{\frac{r}{32}}$, where r is the rope length (in feet). It takes 6 seconds to swing back and forth. How long is the rope? Use 3.14 for π.



46. **GEOMETRY** The formula $s = \sqrt{r^2 + h^2}$ gives the slant height s of a cone, where r is the radius of the base, and h is the height. The slant heights of the two cones are equal. Find the radius of each cone.

47. **CRITICAL THINKING** How is squaring $\sqrt{x + 2}$ different than squaring $\sqrt{x} + 2$?

Solve the equation. Check your solution.

48. $\sqrt{x + 15} = \sqrt{x} + \sqrt{5}$

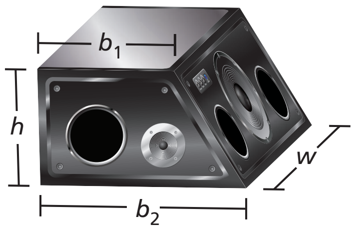
49. $2 - \sqrt{x + 1} = \sqrt{x + 2}$

50. **Modeling** The formula $h = \sqrt{2A - b_2h}$ gives the height h of the speaker box, where A is the area of one trapezoidal side, and b_2 is the length of base 2.

a. Given that $A = 168$ square inches and $b_2 = 16$ inches, find h.

b. What is the length of b_1 (base 1)?

c. Speakers work best when the volume of the speaker box is $\pm 10\%$ of the manufacturer's recommendation. Find the range of the widths w when the manufacturer recommends a volume of 1.5 cubic feet.



Fair Game Review What you learned in previous grades & lessons

Two angle measures of a triangle are given. Find the measure of the missing angle. *(Skills Review Handbook)*

51. 40°, 48°

52. 45°, 55°

53. 36°, 54°

54. **MULTIPLE CHOICE** Which function is represented by the ordered pairs (−1, 0.5), (0, 1), (1, 2), (2, 4), and (3, 8)? *(Section 8.5)*

Ⓐ $y = 0.5x^2$ Ⓑ $y = 2^x$ Ⓒ $y = 2x^2$ Ⓓ $y = 2x$

10 Study Help

Check It Out
Graphic Organizer
BigIdeasMath.com

You can use a **word magnet** to organize information associated with a vocabulary word. Here is an example of a word magnet for square root functions.

Square Root Function

Definition: A function that contains a square root with the independent variable in the radicand.

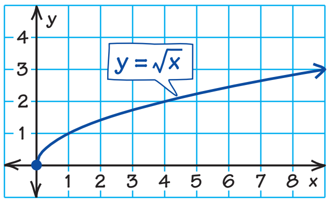
Examples:

$y = \sqrt{x} + 3$

$y = \sqrt{x - 1}$

$y = \sqrt{x + 5} - 4$

Sample Graph:



Domain: The value of the radicand cannot be negative. So, the domain is limited to *x*-values for which the radicand is greater than or equal to 0.

Graph: Make a table of values. Plot the ordered pairs. Draw a smooth curve through the points. Find the domain and range.

Compare: When graphing a square root function $f(x)$:
- $f(x) + k$ is a vertical translation of $f(x)$.
- $f(x + h)$ is a horizontal translation of $f(x)$.
- $-f(x)$ is a reflection of $f(x)$ in the *x*-axis.

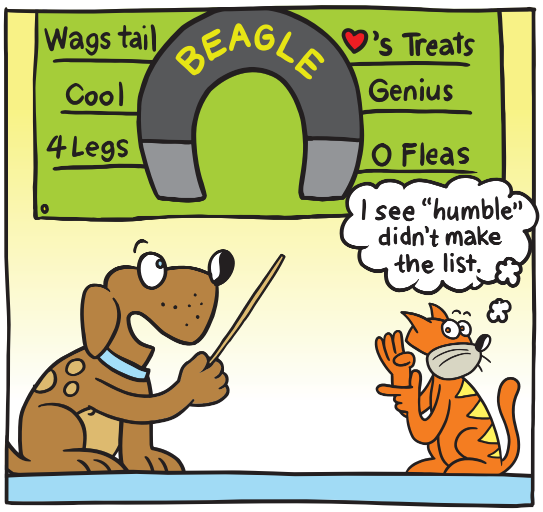
On Your Own

Make word magnets to help you study these topics.

1. rationalizing the denominator
2. solving a square root equation
3. extraneous solution

After you complete this chapter, make word magnets for the following topics.

4. Pythagorean Theorem
5. converse of the Pythagorean Theorem
6. distance formula



"How do you like the **word magnet** I made for 'Beagle'?"

Sample Answers

1.

Rationalizing the Denominator

Rationalizing the denominator is the process of multiplying a fraction by a form of 1 to eliminate radicals from the denominator.

Examples:

$$\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$
$$= \frac{3\sqrt{5}}{5}$$

$$\sqrt{\frac{2}{11}} = \frac{\sqrt{2}}{\sqrt{11}} \cdot \frac{\sqrt{11}}{\sqrt{11}}$$
$$= \frac{\sqrt{22}}{11}$$

Conjugates:
$a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$

The product of two conjugates does not contain a radical.

Example:

$$\frac{6}{\sqrt{7}-2} = \frac{6}{\sqrt{7}-2} \cdot \frac{\sqrt{7}+2}{\sqrt{7}+2}$$
$$= \frac{6(\sqrt{7}+2)}{(\sqrt{7})^2 - 2^2}$$
$$= \frac{6\sqrt{7}+12}{7-4}$$
$$= 2\sqrt{7} + 4$$

2.

Solving a Square Root Equation

A square root equation contains a square root with a variable in the radicand.

Squaring each side of an equation:
If $a = b$, then $a^2 = b^2$.

Example:

$$\sqrt{x} - 6 = 4$$
$$\sqrt{x} = 10$$
$$\left(\sqrt{x}\right)^2 = 10^2$$
$$x = 100$$

Check:

$$\sqrt{100} - 6 \stackrel{?}{=} 4$$
$$10 - 6 = 4 \ \checkmark$$

Squaring each side of an equation sometimes introduces an extraneous solution.

Example:

$$x = \sqrt{x+2}$$
$$x^2 = \left(\sqrt{x+2}\right)^2$$
$$x^2 = x + 2$$
$$x^2 - x - 2 = 0$$
$$(x-2)(x+1) = 0$$
$$x = 2 \text{ or } x = -1$$

Check:

$2 \stackrel{?}{=} \sqrt{2+2}$ $\qquad$ $-1 \stackrel{?}{=} \sqrt{-1+2}$

$2 = \sqrt{4}$ ✓ $\qquad$ $-1 \neq \sqrt{1}$ ✗

The solution is $x = 2$.

3. Available at *BigIdeasMath.com.*

List of Organizers

Available at *BigIdeasMath.com*

Comparison Chart
Concept Circle
Definition (Idea) and Example Chart
Example and Non-Example Chart
Formula Triangle
Four Square
Information Frame
Information Wheel
Notetaking Organizer
Process Diagram
Summary Triangle
Word Magnet
Y Chart

About this Organizer

A **Word Magnet** can be used to organize information associated with a vocabulary word or term. As shown, students write the word or term inside the magnet. Students write associated information on the blank lines that "radiate" from the magnet. Associated information can include, but is not limited to: other vocabulary words or terms, definitions, formulas, procedures, examples, and visuals. This type of organizer serves as a good summary tool because any information related to a topic can be included.

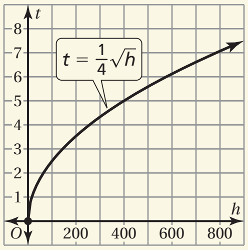
Technology for the Teacher

Editable Graphic Organizer

Answers

1. $x \geq 0$
2. $x \geq 3$
3. $x \leq 3$

4–6. See Additional Answers.

7. $\frac{\sqrt{7}}{7}$
8. $\frac{\sqrt{3}}{3}$
9. $7\sqrt{5} - 14$
10. $x = 65$
11. $x = 4$
12. $x = 3$
13. 13
14. 4
15. a. 

domain: $h \geq 0$; range: $t \geq 0$

b. about 876 ft

16. a. 16°C

b. 5.5 sec

Technology for the Teacher

Online Assessment
Assessment Book
ExamView® Assessment Suite

Alternative Quiz Ideas

100% Quiz
Error Notebook
Group Quiz
Homework Quiz
Math Log
Notebook Quiz
Partner Quiz
Pass the Paper

Math Log

Ask students to keep a math log for the chapter. Have them include diagrams, definitions, and examples. Everything should be clearly labeled. It might be helpful if they put the information in a chart. Students can add to the log as they are introduced to new topics.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Study Help • Practice A and Practice B • Puzzle Time Lesson Tutorials *BigIdeasMath.com*	Resources by Chapter • Enrichment and Extension • School-to-Work Game Closet at *BigIdeasMath.com* Start the next section

10.1–10.2 Quiz

Check It Out
Progress Check
BigIdeasMath.com

Find the domain of the function. *(Section 10.1)*

1. $y = 15\sqrt{x}$ **2.** $y = \sqrt{x - 3}$ **3.** $y = \sqrt{3 - x}$

Graph the function. Describe the domain and range. Compare the graph to the graph of $y = \sqrt{x}$. *(Section 10.1)*

4. $y = \sqrt{x} + 5$ **5.** $y = \sqrt{x - 4}$ **6.** $y = -\sqrt{x - 2} + 1$

Simplify the expression. *(Section 10.1)*

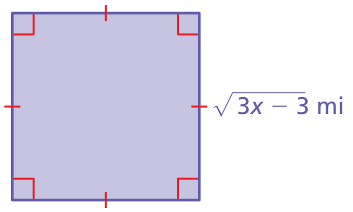
7. $\sqrt{\dfrac{6}{42}}$ **8.** $\dfrac{2}{\sqrt{3}} - \dfrac{1}{\sqrt{3}}$ **9.** $\dfrac{7}{\sqrt{5} + 2}$

Solve the equation. *(Section 10.2)*

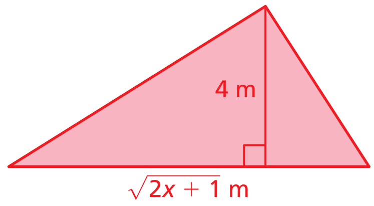
10. $\sqrt{x - 1} + 7 = 15$ **11.** $\sqrt{x} = \sqrt{6x - 20}$ **12.** $x = \sqrt{21 - 4x}$

Find the value of *x*. *(Section 10.2)*

13. Perimeter = 24 mi



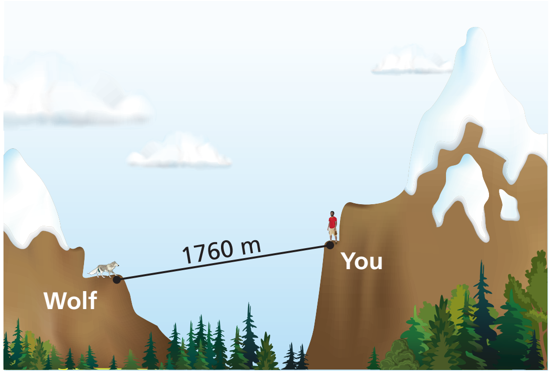
14. Area = $2\sqrt{4x - 7}$ m^2



15. BRIDGE The time t (in seconds) it takes an object to drop h feet is given by $t = \frac{1}{4}\sqrt{h}$. *(Section 10.1)*

a. Graph the function. Describe the domain and range.

b. It takes about 7.4 seconds for a stone dropped from the New River Gorge Bridge in West Virginia, to reach the water below. About how high is the bridge above the New River?



16. SPEED OF SOUND The speed of sound s (in meters per second) through air is given by $s = 20\sqrt{T + 273}$, where T is the temperature in degrees Celsius. *(Section 10.2)*

a. What is the temperature when the speed of sound is 340 meters per second?

b. How long does it take you to hear the wolf howl when the temperature is -17°C?

10.3 The Pythagorean Theorem

Essential Question How are the lengths of the sides of a right triangle related?

Pythagoras was a Greek mathematician and philosopher who discovered one of the most famous rules in mathematics. In mathematics, a rule is called a **theorem**. So, the rule that Pythagoras discovered is called the Pythagorean Theorem.

Pythagoras
(c. 570 B.C.–c. 490 B.C.)

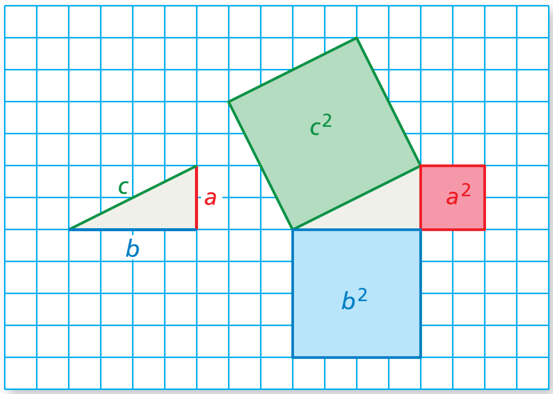
1 ACTIVITY: Discovering the Pythagorean Theorem

Work with a partner.

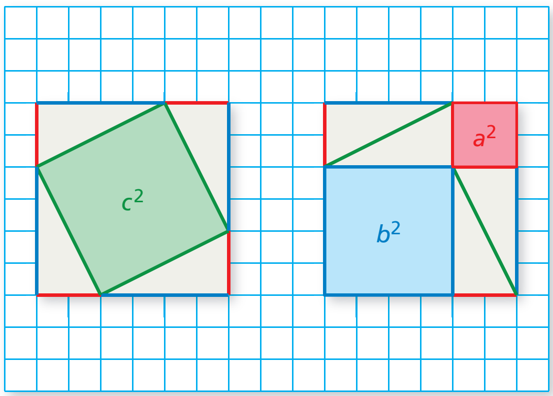
a. On grid paper, draw any right triangle. Label the lengths of the two shorter sides (the **legs**) a and b.

b. Label the length of the longest side (the **hypotenuse**) c.

c. Draw squares along each of the three sides. Label the areas of the three squares a^2, b^2, and c^2.



d. Cut out the three squares. Make eight copies of the right triangle and cut them out. Arrange the figures to form two identical larger squares.



e. What does this tell you about the relationship among a^2, b^2, and c^2?

COMMON CORE

Pythagorean Theorem

In this lesson, you will
- discover the Pythagorean Theorem.
- find missing side lengths of right triangles.
- solve real-life problems.

Learning Standards
8.G.6
8.G.7

Laurie's Notes

Introduction

Standards for Mathematical Practice

- **MP4 Model with Mathematics:** Mathematically proficient students can apply what they know about the Pythagorean Theorem to model real-world phenomena. Sketching a representation of the problem, perhaps on grid paper, may be a helpful first step.

