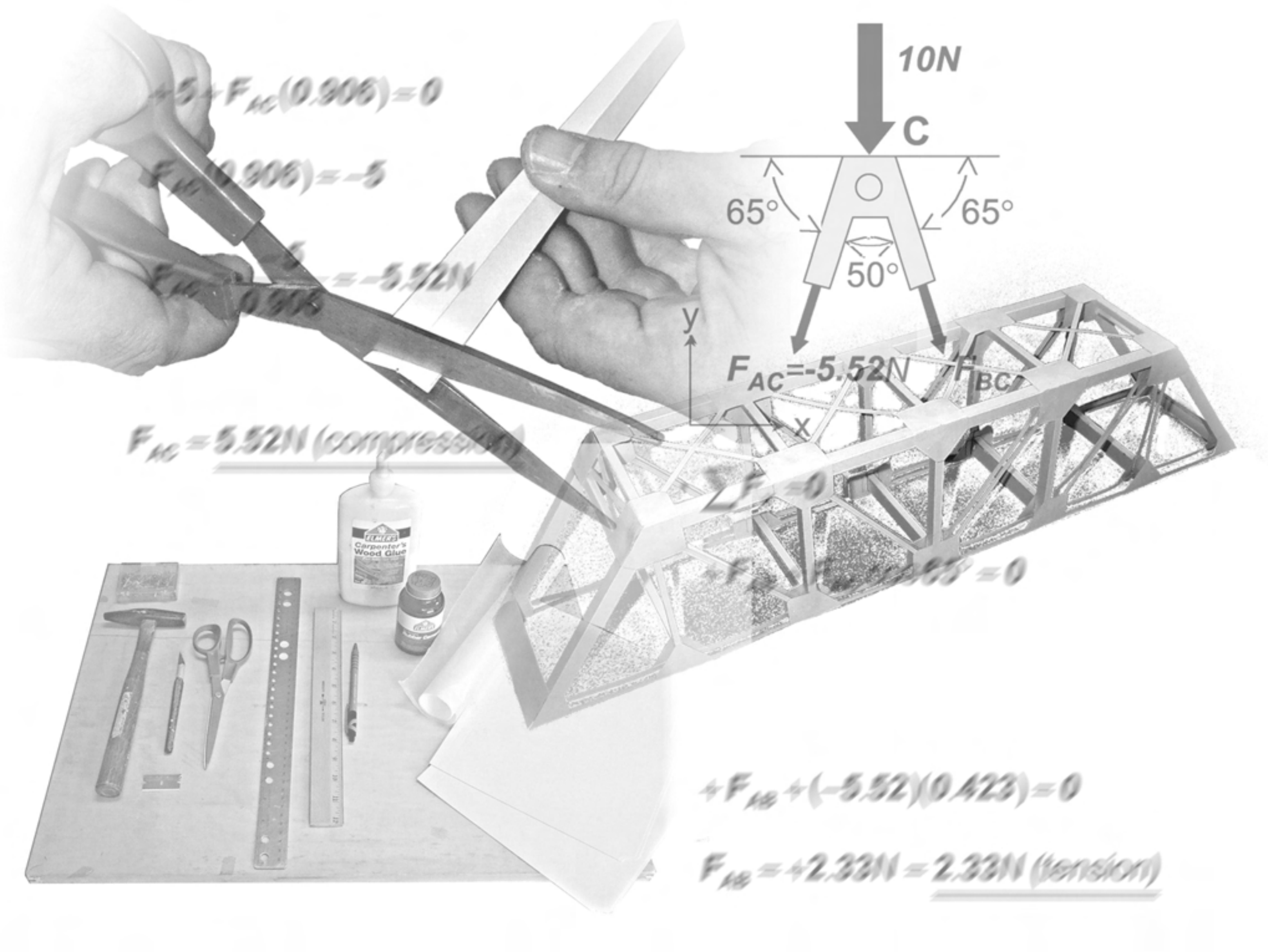
Designing and Building File-Folder Bridges

A Problem-Based Introduction to Engineering



Stephen J. Ressler, P.E., Ph.D.
United States Military Academy

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Every bridge begins in the mind of an engineer.

Contents

Preface:

For the Teacher

About Bridge-Building Projects

A few years ago, I worked with a group of our undergraduate engineering students to run a popsicle stick bridge-building contest for 11th and 12th graders from several local schools. Our purpose was to introduce the high school students to engineering and to stimulate their interest in engineering careers. We also hoped to motivate them to work hard in their math and science courses—to acquire the background necessary to study engineering at the college level.

The format of our contest was typical of the bridge-building projects that have become so popular in secondary school science and technology programs in recent years. We organized the students into teams, and each team received a pile of popsicle sticks and a hot glue gun. Within a specified period of time, each team built a model bridge to span a specified distance. At the end of the construction period, we placed each bridge into a hydraulic testing machine and loaded it to failure, to determine its strength. The bridge with the highest strength-to-weight ratio was declared the winner, and the students who created the winning structure received a nice trophy.

By all accounts, the event was a great success. A large number of students from several different high schools participated, and the inter-school rivalry helped to generate the sort of excitement I normally associate with a championship basketball game. The students certainly enjoyed themselves, and their teachers praised both the content and organization of the contest. Based on the unanimously positive feedback, we concluded that we had accomplished our goal. We had indeed introduced participants to the exciting, creative world of engineering.

Only after the event was over did I begin to question the value of our bridge-building contest. What had the students actually *learned* about engineering from the contest? After much soul-searching, I had to admit that the answer was "not much." Based on what our student participants actually *did* in the contest, they could only have learned three things:

- Engineers build bridges.
- Engineers test structures by loading them to failure.
- Engineers design bridges for maximum strength-to weight ratio.

Unfortunately, all three of these notions are quite wrong; yet they are perpetuated by virtually every model bridge-building project I have ever seen.

The essence of engineering is *design*. Engineering design entails the application of math, science, and technology to create something that meets a human need. The engineering design process is, at the same time, both systematic and creative. And the engineering design process is always iterative: engineers must explore many different alternatives before they can hope to achieve an optimum solution. These are the essential characteristics of engineering; yet our bridge-building contest communicated *none* of these characteristics to the student participants.

- Our students never actually *designed* their bridges. Some simply glued popsicle sticks together without forethought. Others drew sketches before they started building, but their sketches were based on nothing more than vague ideas of what a bridge should look like. We gave participants no basis to decide what might make a bridge design effective or efficient.
- Our students did not apply math or science, nor did we show them any evidence that math and science could have been used to design their bridges more effectively.
- Our students never experienced the iterative nature of design. They built a bridge and broke it—precisely one iteration. They had no opportunity to assess how well the design worked, make appropriate modifications, and test the validity of those modifications in subsequent design iterations.
- We gave our students a totally unrealistic standard for success—maximum strength-to-weight ratio, determined by testing the structure to failure. Engineers design actual structures to *stand up*, not to fail. Actual structures are generally designed to carry a *specified loading* safely, at minimum cost. Actual structures are *never* designed for maximum strength-to-weight ratio. If they were, then a 10-ton bridge that can safely carry a 10-ton load would be just as good as a 50-ton bridge that can carry a 50-ton load. But these two bridges are not equally safe. If you don't believe me, try driving a 20-ton truck across each one.

At the end of the day, our bridge-building contest provided little or no opportunity for students to learn what engineering is or what engineers do. However, it did have one positive impact: it convinced me that there must be a better way.

About the West Point Bridge Designer

I developed the West Point Bridge Designer software in direct response to the inherent limitations of the traditional model bridge-building project. When a student uses the Bridge Designer, he or she designs a real bridge, not a model. The design uses real structural materials, not Popsicle sticks, balsa wood, or pasta. The "simulated load test" is based on a realistic truck loading and actual principles of structural mechanics. More important, the basic design paradigm is realistic. With the West Point Bridge Designer, a bridge must be designed to carry a fixed, code-specified loading safely and at minimum cost. No more maximum strength-to-weight ratio! And the computer simulation provides a reasonably accurate representation of the iterative nature of design. The student is free to explore a nearly limitless range of alternative designs and to observe the cause-effect relationships between design changes and subsequent structural performance. A student who designs a bridge with this software experiences a reasonably authentic simulation of the engineering design process.

Since I first made the West Point Bridge Designer available on the worldwide web three years ago, the response from teachers, students, and engineering practitioners has been overwhelmingly positive and enormously valuable. Many teachers, in particular, have provided insightful suggestion for making the Bridge Designer a more valuable educational tool. I have incorporated these recommendations into subsequent software releases whenever it was feasible to do so.

It is important for me to acknowledge up front that the West Point Bridge Designer also has some serious limitations as an educational tool. It can easily contribute to an unhealthy reliance on the computer as the unquestioned source of the Right Answer. In a sense, it is a "black box"—a computer tool that students can use without really understanding the principles on which the tool is based. Most importantly, the Bridge Designer is only a simulation. Civil engineering involves lots of physical things, like steel, concrete, and soil; and lots of physical concepts, like force, load, and strength. Yet the Bridge Designer provides no opportunity for students to work with any physical object beyond the computer mouse.

These limitations are significant; yet I believe they can be largely overcome by providing appropriate context—by integrating the software into math, science, or technology instruction in a rigorous and meaningful way. This book is intended to help you do it.

About this Book

Many teachers saw the need for this book long before I did. Soon after the West Point Bridge Designer was released, they began asking for information to help integrate the software into their math, science, and technology curricula. With amazing consistency, they asked these two questions:

- How does the software actually analyze a bridge, and how can I teach these mathematical and scientific principles to my students?
- How can my students use the West Point Bridge Designer in conjunction with a hands-on model bridge-building activity?

These requests reflect admirable educational goals. The first seeks to use a practical application as the basis for teaching fundamental principles, the second to connect the design process to the creation of a physical product. These requests provided the inspiration for this book and have guided its development from start to finish.

The purpose of this book is to provide students with an opportunity to learn how engineers use math, science, and technology to design real structures. The book is composed of five separate but closely integrated learning activities. Students who do all five will:

- design, build, and test model bridges;
- use an authentic engineering design process to develop their designs;
- apply math, science, and computer technology as problem-solving tools;
- learn how real bridges are designed and built; and
- learn how real truss bridges work.

This book is necessarily rigorous. Consistent with its purpose, it includes many of the mathematical and scientific concepts that engineers use to analyze and design real structures. I have attempted to present these concepts in their simplest possible form; nonetheless, many students will find them to be quite challenging. And that's good! In my own experience, presenting students with a tough challenge is a powerful way to motivate them to learn.

Many books that introduce students to engineering contain no math at all. A number of these books are wonderfully written, and they all serve an important purpose. Nonetheless, I sometimes wonder if the total exclusion of math from an introductory engineering book doesn't send some students an unhealthy message: *engineering is interesting, but the math behind it is too hard for you to understand*. In this book I have tried to send a different message: *engineering is interesting, and the math behind it is challenging but achievable*.

I should add that every math and science concept presented herein has a direct, practical application in one or more of the five learning activities. Thus, students who do the learning activities in a thoughtful way will also receive an important message about the relevance of math and science in our world.

Overview of the Learning Activities

The five learning activities are as follows:

- ***Learning Activity #1: Build a model of a truss bridge.*** In this activity, we will build a model bridge from cardboard file folders. The bridge has already been designed, and accurate drawings and fabrication instructions are provided. Through this activity, students will learn bridge terminology, construction techniques, and some basic concepts in physics and structural engineering. Students do not need any special knowledge of math or science to do this activity.
- ***Learning Activity #2: Test the strength of structural members.*** In this activity, we will use experimental testing to determine the strength of structural members made of file folder cardboard—the same stuff we used to build our bridge model in Learning Activity #1. The data obtained from these tests will be used extensively in Learning Activities #3 and #5. Students will learn some basic concepts from engineering mechanics, as well as procedures for designing and conducting experiments. To do this activity, students need only basic arithmetic skills and the ability to create a graph. The ability to use a spreadsheet program is helpful but not required. This activity requires the use of a simple wooden testing device. Instructions for building the device are included in Appendix C.
- ***Learning Activity #3: Analyze and evaluate a truss.*** Here we will calculate the internal member forces in our model truss bridge. We will then evaluate the structural safety of the truss by comparing these calculated forces to the member strengths we determined experimentally in Learning Activity #2. Through this activity, students will learn more advanced concepts from physics and engineering mechanics. Students need to apply geometry, algebra, and trigonometry to do the activity successfully. A review of key concepts from trigonometry is included; however, students who have not yet learned geometry or algebra will not be able to do this project.
- ***Learning Activity #4: Design a truss bridge with a computer.*** In this activity, we will design a full-scale highway truss bridge using the West Point Bridge Designer software. The design process includes working through multiple iterations to ensure that the structure will carry the prescribed loads safely and at minimum cost. Through this activity, students will learn the engineering design process and will have an opportunity to reinforce many of the basic structural engineering concepts learned in earlier activities. This activity also includes an overview of how actual bridges are designed and built. Students do not need any special knowledge of math or science to use the West Point Bridge Designer.

Why cardboard?

At first glance, cardboard from manila file folders might seem an odd material to use for bridge-building projects. But in fact, I have found it to be far superior to the more traditional model bridge-building materials—balsa wood, popsicle sticks, toothpicks, and pasta.

Here's why:

- File folders are readily available and very inexpensive.
- Cardboard is easy to work with. It can be easily folded, cut with a scissors, and glued with common household adhesives.
- The behavior of cardboard as a structural material is surprisingly predictable.
- Cardboard provides the capability to build two fundamentally different kinds of structural members—hollow tubes and solid bars. Understanding how these two types of members work is an important part of understanding structural engineering.
- Cardboard provides the capability to build connections that are stronger than the members they join together. I can't overstate the importance of this characteristic. Throughout this book, we will learn how to design structural members so that they are strong enough to carry load safely. But a well-designed member is of little use if its connections fail before the member itself does. A chain is only as strong as its weakest link. If you've ever built and tested a truss bridge made of balsa wood or Popsicle sticks, you know that these structures almost always fail at the connections. As a result, their load-carrying capacity is less than it could be and, more importantly, is almost impossible to predict analytically.

So head for the supply closet; grab a stack of file folders; and let's build some bridges.

- ***Learning Activity #5: Design and build a model truss bridge.*** Here we will apply what we have learned in the previous four activities to design, build, and test a model truss bridge. Students should have completed Learning Activities #1, #2, and #3 to do this project successfully; however, if they do not have adequate math background to complete Learning Activity #3, they can bypass the mathematical structural analysis by using the Gallery of Structural Analysis Results provided in Appendix B. The gallery presents a complete set of computed analysis results for a variety of different truss configurations.

A Gallery of Truss Bridges—a compendium of photographs showing actual truss bridges from all over the United States—is also provided in Appendix A. The gallery is used as part of several learning activities and is also intended to provide students with a resource for ideas about their own bridge designs.

Also included is a Glossary (Appendix D), which provides definitions for mathematical, scientific, and engineering terms used throughout the book. The first appearance of any Glossary term in the text is highlighted in **bold** type.

Organization of Each Activity

This book is organized in a *problem-based learning* format. Each learning activity is presented as a problem to be solved. Information pertinent to the problem solution is provided "just in time"—mathematical, scientific, and technological concepts are included within the specific learning activities in which they are applied. Each activity has a set of learning objectives, which students achieve by (1) working through the problem solution and (2) answering questions that are intended to stimulate critical thought about key concepts.

Each learning activity is organized into the following sections:

- **Overview of the Activity.** This section provides a brief description of the learning activity.
- **Why?** This section explains why the activity is worth doing and how it relates to previous and subsequent learning activities.
- **Learning Objectives.** This section lists the specific knowledge and skills that students can be expected to gain from thoughtful completion of the activity.
- **Information.** This section provides background information pertinent to the activity. In most cases, students would probably be able to complete the activity successfully without this information; however, it is unlikely that they will really learn from the activity without the context that this information provides. For example, a student can certainly build a model bridge without understanding the terms *tension* and *compression*; however, it is highly unlikely that the student will really learn anything meaningful about how structures are designed without some appreciation for these terms.
- **The Problem.** This section presents a fictitious scenario describing a *need* and the student's role in devising a solution that satisfies the need.
- **The Solution.** This section guides the student through the planning and conduct of the problem solution, step by step. At appropriate points throughout the solution, questions are posed, as a means of stimulating critical thinking about important aspects of the project.
- **Answers to the Questions.** Here answers to the critical thinking questions from the preceding section are provided. This section always starts on a new page, so that the teacher can conveniently provide students with copies of the preceding six sections, without revealing the answers to the critical thinking questions.
- **Ideas for Enhancing the Activity.** This final section provides suggestions for enriching or extending the students' learning experience in the activity.

Of these eight sections, the first six should be provided to the students to guide their participation in the learning activity. The seventh—Answers to the Questions—can be provided to students at the end of the activity, if the teacher chooses to do so. The eighth section is intended solely for the teacher.

Some Simpler Bridge-Building Activities

For the teacher who would prefer to do simpler, more qualitative structural engineering activities, there are a number of excellent references available. These include:

Johmann, Carol A. and Elizabeth J. Rieth. ***Bridges! Amazing Structures to Design, Build, and Test.*** Charlotte, Vermont: Williamson Publishing, 1999. *(For ages 7-14.)*

Kaner, Etta. ***Bridges.*** Toronto: Kids Can Press, 1997. *(For ages 8-12.)*

Pollard, Jeanne. ***Building Toothpick Bridges.*** Palo Alto: Dale Seymour Publications, 1985. *(For ages 5-8.)*

Salvadori, Mario. ***The Art of Construction.*** Chicago: Chicago Review Press, 1990. *(For ages 10 and up.)*

WGBH Educational Foundation. ***Building Big Activity Guide.*** Boston: WGBH Educational Foundation, 2000.

Acknowledgements

I am deeply indebted to many people for their invaluable contributions to this book. Colonel Kip Nygren, Head of the Department of Civil and Mechanical Engineering at the U. S. Military Academy, first suggested that a book of learning activities might be an appropriate way to reinforce the educational value of the West Point Bridge Designer software. Mr. Brian Brenner, Dr. Mark Evans, and a team of civil engineers from Parsons Brinckerhoff provided invaluable input to the description of bridge design in Learning Activity #4. Ms. Cathy Bale reviewed the manuscript, provided insightful recommendations for improvement, and did the myriad coordination tasks necessary to get the book into production. The American Society of Civil Engineers (ASCE) provided funding to support the development of the book, and ASCE's Managing Director of Education, Dr. Tom Lenox, was a constant source of guidance, encouragement, and feedback throughout the project. While the book was still being written, Dr. Doug Schmucker used draft versions of Learning Activities #1 and #2 as the basis for a project in his Materials Engineering course at Valparaiso University. Doug and his students provided thoughtful feedback about the book at a very critical time in its development. Many of the photographs in Learning Activity #1 were taken by Mr. Mike Doyle. Finally, the bridge photographs used for each chapter heading were provided by Mr. Jet Lowe of the National Park Service.

Colonel Stephen J. Ressler
West Point, New York
February, 2001

For Claire and Anne.

Learning Activity #1:

Build a Model of a Truss Bridge

Overview of the Activity

In this learning activity, you will build a model truss bridge that has already been designed for you. When construction is complete, you will load the bridge to determine if it performs as its designer intended. With the load in place, you will be able to observe how the structure works—how the various structural members work together to carry the load safely and efficiently. And at the end of the project, you will save the model as evidence of your bridge-building skill. Don't break it! We will be using it again in subsequent learning activities.

Why?

Design is the essence of engineering. The only way to truly appreciate the challenges and rewards of engineering is to actively engage in the creative process of design. So why, in this learning activity, will we devote considerable effort to building a bridge that has already been designed by someone else? It is true that building an existing design will not allow you to exercise a lot of creativity; nonetheless, this activity will provide you with valuable preparation for learning how to design a structure. Building an existing design will allow you to:

- Learn many key concepts about trusses and structural behavior that you'll use when you design your own bridge in Learning Activity #5.
- Familiarize with the engineering characteristics of a rather unique building material—cardboard from a manila file folder.
- Learn some special construction techniques appropriate for this material.
- Work with confidence, knowing that your bridge will carry the prescribed loading successfully, as long as you build the structure with care.
- Learn about the challenges faced by real-world construction contractors, who are often required to build structures that have been designed by someone else.

Learning Objectives

As a result of this learning activity, you will be able to do the following:

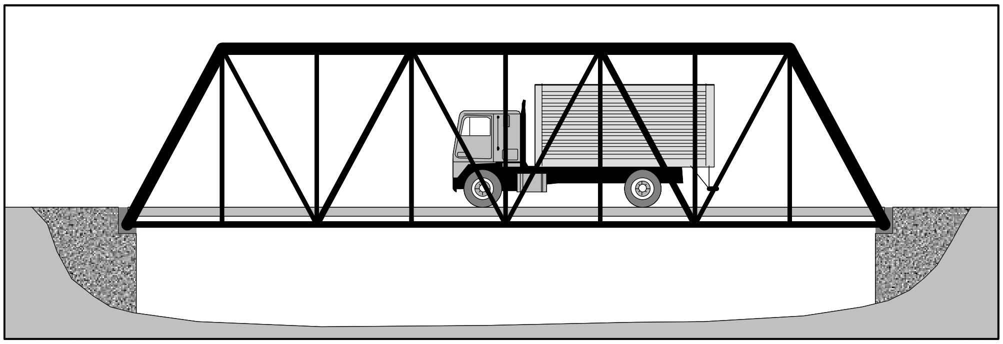
- Explain what a *truss* is.
- Identify the major components of a truss bridge.
- Identify the types of truss bridges.
- Explain the following fundamental structural engineering concepts: *force, load, reaction, equilibrium, tension, compression, and strength*.
- Explain how a truss bridge works—how each individual component contributes to the ability of the entire structure to carry a load.
- Explain the roles of the four key players in the design-construction process—the *Owner*, the *Design Professional*, the *Constructor*, and the *Project Manager*.
- Explain how construction quality affects the performance of a structure.

Information

1. Component Parts of a Truss Bridge

What is a Truss?

A **truss** is a structure composed of members connected together to form a rigid framework. **Members** are the load-carrying components of a structure. In most trusses, members are arranged in interconnected triangles, as shown below. Because of this configuration, truss members carry load primarily in **tension** and **compression.** (We'll discuss these terms in Section 3 below.) Because trusses are very strong for their weight, they are often used to span long distances. They have been used extensively in bridges since the early 19th century; however, truss bridges have become somewhat less common in recent years. Today trusses are often used in the roofs of buildings and stadiums, in towers, construction cranes, and many similar structures and machines.