Motivate

- Share information about Pythagoras who was born in Greece in 569 B.C.
  - He is known as the *Father of Numbers*.
  - He traveled extensively in Egypt, learning math, astronomy and music.
  - Pythagoras undertook a reform of the cultural life of Cretona, urging the citizens to follow his religious, political, and philosophical goals.
  - He created a school where his followers, known as Pythagoreans, lived and worked. They observed a rule of silence called *echemythia*, the breaking of which was punishable by death. One had to remain silent for *five years* before he could contribute to the group.

Activity Notes

Activity 1

- **Suggestions:** Use centimeter grid paper for ease of manipulating the cut pieces. Suggest to students that they draw their original triangle in the upper left of the grid paper and then make a working copy of the triangle towards the middle of the paper. This gives enough room for the squares to be drawn along each side of triangle.
- Vertices of the triangle need to be on lattice points. You do not want every student in the room to use the same triangle. Suggest other leg lengths (3 and 4, 3 and 6, 2 and 4, 2 and 3, and so on).
- **MP4:** Drawing the square along the hypotenuse is the challenging step. Model one technique for accomplishing the task using a right triangle with legs 2 and 5 units.
  - Interpret the slope of the hypotenuse as "right 5 units, up 2 units."
  - Place your pencil on the upper right endpoint of the hypotenuse and rotate the paper 90° clockwise. Move your pencil right 5 units and up 2 units. Mark a point.
  - Repeat rotating and moving according to the slope of the hypotenuse until you end at the other endpoint of the original hypotenuse.
  - Use a straightedge to connect the four points (two that you marked and two on the endpoints of the hypotenuse) to form the square.
- Before students cut anything, check that they have 3 squares of the correct size.
- **MP2 Reason Abstractly and Quantitatively**: The two large squares in part (d) have equal area. Referring to areas, if $c^2 + (4\text{ triangles}) = a^2 + b^2 + (4\text{ triangles})$, then $c^2 = a^2 + b^2$ by subtracting the 4 triangles from each side of the equation.

Common Core State Standards

8.G.6 Explain a proof of the Pythagorean Theorem and its converse.
8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Previous Learning

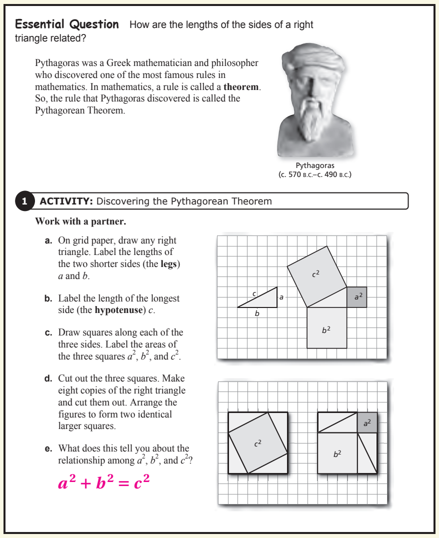
Students should know how to multiply fractions and decimals.

Technology for the Teacher

Dynamic Classroom

Lesson Plans
Complete Materials List

10.3 Record and Practice Journal



Essential Question How are the lengths of the sides of a right triangle related?

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b. Label the length of the longest side (the **hypotenuse**) c.

c. Draw squares along each of the three sides. Label the areas of the three squares a^2, b^2, and c^2.

d. Cut out the three squares. Make eight copies of the right triangle and cut them out. Arrange the figures to form two identical larger squares.

e. What does this tell you about the relationship among a^2, b^2, and c^2?

$a^2 + b^2 = c^2$

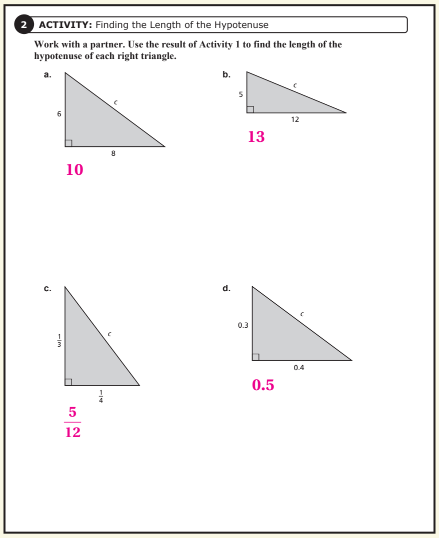
English Language Learners

Vocabulary

Help English learners understand the meanings of the words that make up a definition. Provide students with statements containing blanks and a list of the words used to fill in the blanks.

- In any right __, the __ is the side __ the right __.
  Word list: angle, hypotenuse, opposite, triangle
  triangle, hypotenuse, opposite, angle
- In any right __, the __ are the __ sides and the __ is always the __ side.
  Word list: hypotenuse, legs, longest, shorter, triangle
  triangle, legs, shorter, hypotenuse, longest

10.3 Record and Practice Journal


2 ACTIVITY: Finding the Length of the Hypotenuse

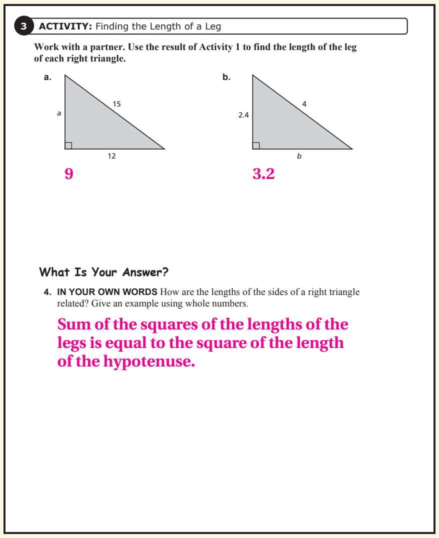
Work with a partner. Use the result of Activity 1 to find the length of the hypotenuse of each right triangle.

a. (legs 6 and 8, hypotenuse c) **10**

b. (legs 5 and 12, hypotenuse c) **13**

c. (legs $\frac{1}{3}$ and $\frac{1}{4}$, hypotenuse c) $\mathbf{\frac{5}{12}}$

d. (legs 0.3 and 0.4, hypotenuse c) **0.5**


3 ACTIVITY: Finding the Length of a Leg

Work with a partner. Use the result of Activity 1 to find the length of the leg of each right triangle.

a. (leg a, leg 12, hypotenuse 15) **9**

b. (leg 2.4, leg b, hypotenuse 4) **3.2**

What Is Your Answer?

4. **IN YOUR OWN WORDS** How are the lengths of the sides of a right triangle related? Give an example using whole numbers.

Sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

Laurie's Notes

Activity 2

- **Part (a):** This triangle is similar to the 3-4-5 right triangle. Using the property from the investigation, $6^2 + 8^2 = 36 + 64 = 100$. Students will recognize that $100 = 10^2$, so the length of the hypotenuse is 10.
- Have students share their work for each of these problems.
- **Common Error:** In part (c), when students square a fraction, they sometimes double the numerator and denominator instead of squaring each number.
  In other words, $\left(\frac{1}{3}\right)^2 \neq \frac{2}{6}$, but $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$.

Activity 3

- The two triangles in this activity have a leg length missing. Building squares on the two legs of the triangle, and finding their areas gives a^2 and 12^2 for part (a). The area of the square built on the hypotenuse is 15^2. The result of Activity 1 says that $a^2 + 12^2 = 15^2$. Students should recognize this as an opportunity to solve an equation.

? "What is the first step in solving the equation $a^2 + 12^2 = 15^2$?" Evaluate 12^2 and 15^2.

? "What is the next step in solving $a^2 + 144 = 225$?" Subtract 144 from each side.

? "Finally, what positive number squared is 81?" 9

What Is Your Answer?

- **Neighbor Check:** Have students work independently and then have their neighbors check their work. Have students discuss any discrepancies.

Closure

- **Exit Ticket:** If you drew a right triangle with legs of 5 and 6 on grid paper, what would be the area of the square drawn on the hypotenuse of the triangle? 61 square units

Math Practice 8

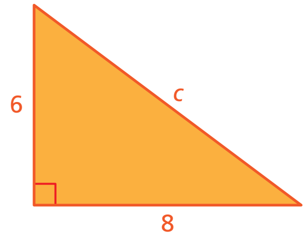
Evaluate Results

How can you verify that your answer is reasonable?

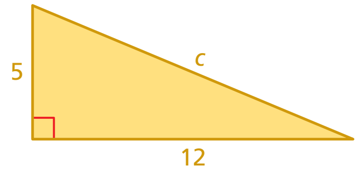
2 ACTIVITY: Finding the Length of the Hypotenuse

Work with a partner. Use the result of Activity 1 to find the length of the hypotenuse of each right triangle.

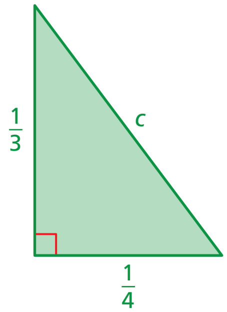
a.



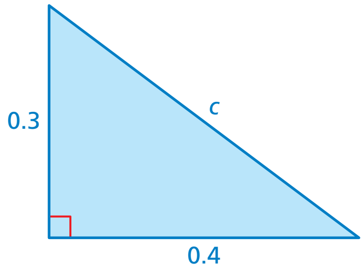
b.



c.



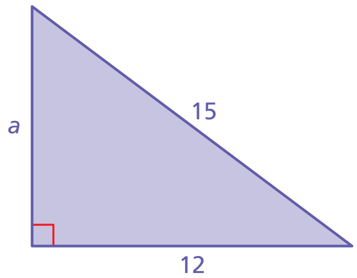
d.



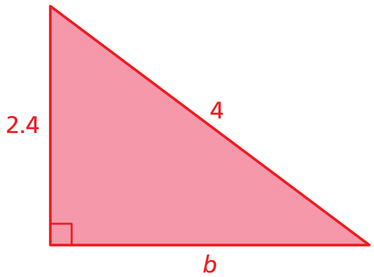
3 ACTIVITY: Finding the Length of a Leg

Work with a partner. Use the result of Activity 1 to find the length of the leg of each right triangle.

a.



b.



What Is Your Answer?

4. **IN YOUR OWN WORDS** How are the lengths of the sides of a right triangle related? Give an example using whole numbers.

Practice Use what you learned about the Pythagorean Theorem to complete Exercises 3–5 on page 524.

10.3 Lesson

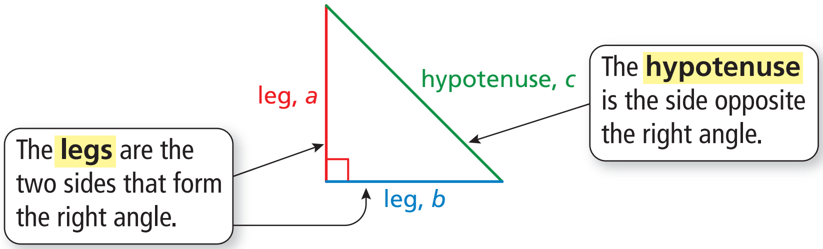
Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
theorem, *p. 520*
legs, *p. 522*
hypotenuse, *p. 522*
Pythagorean Theorem, *p. 522*

Key Ideas

Sides of a Right Triangle

The sides of a right triangle have special names.



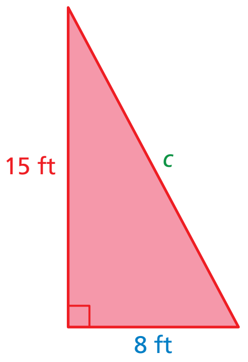
Study Tip
In a right triangle, the legs are the shorter sides and the hypotenuse is always the longest side.

The Pythagorean Theorem

Words In any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

Algebra $a^2 + b^2 = c^2$

EXAMPLE 1 Finding the Length of a Hypotenuse



Find the length of the hypotenuse of the triangle.

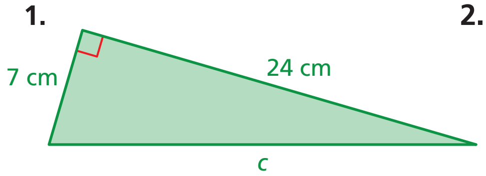
$a^2 + b^2 = c^2$	Write the Pythagorean Theorem.
$15^2 + 8^2 = c^2$	Substitute 15 for *a* and 8 for *b*.
$225 + 64 = c^2$	Evaluate powers.
$289 = c^2$	Add.
$\sqrt{289} = \sqrt{c^2}$	Take positive square root of each side.
$17 = c$	Simplify.

∴ The length of the hypotenuse is 17 feet.

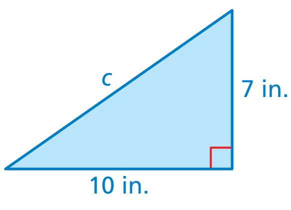
On Your Own

Find the length of the hypotenuse of the triangle.

1.



2.



Multi-Language Glossary at BigIdeasMath.com

Laurie's Notes

Introduction

Connect

- **Yesterday:** Students investigated a visual proof of the Pythagorean Theorem. (MP2, MP4)
- **Today:** Students will use the Pythagorean Theorem to find the missing lengths of a right triangle.

Motivate

- **Preparation:** Cut coffee stirrers (or carefully break spaghetti) so that triangles with the following side lengths can be made: 2-3-4; 3-4-5; 4-5-6.

? "What are consecutive numbers?" numbers in sequential order

- With student aid, use the coffee stirrers to make three triangles: 2-3-4; 3-4-5; and 4-5-6 on the overhead projector. If arranged carefully, all 3 will fit on the screen.
- Ask students to make observations about the 3 triangles. Students may mention that all triangles are scalene; one triangle appears to be acute, one right, one obtuse.
- They should observe that the change in the side lengths seems to have made a big change in the angle measures.

Lesson Notes

Key Ideas

- Draw a right triangle and label the *legs* and *hypotenuse.* The hypotenuse is always opposite the right angle and is the longest side of a right triangle.
- **Teaching Tip:** Try not to have all right triangles in the same orientation.
- Write the Pythagorean Theorem.
- **Common Error:** Students often forget that the Pythagorean Theorem is a relationship that is *only* true for right triangles.

Example 1

- Draw and label the triangle. Review the symbol used to show that an angle is a right angle.

? "What information is known for this triangle?" The legs are 8 ft and 15 ft.

On Your Own

- Give time for students to work the problems. Knowing their perfect squares is helpful.

Goal Today's lesson is using the **Pythagorean Theorem** to solve for lengths of a right triangle.

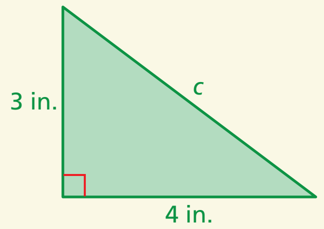
Technology for the Teacher

Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Extra Example 1

Find the length of the hypotenuse of the triangle. 5 in.