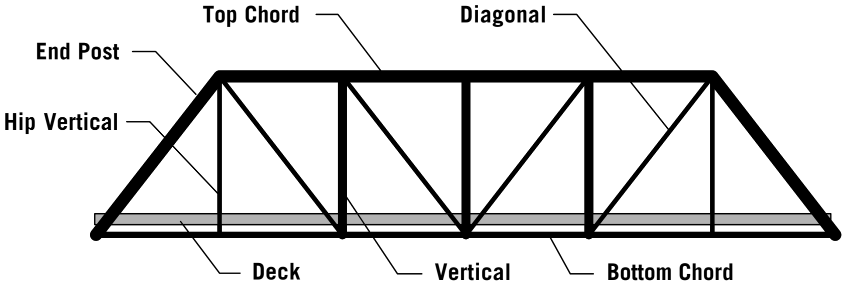


A typical truss bridge. Note that the structure is composed entirely of interconnected triangles.

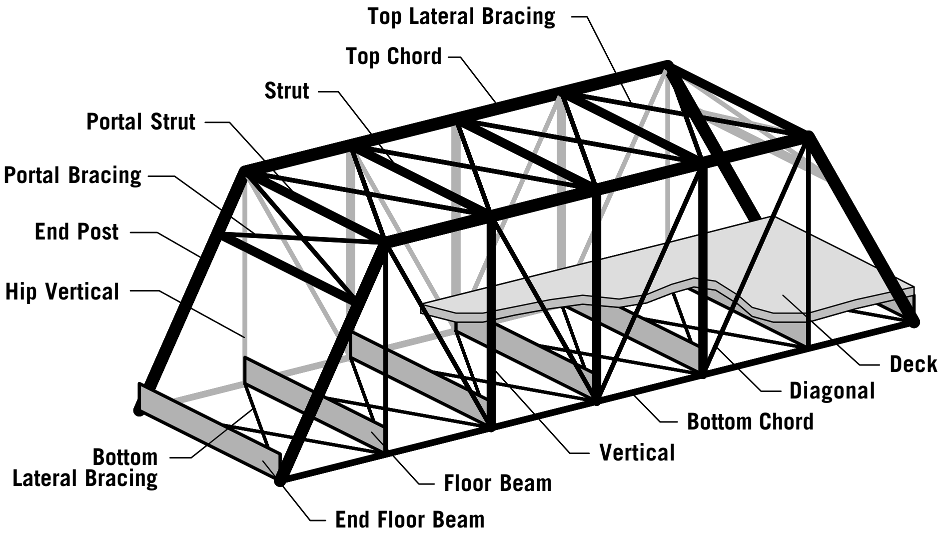
Trusses, like all structures, are designed by civil engineers with special expertise in structural analysis and design. These men and women are called **structural engineers**.

Component Parts

The major components of a typical truss bridge are illustrated in the two diagrams below. The **elevation view** shows the bridge from the side. The **isometric view** is a three-dimensional representation of the structure. Note that certain members are *only* visible in the isometric view.



Component parts of a typical truss bridge - Elevation View



Component parts of a typical truss bridge - Isometric View [1]

The three-dimensional bridge structure has two main load-carrying trusses. Each truss is composed of a **top chord**, a **bottom chord**, and several **verticals** and **diagonals**. The two trusses are connected together by a series of transverse members—**struts**, **lateral bracing**, and **floor beams**.

In early truss bridges, all of these members would have been made of wood or iron. Today they are usually made of steel. Modern steel truss members are manufactured in a wide variety of shapes and sizes. A few common examples are shown on the following page. The model truss we will be building uses both **solid bars** and **hollow tubes.** When we load-test our model, we'll see why one truss often uses two different types of members.

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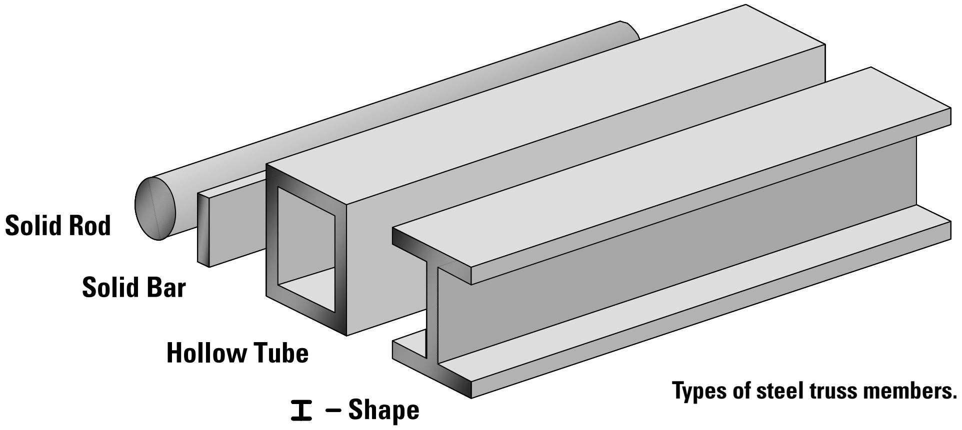
[1] Based on "Truss Identification: Nomenclature," Historic American Engineering Record HAER T1-1, National Park Service, 1976.

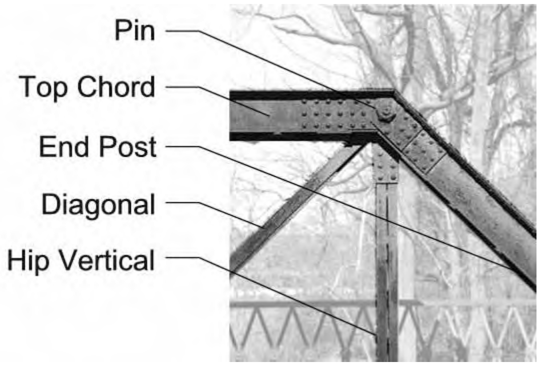


Types of steel truss members.

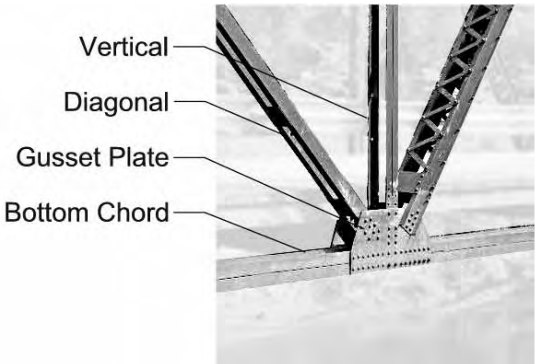
One major component of a truss bridge that is usually *not* made of steel is the **deck**—the flat surface between the two main trusses. (In the isometric drawing, only part of the deck is shown, so the structural members below it can be seen.) Bridge decks are usually made of concrete, but might also be built from wooden planks or steel grating. When vehicles or pedestrians cross a bridge, their weight is directly supported by the deck. The deck, in turn, is supported on the floor beams. The floor beams transmit the weight of the vehicles and pedestrians (and the weight of the deck) to the main trusses.

The truss drawings above do not show the **connections** that are used to join the structural members together. Even though the connections are not shown, they *are* important! They have a big influence on the ability of a structure to carry load. Indeed, inadequately designed connections have been the cause of several catastrophic structural failures in the U.S. [2]

There are two common types of structural connections used in trusses—**pinned connections** and **gusset plate connections.** Examples of each are shown in the photographs below. As the name suggests, the pinned connection uses a single large metal pin to connect two or more members together, much like the pin in a door hinge. In a gusset plate connection, members are joined together by one or two heavy metal **gusset plates**, which are attached to the individual members with rivets, bolts, or welds. Pinned connections were used extensively throughout the 19th century. Most modern bridges—including the model bridge we will be building here—use gusset plate connections.



Typical pinned connection.



Typical gusset plate connection

Each of the bridge components described above has a specific purpose. All of the components work together to ensure that the bridge carries load safely and efficiently. In this learning activity, we will fabricate and assemble these various types of structural members and components, and we will observe how each one works.

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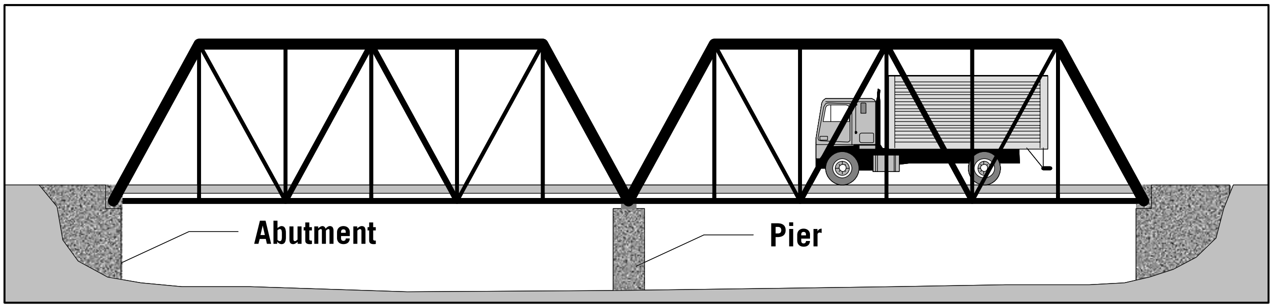
[2] For more information on structural failures, see Why Buildings Fall Down, by Mario Savadori.

Foundations

Every structure must be supported on a firm **foundation**, which distributes the weight of the structure to the soil or rock below it. Bridges use two different types of foundations. The ends of a bridge usually rest on **abutments**, which serve two functions simultaneously—they support the bridge and also hold back the soil that is filled in behind them. If the bridge requires additional support in the middle of the gap, one or more **piers** are used, as shown below. Abutments and piers are normally made of concrete.

All structural foundations are designed by civil engineers with special expertise in soils and foundations. These men and women are called **geotechnical engineers**.



Types of bridge foundations.

Q1

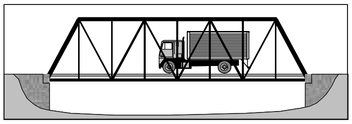
Can you identify the component parts of a truss bridge?

Select any bridge pictured in the Gallery of Truss Bridges (Appendix A), and identify its major component parts—top and bottom chords, verticals, diagonals, floor beams, lateral bracing, struts, portal bracing, deck, abutments, and piers. *(You will not be able to find every one of these components on every pictured bridge.)*

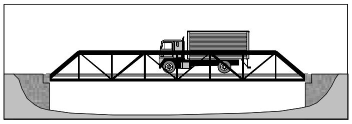
2. Types of Truss Bridges

Truss bridges are grouped into three general categories, based on their deck location. If the deck is located at the level of the bottom chord, the bridge is called a **through truss**. A **pony truss** looks just like a through truss, except it is not as high and has no lateral bracing between the top chords. If the deck is located at the level of the top chord, the bridge is called a **deck truss**.

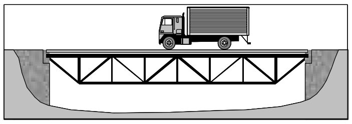
Trusses are also classified according to the geometric arrangement of their chords, verticals, and diagonals. The diagrams on the following page show 15 of the most common truss configurations, many of which were named for the 19th century engineers who developed them. On each diagram, the solid lines represent the main structural members in the truss. The dotted lines shown on some trusses represent supplemental members that may or may not be present on a particular bridge of this type. Designers sometimes use these lightweight diagonal members to more efficiently carry the weight of moving vehicles. The classification of a bridge is not affected by the presence or absence of these supplemental members.



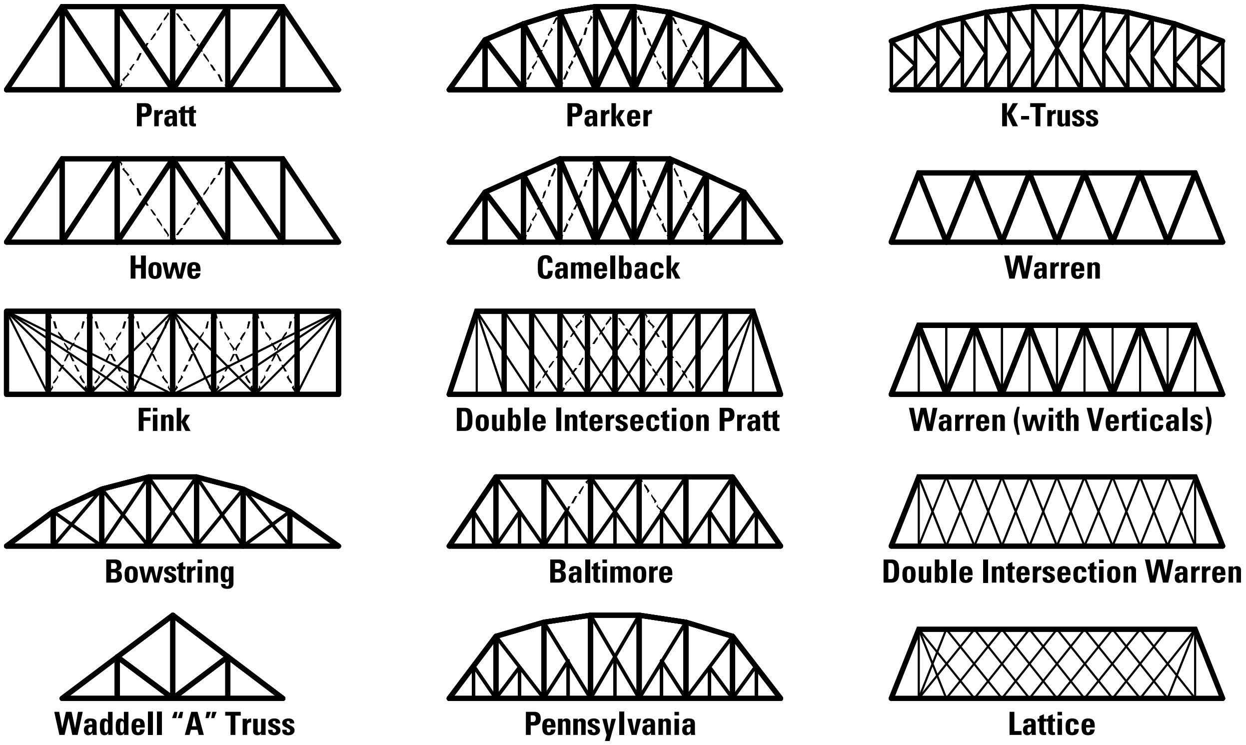
Through truss.



Pony truss.



Deck truss.



Pratt
Parker
K-Truss
Howe
Camelback
Warren
Fink
Double Intersection Pratt
Warren (with Verticals)
Bowstring
Baltimore
Double Intersection Warren
Waddell "A" Truss
Pennsylvania
Lattice

Common truss configurations.[3]

Note that all of these diagrams depict *through trusses*. Many of these configurations are also used in *deck trusses* and *pony trusses* as well.

Q2

Can you identify the configuration of a truss bridge?

Identify the configuration of each bridge pictured in the Gallery of Truss Bridges (Appendix A). Also note whether each bridge is a through truss, deck truss, or pony truss.

3. How a Structure Carries Load

One of the most important learning objectives of this project is to understand how a truss bridge carries load. But what exactly is a "load," and what does it mean for a structure to "carry a load?" To answer these questions, we will need to introduce (or perhaps review) some basic concepts from physics.

Forces

Much of structural engineering deals, in some way, with the concept of *force*. A **force** is simply a push or a pull applied to an object. A force always has both *magnitude* and *direction*. When a truck crosses a bridge, it exerts a force on the bridge. The magnitude of the force is the weight of the truck, and the direction of the force is downward. Mathematically, we represent a force as a **vector.** By definition, a vector is a quantity that

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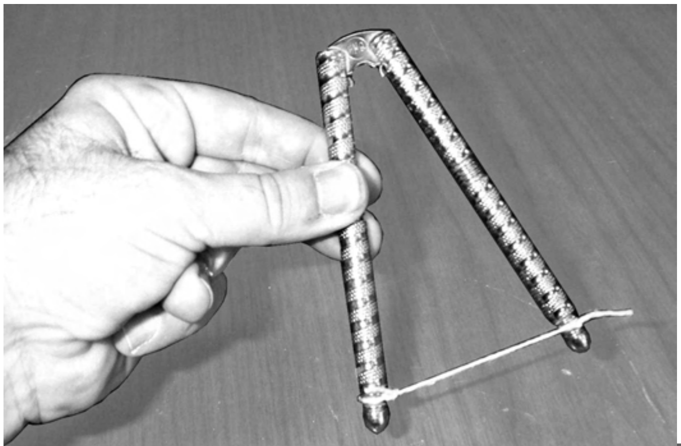
[3] Based on "Truss Identification: Bridge Types," Historic American Engineering Record HAER T1-1, National Park Service, 1976.

has both magnitude and direction. To show a force on a picture or diagram, we normally represent it as an arrow (which shows the direction) and a magnitude (in units of force, such as pounds or newtons), like this:

In structural engineering, it is useful to distinguish between three different kinds of forces—**loads**, **reactions**, and **internal member forces**.

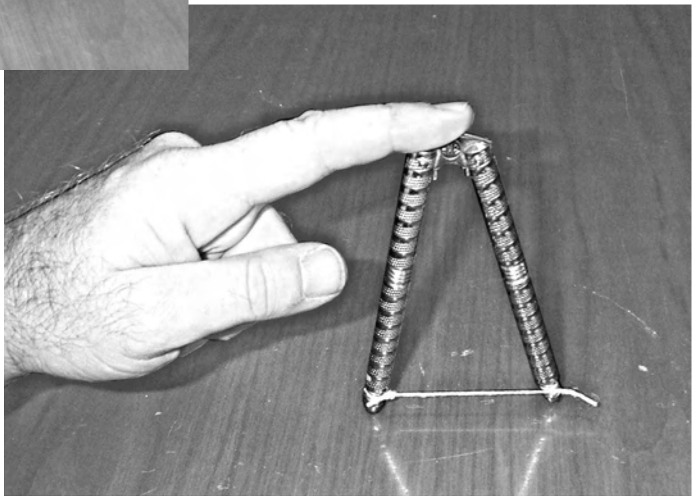
Loads

To illustrate what loads, reactions, and internal member forces are, let's do a simple experiment. Find a nutcracker like the one shown below, and tie the ends of the handles together with a piece of string. Ensure that the string is taut. You have just built a simple truss composed of three members—the two handles and the string. Now put the ends of the nutcracker on a smooth, flat surface, and press down on the center hinge. You are applying a **load** to the nutcracker truss. A load is simply a force applied to a structure.



A simple 3-member truss made from a nutcracker.

Applying a load to the nutcracker truss.



Actual bridges are subjected to many different kinds of loads, including the following:

- Weight of the vehicles and pedestrians crossing the bridge
- Weight of the bridge itself
- Weight of the asphalt or concrete road surface
- Wind pushing sideways on the structure
- Weight of snow, ice, or rainwater
- Forces caused by earthquakes

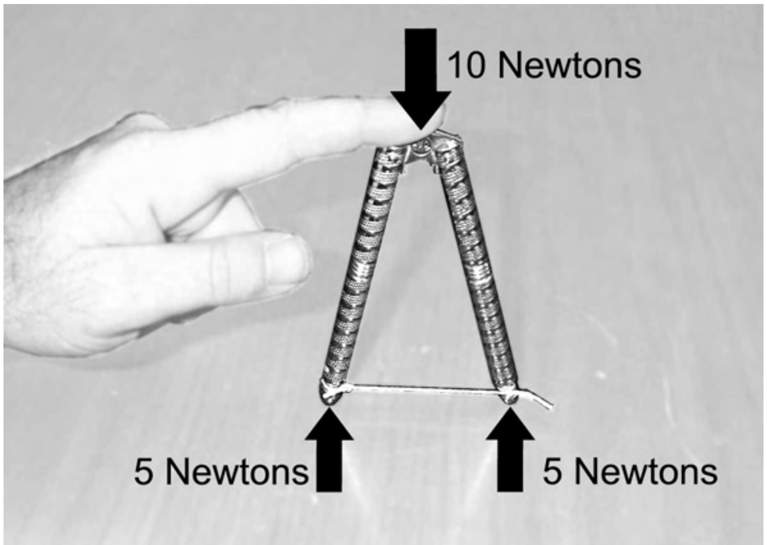
In designing a bridge, the structural engineer must consider the effects of all these loads, including cases where two or more different kinds of loads might occur at the same time.

Reactions

Newton's First Law—one of the basic principles of physics—states that *an object at rest will remain at rest, provided it is not subjected to an unbalanced force*. In other words, if an object is not moving, then the total force acting on it must be zero. When you apply a downward force to your nutcracker truss, it *does not* move; thus, according to Newton's First Law, the total force on the truss must be zero. But how can that be? Suppose you push *down* on the nutcracker with a force of 10 newtons. The nutcracker does not move, because the table pushes back *upward* with a force of 10 newtons. In this particular example, because the structure touches the table at two points, the table actually pushes upward with two forces, each with a magnitude of 5 newtons, as shown below. The structure is said to be in **equilibrium**, because the total upward force equals the total downward force. A structure that is not moving *must* be in equilibrium. Mathematically, the vector sum of all forces acting on the structure is zero. If we assume that the upward direction is positive, then

$$+5+5-10=0$$

In our example, the two upward forces are called **reactions**. Reactions are forces developed at the supports of a structure, to keep the structure in equilibrium. **Supports** are the points where the structure is physically in contact with its surroundings. On our nutcracker truss, the supports are located at the ends of the handles, where the nutcracker touches the table. On an actual bridge, the supports are located at the **abutments** or **piers**. (See Section 1 above.)