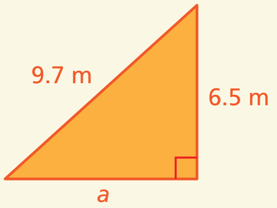


On Your Own

1. 25 cm
2. $\sqrt{149}$ in.

Extra Example 2

Find the missing length of the triangle.



$a = 7.2$ m

Extra Example 3

Ship A is 22 kilometers north and 36 kilometers east of the port. Ship B is 12 kilometers north and 12 kilometers east of the port. How far apart are the ships? 26 km

On Your Own

3. 0.5 m

4. $2\sqrt{39}$ yd

5. 57 ft

Differentiated Instruction

Kinesthetic

Have students verify the Pythagorean Theorem by drawing right triangles with legs of a given length, measuring the hypotenuse, and then calculating the hypotenuse using the Pythagorean Theorem. Use Pythagorean triples so that students work only with whole numbers.

Leg Lengths	*Hypotenuse Length*
3, 4	5
6, 8	10
5, 12	13
8, 15	17

Laurie's Notes

Example 2

- ? "What information is known for this triangle?" One leg is 3.5 kilometers and the hypotenuse is 6.5 kilometers.
- Substitute and solve as shown.
- **Common Error:** Students need to be careful with decimal multiplication. It is very common for students to multiply the decimal by 2 instead of multiplying the decimal by itself.

Example 3

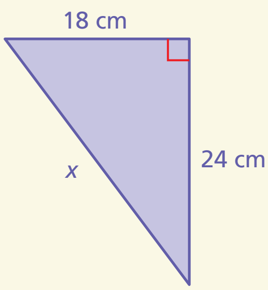
- Ask a student to read the example.
- ? "Given the compass directions stated, what is a reasonable way to represent this information?" coordinate plane
- **MP4 Model with Mathematics:** Explain that east is the positive x-direction and north is the positive y-direction. Draw the situation in a coordinate plane.
- ? "Is there enough information to use the Pythagorean Theorem? Explain." yes; The legs of the triangle can be found and then used to solve for the hypotenuse.
- **FYI:** This example previews the *distance formula,* which will be presented in the next section.

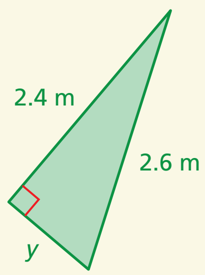
On Your Own

- **Neighbor Check:** Have students work independently and then have their neighbors check their work. Have students discuss any discrepancies.

Closure

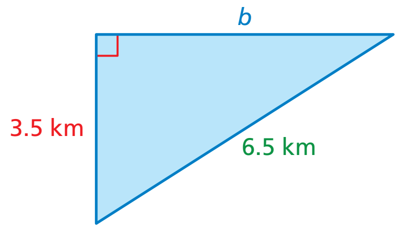
- **Exit Ticket:** Solve for the missing length. $x = 30$ cm, $y = 1$ m





EXAMPLE 2 Finding the Length of a Leg

Find the missing length of the triangle.

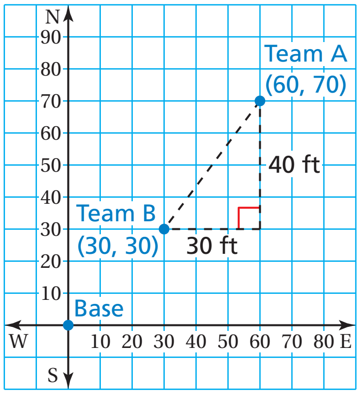


$a^2 + b^2 = c^2$	Write the Pythagorean Theorem.
$3.5^2 + b^2 = 6.5^2$	Substitute 3.5 for *a* and 6.5 for *c*.
$12.25 + b^2 = 42.25$	Evaluate powers.
$b^2 = 30$	Subtract 12.25 from each side.
$b = \sqrt{30}$	Take positive square root of each side.

The length of the leg is $\sqrt{30} \approx 5.5$ kilometers.

EXAMPLE 3 Real-Life Application

Paintball Team A is located 70 feet north and 60 feet east of the base. Team B is located 30 feet north and 30 feet east of the base. How far apart are the teams?



Step 1: Draw the situation in a coordinate plane. Let the base be at the origin. From the descriptions, you can plot Team A at (60, 70) and Team B at (30, 30).

Step 2: Draw a right triangle with a hypotenuse that represents the distance between the teams. The lengths of the legs are 30 feet and 40 feet.

Step 3: Use the Pythagorean Theorem to find the length of the hypotenuse.

$a^2 + b^2 = c^2$	Write the Pythagorean Theorem.
$30^2 + 40^2 = c^2$	Substitute 30 for *a* and 40 for *b*.
$900 + 1600 = c^2$	Evaluate powers.
$2500 = c^2$	Add.
$50 = c$	Take positive square root of each side.

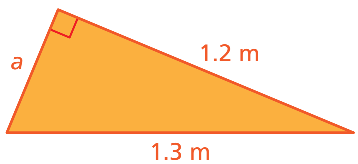
The teams are 50 feet apart.

On Your Own

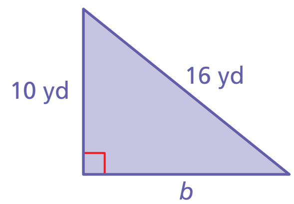
Now You're Ready
Exercises 3–8

Find the missing length of the triangle.

3.



4.



5. **WHAT IF?** In Example 3, Team B moves 10 feet to the west. How far apart are the teams to the nearest foot?

10.3 Exercises

Vocabulary and Concept Check

1. **VOCABULARY** You are given the lengths of the hypotenuse and one leg of a right triangle. Describe how you can find the length of the other leg.

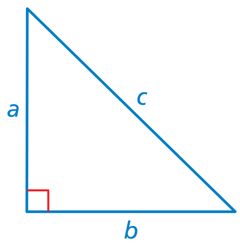
2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

Which side is a leg?

Which side is shortest?

Which side is longest?

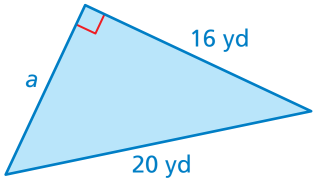
Which side is part of a right angle?



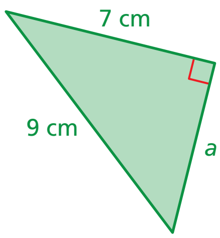
Practice and Problem Solving

Find the missing length of the triangle.

1 2 **3.**



4.



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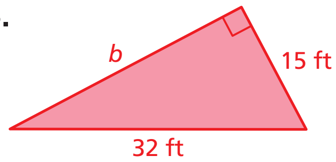
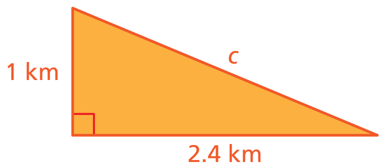
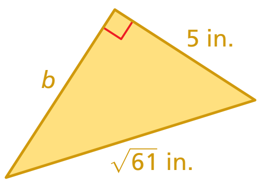


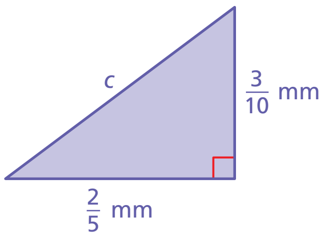
6.

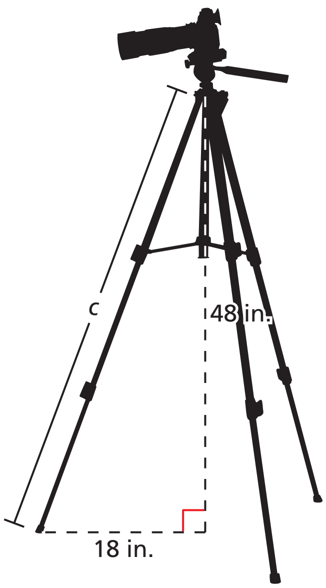


7.

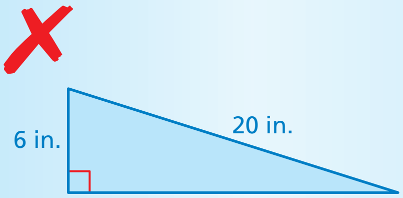


8.





9. **ERROR ANALYSIS** Describe and correct the error in finding the missing length of the triangle.



$$a^2 + b^2 = c^2$$
$$6^2 + 20^2 = c^2$$
$$436 = c^2$$
$$\sqrt{436} = c$$

10. **TRIPOD** The center of the tripod forms a 90° angle with the ground. Find the length of the support leg to the nearest tenth of an inch.

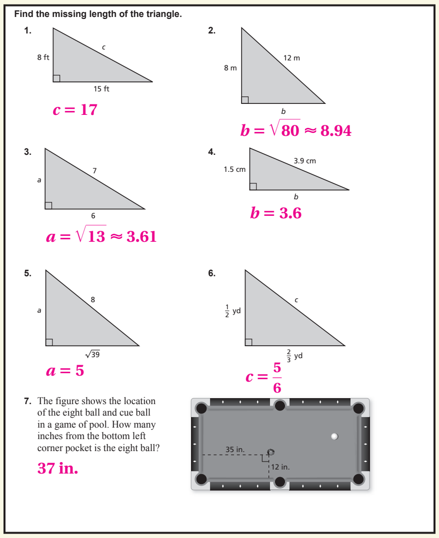
Assignment Guide and Homework Check

Level	Assignment	Homework Check
Average	1, 2, 3–12, 14, 17–20	5, 9, 10, 14
Advanced	1, 2, 6–16, 17–20	7, 10, 14, 16

Common Errors

- **Exercises 3–8** Students may substitute the given lengths in the wrong part of the formula. For example, if they are finding one of the legs, they may write $5^2 + 13^2 = c^2$ instead of $5^2 + b^2 = 13^2$. Remind them that the side opposite the right angle is the hypotenuse c.
- **Exercises 3–8** Students may multiply each side length by two instead of squaring the side length. Remind them of the definition of exponents.

10.3 Record and Practice Journal



Find the missing length of the triangle.

1. $c = 17$
2. $b = \sqrt{80} \approx 8.94$
3. $a = \sqrt{13} \approx 3.61$
4. $b = 3.6$
5. $a = 5$
6. $c = \frac{5}{6}$
7. The figure shows the location of the eight ball and cue ball in a game of pool. How many inches from the bottom left corner pocket is the eight ball?

37 in.

Vocabulary and Concept Check

1. Use the Pythagorean Theorem to find the missing side length.
2. Which side is longest?; c; a or b

Practice and Problem Solving

3. 12 yd
4. $4\sqrt{2}$ cm
5. $\sqrt{799}$ ft
6. 2.6 km
7. 6 in.
8. $\frac{1}{2}$ mm
9. 20 should have been substituted for c, not b. The missing length is $2\sqrt{91}$ inches.
10. about 51.3 in.

Practice and Problem Solving

11. about 60 in.

12. yes; The distance between the tennis player's mouth and the referee's ear is about 41 feet.

13. See *Taking Math Deeper.*

14. $10\sqrt{2}$ ft or about 14.1 ft

15. See Additional Answers.

16. **a.** $x, x + 1, x + 2; x + 2$

b. $x^2 + (x + 1)^2 = (x + 2)^2$; integers: 3, 4, 5

Fair Game Review

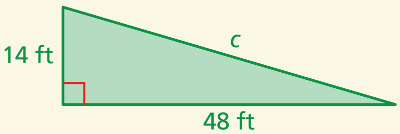
17–19. See Additional Answers.

20. C

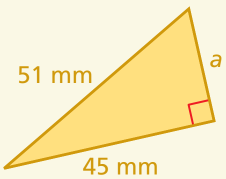
Mini-Assessment

Find the missing length of the triangle.

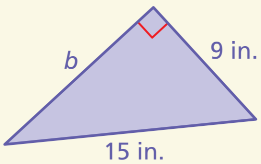
1. 50 ft



2. 24 mm



3. 12 in.

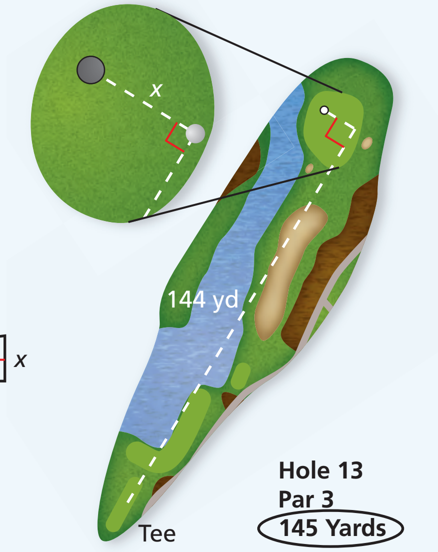


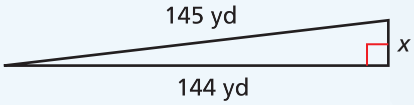
Taking Math Deeper

Exercise 13

The challenging part of this problem is realizing that the length of the hypotenuse of the right triangle is given as 145 yards at the bottom of the diagram.

1 Find the length of the hypotenuse of the right triangle.





2 Use the Pythagorean Theorem.

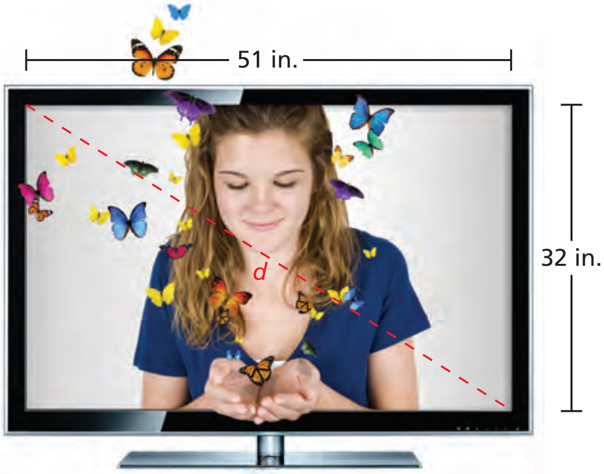
$$144^2 + x^2 = 145^2$$
$$20{,}736 + x^2 = 21{,}025$$
$$x^2 = 289$$
$$x = 17$$

3 Answer the question.
The ball is 17 yards from the hole. Using the relationship of $3 \text{ feet} = 1 \text{ yard}$, $17 \text{ yd} \times \frac{3 \text{ ft}}{1 \text{ yd}} = 51$ ft. So, the ball is 51 feet from the hole.

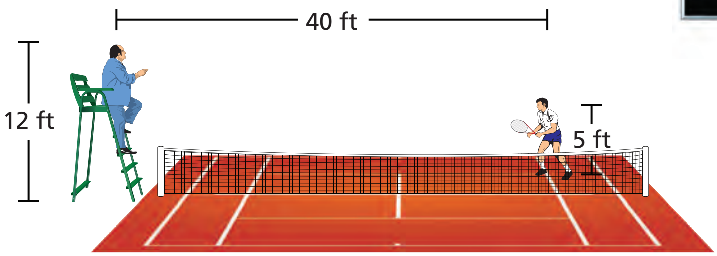
Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension • School-to-Work Start the next section

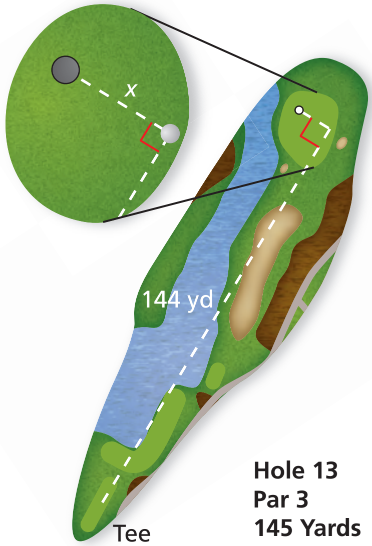
11. **TELEVISIONS** Televisions are advertised by the lengths of their diagonals. Approximate the length of the diagonal of the television to the nearest inch.



12. **TENNIS** A tennis player asks the referee a question. The sound of the player's voice only travels 50 feet. Can the referee hear the question? Explain.



13. **GOLF** The figure shows the location of a golf ball after a tee shot. How many feet from the hole is the ball?



14. **SNOWBALLS** You and a friend throw snowballs at each other. You are 20 feet north and 15 feet east of your house. Your friend is 25 feet east and 10 feet north of your house. How far apart are you and your friend?

15. **PRECISION** The legs of a right triangle have lengths of 28 meters and 21 meters. The hypotenuse has a length of $5x$ meters.
   a. Write an equation to solve for x.
   b. Describe how to solve the equation by factoring and by taking a square root. Which method do you prefer? Explain.
   c. What is the value of x?

16. **Structure** The side lengths of a right triangle are three consecutive integers.
   a. Write an expression that represents each side length. Which side length represents the hypotenuse?
   b. Write and solve an equation to find the three integers.

Fair Game Review What you learned in previous grades & lessons

Graph the function. Compare the graph to the graph of $y = x^2$. *(Section 8.3)*

17. $y = -2x^2 + 4$ 18. $y = -x^2 - 6$ 19. $y = 3x^2 + 8$

20. **MULTIPLE CHOICE** Which polynomial is equivalent to $(x^2 - 3x + 1) - (-2x^2 + x - 4)$? *(Section 7.2)*

Ⓐ $3x^2 - 4x - 3$ Ⓑ $-x^2 - 2x - 5$ Ⓒ $3x^2 - 4x + 5$ Ⓓ $-x^2 + 4x + 3$

10.4 Using the Pythagorean Theorem

Essential Question In what other ways can you use the Pythagorean Theorem?

The *converse* of a statement switches the hypothesis and the conclusion.

Statement:	Converse of the statement:
If p, then q.	If q, then p.

1 ACTIVITY: Analyzing Converses of Statements

Work with a partner. Write the converse of the true statement. Determine whether the converse is true or false. If it is false, give a counterexample.

a. **Sample:** If $a = b$, then $a^2 = b^2$.
Converse: If $a^2 = b^2$, then $a = b$.
The converse is false. A counterexample is $a = -2$ and $b = 2$.

b. If two nonvertical lines have the same slope, then the lines are parallel.

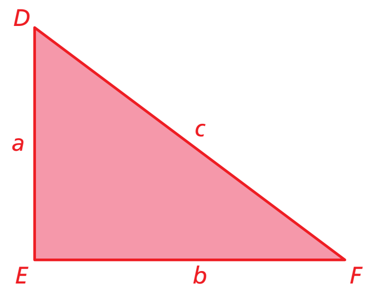
c. If a sequence has a common difference, then it is an arithmetic sequence.

d. If a and b are rational numbers, then $a + b$ is a rational number.

Is the converse of a true statement always true? always false? Explain.

2 ACTIVITY: The Converse of the Pythagorean Theorem

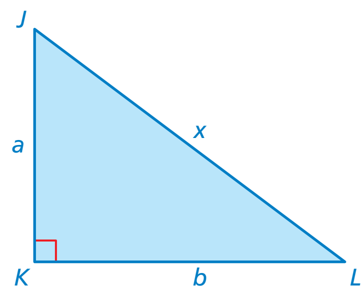
Work with a partner. The converse of the Pythagorean Theorem states: "If the equation $a^2 + b^2 = c^2$ is true for the side lengths of a triangle, then the triangle is a right triangle."



a. Do you think the converse of the Pythagorean Theorem is true or false? How could you use deductive reasoning to support your answer?

b. Consider $\triangle DEF$ with side lengths a, b, and c, such that $a^2 + b^2 = c^2$. Also consider $\triangle JKL$ with leg lengths a and b, where $\angle K = 90°$.

- What does the Pythagorean Theorem tell you about $\triangle JKL$?
- What does this tell you about c and x?
- What does this tell you about $\triangle DEF$ and $\triangle JKL$?
- What does this tell you about $\angle E$?
- What can you conclude?



COMMON CORE

Pythagorean Theorem

In this lesson, you will
- identify right triangles.
- find distances between two points.
- solve real-life problems.

Learning Standards
8.G.6
8.G.7
8.G.8

Laurie's Notes

Introduction

Standards for Mathematical Practice

- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** Students will develop a "proof" of the converse of the Pythagorean Theorem, and they will derive the distance formula. It is important that students be able to explain the steps in their work and compare it to the reasoning of their classmates.

Motivate

- Explain what the converse of a statement is.
- **Example:** If I live in Moab, then I live in Utah.
  The converse is: If I live in Utah, then I live in Moab.
  The original statement is true, but the converse is false.
- Ask students to write two if-then statements with converses that are true and two with converses that are false. The original statements must be true.
- Give students a few minutes and then share some of their examples.

Activity Notes

Activity 1

- Review the meaning of the word counterexample.
- It is important for students to understand that a statement might be false even if they cannot think of a counterexample.
- In part (d), remind students to think about irrational numbers.
- **Big Idea:** Even when a conditional statement is true, its converse does not have to be true. Students should keep this in mind for the next activity.

Activity 2

- Students often say that the Pythagorean Theorem is simply $a^2 + b^2 = c^2$. Tell them that it is actually a conditional statement. If a and b are the lengths of the legs and c is the length of the hypotenuse of a right triangle, then $a^2 + b^2 = c^2$.
- ? "What is the converse of the Pythagorean Theorem?" If $a^2 + b^2 = c^2$, then a triangle with side lengths a, b, and c is a right triangle.
- ? "Do you think the converse of the Pythagorean Theorem is true?" Students may simply guess at this point.
- **MP3:** In this activity, students use deductive reasoning to show that the converse is true. They may need guidance in linking their reasoning.
- **FYI:** Strategies for proofs are taught in later courses, so the framework of a proof is provided.
- In the second bullet, we are looking for $c^2 = x^2$ and then $c = x$. Students found this was not always true in Activity 1 part (a), but point out that it is true here because c and x must be positive.
- In Grade 7 Accelerated Additional Topic 2, students discovered that you can only construct one triangle given three side lengths. Review this concept. It will help them with the third bullet.

Common Core State Standards

8.G.6 Explain a proof of the Pythagorean Theorem and its converse.

8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Previous Learning

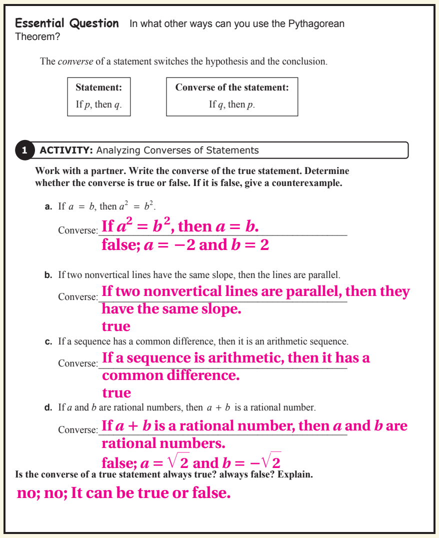
Students should know how to use the Pythagorean Theorem.

Technology for the Teacher

Dynamic Classroom

Lesson Plans
Complete Materials List

10.4 Record and Practice Journal



Essential Question In what other ways can you use the Pythagorean Theorem?

The *converse* of a statement switches the hypothesis and the conclusion.

Statement:	Converse of the statement:
If p, then q.	If q, then p.

1 ACTIVITY: Analyzing Converses of Statements

Work with a partner. Write the converse of the true statement. Determine whether the converse is true or false. If it is false, give a counterexample.

a. If $a = b$, then $a^2 = b^2$.
Converse: If $a^2 = b^2$, then $a = b$.
false; $a = -2$ and $b = 2$

b. If two nonvertical lines have the same slope, then the lines are parallel.
Converse: If two nonvertical lines are parallel, then they have the same slope.
true

c. If a sequence has a common difference, then it is an arithmetic sequence.
Converse: If a sequence is arithmetic, then it has a common difference.
true

d. If a and b are rational numbers, then $a + b$ is a rational number.
Converse: If $a + b$ is a rational number, then a and b are rational numbers.
false; $a = \sqrt{2}$ and $b = -\sqrt{2}$

Is the converse of a true statement always true? always false? Explain.

no; no; It can be true or false.

English Language Learners

Comprehension

English language learners may struggle with the concept of the converse of a statement. Some may think that the converse is always true. Give an example where the converse of a statement is not true.

Statement: *If a figure is a square, then it has four right angles.*

Converse of the Statement: *If a figure has four right angles, then the figure is a square.*

The converse of the statement is not always true. The figure could be a rectangle.

10.4 Record and Practice Journal

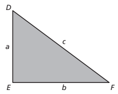
2 ACTIVITY: The Converse of the Pythagorean Theorem

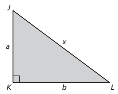
Work with a partner. The converse of the Pythagorean Theorem states: "If the equation $a^2 + b^2 = c^2$ is true of the side lengths of a triangle, then the triangle is a right triangle."

a. Do you think the converse of the Pythagorean Theorem is true or false? How could you use deductive reasoning to support your answer?

true

b. Consider $\triangle DEF$ with side lengths a, b, and c, such that $a^2 + b^2 = c^2$. Also consider $\triangle JKL$ with leg lengths a and b, where $\angle K = 90°$.





- What does the Pythagorean Theorem tell you about $\triangle JKL$?

  $a^2 + b^2 = x^2$

- What does this tell you about c and x?

  $c = x$

- What does this tell you about $\triangle DEF$ and $\triangle JKL$?

  $\triangle DEF \cong \triangle JKL$

- What does this tell you about $\angle E$?

  $m\angle E = 90°$

- What can you conclude?

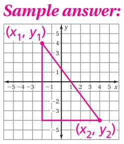
  $\triangle DEF$ is a right triangle.

3 ACTIVITY: Developing the Distance Formula

Work with a partner. Follow the steps below to write a formula that you can use to find the distance between and two points in a coordinate plane.

Step 1: Choose two points in the coordinate plane that do not lie on the same horizontal or vertical line. Label the points (x_1, y_1) and (x_2, y_2).

Sample answer:



Step 2: Draw a line segment connecting the points. This will be the hypotenuse of a right triangle.

Step 3: Draw horizontal and vertical line segments from the points to form the legs of the right triangle.

Step 4: Use the x-coordinates to write an expression for the length of the horizontal leg.

$|x_2 - x_1|$

Step 5: Use the y-coordinates to write an expression for the length of the vertical leg.

$|y_2 - y_1|$

Step 6: Substitute the expressions for the lengths of the legs into the Pythagorean Theorem.

$(x_2 - x_1)^2 + (y_2 - y_1)^2 = c^2$

Step 7: Solve the equation in Step 6 for the hypotenuse c.

$c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

What does the length of the hypotenuse tell you about the two points?

It is the distance between the two points.

What Is Your Answer?

4. **IN YOUR OWN WORDS** In what other ways can you use the Pythagorean Theorem?

   Show a triangle is a right triangle; Find the distance between two points.

5. What kind of real-life problems do you think the converse of the Pythagorean Theorem can help you solve?

   Check students' work.

Laurie's Notes

Activity 3

- Visually, it will be helpful for students to select lattice points.
- In Step 3, point out that there are two distinct ways they can draw the legs. They can draw them either way because the two possible triangles are congruent.
- **MP7 Look for and Make Use of Structure:** In Steps 4 and 5, students may ask about the order in which the subtraction is performed. Tell them that in Step 6 these expressions are squared. So, the order in which the subtraction is performed does not matter.
- Give students adequate time to read carefully and work through the steps on their own. Resist the temptation to jump in and solve it for them.

What Is Your Answer?

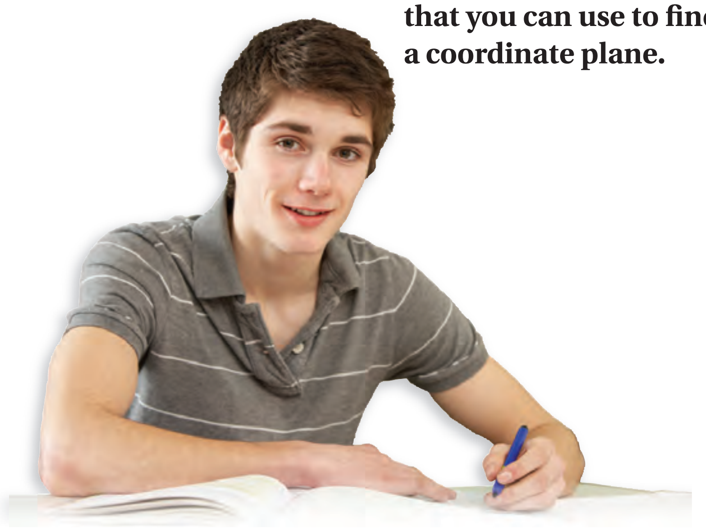
- **Think-Pair-Share:** Students should read each question independently and then work in pairs to answer the questions. When they have answered the questions, the pair should compare their answers with another group and discuss any discrepancies.

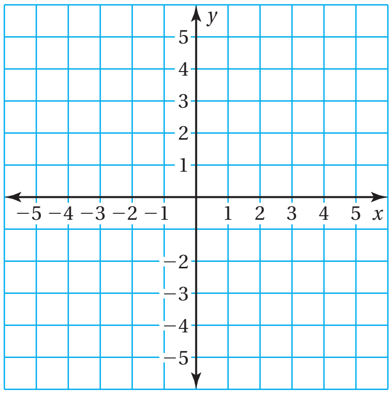
Closure

- **Writing Prompt:** The Pythagorean Theorem can be used to . . . find missing side lengths of right triangles, find the distances between points in a coordinate plane, determine whether given side lengths form a right triangle, etc.