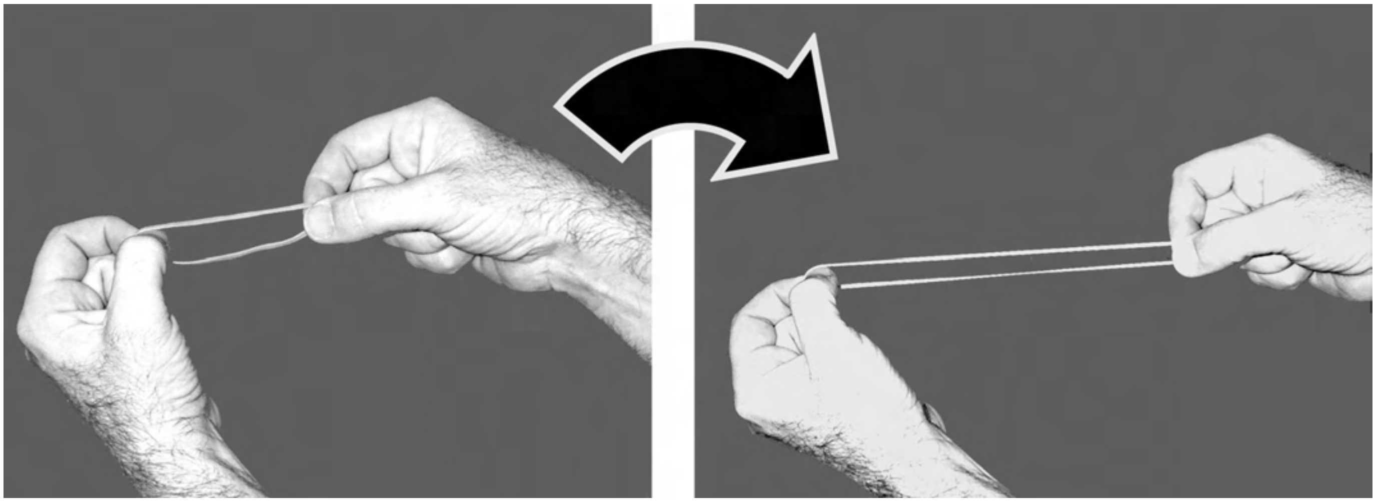


The two 5-newton reactions keep the nutcracker truss in equilibrium with the 10-newton load.

Geotechnical engineers are particularly interested in the reactions of a structure, because the foundations must be designed to carry these forces safely and efficiently.

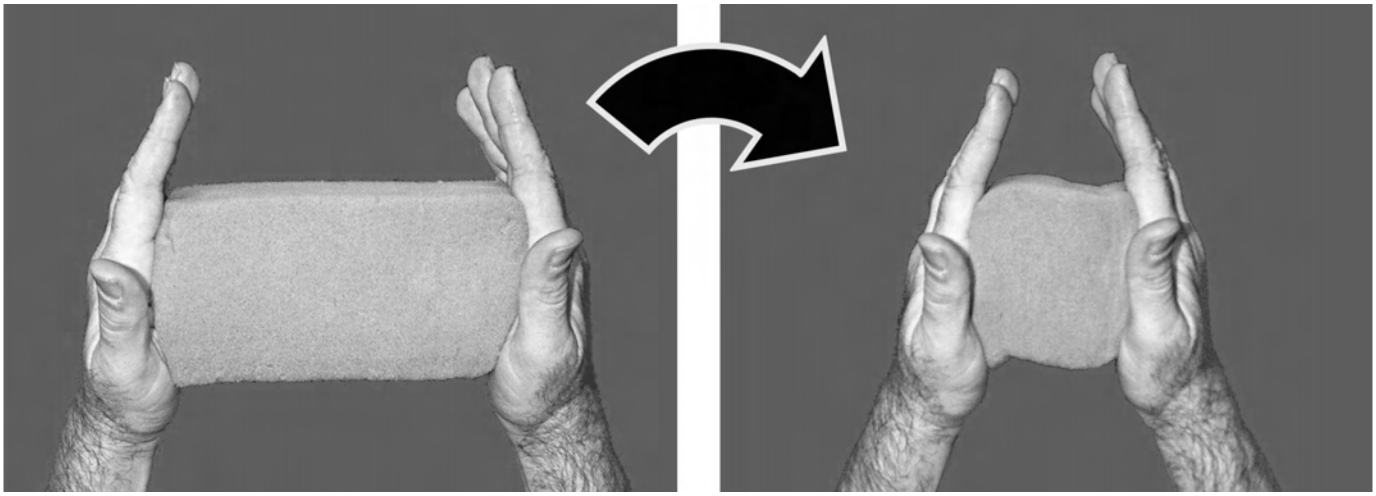
Internal Member Forces

When you apply *external* loads to a structure, *external* reactions occur at the supports. But *internal* forces are also developed within each structural member. In a truss, these internal member forces will always be either **tension** or **compression**. A member in tension is being stretched, like the rubber band in the picture below. Tension force tends to make a member longer.



Tension is an internal force that tends to make a member longer.

A member in compression is being squashed, like the block of foam in the picture below. Compression force makes a member shorter.

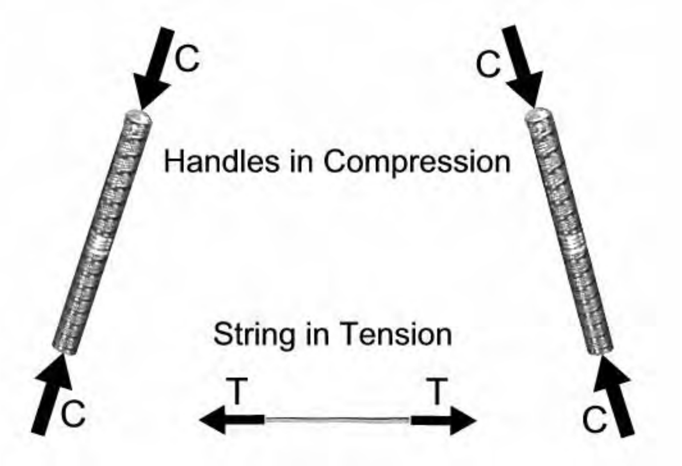


Compression is an internal force that tends to make a member shorter.

Tension and compression are among the most important concepts in structural engineering. A renowned engineer and author named Mario Salvadori once wrote,

> **All structures,** ***in a one-family house or a skyscraper, in an arch or a suspension bridge, in a large dome or a small flat roof, are*** **always** ***either in tension or compression. Structures can only pull or push. If you understand how tension and compression work, you understand why structures stand up.***

In our nutcracker truss example, the two handles are in compression, while the string is in tension, as shown here. If you push down hard enough on the nutcracker, you can actually see the string stretching in tension. Unfortunately, you can't see the nutcracker handles shortening in compression—steel is so stiff that the shortening of the handles is too small to be seen with the naked eye. But the handles actually do get shorter!



Tension and compression in the nutcracker truss.

Like loads and reactions, internal member forces must obey the laws of physics. Internal forces must be in equilibrium with each other *and* with the loads and reactions. By applying the concept of equilibrium and some relatively simple math, we can actually calculate the internal force in every member of a truss. We'll see how to do this Learning Activity #3.

Strength

Let's return once again to our nutcracker truss example. As we have already seen, if you press down on the hinge at the top of the structure, a tension force is developed in the string. If you press down harder (that is, if you increase the load), the tension force in the string increases. If you are very strong, or if the string is very weak, you should be able to apply a downward force that is large enough to break the string.

What causes the string to break? The string breaks *when its internal member force becomes larger than its strength*. This observation leads us to two closely related definitions:

(1) The **strength** of a structural component is the largest internal force the component can experience before it fails.

(2) **Failure** occurs when the internal force in a structural component becomes larger than its strength.

If you have ever bought fishing line, you might have noticed the words "100 pound test" or something similar on the label. "100 pound test" means that the line is guaranteed not to fail, as long as the internal force in the line is less than 100 pounds. To put this in structural engineering terms, the *strength* of the line is 100 pounds. [4]

How Does a Structure Carry Load?

Having discussed loads, reactions, internal member forces, and strength, we can now answer the important question posed at the beginning of this section: *what does it mean for a structure to carry load?*

In this learning activity, you will build and load-test a model bridge. If you build the bridge well, it will carry the load successfully, and you will have an opportunity to observe how the structure works.

> ***When you apply a load to a structure, internal forces—tension and compression—occur in each member. If the strength is greater than the internal force for every member in the structure, then the structure will carry the load successfully.***

4. The Project Team: Key Players and Contributions

Building a model bridge is a great way to start learning about the process used to design and construct actual structures. But before we can fully understand this process, we need to meet the key players in the design-construction process and learn about how they contribute to its ultimate product—a completed structure or facility. This section applies equally well to *any* civil engineering project, not just to a bridge project.

The Project Team

Major construction projects are always performed by a **project team**, composed of many different specialists. Each member of the team contributes unique capabilities or resources to the project, and all must work together to make the project successful. The team has four key players—the Owner, the Design Professional, the Constructor, and the Project Manager—organized as shown in the diagram on the following page.

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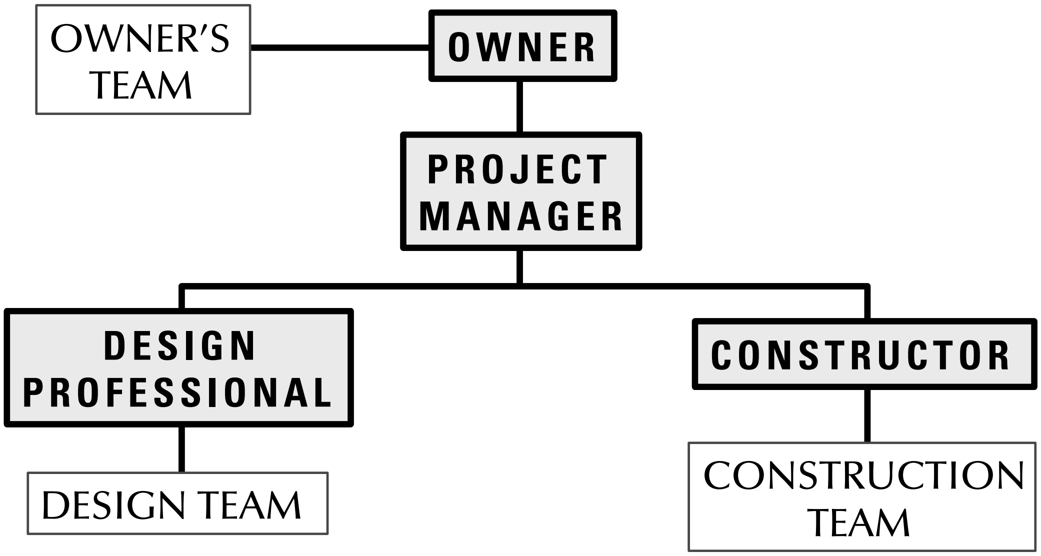
[4] Actually, the true strength of the line is probably somewhat higher than 100 pounds. In order to guarantee a strength of 100 pounds, the manufacturer would normally design the line to be somewhat stronger than that–say 150 pounds. Small variations in the dimensions or material characteristics of the line will always occur during manufacturing. By building in some extra strength, the manufacturer ensures that the "100-pound guarantee" will not be violated. This extra margin of error is called a factor of safety. We will discuss the factor of safety at the end of this learning activity, and we will actually calculate it in Learning Activity #3.



The organization of the Project Team [5]

The Owner

The **Owner** is the person or organization that initiates the project and ultimately will take ownership of the facility after it is built. The Owner might be a private developer, a corporation, a public agency, a municipal government, or simply an individual. Regardless of who the Owner is, he or she makes four vitally important contributions to the project:

- ***Identify the need for a new facility.*** For example, a state Department of Transportation might identify the need for a new highway bridge across a river. Without the need, there can be no project; thus identifying the need is probably the Owner's most important contribution.
- ***Provide funding to pay for the project.*** The Owner provides the money or arranges for financing to fund the project. Often the Owner also provides the land on which the new facility will be built.
- ***Put together the Project Team.*** The Owner selects and hires the Design Professional and the Project Manager, usually based on their professional qualifications and experience. The Owner does not necessarily select the Constructor but always decides *how* the Constructor will be chosen. Often this is done by a competitive bidding process. No matter how the Design Professional, Contractor, and Project Manager are selected, they work for the Owner—either as employees or by contract.
- ***Establish the design requirements***. The **design requirements** include **functional requirements**, **aesthetic requirements**, and any **constraints** that will affect the design or construction of the facility. The Owner often develops the design requirements in close coordination with the Design Professional. On a bridge project, for example, the Owner probably knows generally what purpose the proposed structure will serve—what kind of traffic it will carry, what body of water it will cross, how much money is available to build the bridge, and when it must be completed. The Owner might have some general ideas about how the structure should look—perhaps the appearance of the bridge needs to be consistent with other nearby structures. But the Owner probably does not know what type of bridge will be best for the chosen site, which environmental regulations and safety standards will govern the design, or exactly how much traffic the new bridge must be able to handle. These sorts of technical requirements are provided by a technical expert—the Design Professional.

To perform these functions, the Owner usually has a team of experts to assist with specialized functions such as financial management, real estate, legal assistance, insurance, and facilities management. These specialists—often members of the Owner's own staff—are shown as the **Owner's Team** on the organization chart above.

[5] Quality in the Constructed Product: A Guide for Owners, Designers, and Constructors, American Society of Civil Engineers Manual No. 73, ASCE, New York, 1990.

The Design Professional

The **Design Professional** is responsible for conceiving, planning, and providing a high-quality design solution to the Owner. The Design Professional may be an engineer or an architect, depending on the nature of the project. In either case, the Design Professional rarely has sufficient expertise to perform all aspects of the design. He or she puts together and supervises a team of specialists called the Design Team. The composition of this team will vary depending on the nature of the project, but it is likely to include engineers from many different disciplines—structural, geotechnical, transportation, environmental, mechanical, and electrical—as well as surveyors, draftsmen, and other technicians. The Design Team might be composed entirely of employees from a single, full-service engineering company, or it might include consultants hired for just one particular project.

The Design Professional's principal contribution to the project is a set of **plans and specifications**. *Plans* are drawings, and *specifications* are highly detailed written descriptions of every aspect of the project, including all *quality standards* the completed facility must meet. Plans and specifications often include detailed lists of structural members and components. These lists are called **schedules**. For example, structural drawings often include a Column Schedule and a Beam Schedule. We will be working with several such schedules in this learning activity.

Often the Design Professional has little direct involvement in the construction process. For this reason, the plans and specifications must be sufficiently clear, unambiguous, and thorough that a Constructor *who has had little or no involvement with the design* can build the facility correctly. Engineers must know how to write well and must be able to communicate effectively with drawings.

The Constructor

The **Constructor** is responsible for planning, managing, and performing the construction of a facility, after it has been designed. The Constructor is usually a **construction contractor**—a company that assumes full responsibility for building the facility, under the terms of a formal contract with the Owner. Like the Design Professional, the Constructor assembles and supervises a team of specialists with skills appropriate for the project. When these specialists are hired by the construction contractor, they are called **subcontractors**. On a typical project, the subcontractors might include carpenters, masons, ironworkers, electricians, material suppliers, steel fabricators, equipment operators, surveyors, material testing companies, quality control inspectors, and many others.

The Constructor's contribution to the project is the completed facility. A constructed facility is generally considered to be successful if it is delivered (1) on time, (2) within budget, and (3) to the standard of quality spelled out in the plans and specifications. These three criteria suggest that the Constructor must do much more than just construct. He or she must also (1) develop and manage an accurate project schedule, (2) carefully manage project costs and payments, and (3) perform effective quality control. **Quality control** is the process of routinely inspecting and testing materials and workmanship on a project and taking corrective action when problems are found. Effective quality control happens when the Constructor monitors the work *continuously*, rather than waiting until the end of the project, when it might be too late to correct mistakes.

The Project Manager

The **Project Manager** has overall responsibility for managing both the design and construction of the facility. The Project Manager represents the owner and looks after the Owner's interests on all aspects of the project, to include scheduling, financial management, and construction quality. For buildings, bridges, and other infrastructure projects, the Project Manager is usually a civil engineer. He or she might be an employee of the Owner or a consultant hired for the specific project. Because the Constructor is rarely involved in the design phase of a project, and the Design Professional is often minimally involved in the construction phase, the Project Manager often must provide management continuity from project initiation through completion.

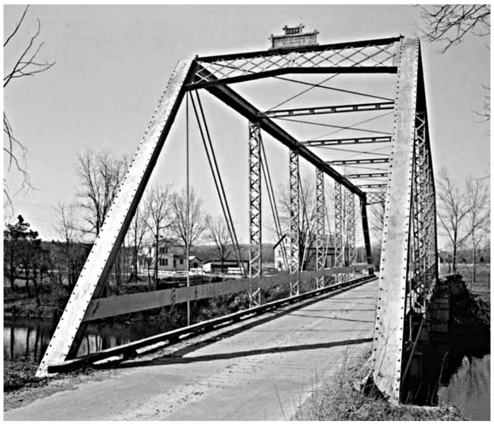
Your Role in this Project

During this learning activity, you will serve primarily as the Constructor. On an actual project, the Constructor often has little or no involvement in the design process and only receives the plans and specifications after the design is complete; thus, your role as the Constructor in this project is actually quite authentic.

You'll play other roles in subsequent learning activities.

The Learning Activity

The Problem



The Need

Just outside the small town of Hauptville, New York, Grant Road crosses Union Creek via a beautiful old 19th Century Pratt truss bridge similar to the one shown here. Recently, the Town Engineer determined that the structure is no longer safe for modern truck traffic and must be replaced. Because of its historic value, the old bridge will be disassembled, moved to a nearby public park, and rebuilt as a pedestrian bridge. A new highway bridge for Grant Road must be built on the existing site.

Design Requirements

The Owner for this project is the Town of Hauptville. Several months ago, the Town Council selected Thayer Associates, a respected local engineering firm, as the Design Professional for this project. The Hauptville Town Engineer worked closely with civil engineers from Thayer Associates to develop three functional requirements for the bridge:

- The new bridge must be constructed on the abutments from the old structure. These existing supports are 24 meters apart.
  [Our 1/40 scale model bridge will actually have a span of 60 centimeters.]

- The bridge must carry two lanes of traffic.
  [Our model bridge must have a roadway width of at least 9 centimeters and at least 9 centimeters of overhead clearance above the deck.]

- The bridge must meet the structural safety requirements of the AASHTO bridge design code.[6]
  [Our model bridge must carry a "traffic load" consisting of a 5 kilogram mass placed on the structure at mid-span.]

The Town Council also added an important **aesthetic requirement.** To preserve the town's historical character, the new Grant Road Bridge should look similar to the old one—a Pratt through truss. The old bridge was made of wrought iron, but the Town Engineer has decided that the new structure will be safer and more practical if it is made of steel.
[For our model, steel will be represented by cardboard from manila file folders.]

The Design

Based on these design requirements, a team of engineers from Thayer Associates has developed **plans and specifications** for the new Grant Road Bridge over Union Creek. The plans and specifications include a structural drawing, isometric drawings of two typical connections, a schedule of truss members, a schedule of connections, and full-scale shop drawings of the structure.

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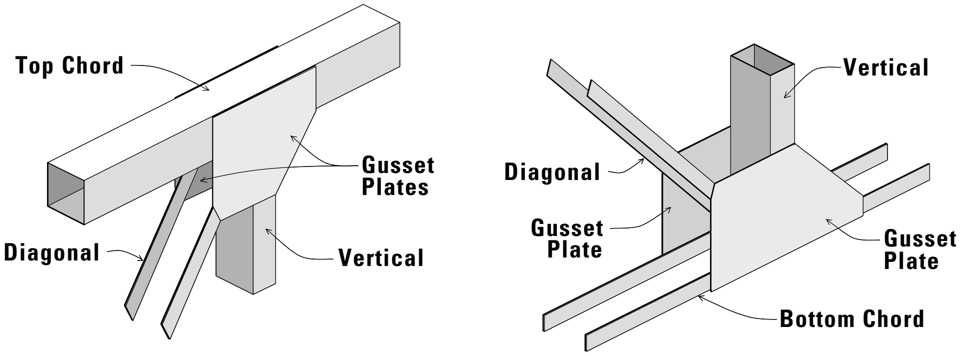
[6] AASHTO is the American Association of State Highway and Transportation Officials, the organization that develops and publishes standard design specifications for bridges in the United States.

Structural Drawing

The structural drawing of the new Grant Road Bridge is designated as Drawing S-1 and is provided on page 1-16. The drawing includes a side elevation, a front elevation, and a plan view. Note that every connection in the structure is designated with a letter—A through N for one main truss and A’ through N’ for the other. These letters are used to identify the members and gusset plates.

Typical Connections

The two isometric drawings below are typical gusset-plate connections found at the top and bottom chords of the main trusses. These drawings illustrate the types of structural members used throughout the Grant Road bridge—hollow tubes for the top chords and verticals; doubled bars for the bottom chords and diagonals. The drawings also show how two gusset plates are used at each connection to hold all of the structural members together. The drawings do not show lateral bracing, struts, or floor beams, which have been omitted for clarity.



Typical top chord and bottom chord connections for the Grant Road Bridge.

Schedule of Truss Members

The Schedule of Truss Members identifies every member required to build the bridge. Note that each member is identified by the two letters corresponding to its endpoints. For example, Member AD is a segment of the bottom chord that goes from Connection A to Connection D.

Component	Members	Type	Approx. Length	# Req’d
Bottom Chords	AD, DG, A’D’, D’G’	4mm bar (double)	30cm	8
Diagonals	CI, DJ, DL, EM C’I’, D’J’, D’L’, E’M’	4mm bar (double)	15cm	16
Verticals	BI, FM, B’I’, F’M’	4mm bar (double)	11cm	8
Top Lateral Bracing	IJ’, I’J, JK’, J’K, KL’, K’ L, LM’, L’M	4mm bar (single)	12cm	8
Portal Bracing	HI’, H’I, MN’, M’N	4mm bar (single)	10cm	4
Top Chords	IK, KM, I’K’, K’M’	10mm x 10mm tube	21cm	4
End Posts	AI, GM, A’I’, G’M’	10mm x 10mm tube	17cm	4
Verticals	CJ, DK, EL, C’J’, D’K’, E’L’	6mm x 10mm tube	12 cm	6
Top Struts	HH’, II’, JJ’, KK’, LL’, MM’, NN’	6mm x 6mm tube	9cm	7
Floor Beams	BB’, CC’, DD’, EE’, FF’	6mm x 15mm tube	10cm	5
Floor Beams	AA’, GG’	28mm x 13mm angle	11cm	2

For this model, a member designated as a 4mm bar is actually a 4mm-wide strip of cardboard. The designation 6mm x 10mm refers to a hollow tube measuring 6mm by 10mm. The lengths shown in the schedule are approximate.

Full-scale Shop Drawings

A full-scale layout drawing of the main trusses (Drawing SD-1) is provided along with this book. This drawing will be used to assemble the main trusses. A full-scale layout drawing of the gusset plates (Drawing SD-2) is also provided on page 1-17. This drawing shows exactly half of the gusset plates required for the bridge.

On an Actual Bridge Project

What are shop drawings?