3 ACTIVITY: Developing the Distance Formula

Work with a partner. Follow the steps below to write a formula that you can use to find the distance between any two points in a coordinate plane.





Math Practice 6

Communicate Precisely

What steps can you take to make sure that you have written the distance formula accurately?

Step 1: Choose two points in the coordinate plane that do not lie on the same horizontal or vertical line. Label the points (x_1, y_1) and (x_2, y_2).

Step 2: Draw a line segment connecting the points. This will be the hypotenuse of a right triangle.

Step 3: Draw horizontal and vertical line segments from the points to form the legs of the right triangle.

Step 4: Use the x-coordinates to write an expression for the length of the horizontal leg.

Step 5: Use the y-coordinates to write an expression for the length of the vertical leg.

Step 6: Substitute the expressions for the lengths of the legs into the Pythagorean Theorem.

Step 7: Solve the equation in Step 6 for the hypotenuse c.

What does the length of the hypotenuse tell you about the two points?

What Is Your Answer?

4. **IN YOUR OWN WORDS** In what other ways can you use the Pythagorean Theorem?

5. What kind of real-life problems do you think the converse of the Pythagorean Theorem can help you solve?

Practice Use what you learned about the converse of a true statement to complete Exercises 3 and 4 on page 530.

10.4 Lesson

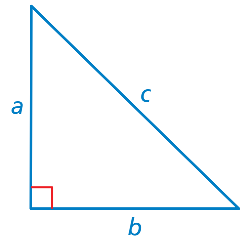
Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
distance formula, *p. 528*

Key Ideas

Converse of the Pythagorean Theorem

If the equation $a^2 + b^2 = c^2$ is true for the side lengths of a triangle, then the triangle is a right triangle.

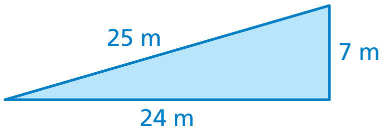


EXAMPLE 1 Identifying a Right Triangle

Study Tip
A *Pythagorean triple* is a set of three positive integers *a*, *b*, and *c*, where $a^2 + b^2 = c^2$.

Tell whether each triangle is a right triangle.

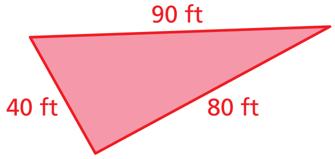
a.



$$a^2 + b^2 = c^2$$
$$7^2 + 24^2 \stackrel{?}{=} 25^2$$
$$49 + 576 \stackrel{?}{=} 625$$
$$625 = 625 \ \checkmark$$

It *is* a right triangle.

b.



$$a^2 + b^2 = c^2$$
$$40^2 + 80^2 \stackrel{?}{=} 90^2$$
$$1600 + 6400 \stackrel{?}{=} 8100$$
$$8000 \neq 8100 \ \times$$

It *is not* a right triangle.

Common Error
When using the converse of the Pythagorean Theorem, always substitute the length of the longest side for c.

On Your Own

Now You're Ready
Exercises 5–10

Tell whether the triangle with the given side lengths is a right triangle.

1. 15 cm, 10 cm, 18 cm
2. 50 yd, 40 yd, 30 yd

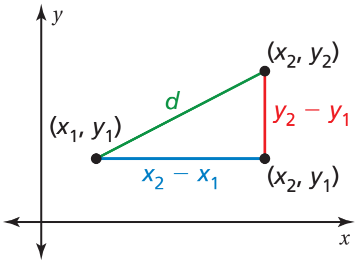
On page 527, you used the Pythagorean Theorem to develop the *distance formula*. You can use the **distance formula** to find the distance between any two points in a coordinate plane.

Key Idea

Distance Formula

The distance d between any two points (x_1, y_1) and (x_2, y_2) is given by the formula

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.



Laurie's Notes

Introduction

Connect

- **Yesterday:** Students proved the converse of the Pythagorean Theorem and developed the distance formula. (MP3, MP7)
- **Today:** Students will use the converse of the Pythagorean Theorem and the distance formula.

Motivate

- Write the following numbers on the board: 3-4-5, 5-12-13, and 8-15-17. Tell students that these are called *Pythagorean triples*. They are positive integers that satisfy the Pythagorean Theorem.
- Multiples of these examples also satisfy the Pythagorean Theorem. Have students try to name a few more Pythagorean triples.

Lesson Notes

Key Idea

- Write the Key Idea stating the converse of the Pythagorean Theorem. It is one way of determining whether given side lengths form a right triangle.

Example 1

- Draw the first triangle.
- Say, "If this is a right triangle, then the side lengths must satisfy the Pythagorean Theorem."
- "Satisfying the theorem" simply means that the sum of the squares of the two lesser numbers must equal the square of the greatest number.
- Write $a^2 + b^2 = c^2$. Substitute for each value and simplify.
- **Common Error:** Students may forget to substitute the longest side for c.
- Work through each example.
- **Extension:** If it is not a right triangle, it is possible to determine whether the triangle is acute or obtuse. Interested students can investigate this using dynamic software or by traditional research.

On Your Own

- The side lengths of the triangles are not listed from least to greatest. Make sure that students substitute correctly.

Key Idea

- Write the Key Idea that states the distance formula, derived from the Pythagorean Theorem.
- Note the use of colors in the diagram. The same colors can be used when writing the formula.
- **MP7 Look for and Make Use of Structure:** Discuss with students that the order in which the subtraction is performed is not important because the difference is squared.

Goal Today's lesson is using the converse of the Pythagorean Theorem and the **distance formula**.

Technology for the Teacher

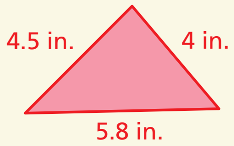
Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Extra Example 1

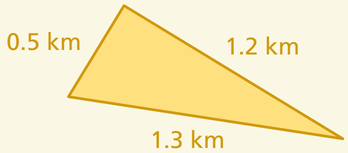
Tell whether each triangle is a right triangle.

a.



not a right triangle

b.



right triangle

On Your Own

1. no
2. yes

Laurie's Notes

Extra Example 2

Find the distance between (1, −5) and (7, 4). $3\sqrt{13}$

Extra Example 3

In Example 3, the receiver starts at (40, 40). Did the receiver run the play as designed? yes

On Your Own

3. $\sqrt{29}$

4. $\sqrt{146}$

5. $8\sqrt{5}$

6. yes; The points (60, 50), (30, 20), and (80, −30) form a right triangle.

Differentiated Instruction

Inclusion

Encourage students to learn the Pythagorean Theorem using the language of the triangle, $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$. Have them label each side as a leg or hypotenuse before substituting the numbers into the equation.

Example 2

- Although it is not necessary, you can plot the two points as a visual aid.
- The choice of which point is (x_1, y_1) and which point is (x_2, y_2) is arbitrary. If time permits, do the problem both ways to show that the result is the same.
- Caution students to be careful with the subtraction. It is easy to make a careless calculation mistake.
- There are no units associated with the answer. In a real-life example, students would need to label the units in their answer.

Example 3

- Ask a student to read the example.
- **MP4 Modeling with Mathematics:** Sketch the coordinate plane shown with the ordered pairs identified.
- ? "How do you determine if the receiver ran the play as designed?" Check whether the triangle is a right triangle.
- ? "How do you determine if the triangle is a right triangle?" Use the converse of the Pythagorean Theorem.
- Use the distance formula to find the length of each side of the triangle.
- **MP7:** The distances found could be simplified. However, you will be using the converse and squaring again, so it is easier to leave the results as shown in the example.
- Use the converse of the Pythagorean Theorem to show that it is not a right triangle.
- Note that students could also use what they know about slopes of perpendicular lines to solve this problem.

On Your Own

- There are multiple steps required in each question. You may wish to divide the class into groups and assign a different question to each group.

Closure

- **Exit Ticket:** Find the distance between (−4, 6) and (3, −2). $\sqrt{113}$

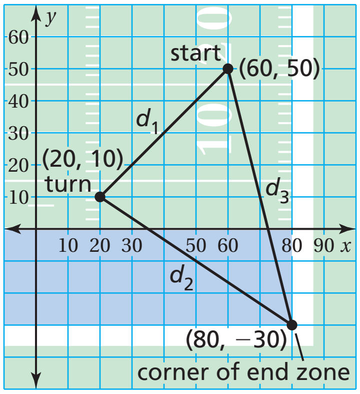
EXAMPLE 2 Finding the Distance Between Two Points

Find the distance between (−3, 5) and (2, −1).

Let $(x_1, y_1) = (-3, 5)$ and $(x_2, y_2) = (2, -1)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Write the distance formula.
$= \sqrt{[2 - (-3)]^2 + (-1 - 5)^2}$	Substitute.
$= \sqrt{5^2 + (-6)^2}$	Simplify.
$= \sqrt{25 + 36}$	Evaluate powers.
$= \sqrt{61}$	Add.

EXAMPLE 3 Real-Life Application



A football coach designs a passing play in which a receiver runs down the field, makes a 90° turn, and runs to the corner of the end zone. A receiver runs the play as shown. Did the receiver run the play as designed? Each unit of the grid represents 10 feet.

Use the distance formula to find the lengths of the three sides.

$$d_1 = \sqrt{(60 - 20)^2 + (50 - 10)^2} = \sqrt{40^2 + 40^2} = \sqrt{3200} \text{ feet}$$

$$d_2 = \sqrt{(80 - 20)^2 + (-30 - 10)^2} = \sqrt{60^2 + (-40)^2} = \sqrt{5200} \text{ feet}$$

$$d_3 = \sqrt{(80 - 60)^2 + (-30 - 50)^2} = \sqrt{20^2 + (-80)^2} = \sqrt{6800} \text{ feet}$$

Use the converse of the Pythagorean Theorem to determine if the side lengths form a right triangle.

$$(\sqrt{3200})^2 + (\sqrt{5200})^2 \stackrel{?}{=} (\sqrt{6800})^2$$

$$3200 + 5200 \stackrel{?}{=} 6800$$

$$8400 \neq 6800 \quad ✗$$

It is not a right triangle. So, the receiver did not make a 90° turn.

The receiver did not run the play as designed.

On Your Own

Now You're Ready
Exercises 11–16

Find the distance between the two points.

3. (0, 4), (5, 2) **4.** (−1, 3), (4, −8) **5.** (−10, −6), (6, 2)

6. WHAT IF? In Example 3, the receiver made the turn at (30, 20). Did the receiver run the play as designed? Explain.

10.4 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **WRITING** Describe two ways to find the distance between two points in a coordinate plane.

2. **WHICH ONE DOESN'T BELONG?** Which set of numbers does not belong with the other three? Explain your reasoning.

3, 4, 5	8, 15, 17	18, 22, 29	9, 40, 41

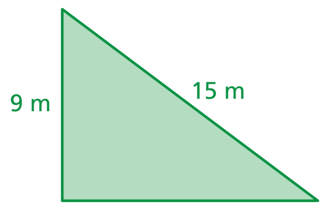
Practice and Problem Solving

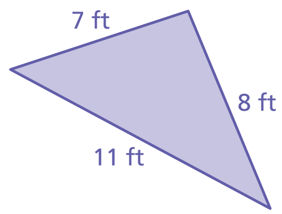
Write the converse of the true statement. Determine whether the converse is true or false.

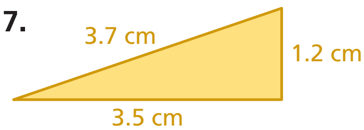
3. If a is an even number, then a^2 is even.

4. If a is positive, then $|a| = a$.

Tell whether the triangle with the given side lengths is a right triangle.

1 5. 

6. 

7. 

8. 16 in., 18 in., 24 in.

9. 30 yd, 22 yd, 15 yd

10. 8 mm, 15 mm, 17 mm

Find the distance between the two points.

2 11. $(4, -3), (-2, -5)$

12. $(1, 1), (4, 5)$

13. $(-7, -1), (-4, 8)$

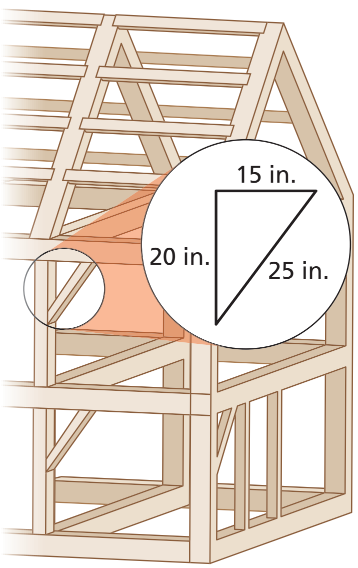
14. $(1, 3), (7, 7)$

15. $(2, -8), (4, -1)$

16. $(-7, 4), (-3, 2)$

17. **ERROR ANALYSIS** Describe and correct the error in finding the distance between the points $(-4, -3)$ and $(2, -1)$.

$$\begin{aligned} d &= \sqrt{[2-(-4)]^2 - [-1-(-3)]^2} \\ &= \sqrt{36-4} \\ &= 4\sqrt{2} \end{aligned}$$



18. **CONSTRUCTION** A post and beam frame for a shed is shown in the diagram. Does the brace form a right triangle with the post and beam? Explain.

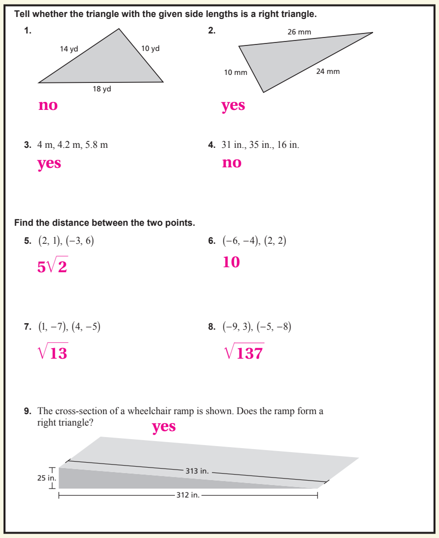
Assignment Guide and Homework Check

Level	Assignment	Homework Check
Average	1–4, 5–25 odd, 18, 27–30	9, 13, 17, 23, 25
Advanced	1–4, 6–20 even, 22–26, 27–30	8, 16, 23, 25, 26

Common Errors

- **Exercises 5–10, 18** Students may substitute the wrong value for c in the Pythagorean Theorem. Remind them that c will be the longest side, so they should substitute the greatest value for c.
- **Exercises 11–16** Students may mismatch the x-values and y-values when using the distance formula. This will result in students subtracting an x from a y, or vice versa. Encourage students to pair the numbers properly.
- **Exercises 11–16** Students may get careless when squaring negative numbers. Remind them that everything inside the parentheses is squared, including the minus sign, and that the square of a negative is a positive.
- **Exercise 25** Students may pick Plane A because it appears to be closer. Remind students that the drawing is not to scale. Tell them to calculate the distances before answering the question.