Shop drawings are detailed drawings of every component that will be part of the completed structure. These drawings are normally prepared by the Constructor and approved by the Design Professional.

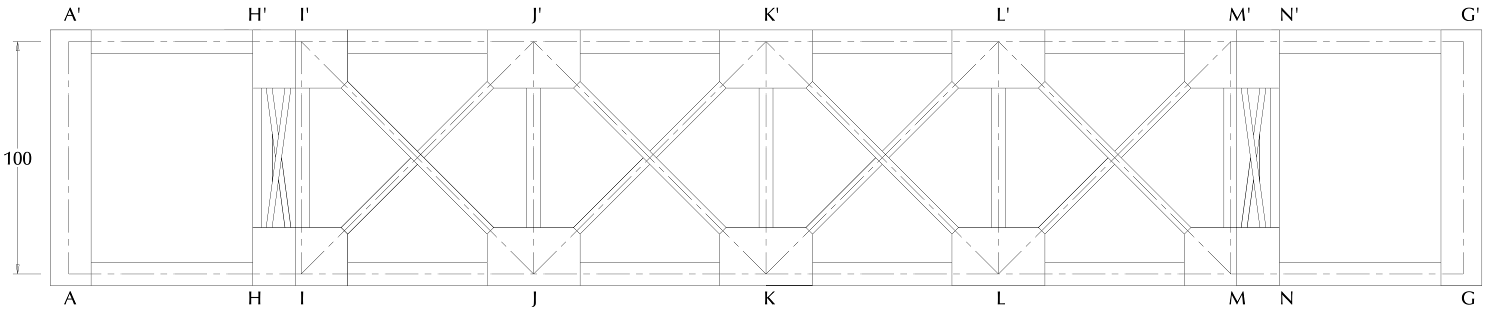
Schedule of Gusset Plates

The Schedule of Gusset Plates designates the gusset plates that will be used for each truss connection in the bridge. The numbers in the "Gusset Plate" column are from Drawing SD-2. The letters in the "Connections" column are from the structural drawing, Drawing S-1.

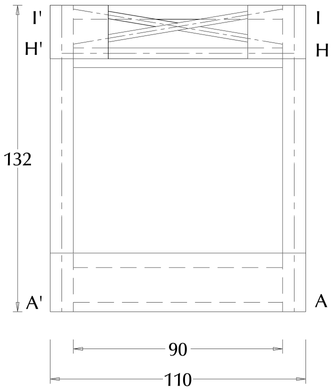
Gusset Plate	Connections	# Req'd
1	A, A′, G, G′	8
2	B, B′, F, F′	8
3	C, C′, E, E′	8
4	D, D′	4
5	I, I′, M, M′	8
6	J, J′, L, L′	8
7	K, K′	4
8	I, I′, M, M′ (top)	4
9	J, J′, K, K′, L, L′ (top)	6

Your Job

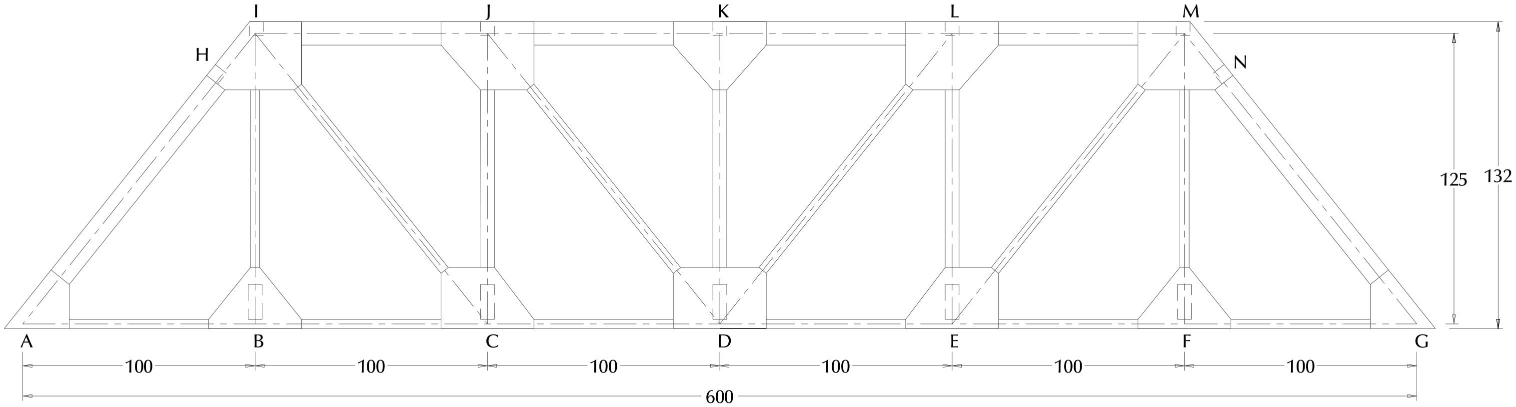
Your construction company has been selected as the Constructor for the Grant Road Bridge project. Your job is to fabricate and construct the Grant Road Bridge, as specified in the Thayer Associates design. As Constructor, you are responsible for completing the project on time, within budget, and to the level of quality specified in the plans and specifications.



Plan View



Front Elevation



Side Elevation

Notes:

1. All dimensions are in millimeters.
2. All structural members and gusset plates are made from standard file folder material.
3. See Schedule of Truss Members for specific member sizes.

Thayer Associates, Inc.
Architects & Engineers

Designed by:

Drawn by:

Checked by:

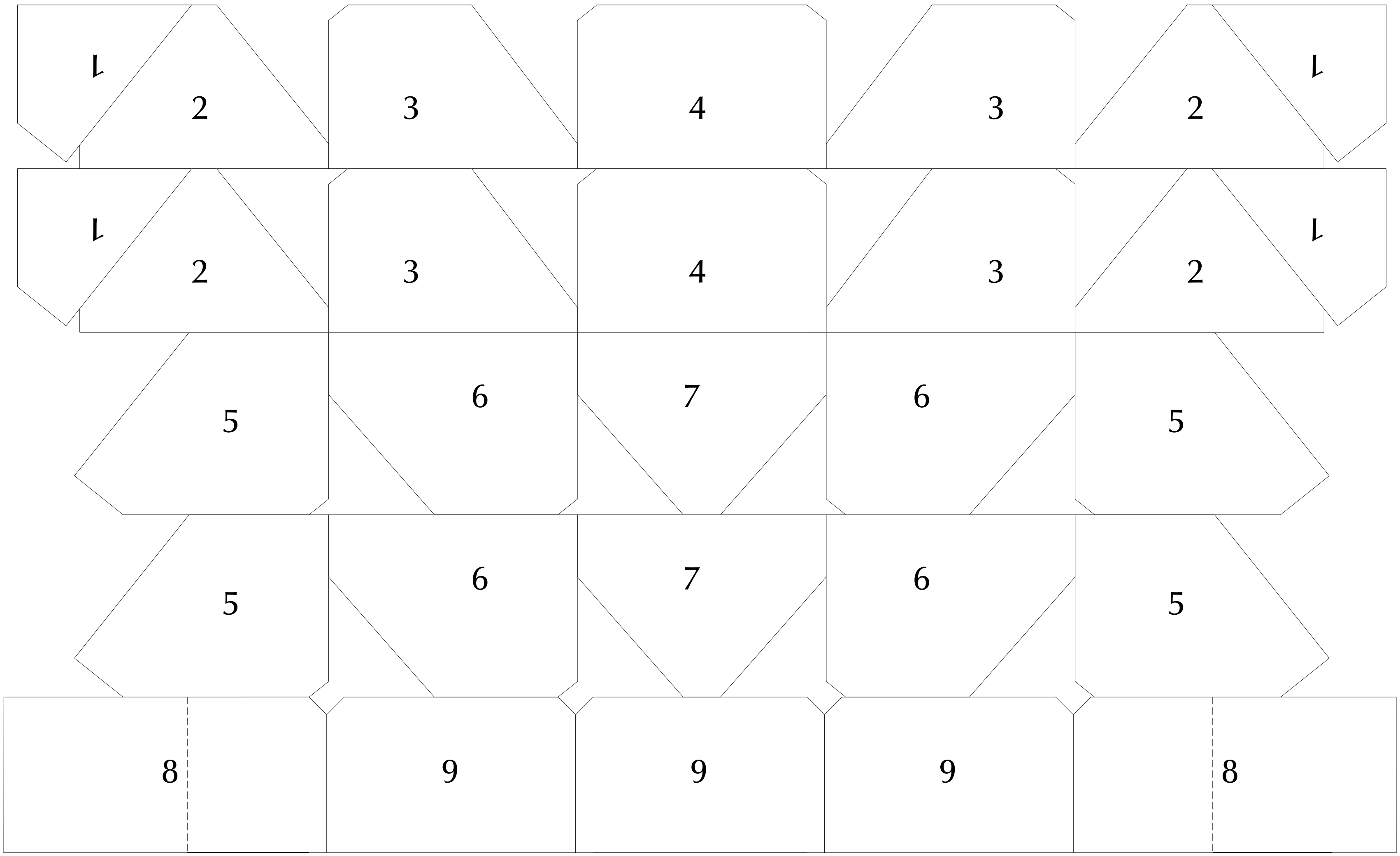
HAUPTVILLE, NEW YORK
GRANT ROAD BRIDGE
OVER UNION CREEK

Reviewed by:

Approved by:

Date: NOVEMBER 10, 2000

Sheet Reference Number S-1



Notes:

1. All gusset plates are shown full scale.
2. This layout shows exactly half the number of gusset plates required for the bridge.
3. See the Schedule of Gusset Plates for the location of each plate.

Thayer Associates, Inc.
Architects & Engineers

GRANT ROAD BRIDGE

GUSSET PLATE LAYOUT

Designed by:

Drawn by:

Checked by:

Reviewed by:

Approved by:

Date: NOVEMBER 10, 2000

Sheet Reference Number SD-2

The Plan

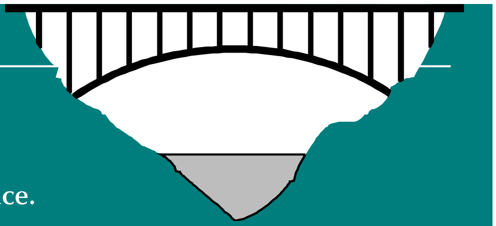
Congratulations on your selection as the Constructor for the Grant Road Bridge project! You have received the plans and specifications, and the Owner has given you the **notice to proceed**—an official authorization to start work on the project.

At the start of any construction project, the Constructor's first priority is to develop a detailed plan for building the facility. For this project, our construction management plan is relatively simple. It consists of the following six major steps, which should be performed in sequence:

- Obtain the necessary supplies and tools.
- Prefabricate the structural members and connections.
- Set up the construction site.
- Build the structure.
- Perform a quality control inspection.
- Put the bridge into service.

Put on your hard hat and your steel-tipped boots. It's time to get to work!

On an Actual Bridge Project



Safety comes first.

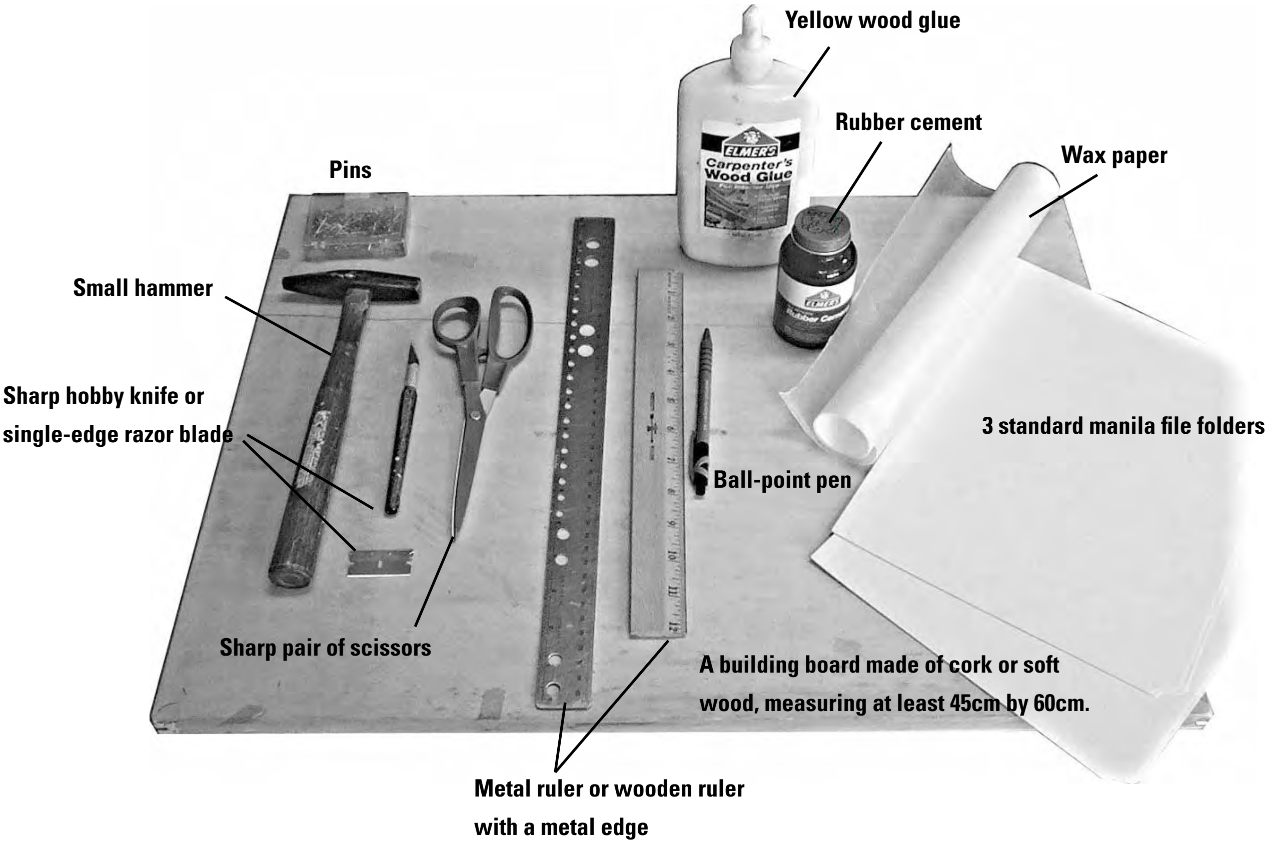
A construction site can be a very dangerous place.

Everyone on the site is responsible for ensuring that the work gets done without serious injury or loss of life. The Design Professional must ensure that the structure can be built safely. The Constructor must ensure that every worker has the necessary safety equipment and that everyone on the site follows appropriate safety procedures. The Project Manager oversees the safety of the entire project. Getting a project done can never be as important as the life of one worker.

Obtain the Necessary Supplies and Tools

To build the Grant Road Bridge, you will need the supplies and tools shown below. In addition, you'll need the full-size bridge plans that were included with this book.

- 3 standard manila file folders
- A building board made of cork or soft wood, measuring at least 45cm by 60cm.
- Wax paper
- Pins
- Small hammer
- Sharp pair of scissors
- Sharp hobby knife or single-edge razor blade
- Metal ruler or wooden ruler with a metal edge
- Ball-point pen
- Yellow wood glue
- Rubber cement



Some of the layout work will be easier if you use a draftsman's T-square and a drawing board. You'll also need a metric scale to weigh the books that we will be using to load-test the completed structure.

Prefabricate the Structural Members and Connections

Prefabricate the Bars

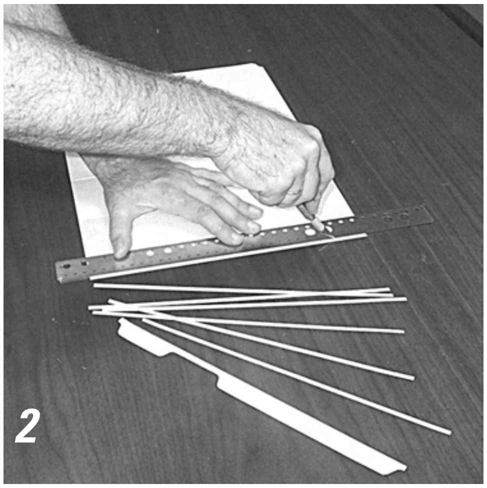
The bottom chords, diagonals, and hip verticals specified in the Schedule of Truss Members are all 4mm-wide bars—actually strips of cardboard sliced from a manila file folder. If you are using a standard 30cm-wide file folder, you'll need to slice 30 strips to make all of these members. Here's how to do it:

1) Using a ruler and a pen, carefully measure and draw 31 parallel lines exactly 4mm apart. Draw the lines parallel to the longer dimension of the folder, so that each line is 30cm long. (You can draw these lines more easily and more precisely if you use a T-square and a drawing board. Step 4 shows how the T-square is used.)

2) Place the marked file folder on your building board, and use a sharp knife and a metal ruler to cut along each line. Don't press too hard, or you'll have trouble making a straight cut. It will probably take two or three passes with the knife to cut all the way through the cardboard. (You could also use a scissors for this job, but you would find it much more difficult to make straight cuts.)

Don't cut the strips to length at this time. You'll do this when you build the trusses.


1


2

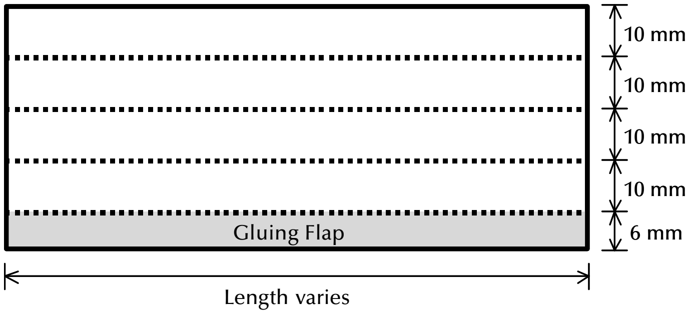
Prefabricate the Tubes

Tubes are a bit harder to make, because each one must be cut out, then folded four times and cemented together. It may take a few minutes to make each of these members, but do the job carefully—the load-carrying capacity of your bridge depends on it!

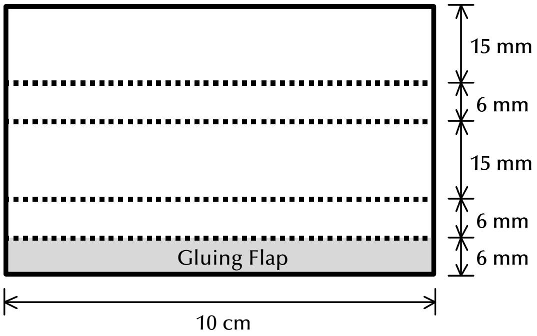
Here's how to build the tubes:

3) These diagrams show how each of the four different types of tubes should be laid out. The solid lines indicate the outline of the member—you will cut along these lines. The dotted lines indicate where the member will be folded.

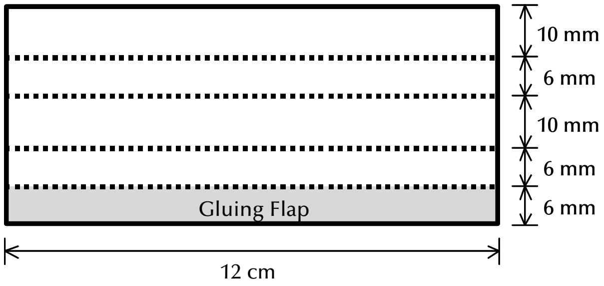
10 mm x 10 mm Tube (Top Chords & End Posts)



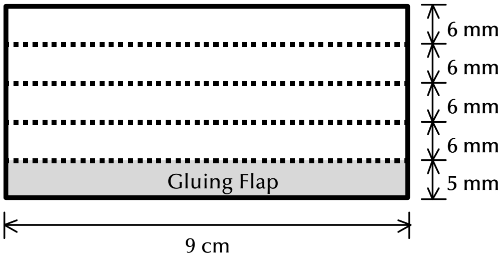
15 mm x 6 mm Tube (Floor Beams)



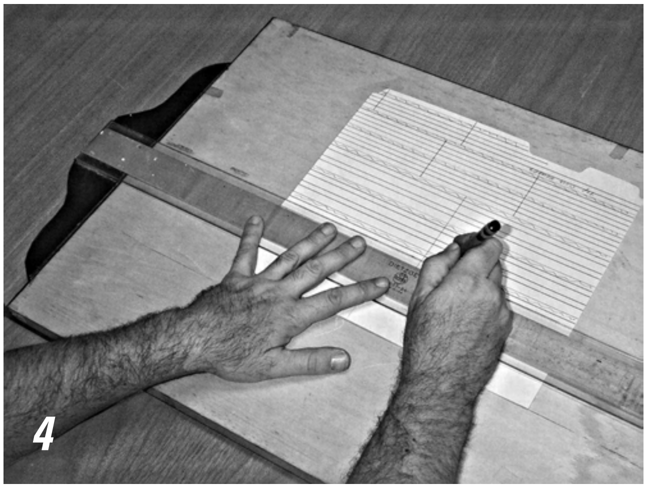
10 mm x 6 mm Tube (Verticals)



6 mm x 6 mm Tube (Struts)



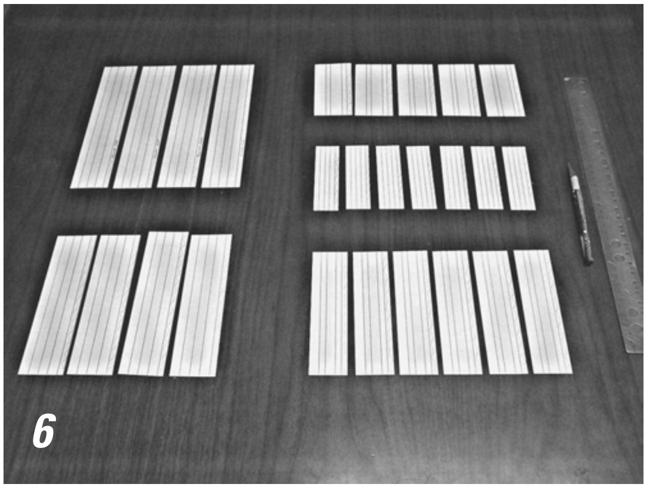
4) Using a ball-point pen and a ruler (or a T-square, as shown in the photo), lay out all the tubes specified in the Schedule of Truss Members. Start by measuring and marking the parallel edges and fold-lines as accurately as possible on the file-folder cardboard. When you draw the fold-lines, *press hard* with the pen. The indentations made by the pen will make the cardboard easier to fold. Now draw perpendicular lines to indicate the lengths of the members, as specified in the Schedule of Truss Members. It is best to mark the lengths a bit oversize now, then trim them to the exact length later, when you build the trusses. In planning your layout, note that you can get two verticals, three struts, or three floor beams from each 30cm length of material.