10.4 Record and Practice Journal



Tell whether the triangle with the given side lengths is a right triangle.

1. 14 yd, 10 yd, 18 yd
no

2. 26 mm, 10 mm, 24 mm
yes

3. 4 m, 4.2 m, 5.8 m
yes

4. 31 in., 35 in., 16 in.
no

Find the distance between the two points.

5. $(2, 1), (-3, 6)$
$5\sqrt{2}$

6. $(-6, -4), (2, 2)$
10

7. $(1, -7), (4, -5)$
$\sqrt{13}$

8. $(-9, 3), (-5, -8)$
$\sqrt{137}$

9. The cross-section of a wheelchair ramp is shown. Does the ramp form a right triangle?
yes

25 in. 313 in. 312 in.

Vocabulary and Concept Check

1. the Pythagorean Theorem and the distance formula
2. 18, 22, 29; This set does not form a right triangle.

Practice and Problem Solving

3. If a^2 is even, then a is an even number; true
4. If $|a| = a$, then a is positive; false (counterexample: $a = 0$)
5. yes
6. no
7. yes
8. no
9. no
10. yes
11. $2\sqrt{10}$
12. 5
13. $3\sqrt{10}$
14. $2\sqrt{13}$
15. $\sqrt{53}$
16. $2\sqrt{5}$
17. The squared quantities under the radical should be added, not subtracted; $2\sqrt{10}$
18. yes; The side lengths satisfy the converse of the Pythagorean Theorem.

Practice and Problem Solving

19. yes **20.** no

21. yes

22. yes; Use the distance formula to find the lengths of the three sides. Use the converse of the Pythagorean Theorem to show they form a right triangle.

23. See *Taking Math Deeper.*

24. yes; $\sqrt{41}$; Because you square the differences $(x_2 - x_1)$ and $(y_2 - y_1)$, it does not matter if the differences are positive or negative. The squares of opposite numbers are equivalent.

25. Plane B; Plane A is about 8.35 kilometers away and Plane B is about 8.27 kilometers away.

26. See Additional Answers.

Fair Game Review

27. $x = \frac{3}{2}, x = 1$

28. $x = -\frac{1}{2}, x = 1$

29. no real solutions

30. A

Mini-Assessment

Tell whether the triangle with the given side lengths is a right triangle.

1. 32 m, 56 m, 64 m no

2. 1.8 mi, 8 mi, 8.2 mi yes

Find the distance between the two points.

3. $(-3, -1), (6, 2)$ $3\sqrt{10}$

4. $(2, 10), (5, -4)$ $\sqrt{205}$

Taking Math Deeper

Exercise 23

You can use what you learned about slopes of perpendicular lines to solve this problem.

1 Interpret the diagram.

From the diagram, you can see that your path forms a right triangle when the angle at Container 1 is a right angle.

2 Find the slopes of the line segments that meet at Container 1.

Car to Container 1

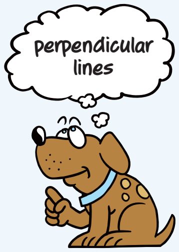
(10, 50) to (20, −10)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-10 - 50}{20 - 10} = \frac{-60}{10} = -6$$

Container 1 to Container 2

(20, −10) to (80, 0)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-10)}{80 - 20} = \frac{10}{60} = \frac{1}{6}$$



3 Interpret the slopes.

Because $-6 \cdot \frac{1}{6} = -1$, the line segments are perpendicular. So, the angle at Container 1 is a right angle, meaning that the path formed is a right triangle.

Project

Research global positioning systems. How do they work?

Reteaching and Enrichment Strategies

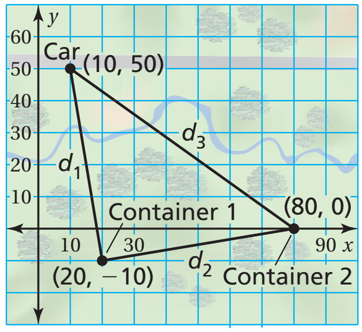
If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension • School-to-Work Start the next section

Tell whether the set of measurements can be the side lengths of a right triangle.

19. $5\sqrt{5}$, 10, 15 **20.** 7, $3\sqrt{10}$, 6 **21.** 21, 72, 75

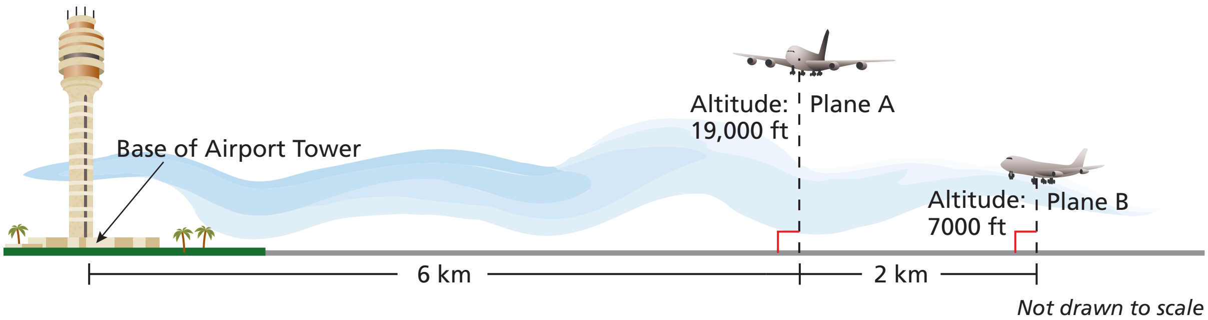
22. **REASONING** Plot the points $(-1, -2)$, $(2, 1)$, and $(-3, 6)$ in a coordinate plane. Are the points the vertices of a right triangle? Explain.

23. **GEOCACHING** You spend the day looking for hidden containers in a wooded area using a global positioning system (GPS). You park your car on the side of the road and then locate Container 1 and Container 2 before going back to the car. Does your path form a right triangle? Explain. Each unit of the grid represents 10 yards.

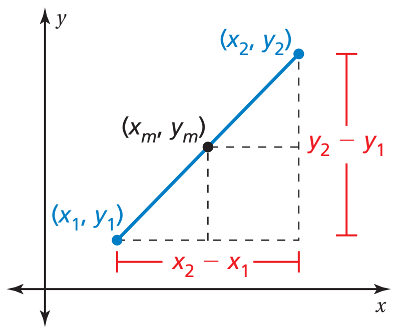


24. **REASONING** Your teacher wants the class to find the distance between the two points (3, 2) and (8, 6). You choose (3, 2) for (x_1, y_1) and your friend chooses (8, 6) for (x_1, y_1). Do you and your friend obtain the same answer? Justify your answer.

25. **AIRPORT** Which plane is closer to the base of the airport tower? Explain.



26. **Structure** Consider the two points (x_1, y_1) and (x_2, y_2) in the coordinate plane. How can you find the point (x_m, y_m) located in the middle of the two given points? Justify your answer using the distance formula.



Fair Game Review What you learned in previous grades & lessons

Solve the equation using the quadratic formula. *(Section 9.4)*

27. $2x^2 - 5x + 3 = 0$ **28.** $2x^2 - x - 1 = 0$ **29.** $x^2 + 3x + 5 = 0$

30. **MULTIPLE CHOICE** Which point is the focus of the graph of $y = 2x^2$? *(Section 8.2)*

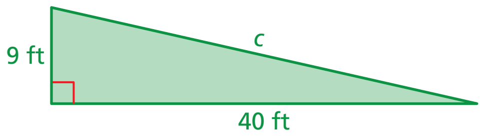
Ⓐ $\left(0, \frac{1}{8}\right)$ Ⓑ $\left(0, \frac{1}{4}\right)$ Ⓒ $\left(0, \frac{1}{2}\right)$ Ⓓ $\left(0, -\frac{1}{8}\right)$

10.3 – 10.4 Quiz

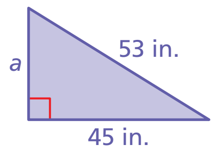
Check It Out
Progress Check
BigIdeasMath.com

Find the missing length of the triangle. *(Section 10.3)*

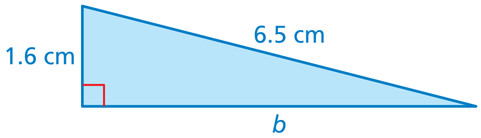
1.



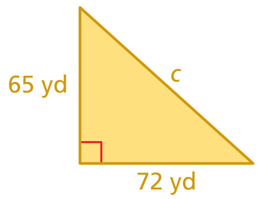
2.



3.

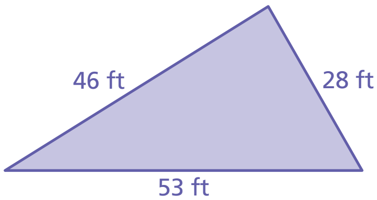


4.

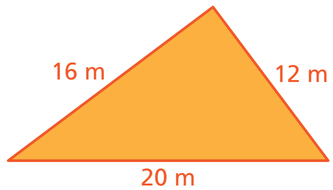


Tell whether the triangle with the given side lengths is a right triangle. *(Section 10.4)*

5.



6.



Find the distance between the two points. *(Section 10.4)*

7. $(-3, -1), (-1, -5)$

8. $(-4, 2), (5, 1)$

9. $(1, -2), (4, -5)$

10. $(-1, 1), (7, 4)$

11. $(-6, 5), (-4, -6)$

12. $(-1, 4), (1, 3)$

13. **FABRIC** You cut a rectangular piece of fabric in half along the diagonal. The fabric measures 28 inches wide and $1\frac{1}{4}$ yards long. What is the length (in inches) of the diagonal? *(Section 10.3)*

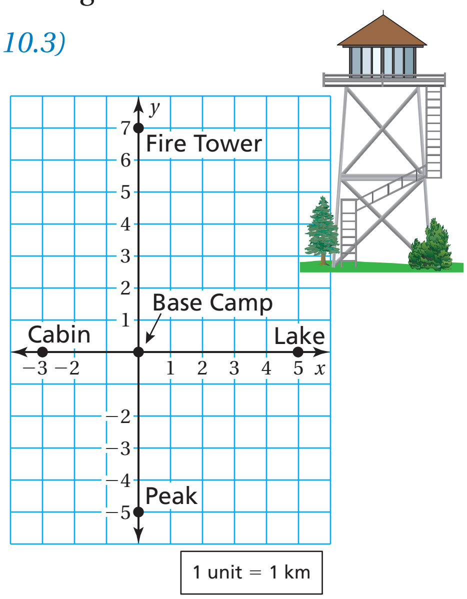
Use the figure to answer Exercises 14–17. *(Section 10.4)*

14. How far is the cabin from the peak?

15. How far is the fire tower from the lake?

16. How far is the lake from the peak?

17. You are standing at $(-5, -6)$. How far are you from the lake?



Alternative Assessment Options

Math Chat	Student Reflective Focus Question
Structured Interview	Writing Prompt

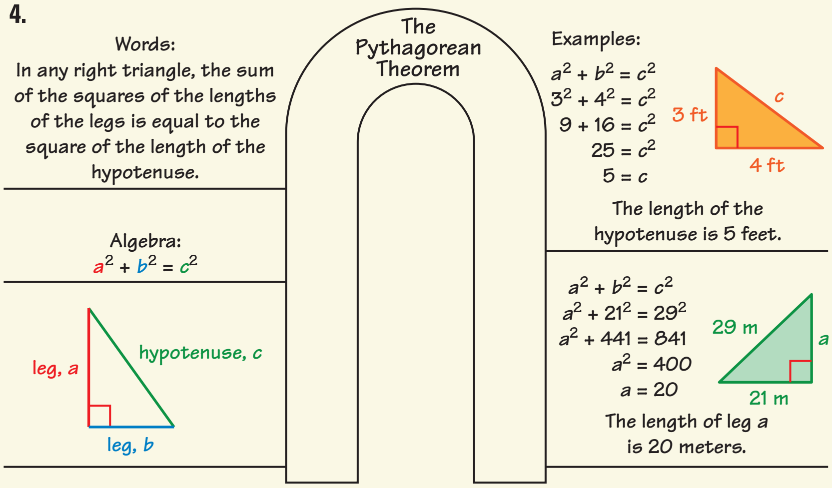
Math Chat

- Have students work in pairs. Assign Quiz Exercises 14–17 to each pair. Each student works through all four problems. After the students have worked through the problems, they take turns talking through the processes that they used to get each answer. Students analyze and evaluate the mathematical thinking and strategies used.
- The teacher should walk around the classroom listening to the pairs and ask questions to ensure understanding.

Study Help Sample Answers

Remind students to complete Graphic Organizers for the rest of the chapter.

4.



5–6. Available at *BigIdeasMath.com.*

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Study Help • Practice A and Practice B • Puzzle Time Lesson Tutorials *BigIdeasMath.com*	Resources by Chapter • Enrichment and Extension • School-to-Work Game Closet at *BigIdeasMath.com* Start the Chapter Review

Answers

1. 41 ft
2. 28 in.
3. 6.3 cm
4. 97 yd
5. no
6. yes
7. $2\sqrt{5}$
8. $\sqrt{82}$
9. $3\sqrt{2}$
10. $\sqrt{73}$
11. $5\sqrt{5}$
12. $\sqrt{5}$
13. 53 in.
14. $\sqrt{34}$ km
15. $\sqrt{74}$ km
16. $5\sqrt{2}$ km
17. $2\sqrt{34}$ km

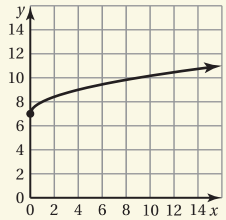
Technology for the Teacher

Online Assessment
Assessment Book
ExamView® Assessment Suite

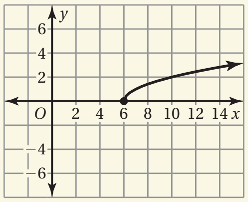
For the Teacher
Additional Review Options

- *BigIdeasMath.com*
- Online Assessment
- Game Closet at *BigIdeasMath.com*
- Vocabulary Help
- Resources by Chapter

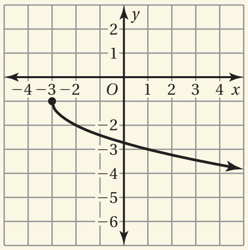
Answers

1. 

domain: $x \geq 0$; range: $y \geq 7$; The graph of $y = \sqrt{x} + 7$ is a translation 7 units up of the graph of $y = \sqrt{x}$.

2. 

domain: $x \geq 6$; range: $y \geq 0$; The graph of $y = \sqrt{x - 6}$ is a translation 6 units to the right of the graph of $y = \sqrt{x}$.

3. 

domain: $x \geq -3$; range: $y \leq -1$; The graph of $y = -\sqrt{x + 3} - 1$ is a reflection in the x-axis of the graph of $y = \sqrt{x}$ and then a translation 3 units to the left and 1 unit down.

Review of Common Errors

- **Exercises 1–3** Students may treat constants added to a radical expression as part of the radicand, or vice versa. Encourage them to identify the radicand before solving the exercise.
- **Exercises 4 and 5** Students may square each side of the equation before isolating the square root on one side of the equation.
- **Exercises 8 and 9** Students may substitute the given lengths in the wrong part of the formula. For example, if they are finding one of the legs, they may write $5^2 + 13^2 = c^2$ instead of $5^2 + b^2 = 13^2$. Remind them that the side opposite the right angle is the hypotenuse c.
- **Exercises 8 and 9** Students may multiply each side length by two instead of squaring the side length. Remind them of the definition of exponents.
- **Exercises 10 and 11** Students may substitute the wrong value for c in the Pythagorean Theorem. Remind them that c will be the longest side, so they should substitute the greatest value for c.
- **Exercises 12 and 13** Students may mismatch the x-values and y-values when using the distance formula. This will result in students subtracting an x from a y, or vice versa. Encourage students to pair the numbers properly.

10 Chapter Review

Check It Out
Vocabulary Help
BigIdeasMath.com

Review Key Vocabulary

square root function, *p. 504*
simplest form of a radical expression, *p. 508*
rationalizing the denominator, *p. 508*
conjugates, *p. 509*
square root equation, *p. 512*
extraneous solution, *p. 513*
theorem, *p. 520*
legs, *p. 522*
hypotenuse, *p. 522*
Pythagorean Theorem, *p. 522*
distance formula, *p. 528*

Review Examples and Exercises

10.1 Graphing Square Root Functions *(pp. 502–509)*

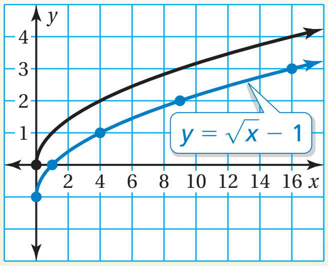
a. Graph $y = \sqrt{x} - 1$. Describe the domain and range. Compare the graph to the graph of $y = \sqrt{x}$.

Step 1: Make a table of values.

x	0	1	4	9	16
y	-1	0	1	2	3

Step 2: Plot the ordered pairs.

Step 3: Draw a smooth curve through the points.



The domain is $x \geq 0$. The range is $y \geq -1$. The graph of $y = \sqrt{x} - 1$ is a translation 1 unit down of the graph of $y = \sqrt{x}$.

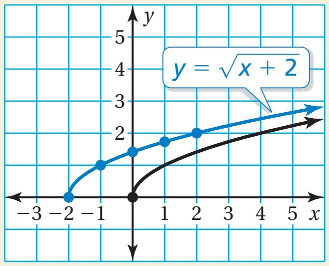
b. Graph $y = \sqrt{x+2}$. Describe the domain and range. Compare the graph to the graph of $y = \sqrt{x}$.

Step 1: Make a table of values.

x	-2	-1	0	1	2
y	0	1	1.4	1.7	2

Step 2: Plot the ordered pairs.

Step 3: Draw a smooth curve through the points.



The domain is $x \geq -2$. The range is $y \geq 0$. The graph of $y = \sqrt{x+2}$ is a translation 2 units to the left of the graph of $y = \sqrt{x}$.

Exercises

Graph the function. Describe the domain and range. Compare the graph to the graph of $y = \sqrt{x}$.

1. $y = \sqrt{x} + 7$

2. $y = \sqrt{x - 6}$

3. $y = -\sqrt{x + 3} - 1$

10.2 Solving Square Root Equations (pp. 510–517)

Solve $\sqrt{12 - x} = x$.

$\sqrt{12 - x} = x$ — Write the equation.

$(\sqrt{12-x})^2 = x^2$ — Square each side of the equation.

$12 - x = x^2$ — Simplify.

$0 = x^2 + x - 12$ — Rewrite equation.

$0 = (x - 3)(x + 4)$ — Factor.

$(x - 3) = 0 \quad or \quad (x + 4) = 0$ — Use Zero-Product Property.

$x = 3 \quad or \quad x = -4$ — Solve for x.

Check $\sqrt{12 - 3} \stackrel{?}{=} 3$ — Substitute for x. — $\sqrt{12 - (-4)} \stackrel{?}{=} -4$

$\sqrt{9} \stackrel{?}{=} 3$ — Simplify. — $\sqrt{16} \stackrel{?}{=} -4$

$3 = 3$ ✓ — $4 \neq -4$ ✗

Because $x = -4$ does not check in the original equation, it is an extraneous solution. The only solution is $x = 3$.

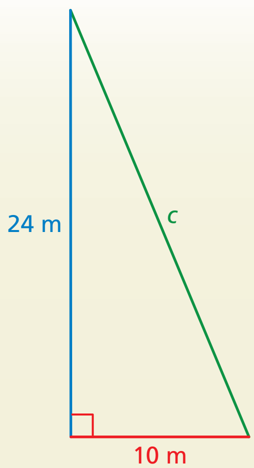
Exercises

Solve the equation. Check your solution.

4. $8 + \sqrt{x} = 18$
5. $\sqrt{x - 1} + 9 = 15$
6. $\sqrt{5x - 9} = \sqrt{4x}$
7. $x = \sqrt{3x + 4}$

10.3 The Pythagorean Theorem (pp. 520–525)

Find the length of the hypotenuse of the triangle.



$a^2 + b^2 = c^2$ — Write the Pythagorean Theorem.

$10^2 + 24^2 = c^2$ — Substitute.

$100 + 576 = c^2$ — Evaluate powers.

$676 = c^2$ — Add.

$\sqrt{676} = \sqrt{c^2}$ — Take positive square root of each side.

$26 = c$ — Simplify.

The length of the hypotenuse is 26 meters.

Review Game

Pythagorean Theorem

Materials per group:

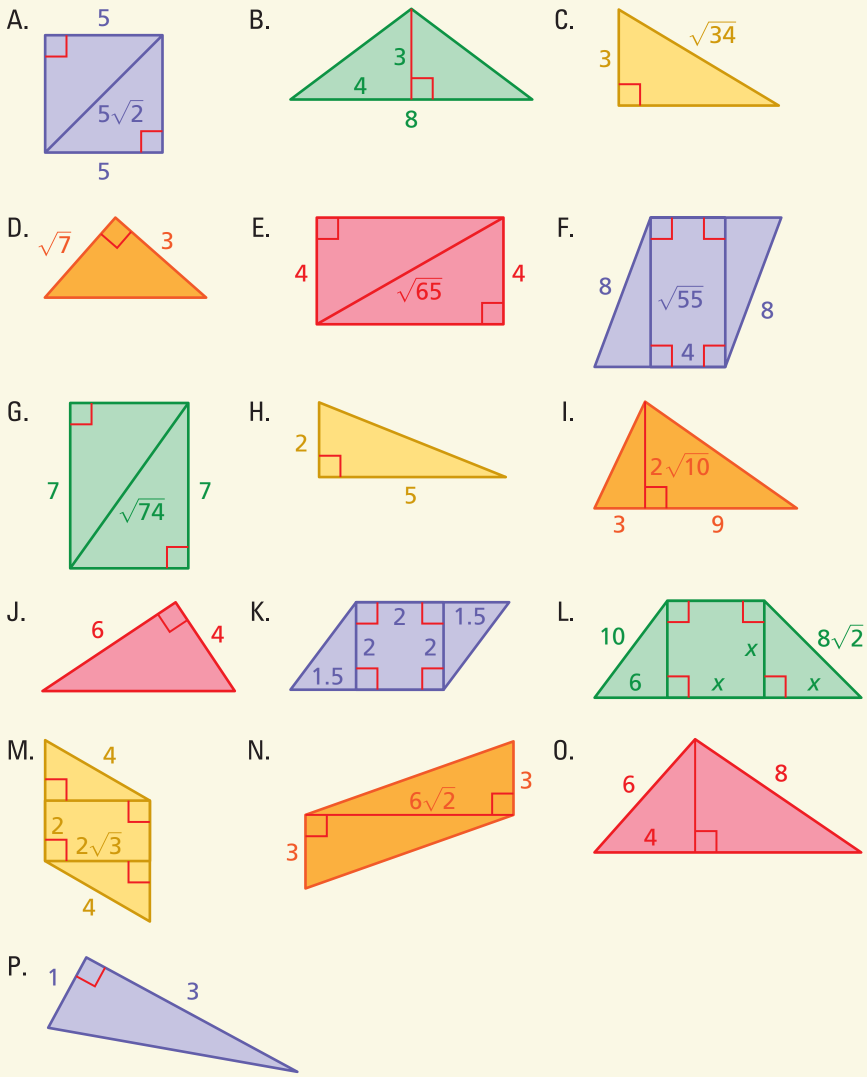
- copies of the polygons below
- pencil
- paper
- calculator

Directions

Play in pairs. Students take turns picking a polygon and finding its perimeter. (Round the perimeter to the nearest tenth, if necessary.) Play continues as students add each new perimeter to their previous perimeters, until all polygons are used. Polygons may only be used once. All measurements are in inches.

Who wins?

The student with the greatest perimeter total wins.



For the Student
Additional Practice

- Lesson Tutorials
- Multi-Language Glossary
- Self-Grading Progress Check
- *BigIdeasMath.com*
  Dynamic Student Edition
  Student Resources

Answers

4. $x = 100$

5. $x = 37$

6. $x = 9$

7. $x = 4$

8. 50 in.

9. 0.8 cm

10. yes

11. no

12. $5\sqrt{5}$

13. $\sqrt{113}$

My Thoughts on the Chapter

Teacher Tip

Not allowed to write in your teaching edition? Use sticky notes to record your thoughts.

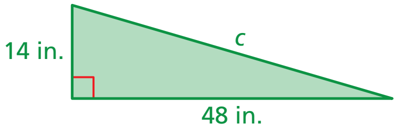
What worked. . .

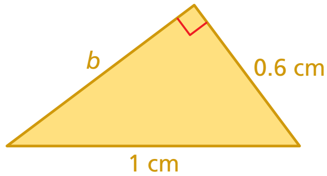
What did not work. . .

What I would do differently. . .

Exercises

Find the missing length of the triangle.

8. 

9. 

10.4 Using the Pythagorean Theorem *(pp. 526–531)*

a. Is the triangle formed by the rope and the tent a right triangle?

$$a^2 + b^2 = c^2$$

$$64^2 + 48^2 \stackrel{?}{=} 80^2$$

$$4096 + 2304 \stackrel{?}{=} 6400$$

$$6400 = 6400 \checkmark$$

∴ It *is* a right triangle.

b. Find the distance between (−3, 1) and (4, 7).

Let $(x_1, y_1) = (-3, 1)$ and $(x_2, y_2) = (4, 7)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Write the distance formula.}$$

$$= \sqrt{[4 - (-3)]^2 + (7 - 1)^2} \quad \text{Substitute.}$$

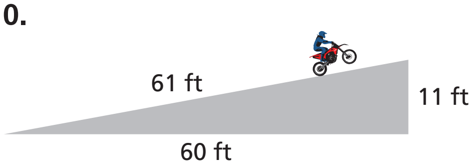
$$= \sqrt{7^2 + 6^2} \quad \text{Simplify.}$$

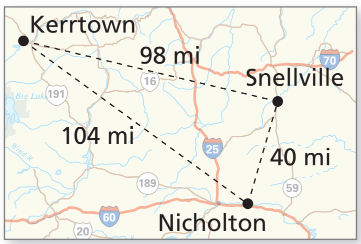
$$= \sqrt{49 + 36} \quad \text{Evaluate powers.}$$

$$= \sqrt{85} \quad \text{Add.}$$

Exercises

Tell whether the triangle is a right triangle.

10. 

11. 

Find the distance between the two points.

12. $(-2, -5), (3, 5)$

13. $(-4, 7), (4, 0)$

10 Chapter Test

Check It Out
Test Practice
BigIdeasMath.com

Graph the function. Describe the domain and range. Compare the graph to the graph of $y = \sqrt{x}$.

1. $y = \sqrt{x} - 6$ **2.** $y = \sqrt{x + 10}$ **3.** $y = -\sqrt{x - 2} + 3$

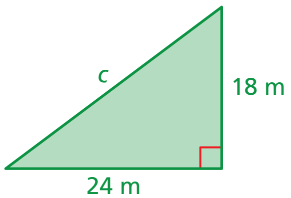
Solve the equation.

4. $9 - \sqrt{x} = 3$ **5.** $\sqrt{2x - 7} - 3 = 6$

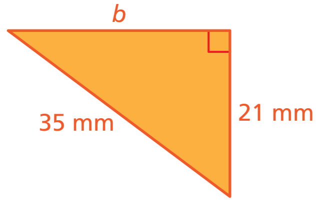
6. $\sqrt{8x - 21} = \sqrt{18 - 5x}$ **7.** $x + 5 = \sqrt{7x + 53}$

Find the missing length of the triangle.

8.

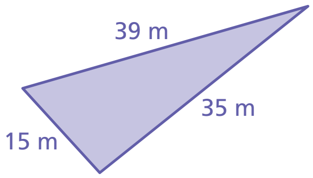


9.

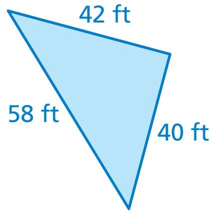


Tell whether the triangle is a right triangle.

10.



11.



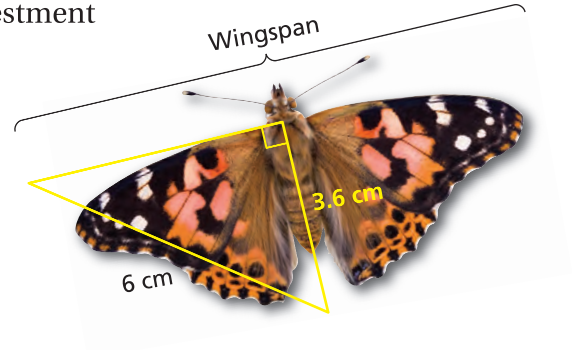
Find the distance between the two points.

12. $(-2, 3), (6, 9)$ **13.** $(0, -5), (4, 1)$ **14.** $(-3, -4), (2, -7)$

15. ROLLER COASTER The velocity v (in meters per second) of a roller coaster at the bottom of a hill is given by $v = \sqrt{19.6h}$, where h is the height of the hill (in meters). (a) Graph the function. Describe the domain and range. (b) How tall must the hill be for the velocity of the roller coaster at the bottom of the hill to be at least 28 meters per second?

16. FINANCE The average annual interest rate r (in decimal form) that an investment earns over 2 years is given by $r = \sqrt{\frac{V_2}{V_0}} - 1$, where V_0 is the initial investment and V_2 is the value of the investment after 2 years. You initially invest \$800 which earns an average annual interest of 6% over 2 years. What is the value of V_2?