4

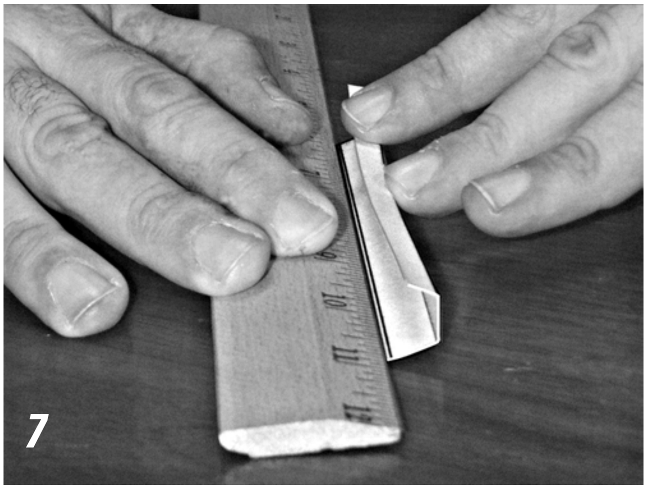
5) Once you have drawn all the tubes, cut them out with a scissors or knife. Remember *not* to cut the fold lines.

5

6) When all of the members have been cut out, you are ready to begin folding them into tubes.


6

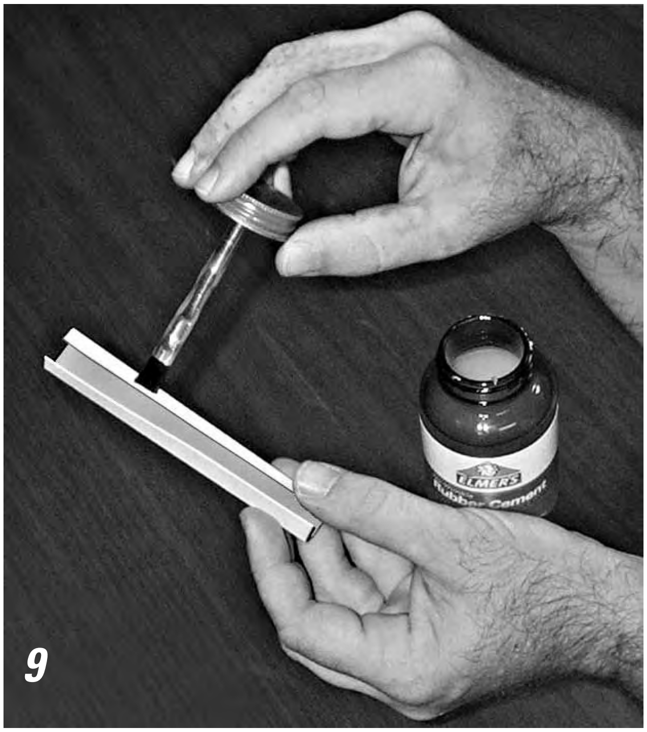
7) Starting with one of the 10mm x 10mm members, fold and crease each of the four fold-lines. To help you make the folds straight, lay a ruler along each fold-line as you bend the cardboard.


7

8) Once you have made all four folds, the member will form a square tube, as shown here.

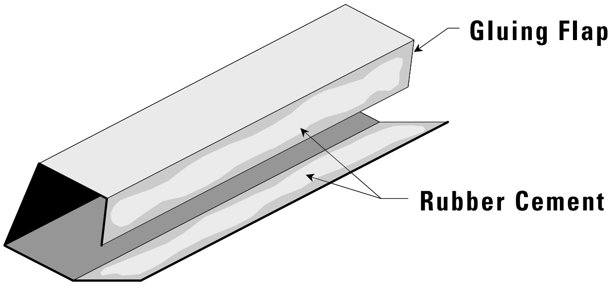

8

9) We will use rubber cement to attach the "gluing flap" to the other free edge of the tube. Rubber cement works well for his job, because it dries very quickly and because these joints do not require the greater strength provided by wood glue.



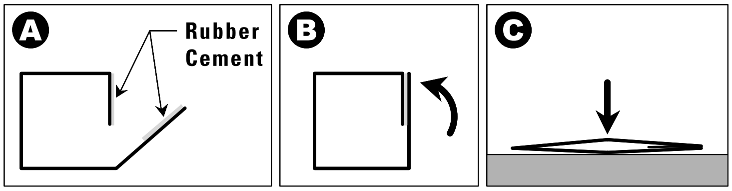
9

10) Apply an even coat of rubber cement to both surfaces that will be bonded together, as shown here. Wait 2-3 minutes, until the cement is tacky. (If the cement is still wet when you put the glued surfaces together, you'll have to hold them together as the cement dries. If the cement is sticky, but not quite dry, the two surfaces will bond together as soon as they touch. To get the timing right, you might want to practice on a scrap piece of cardboard before actually gluing the tubes together.)



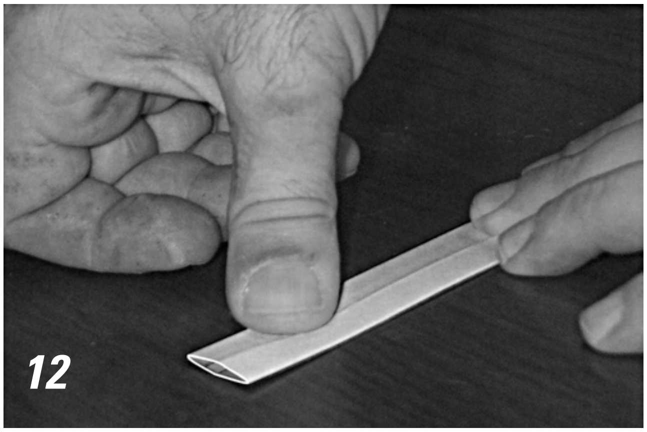
10

11) Now carefully bring the two cemented surfaces together **(A)** to form a square cross-section **(B)**. Note that the gluing flap goes on the *inside* of the completed tube. Immediately flatten the tube on you building board **(C)** and hold it flat for a few seconds, until the cement sets.



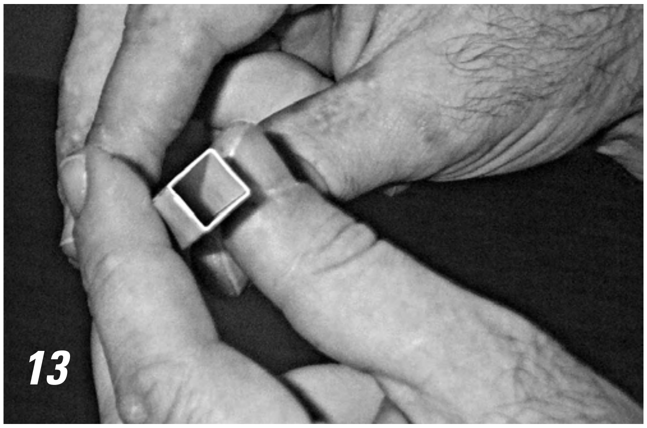
11

12) Flattening the tube in this manner does two things. First, it ensures that the cemented surfaces are firmly in contact with each other; and second, it ensures that the completed member is not curved or twisted.



12

13) After the rubber cement has dried, pick up the tube and reshape it back into a square cross-section.



13

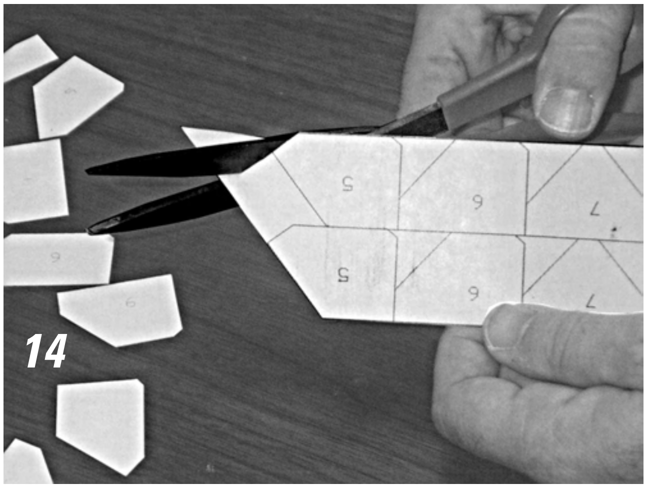
Now repeat Steps 7 through 13 for all of the remaining tube members.

Prefabricate the Gusset Plates

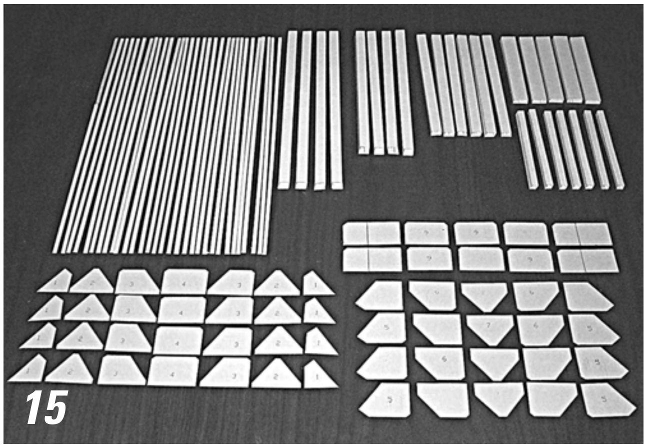
Our example bridge uses a total of 58 gusset plates to connect the structural members together. If you wanted to make these components in the easiest possible way, you could simply cut out 58 cardboard rectangles, each measuring 40mm x 30mm. These would work fine; however, they would not look authentic, and they would use a lot more cardboard than necessary. The gusset plates shown in Drawing SD-2 more accurately depict real truss connections.

To make the authentic gusset plates, the full-scale layout provided on Drawing SD-2 must be transferred to a file folder. There are three possible ways to accomplish this: (1) you can lay a sheet of carbon paper on the file folder, then lay Drawing SD-2 on top of the carbon paper, and trace over the outline of each gusset plate with a pen or pencil; (2) you can cut out the gusset plates from Drawing SD-2, lay them on the file folder, and trace around each one with a sharp pencil or pen; or (3) if you have access to a copy machine with a "single sheet feeder," you can photocopy the patterns directly onto file-folder card board. You'll need to cut the folders to 8 ½" x 11" size, so they will run through the single sheet feeder without jamming. Clearly this third option is easiest and most accurate. No matter which method you use, remember that you will need to do the process twice, because Drawing SD-2 only shows half of the required gusset plates.

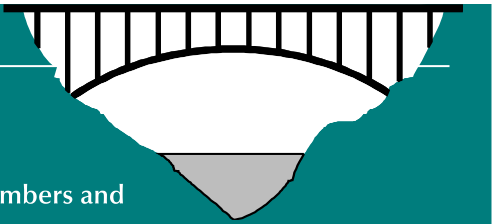
14) Once you have transferred the gusset plate patterns onto the cardboard, carefully cut out each gusset plate with a sharp scissors.



15) The prefabrication of structural members is now complete. We're ready to start building!



On an Actual Bridge Project

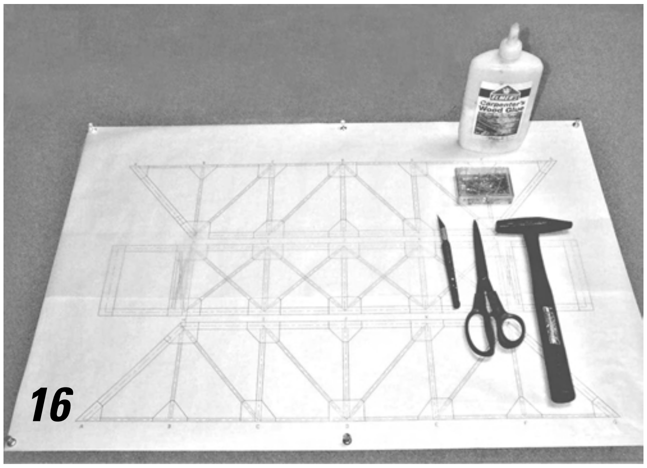


Who prefabricates structural components?

On an actual bridge project, steel structural members and connections are made by a **steel fabricator**. The steel fabricator is a member of the Construction Team and is usually a subcontractor. The fabricator's work is done in a shop, not on the construction site, in order to achieve the highest possible quality.

Set up the Construction Site

16) Find a large, flat tabletop to use as your construction site. Place your building board on the tabletop. Place Drawing SD-1 on top of the board, and put a layer of wax paper over it. Use a few pins or staples to keep everything in place.



On an Actual Bridge Project

How does the Constructor set up the construction site?

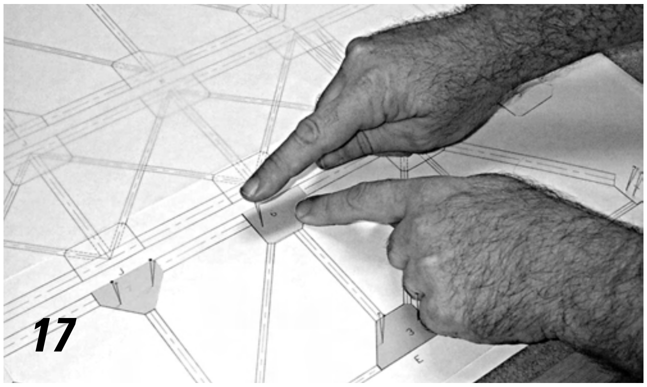
The setup of a construction site varies from project to project. However, on just about any major project site, the Constructor will provide the following:

- A construction office with telephones, computers, storage fo important documents, and workspace for the Superintendent—the on-site construction supervisor.
- Space for on-site storage of construction materials.
- Access to the site for construction vehicles.
- A location for the crane that will be used to lift pieces of the structure into place.
- Traffic control and safety fencing to keep unauthorized vehicles and pedestrians away from the site.
- Electrical power for tools and equipment.

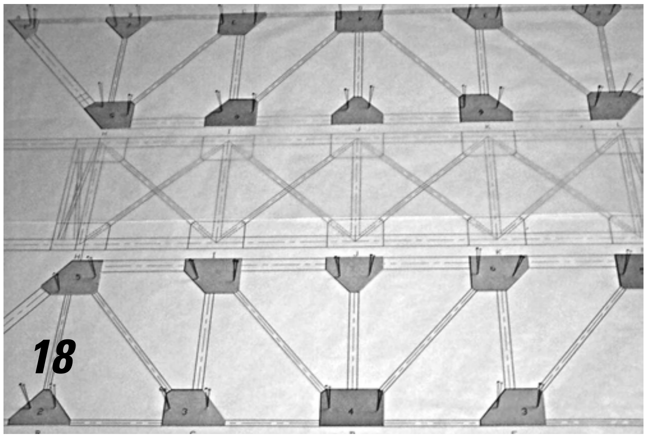
Build the Structure

Build the Main Trusses

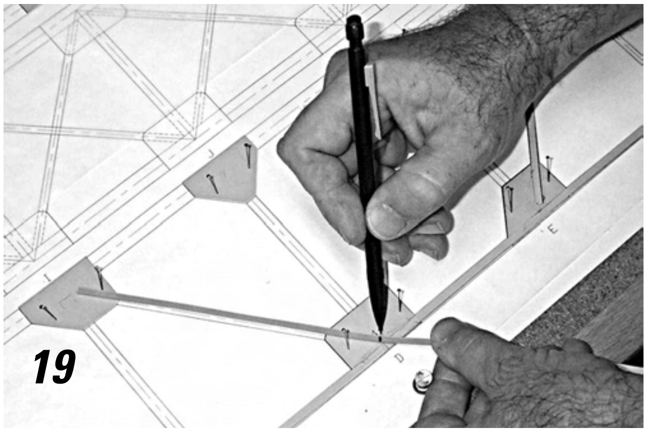
17) Start by placing the appropriate gusset plates directly onto the drawings of the two main trusses, at the locations designated in the Schedule of Gusset Plates. Hold each gusset plate in place with two pins. (If you use only one pin, the plate will be able to rotate out of position.) Put each pin through a point on the gusset plate where no members will be attached; otherwise, the pins will be in the way when you glue the members to the gusset plates.



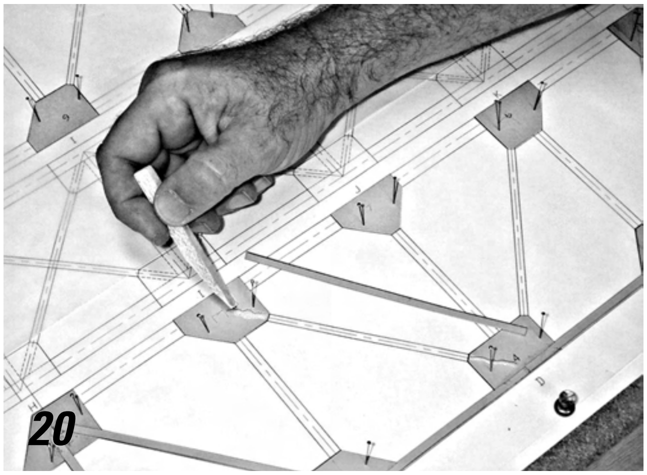
18) When you're done, you will have 24 gusset plates pinned to the board.



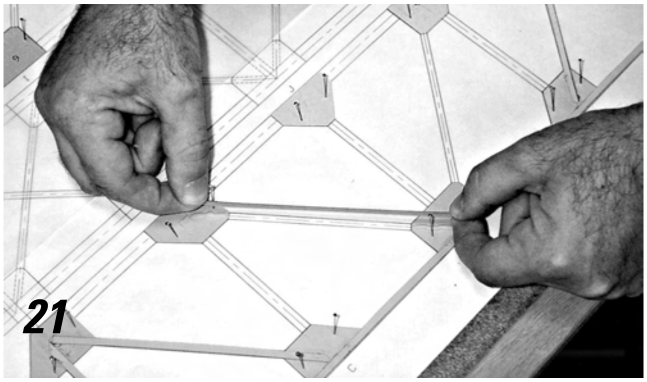
19) Now add the 4mm bars. Select one of the cardboard strips you prefabricated in Step 2. Place it in the correct position, and mark its length with a pencil or pen. (The photo shows Member DJ, a diagonal, being marked. Note that the bottom chord members are already in place.)



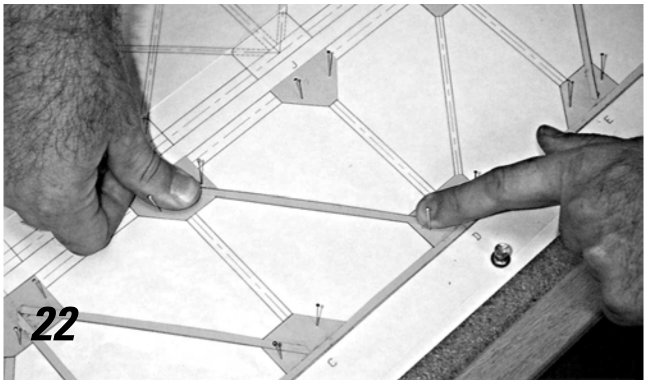
20) Spread wood glue on the two gusset plates to which the member will be attached. You can apply the glue directly from the bottle; however, you will find that you can do a neater job if you use a scrap piece of cardboard or wood as an applicator, as shown here. Do not use rubber cement for these joints! It's not strong enough.



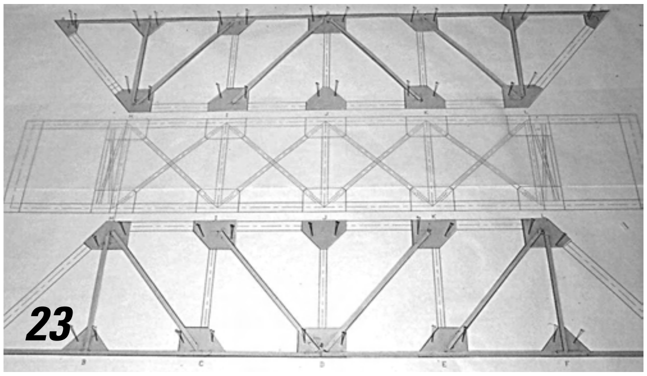
21) Now place the 4mm strip onto the gusset plates, using the drawing beneath to ensure that it is in the correct position.



22) Press firmly on both ends of the member, and hold it in place for about 30 seconds, until the glue starts to set. Be careful not to glue your fingers to the gusset plates!

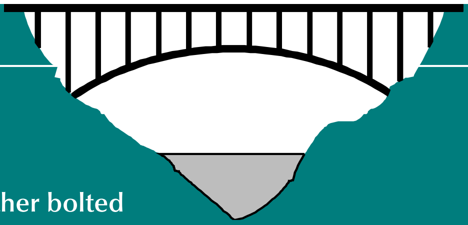


23) Repeat Steps 19 through 22 for each of the 16 bars on the two trusses. Note that the two outermost verticals on each truss are bars. The remaining three verticals are tubes, which will be attached in the next series of steps.



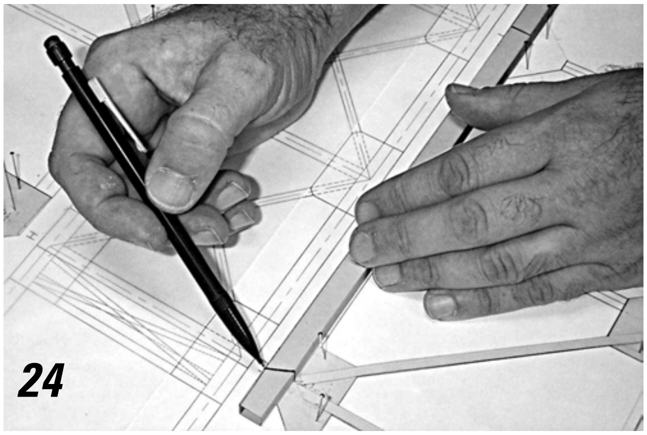
On an Actual Bridge Project

How are members attached to gusset plates?

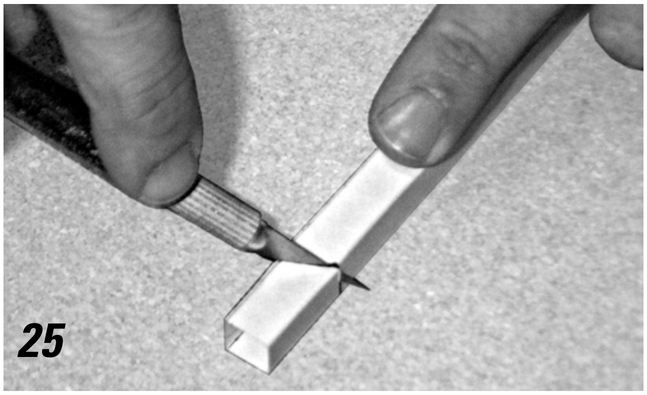


On modern bridges, structural members are either bolted or welded to the gusset plates. When two pieces of steel are welded, they are actually fused together to make a single piece of steel. Welds are strong and relatively inexpensive, but they require highly skilled workers and specialized equipment to make. Thus welded connections are often used for portions of a structure that can be assembled in the shop by the steel fabricator. Bolted connections do not require specialized equipment and are relatively easy to assemble; thus, they are often used for field connections—those that are assembled on the construction site, rather than in the fabrication shop.

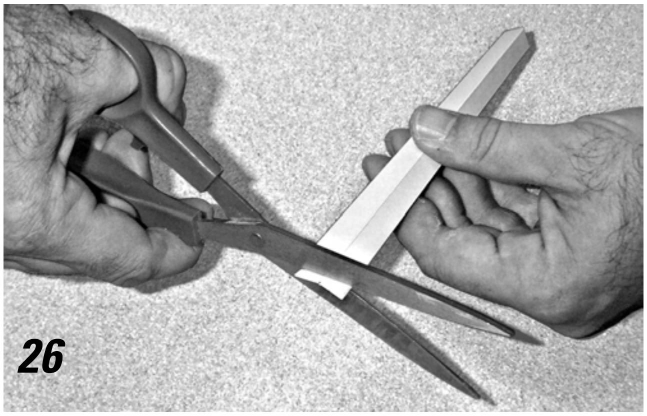
24) Now you will attach the tubes to the gusset plates, but only on one of the two trusses—the one closer to you. Start by placing one of the 10mm x 10mm tubes into position on the top chord and marking its length with a pencil or pen. (Member IK is shown in the photo.)


24

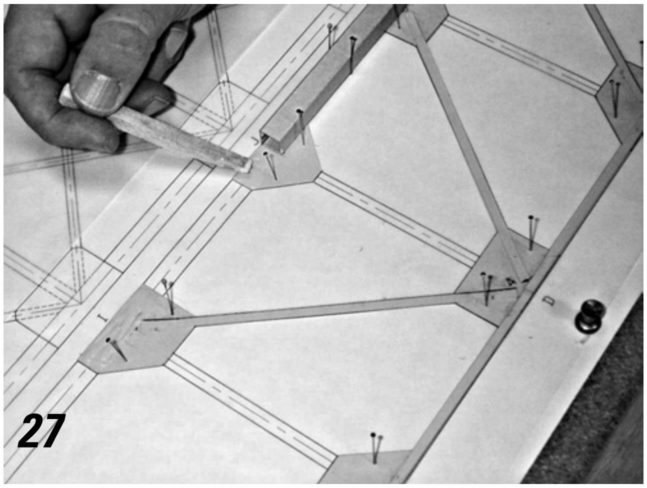
25) Cut this member to the correct length. Some of the tubes—like this one—must be cut at an angle. You'll need to use your knife or razor blade to make these cuts.


25

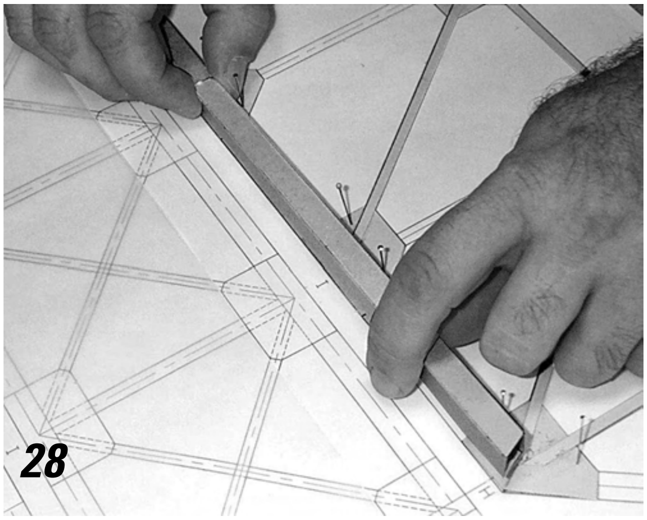
26) Other members must be cut square. For these cuts, it is easier to flatten the end of the tube, and use a scissors.


26

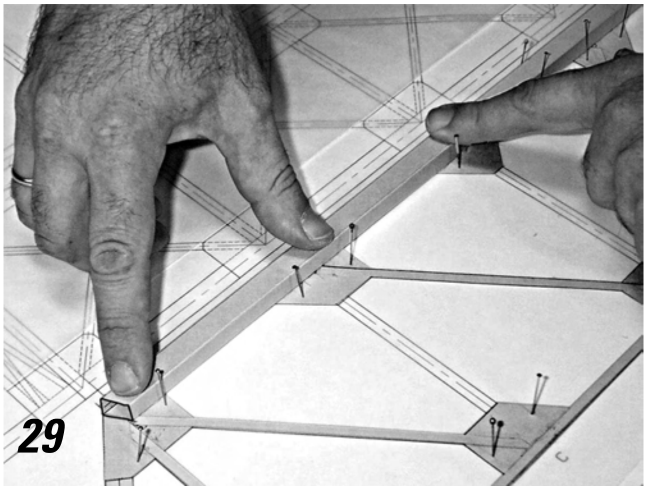
27) Now apply glue to the two gusset plates.


27

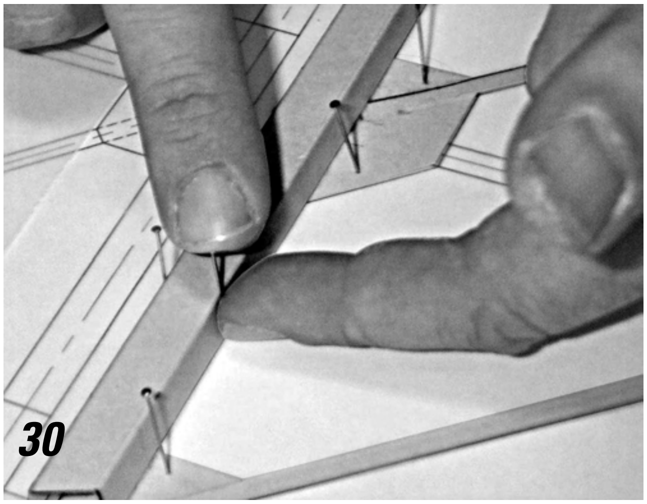
28) Put the member into position.



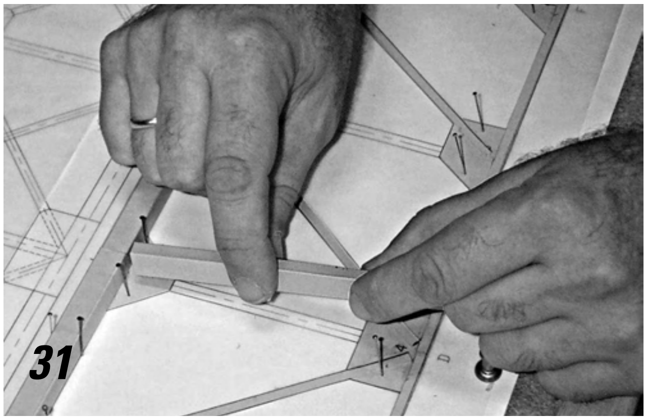
29) And hold the member in place until the glue sets.



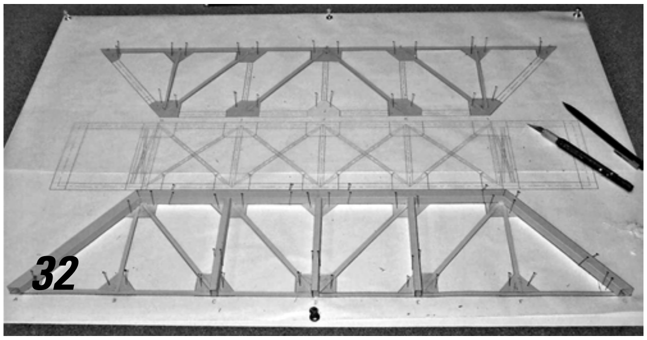
30) As you assemble the main trusses, it is very important to keep the cross-section of each tube as close to square as possible. Do this by pushing a pin into the building board on either side of each tube, as shown. The pins will keep the sides of the tube vertical as you assemble the remainder of the truss.



31) Repeat Steps 24 through 30 for the remaining tubes. Be sure to use the correct sizes—10mm x 10mm tubes for the top chords and end posts, and 10mm x 6mm tubes for the three interior verticals. The photo shows the vertical member DK being glued into position. Note that the verticals should be placed with their shorter (6mm) side flat on the building board and the longer (10mm) side standing upright.

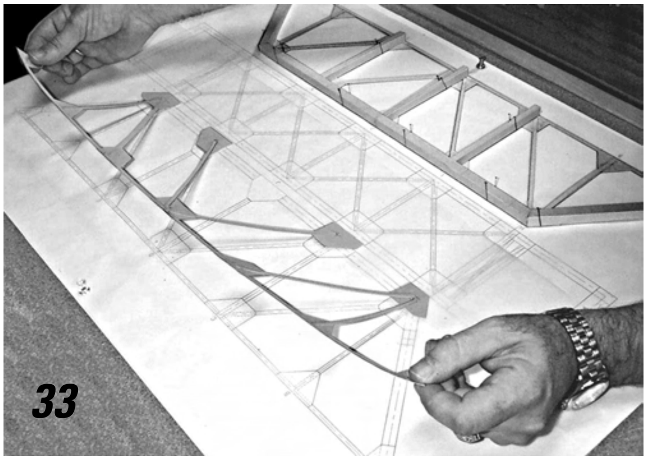


32) When all of the tubes are glued in place, the result should look like this. Note again that the tubes are only glued to one of the two trusses on the building board.

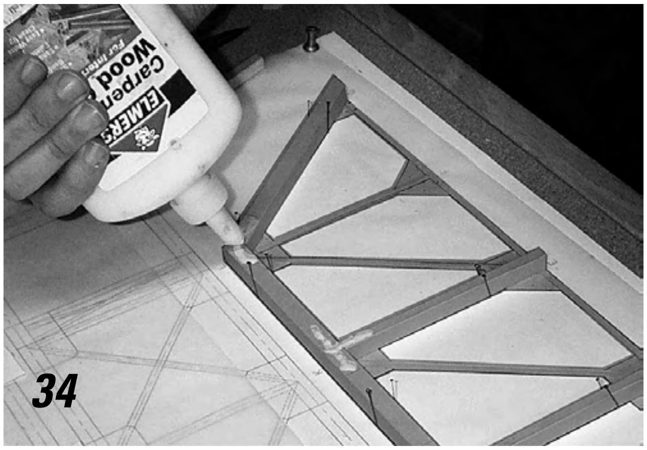


33) The two subassemblies currently on the board are actually two halves of the *same* truss. Now we will put them together. Begin with the "upper half"—the one composed only of gusset plates and bars. Remove all of the pins holding the upper half to the building board, and lift it free. *Carefully* separate this subassembly from the wax paper. If you tear one of these members, your bridge will probably not be able to carry its specified 5 kilogram load. Once you have separated the subassembly from the wax paper, put it aside.

On the "lower half"—the one closest to you—pull out all of the pins in the gusset plates, but do not remove the pins you added in Step 30.



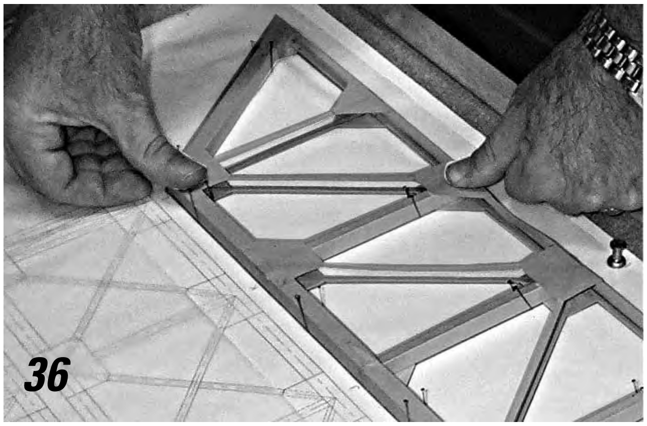
34) Put glue at the appropriate locations on the lower half of the truss—the places where the upper gusset plates will connect to the tubes on the lower half.



35) Carefully place the upper half directly on top of the lower half. Ensure that the two halves are aligned before the glue starts to set.

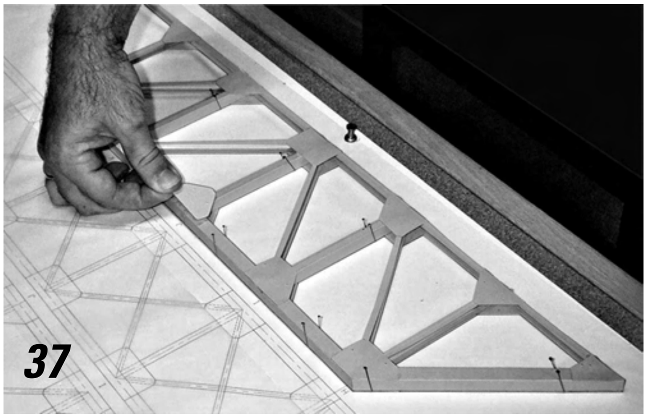

35

36) Note that, when you assemble the two halves, you are creating the *doubled bars* specified in the Schedule of Truss Members. It is very important that both of the bars in each pair are stretched tightly between gusset plates. If one is tight and the other is loose, the two will not share load equally and may fail prematurely. Use your thumbs to pull the upper bars tight, as shown.


36

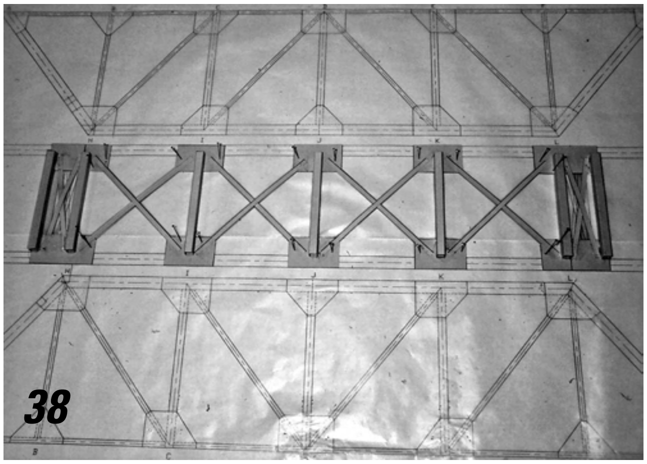
37) At this point, the assembled truss is missing only the upper gusset plate at Connection K. This one was left behind on the building board, because it is not attached to any bars on the upper half of the truss. Glue this final gusset plate in place.

Now remove the truss from the board, and set it aside. Using exactly the same procedure (Steps 17 through 37) build a second *identical* main truss.

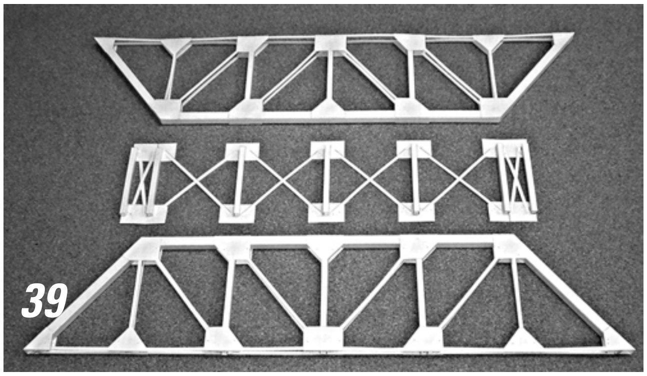

37

Build the Lateral Bracing Subassembly

38) Following the same procedure you used for the main trusses, assemble the lateral bracing subassembly, which will connect the two main trusses together. Start by pinning the gusset plates to the board; then add the lateral bracing, made of *single* 4mm bars. Finally add the struts, which are made of 6mm x 6mm tubes, all exactly 9 cm long.


38

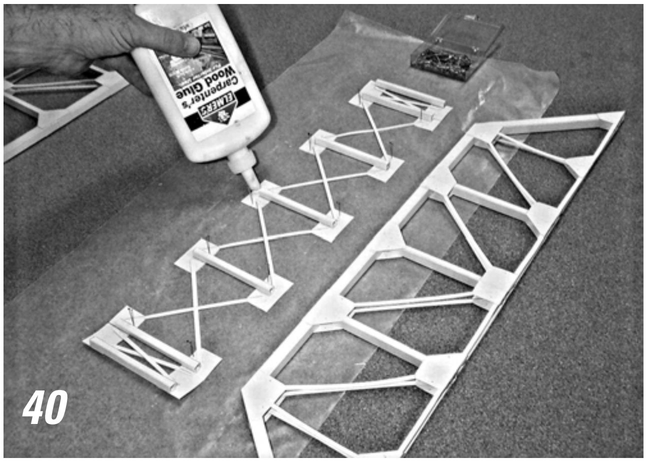
39) When the lateral bracing subassembly is complete, remove it from the building board. The three major subassemblies are now done. We are ready to assemble the three-dimensional structure.



39

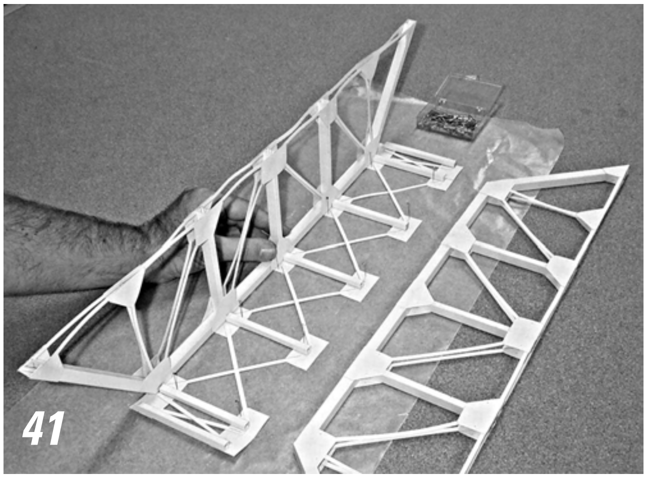
Connect the Two Trusses Together

40) Place the lateral bracing subassembly flat on the building board. Apply glue to all five gusset plates on one side.



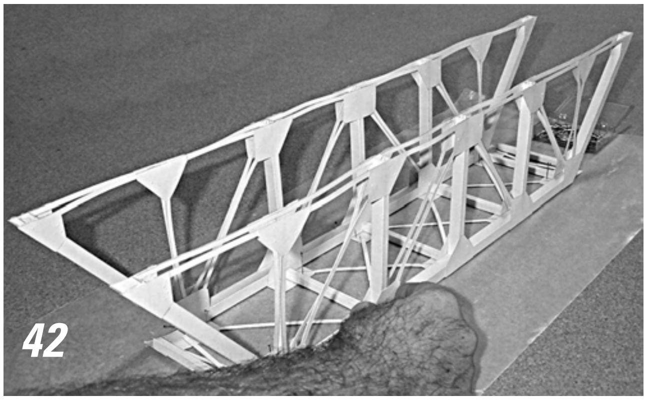
40

41) Position one of the two main trusses. (Note that you are assembling the bridge upside down.) Once the truss is in position, check to ensure that it is vertical, and hold it in place for a minute or so.



41

42) Now add the second truss in the same manner.



42

43) You may find it helpful to use pins on the outsides of the two trusses to hold the subassemblies together while the glue sets. Do not remove any pins or move the bridge until the glue has dried completely.


43

Q 3

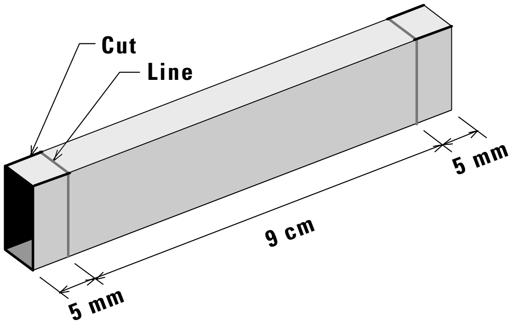
What are the purposes of the struts and lateral bracing?

As you can see from Steps 40 – 42, the struts and lateral bracing help connect the two main trusses together. What other purposes do you think these members serve?

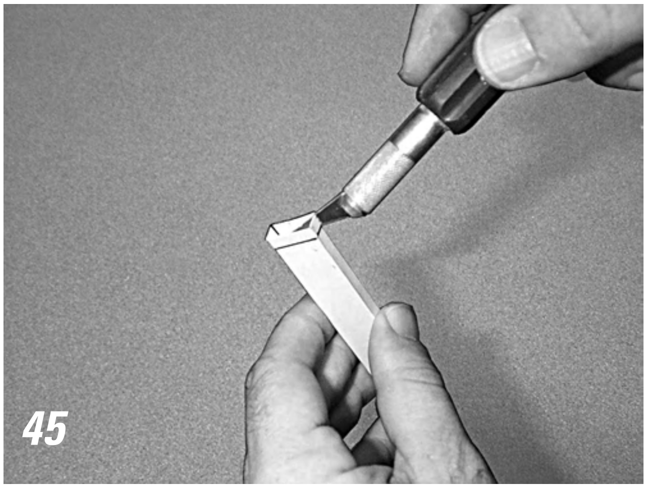
Add Floor Beams

Just as the struts connect the two trusses together at their top chords, the floor beams connect the trusses together at their bottom chords. As you might have noticed when you prefabricated these members, the floor beams are one centimeter longer than the struts. Why? Since there are no gusset plates to connect the floor beams to the bottom chord, we will use the outer 5mm on each end of the floor beams to form *connecting angles*. These angles will connect the floor beams to the gusset plates on the inside of each truss.