17. BUTTERFLY Approximate the wingspan of the butterfly.



Test Item References

Chapter Test Questions	Section to Review	Common Core State Standards
1–3, 15	10.1	F.IF.4, F.IF.7b
4–7, 16	10.2	N.RN.2
8, 9, 17	10.3	8.G.6, 8.G.7
10–14	10.4	8.G.6, 8.G.7, 8.G.8

Test-Taking Strategies

Remind students to quickly look over the entire test before they start so that they can budget their time. Students should estimate and check their answers for reasonableness as they work through the test. Teach students to use the Stop and Think strategy before answering. **Stop** and carefully read the question, and **Think** about what the answer should look like.

Common Errors

- **Exercises 1–3** Students may treat constants added to a radical expression as part of the radicand, or vice versa. Encourage them to identify the radicand before solving the exercise.
- **Exercises 4 and 5** Students may square each side of the equation before isolating the square root on one side of the equation.
- **Exercises 8 and 9** Students may substitute the given lengths in the wrong part of the formula. For example, if they are finding one of the legs, they may write $5^2 + 13^2 = c^2$ instead of $5^2 + b^2 = 13^2$. Remind them that the side opposite the right angle is the hypotenuse c.
- **Exercises 8 and 9** Students may multiply each side length by two instead of squaring the side length. Remind them of the definition of exponents.
- **Exercises 10 and 11** Students may substitute the wrong value for c in the Pythagorean Theorem. Remind them that c will be the longest side, so they should substitute the greatest value for c.
- **Exercises 12–14** Students may mismatch the x-values and y-values when using the distance formula. This will result in students subtracting an x from a y, or vice versa. Encourage students to pair the numbers properly.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials *BigIdeasMath.com* Skills Review Handbook	Resources by Chapter • Enrichment and Extension • School-to-Work Game Closet at *BigIdeasMath.com* Start Standards Assessment

Answers

1–3. See Additional Answers.

4. $x = 36$ **5.** $x = 44$

6. $x = 3$ **7.** $x = 4$

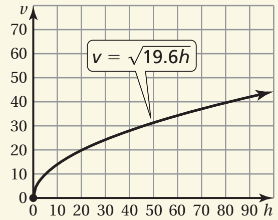
8. 30 m **9.** 28 mm

10. no **11.** yes

12. 10 **13.** $2\sqrt{13}$

14. $\sqrt{34}$

15. a.



domain: $h \geq 0$; range: $v \geq 0$

b. 40 meters

16. \$898.88

17. 9.6 cm

Technology for the Teacher

Online Assessment
Assessment Book
ExamView® Assessment Suite

Test Taking Strategies

Available at *BigIdeasMath.com*

After Answering Easy Questions, Relax
Answer Easy Questions First
Estimate the Answer
Read All Choices before Answering
Read Question before Answering
Solve Directly or Eliminate Choices
Solve Problem before Looking at Choices
Use Intelligent Guessing
Work Backwards

About this Strategy

When taking a multiple choice test, be sure to read each question carefully and thoroughly. Sometimes it is easier to solve the problem and then look for the answer among the choices.

Answers

1. B
2. G
3. 4
4. C
5. G

Technology for the Teacher

Common Core State Standards Support
Performance Tasks
Online Assessment
Assessment Book
ExamView® Assessment Suite

Item Analysis

1. **A.** The student shifts the graph right instead of left.

B. Correct answer

C. The student confuses the vertical and horizontal shifts and shifts the graph right instead of left.

D. The student confuses the vertical and horizontal shifts.

2. **F.** The student adds the exponents for the powers of powers.

G. Correct answer

H. The student rewrites the expression as $\frac{(2^3)^{10} \cdot 2^{-5}}{2^{16}}$ and then adds all of the exponents.

I. The student rewrites $\frac{8^{10}}{16^4}$ as $\left(\frac{1}{2}\right)^{10-4} = \left(\frac{1}{2}\right)^6$.

3. **Gridded response:** Correct answer: 4

Common error: The student fails to check the answers and chooses $x = -2$ as the answer.

4. **A.** The student makes an error either in finding the slope-intercept form of the line, or in checking the point in the equation.

B. The student makes an error either in finding the slope-intercept form of the line, or in checking the point in the equation.

C. Correct answer

D. The student makes an error either in finding the slope-intercept form of the line, or in checking the point in the equation.

5. **F.** The student incorrectly determines that the solutions of $(x - 2)(x - 4) = 0$ are $x = -2$ and $x = -4$, and assumes that $x = -4$ is extraneous.

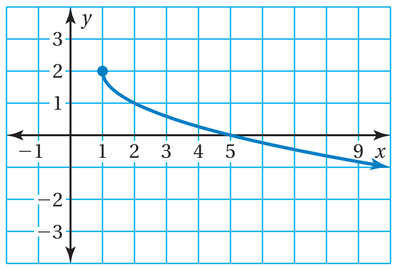
G. Correct answer

H. The student incorrectly assumes that $x = 2$ is extraneous.

I. The student incorrectly determines the solutions of $(x - 2)(x - 4) = 0$.

10 Standards Assessment

1. Which function is shown in the graph? *(F.IF.7b)*



A. $y = -\sqrt{x+1} + 2$

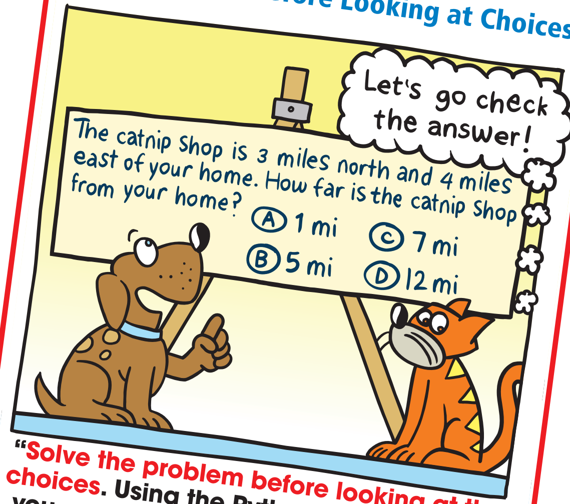
B. $y = -\sqrt{x-1} + 2$

C. $y = -\sqrt{x+2} + 1$

D. $y = -\sqrt{x-2} + 1$

Test-Taking Strategy

Solve Problem Before Looking at Choices



"Solve the problem before looking at the choices. Using the Pythagorean Theorem, you can determine that the catnip shop is 5 miles from your home."

2. Which number is equivalent to $\dfrac{8^{10} \cdot 2^{-5}}{16^4}$? *(N.RN.2)*

F. 1

H. 2^{24}

G. 2^9

I. 2^{-11}

3. What value of x makes the equation below true? *(N.RN.2)*

$$x = \sqrt{2x + 8}$$

4. A line with a slope of -2 passes through the point $(1, -6)$. Which of the following is not a point on the line? *(A.CED.2)*

A. $(-8, 12)$

C. $(4, -4)$

B. $(-4, 4)$

D. $(8, -20)$

5. What value(s) of x make the equation below true? *(N.RN.2)*

$$\sqrt{2x - 4} = x - 2$$

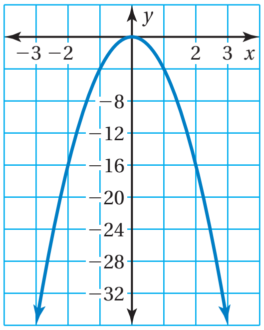
F. Only -2

H. Only 4

G. 2 and 4

I. -2 and -4

6. What is the focus of the parabola? *(F.IF.4)*



A. $\left(0, \frac{1}{16}\right)$

B. (0, 16)

C. $\left(0, -\frac{1}{16}\right)$

D. (0, −16)

7. The range of the function $y = 6x - 8$ is all real numbers from 1 to 10. What is the domain of the function? *(F.IF.1)*

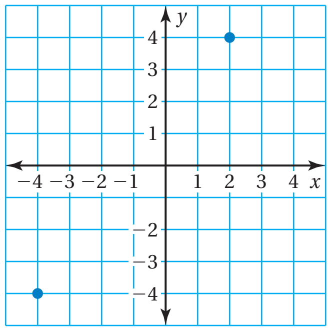
F. all real numbers from 1.5 to 3

G. all real numbers from −2 to 52

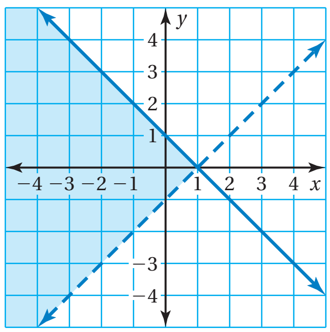
H. all integers from −2 to 52

I. all real numbers

8. What is the distance between the two points in the coordinate plane? *(8.G.8)*



9. Which ordered pair is a solution of the system of inequalities shown in the graph? *(A.REI.12)*



A. (1, 0)

B. (0, 1)

C. (0, −1)

D. (2, 0)

Item Analysis (continued)

6. **A.** The student forgets the negative sign at some point in the calculations.

   B. The student uses $-4a$ instead of $\frac{1}{4a}$ to find the y-value of the focus.

   C. Correct answer

   D. The student uses $4a$ instead of $\frac{1}{4a}$ to find the y-value of the focus.

7. **F.** Correct answer

   G. The student finds the range for a domain of 1 to 10 instead of the domain for a range of 1 to 10.

   H. The student specifies the integer elements of the range for a domain of 1 to 10 instead of the domain for a range of 1 to 10.

   I. The student ignores the range restriction.

8. **Gridded response:** Correct answer: 10

   Common error: The student applies the distance formula incorrectly, by adding instead of subtracting the x-terms and y-terms, and gets an answer of 2.

9. **A.** The student incorrectly thinks that a point on a dashed boundary line represents a solution of the system.

   B. Correct answer

   C. The student incorrectly thinks that a point on a dashed boundary line represents a solution of the system.

   D. The student does not realize that the solutions of the system of inequalities are in the shaded region.

10. **2 points** The student demonstrates a thorough understanding of the Pythagorean Theorem. The student correctly finds each missing distance and the total distance of each trail. The student compares the trails correctly, and provides an adequate explanation.

    1 point The student's work and explanation demonstrate an understanding of the Pythagorean Theorem, although one or more calculations are incorrect, leading to incorrect or poorly supported answers to the questions.

    0 points The student provides no response, a completely incorrect or incomprehensible response, or a response that demonstrates insufficient understanding of the Pythagorean Theorem.

Answers

6. C

7. F

8. 10

9. B

Answers

10. Trail B is longer by 10 miles; Using the Pythagorean Theorem, x and y are each 5 miles and z is 10 miles. So, Trail A is 30 miles and Trail B is 40 miles.

11. I

12. B

13. H

Answer for Extra Example

1. **A.** The student makes a computation error.

B. The student makes a computation error.

C. Correct answer

D. The student makes a computation error.

Item Analysis (continued)

11. **F.** The student shades in the wrong direction and incorrectly uses an open circle.

G. The student incorrectly uses an open circle.

H. The student shades in the wrong direction.

I. Correct answer

12. **A.** The student incorrectly translates the graph of $y = \sqrt{x}$ one unit to the right and two units down.

B. Correct answer

C. The student incorrectly translates the graph of $y = \sqrt{x}$ one unit to the left and two units up.

D. The student incorrectly translates the graph of $y = \sqrt{x}$ one unit to the left and two units down.

13. **F.** The student makes a sign error in finding the x-coordinate of the vertex.

G. The student makes a calculation error in finding the x-coordinate of the vertex.

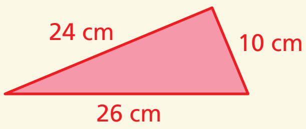
H. Correct answer

I. The student uses $x = -\frac{b}{a}$ to find the x-coordinate of the vertex.

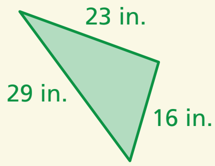
Extra Example

1. Which triangle is *not* a right triangle? *(8.G.7)*

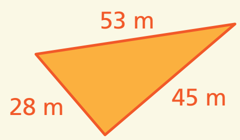
A.



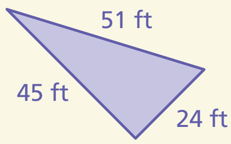
C.



B.

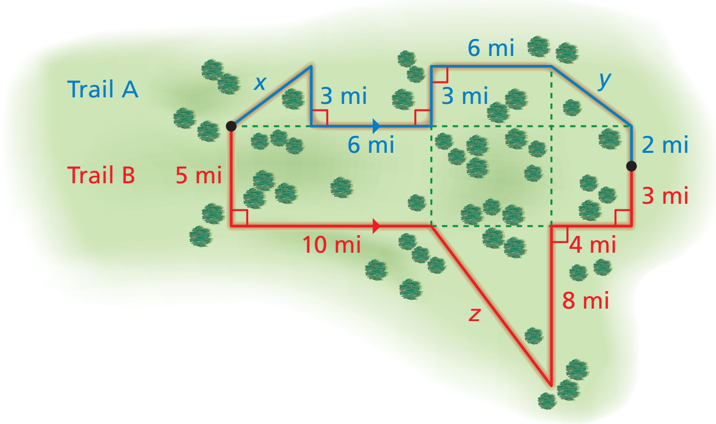


D.

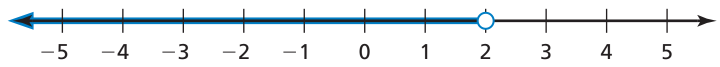


10. Two nature trails are shown below. Which trail is longer? By how much? Explain your reasoning. *(8.G.7)*

Think
Solve
Explain



11. The solution of which inequality is shown in the graph below? *(A.REI.3)*



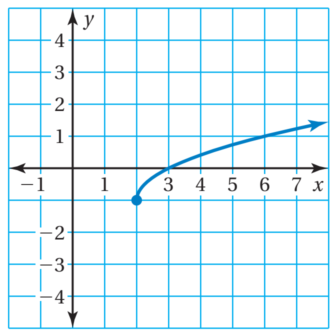
F. $5x - 7 \geq 3$

G. $4x + 3 \leq 11$

H. $12 - 3x < 6$

I. $10 - 2x > 6$

12. Tom was graphing $y = \sqrt{x + 2} - 1$. His work is shown below. *(F.IF.7b)*



What should Tom do to correct the error that he made?

A. Shift the graph 1 unit down and 1 unit left.

B. Shift the graph 4 units left.

C. Shift the graph 3 units up and 3 units left.

D. Shift the graph 1 unit down and 3 units left.

13. What is the vertex of the graph of $y = 2x^2 - 4x + 6$? *(F.IF.4)*

F. $(-1, 12)$

G. $(-2, 22)$

H. $(1, 4)$

I. $(2, 6)$