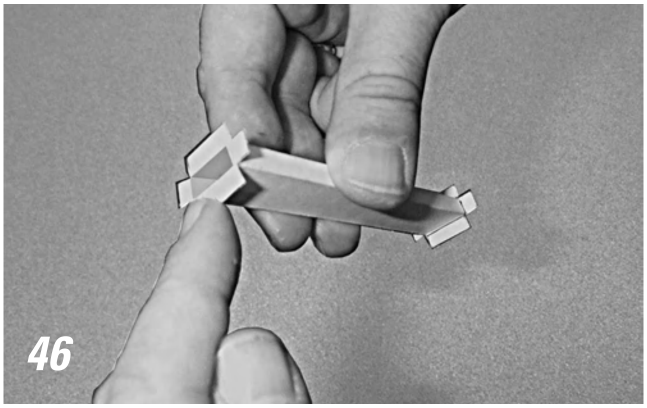
44) Draw two lines completely around the tube, as shown on this diagram. (An easy way to do this is to flatten the tube, then use a ruler to draw the lines.) The two lines should be *exactly* 9cm apart.


44

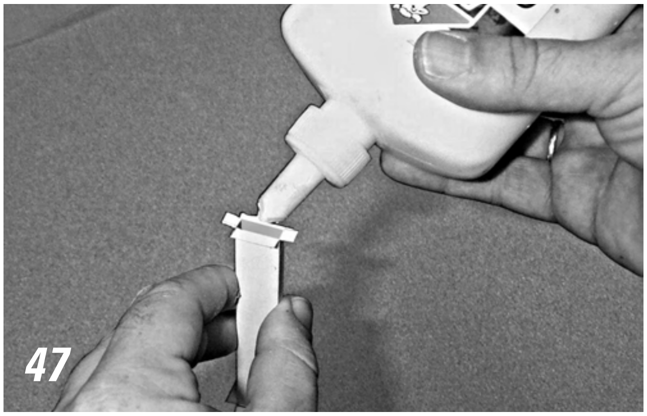
45) Now use your knife to slice through all four corners of the beam, as shown in the photo. Cut *only* as far as the line you drew in Step 44.


45

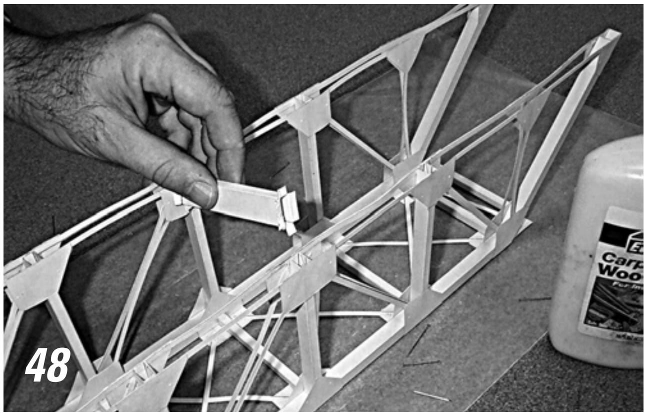
46) At each end of the beam, fold all four sides outward along the line. These four flaps form the connecting angles that we will use to attach the beam to the bottom chord.



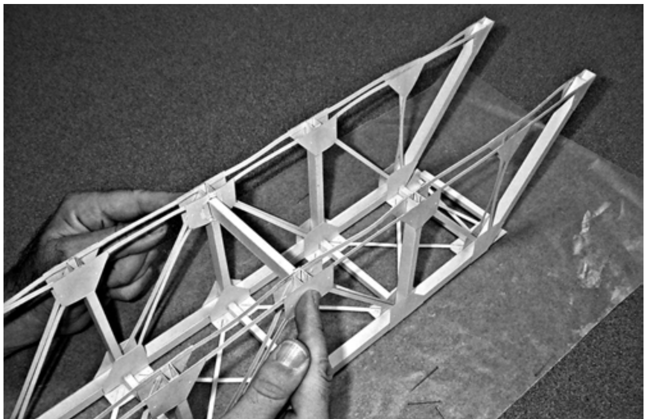
47) Apply glue to all four flaps.



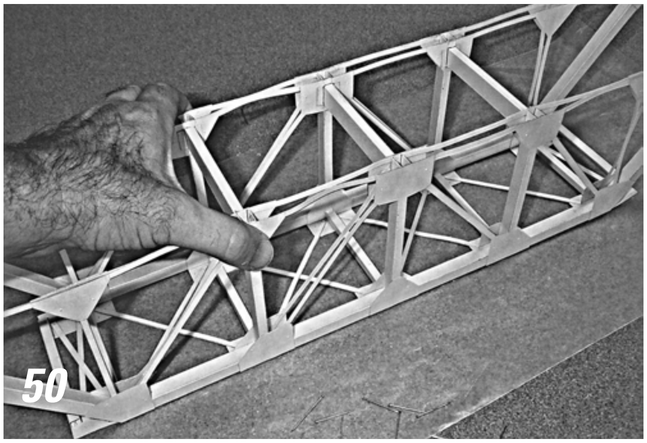
48) Place the beam between Connections D and D′, at the center of the bottom chords. Note the orientation of the beam—the larger dimension is vertical.



49) Press inward on the two gusset plates until the glue sets.

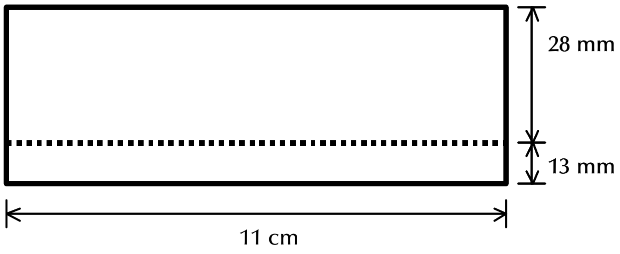


50) Now repeat the process for floor beams CC′ and EE′.


50

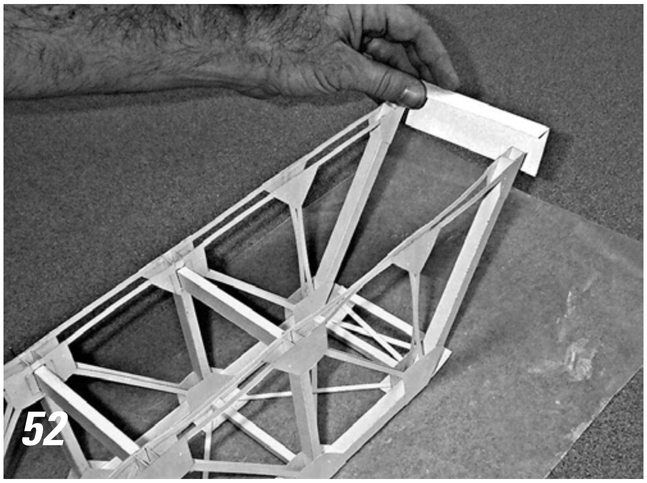
51) Cut out two rectangles of file-folder cardboard with these dimensions, and fold each one sharply along the dotted line. These are the two end floor beams AA′ and GG′.

28 mm x 13 mm L-angle (Floor Beams)

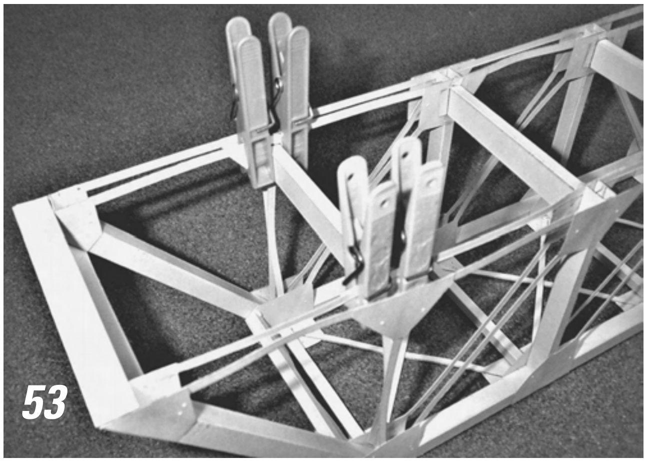


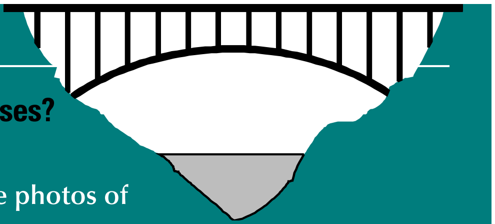
51

52) Glue the end floor beams in place, as shown here.


52

53) Glue the two remaining floor beams (BB′ and FF′) in place. Because there is no tube between the pairs of gusset plates at Connections B, B′, F, and F′, you may find it helpful to clamp the beam in place with four clothespins, as shown.


53



On an Actual Bridge Project

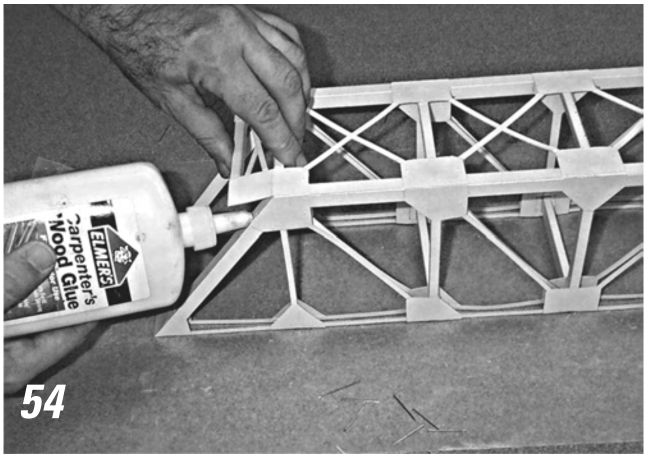
How are the floor beams attached to the main trusses?

In the Gallery of Truss Bridges (Appendix A), the photos of Bridges 9 and 12 show a typical arrangement used to attach the floor beams to the main trusses in older pin-connected bridges. In both of these bridges, the floor beam is suspended from the pin by two U-shaped bolts. Modern trusses generally use connecting angels very similar to the ones we used in our model.

Add the Portal Bracing

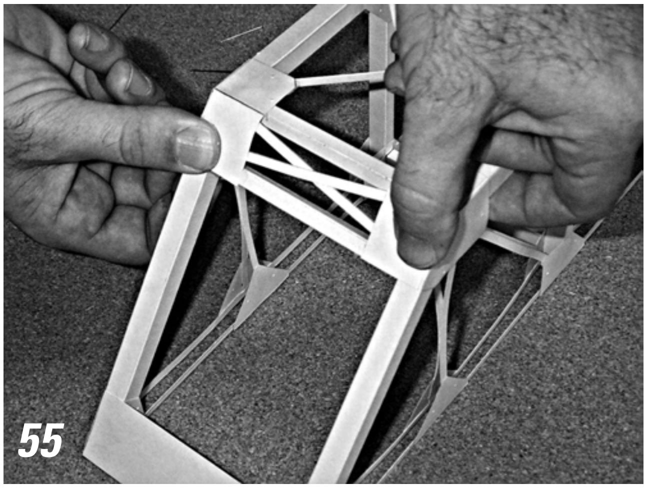
You have already created the portal bracing. You did it when you made the lateral bracing subassembly in Step 38. Now all you need to do is glue it into place.

54) Apply glue to the top front of the two end posts.



54

55) Fold the portal bracing down onto the end posts and hold in place until the glue sets. Do the same for the portal bracing on the opposite end of the bridge.



55

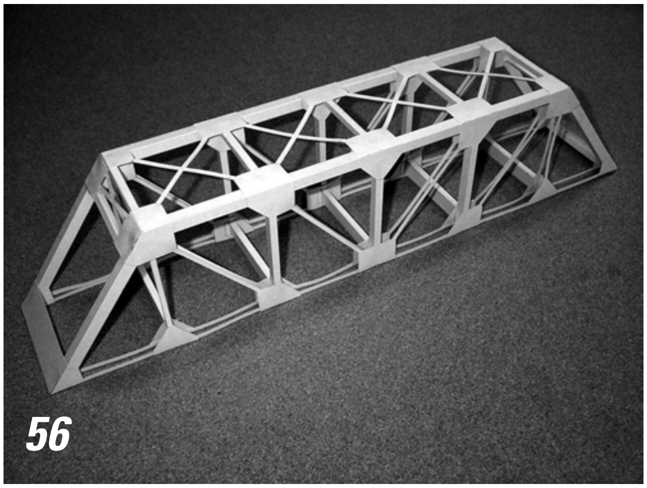
Q4

What is the purpose of the portal bracing?

The portal bracing serves an important purpose in a through truss bridge. What do you think that purpose is? To answer this question, look at your own model bridge, and try to visualize how it might fail if the portal bracing were not present.

Perform a Quality Control Inspection

56) The bridge is finished! But before you place it into service, you should carefully inspect the structure to ensure that it has been built according to the plans and specifications. Are all the structural members positioned correctly? Are any of them cut, torn, or dented? If so, repair or reinforce these points before you load the structure. Are all of the members firmly attached to their gusset plates? If not, add more glue to these joints. It is important that the connections are stronger than the members themselves.


56

Q5

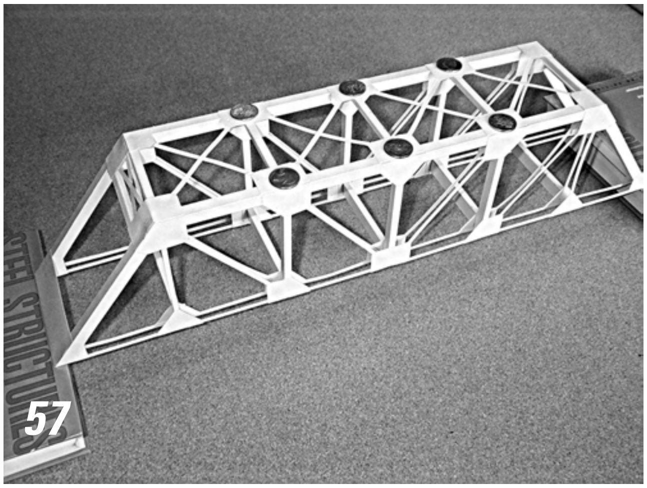
Why are truss bridges less common today?

Trusses are used much less commonly in bridge structures today than they were a hundred years ago. Based on your own experience with the Grant Road Bridge project, why do you think truss bridges are less common today?

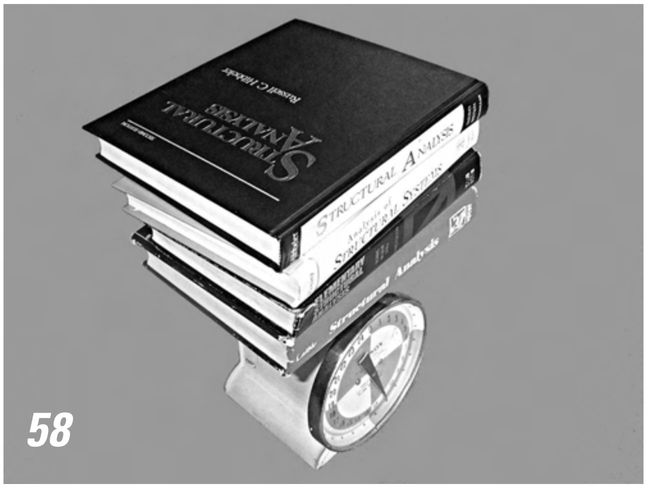
Place the Bridge Into Service

If this were a real bridge, we would be ready to hold a ribbon-cutting ceremony and open the bridge to traffic. For our model bridge, we'll simulate the grand opening by applying the prescribed 5-kilogram load to the structure. We will use a stack of books as the load.

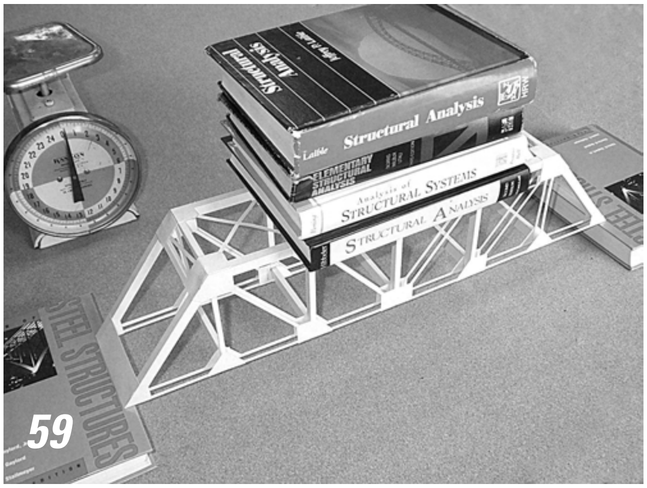
57) To prepare for the load test, set up two books as abutments approximately 59 cm apart, and put the bridge on top of them. Place a coin on top of each gusset plate at Connections J, J', K, K', L, and L'. Trusses are designed to be loaded *only* at the joints; the coins will ensure that the weight of the books is transmitted to the main trusses only at these locations.



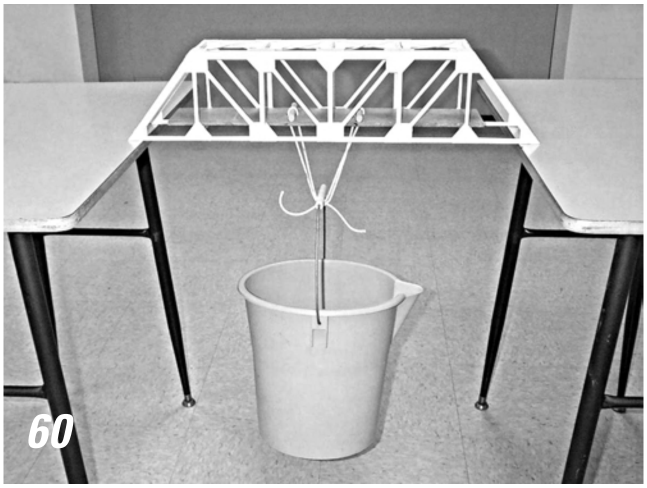
58) Using a metric scale, experiment with various books until you have assembled a stack with a mass of approximately 5 kilograms.



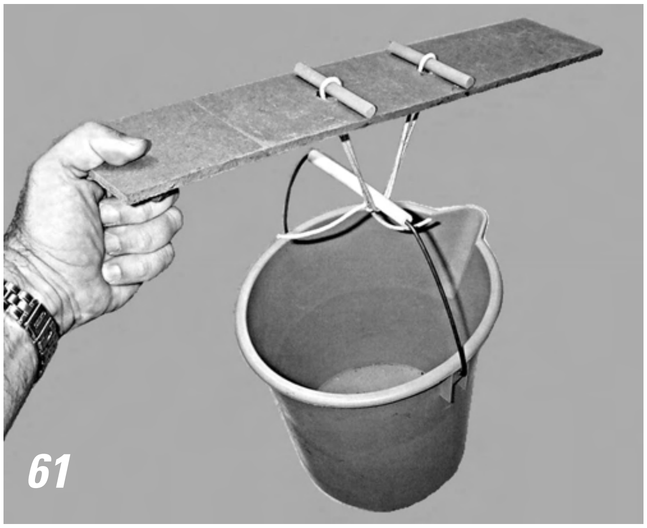
59) Now place the books gently, one at a time onto the top of the truss. Keep them centered. Leave the full 5-kilogram stack of books is in place for a few minutes.



60) Another way to load-test the bridge is to suspend a bucket filled with sand from the floor beams. The total mass of the bucket, sand, and loading platform should be 5 kilograms.


60

61) You'll need to build a wooden platform like this one to support the bucket. The platform is 8 cm wide and 50 cm long. It has two 1.5 cm holes drilled 10 cm apart in the center. The bucket hangs from a single loop of heavy cord. The two ends of the loop are pushed through the holes in the platform and held in place with two pencils or wooden dowels. The platform rests on all five interior floor beams of the bridge. Don't just hang the bucket from floor beam DD'; the concentrated weight will probably rip the floor beam out of the structure.


61

Q6

Which method of loading is better?

Compare the two methods of loading described in Steps 59 and 60. What are the advantages of each method?

If your bridge carries the prescribed 5-kilogram loading, congratulations! Your bridge construction project is a success! Weigh the bridge. You'll find that it has a mass of about 55 grams—a strength-to-weight ratio of over 90. That's not bad for a structure made of paper.

If you built your bridge well, it should actually be able to carry about 10 kilograms before it collapses. But resist the temptation to load it to failure! Engineers take great pride in the physical products of their work, and you should too. You've put a lot of time and effort into this project. Save your bridge; don't destroy it. And remember that engineers *always* design structures to stand up, not to collapse!

Q7

Why is the bridge “too strong?”

The functional requirements for the Grant Road Bridge specify that the structure must be capable of carrying a 5 kilogram mass. The actual capacity of the structure is about 10 kilograms. Why did the structural engineer design the bridge with so much extra capacity?

While your bridge still has the load in place, take a moment to examine how the structure is carrying the load. You can learn a lot about structural engineering just by carefully observing how the members in this bridge behave when a load is applied. Note that some members are stretched tightly in tension. Some are slack—they appear to have no internal force at all. Others are in compression, though these are a bit harder to identify.

Q8

How does your model bridge carry load?

Which members of the Grant Road Bridge are in tension? Which are in compression?

Q9

Why tubes and bars?

Why did the structural engineer specify tubes for some members and bars for others?

Q10

How does construction quality affect structural performance?

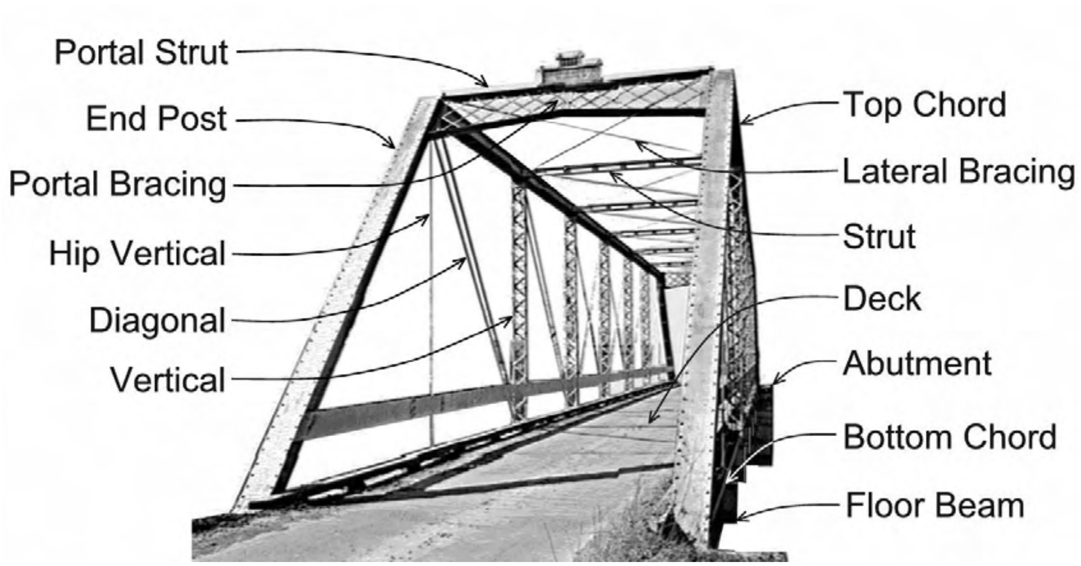
Based on your own experience on this project, explain how the quality of the Constructor’s work affects the performance of a structure. If you make errors in construction, or if your glue joints are not strong enough, or if parts of the structure are damaged during assembly, how are the function and appearance of the structure affected?

Conclusion

The bridge construction project is complete. In doing the project, you had the opportunity to learn a lot about bridges, about construction, and about some basic principles of structural engineering. You also practiced some techniques for working with a rather unusual construction material. Perhaps you had some fun too. But you haven’t actually *done* any engineering yet. Stay tuned for Learning Activity #2, where we will design and conduct a series of experiments to determine the strengths of structural members—the first step in the process of designing your own bridge.

Answers to the Questions

1) **Can you identify the component parts of a truss bridge?** The component parts of a typical Pratt through truss bridge are annotated on the photo below.

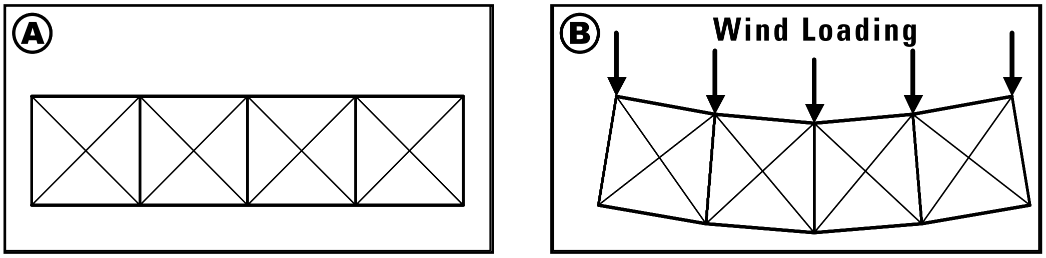


2) **Can you identify the configuration of a truss bridge?** The configurations of the bridges pictured in the Gallery of Truss Bridges (Appendix A) are as follows:

#	Truss Configuration	Through, Deck, or Pony
1	Warren with Verticals	Through
2	Baltimore	Through
3	Pratt	Pony
4	Waddell "A" Truss	Through
5	Fink	Through
6	Pratt	Through
7	Pratt	Deck
8	Camelback	Through
9	Double Intersection Pratt	Through
10	Pennsylvania	Through
11	Non-standard "A" Truss	Through
12	Pratt	Through
13	Lattice	Through
14	Warren with Verticals (background)	Deck
15	Parker	Through

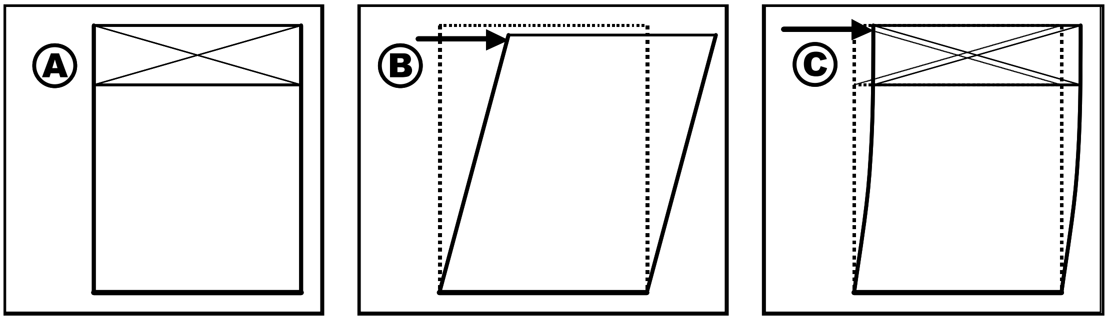
#	Truss Configuration	Through, Deck, or Pony
16	Baltimore (left) & Pratt (right)	Through
17	Pratt	Through
18	Double Intersection Warren	Deck
19	Parker	Through
20	Pennsylvania	Through
21	Parker	Pony
22	Bowstring	Through
23	Warren with Verticals	Pony
24	Warren with Verticals	Through
25	Bowstring	Pony
26	Pratt	Deck
27	Warren with Verticals	Pony
28	Parker	Through
29	Pratt Arch	Deck
30	Pratt Arch	Deck

3) What are the purposes of the struts and lateral bracing? If you look at the Grant Road Bridge from directly above (plan view), you notice that the two top chords, the struts, and the lateral bracing form a truss, as shown in **A** below:



This truss lies in a horizontal plane, rather than standing vertically, as the two main trusses do; but it is a truss nonetheless. And if you view this subassembly as a truss, you'll probably be able to see one of its major purposes—carrying loads caused by wind striking the bridge from the side, as shown in **B**. The struts and lateral bracing also prevent the top chords from buckling sideways, as a result of compression in those members. We'll learn more about buckling in Learning Activity #2.

4) What is the purpose of the portal bracing? Portal bracing keeps the two main trusses from tipping over sideways when the bridge is loaded. Diagram **A** below shows a front view of the Grant Road Bridge. If the portal bracing were not present, wind striking the structure from the side would easily knock it over, as shown in **B**. The portal bracing adds rigidity to the structure by ensuring that the tops of the end posts remain vertical, as in **C**. Like the top lateral bracing, the portal bracing is really just another truss, which uses members arranged in interconnected triangles to add rigidity to the structure.



In actual bridges, the portal bracing takes many different forms, as you can readily see in the Gallery of Truss Bridges (Appendix A). In older bridges, this portion of the structure was often decorated with ornate ironwork, as in Bridges 6, 8, and 12 in the Gallery.

5) Why are truss bridges less common today? Today most short-span and medium-span bridges use the **beam** as their principal structural element. Yet, in comparison with the beam, the truss is a far more efficient structure. For a given span and loading, a well-designed truss uses far less material than a beam. So why are beams used more often in modern bridge construction? To answer this question, you must recognize that the material cost is only a portion of the total cost of building a bridge. The costs of fabricating structural elements and the cost of actually building the structure also contribute significantly to the total cost. As you probably now recognize from your experience in this project, truss bridges are relatively complex structures. A truss has many more members and connecting elements and thus requires considerably greater effort to fabricate and assemble than a beam. In the 19th century, when skilled labor was relatively cheap, and construction materials were relatively expensive, truss bridges could generally be built for the lower total cost. But in recent times, the cost of skilled labor has increased in comparison with the cost of materials. Fabrication and construction costs have become at least as important as material cost in bridge construction. As a result, beam bridges now can generally be built at a lower total cost than truss bridges.

6) **Which method of loading is better?** Of the two possible methods of loading your bridge, hanging the load from the floor beams is more realistic. On an actual truss bridge, traffic loads are applied to the deck, which is supported by the floor beams. Thus most of the load is transmitted to the main trusses via the floor beams. Loading the bridge by stacking books on top of it is considerably less realistic, but it is much easier to do. If we were to compare the internal member forces developed by these two different types of loading, we would find surprisingly little difference between the two. We'll see this comparison in Learning Activity #3.

7) **Why is the bridge "too strong?"** The structural engineer who designed the Grant Road Bridge didn't know if you would put *exactly* five kilograms of books on the bridge. She wanted to ensure that the bridge would not collapse if you used a slightly larger load. She also didn't know if you would be using the same type of file folders she used as the basis for her design. File folders made by a different manufacturer might be considerably weaker. The engineer knew that a relatively inexperienced Constructor would build the bridge, so she assumed that some minor errors in fabrication or construction might reduce the strength of the bridge somewhat. Because of all this uncertainty, she designed the structure to carry twice as much load as the design requirements specify. This margin of error is called a *factor of safety*. The factor of safety for this structure is approximately 2.

We will work extensively with the factor of safety in Learning Activities #3 and #5.

8) **How does you model bridge carry load?** With the load in place at mid-span, the bottom chords and the diagonals are all in tension. The hip verticals are in tension only if load is applied to the floor beams BB' and FF'. When there is no load on these members, the internal force in the hip verticals is essentially zero. The top chords, the end posts, and all of the other verticals are in compression. We will calculate the actual magnitudes of these tension and compression forces in Learning Activity #3.

9) **Why tubes and bars?** Hollow tubes are much more efficient in carrying compression than are solid bars, because tubes resist buckling more efficiently. (We'll learn more about buckling in Learning Activity #2.) So the engineer has specified tubes for all members that carry compression. For members that carry tension, tubes and bars are equally effective. But, as you know from this project, tubes are quite a bit harder to make than bars. If you were a steel fabricator, you would charge more money to make them. Thus the engineer has specified the less expensive bars for all members that carry tension.

A quick review of the Gallery of Truss Bridges (Appendix A) indicates that many designers of actual truss bridges have made the same design decision—many of these structures use tubes for compression members and bars for tension members.

To see why members of a truss must be specially designed to carry tension and compression, try turning the bridge upside down, and supporting it by its ends. In this configuration, if you apply just a slight downward load at mid-span, the bars will buckle almost immediately. Turning a truss upside down reduces its load-carrying capacity to nearly zero, because members designed for tension are now subjected to compression.

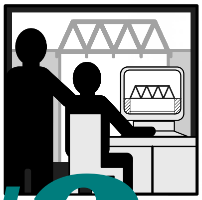
10) **How does construction quality affect structural performance?** *Any* construction error and *any* instance of poor workmanship might adversely affect the ability of a structure to carry load. Suppose one of your bottom chord members is 3 mm wide, rather than the 4 mm width required by the plans and specifications. As a result of this "small" fabrication error, the undersized member would be 25% weaker than the designer intended and might fail when the structure is loaded. Suppose you didn't put enough glue on one of your gusset plate connections. If the glue joint is weaker than the member it is connected to, then the full strength of the member can never be achieved, and a premature connection failure could cause the structure to collapse. Construction quality can mean the difference between a successful structure and a catastrophic failure.

Poor workmanship can also adversely affect the appearance of a completed structure. Sloppy glue joints and unevenly cut gusset plates don't look good and will probably cause the Owner to be dissatisfied with the finished product, even if it is structurally safe.

Some Ideas for Enhancing this Learning Activity

Given that actual bridges are always built by teams of construction specialists, it would be appropriate to organize students into teams to do this project. The authenticity of the experience would be further enhanced by assigning students specific roles within each team. One student might be the *project manager*, with overall responsibility for supervising the activity and conducting quality control inspections. One or two might be designated as *steel fabricators*, with responsibility to cut out all members and gusset plates. One might be assigned as the *steel erector*, responsible for trimming each member to size and positioning it on the building board. One might be the *welder*, with responsibility for making all glue joints. The teacher should act in the role of the *Owner*, by assigning the project, accepting the completed structure, placing it into service (by applying the 5 kilogram load), and paying the Constructor (by assigning a grade). At the conclusion of the project, students might be asked to write a reflective essay on what they learned or on how the various team members contributed to the final product.

On actual projects, the Constructor is often selected by a competitive bidding process. To simulate this process, student teams could be asked to begin the project by submitting bids. Each team could be asked to prepare a bid for the least possible number of file folders required to build the bridge. The team that submits the lowest bid (the smallest number of file folders) would win the contract—and perhaps a bonus grade for the project. The team would only earn their bonus, however, if they are actually able to build the structure with the number of folders requested in their bid. This enhancement to the project would require each team to carefully plan the layout of structural members and gusset plates prior to submitting its bid—a good exercise in planning and geometry.



#2

Learning Activity #2:

Test the Strength of Structural Members

Overview of the Activity

In this learning activity, we will test the strengths of the cardboard structural members we used in our model of the Grant Road Bridge. We will design a series of experiments to determine the strengths of these members in both tension and compression. The experiments will be conducted with a simple testing machine that uses a lever to apply a controlled force to a test specimen. [1]

To analyze the experimental data obtained from the testing machine, we will learn and apply the *principle of the lever*. Finally, we will use a computer spreadsheet to graph the results of our tests. These graphs will help us to observe how various physical properties affect the strength of a structural member. We will also use these graphs as a tool for analyzing and designing bridges in Learning Activities #3 and #5.

Why?

To design a structure, an engineer must be able to determine the strengths of the structural members that comprise it. In Learning Activity #1, we saw that external loads cause internal forces to develop in a structure. We also observed that a structure can successfully carry its external loads only if the internal member forces are less than the corresponding member strengths. Thus an engineer can't evaluate the load-carrying ability of the structure without first being able to determine member strengths.

Engineers determine the strengths of members in two different ways—through experimentation and through the application of scientific principles. The scientific study of structural members and materials is called the *mechanics of materials*. The mechanics of materials is typically taught as an entire undergraduate

[1] It is not intended that students build the testing machine as part of this learning activity. The device is quite simple, but it will require some woodworking skill to build. Detailed drawings and dimensions are provided In Appendix C. One device will be adequate for a class of students; however, one device per four or five students will greatly enhance the opportunities for hands-on participation in the learning activity. Once built, the testing machine can be re-used indefinitely.

engineering course and thus is beyond the scope of this book. In this learning activity, we will determine member strengths primarily through experimentation. However, by carefully examining trends in our experimental data, we will discover some of the fundamental principles on which the mechanics of materials is based.

We will also learn how to design a series of experiments to obtain the data required to solve a problem. The ability to design and conduct experiments is not just important to structural engineers—it is a critical skill in a wide variety of engineering and scientific fields.

Learning Objectives

As a result of this learning activity, you will be able to do the following:

- Calculate the *cross-sectional area* of a structural member.
- Describe the *yielding, rupture,* and *buckling* failure modes.
- Explain the factors that affect the *tensile strength* and the *compressive strength* of a structural member.
- Design a testing program to determine the strength of structural members.
- Determine the *tensile strength* and the *compressive strength* of structural members through experimentation.
- Explain the *principle of the lever,* and apply this principle to the analysis of experimental data.
- Use a computer spreadsheet to analyze and graph experimental data.

Key Terms

To successfully complete this learning activity, you must understand the following key terms and concepts from Learning Activity #1:

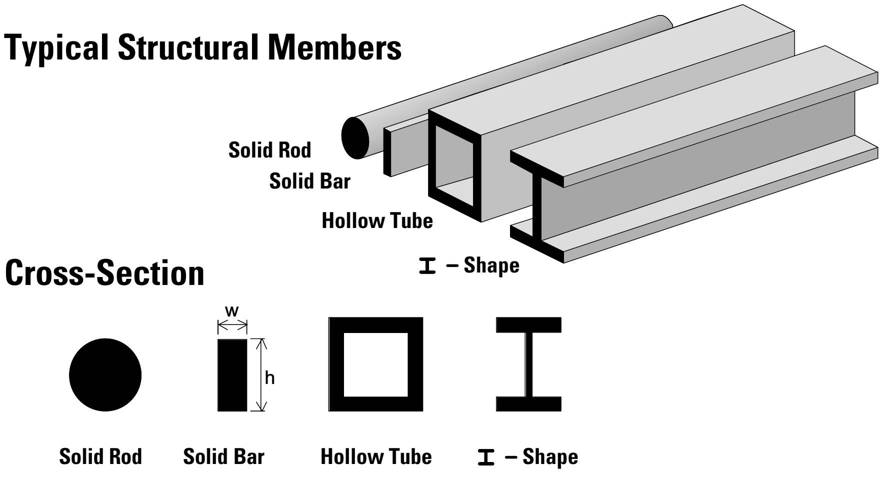
member
force
load
internal force
tension
compression
strength
failure

Information

1. Cross-Section and Cross-Sectional Area

One of the most important characteristics affecting the strength of a structural member is its *cross-section*.

A **cross-section** is the two-dimensional shape you see when you look at the end of a member. In the illustration below, for example, the solid rod has a *circular cross-section*, and the solid bar has a *rectangular cross-section*.



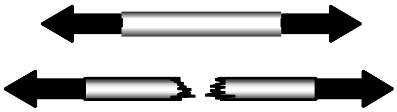
Typical structural members and their cross-sections.

The **cross-sectional area** is the surface area of the cross-section. For example, the cross-sectional area of the solid bar on the previous page is the area of the black rectangle. To calculate it, you would multiply the width **w** by the height **h**. The cross-sectional area is always expressed in units of length squared—for example, square inches or square millimeters.

2. Tensile Strength

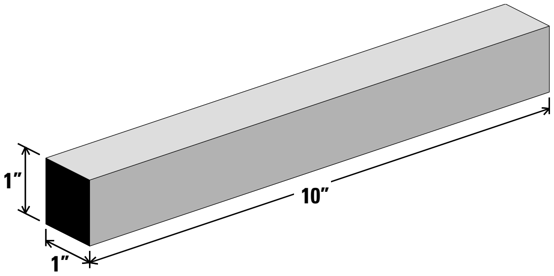
In Learning Activity #1, we defined *strength* as the maximum internal force a member can carry before it fails. The internal force in a structural member can be either *tension* or *compression*. Because the failure of a structural member in tension is very different from its failure in compression, we must consider the *tensile strength* and *compressive strength* separately.

Tensile strength is the maximum tension force a member can carry before it fails. As this definition suggests, one way to determine the tensile strength of a member is to load it in tension until it fails—that is, pull on the member from both ends until it physically breaks in two—then measure the amount of force that caused the failure.

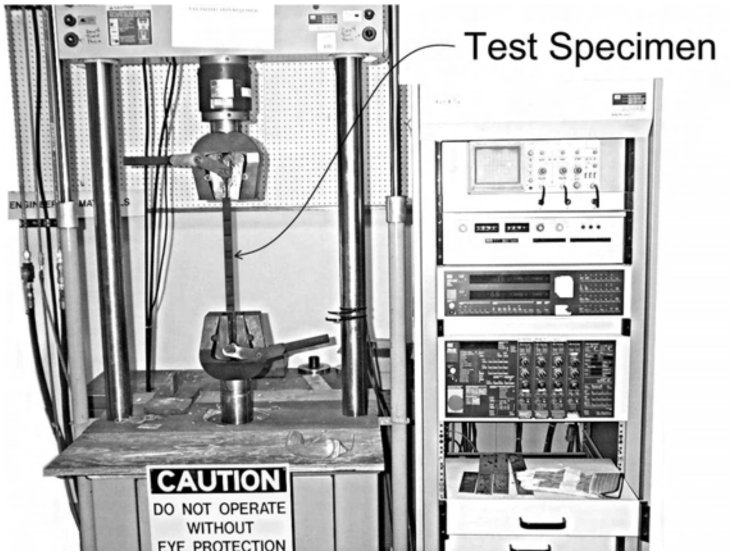


To determine the tensile strength of a member, pull on it from both ends until it breaks in two.

Suppose we wanted to test the tensile strength of a carbon steel bar. **Carbon steel** is one of the most common materials used in structures. It is a mixture of iron and a very small amount of carbon—less than 1%. For our carbon steel test specimen, we will use the bar shown here. It has a square cross-section measuring 1 inch on each side. This cross-section is typically designated 1″ x 1″ (“one inch by one inch”), and its cross-sectional area is 1 square inch.

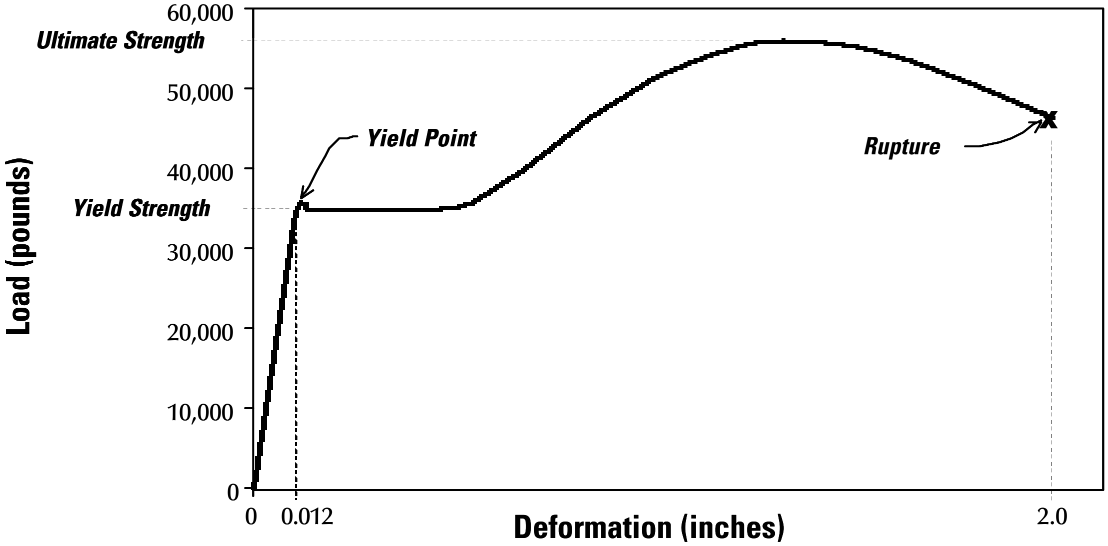


Steel is quite strong. To break a steel bar—even this relatively small one—we will need a special machine like the one pictured below. This hydraulic testing machine is capable of stretching a test specimen with many thousands of pounds. The machine can measure both the **load** on the specimen and its corresponding **deformation**—the increase in the length of the bar as it is stretched.



A hydraulic testing machine.

To test the bar, we will clamp its ends into the machine and gradually increase the load until the steel fails. As the load is applied, the machine will continuously measure and record both the load and the deformation of the specimen. If we plot these data on a graph, the result will look something like this.



Load-deformation curve for a 1″ x 1″ carbon steel bar.

This graph is called a **load-deformation curve**. It shows us how the member deforms—and ultimately how it fails—as the load is increased.[2] A careful examination of the load-deformation curve will tell us a lot about carbon steel. Let's examine the curve from left to right. The load-deformation curve originates in the lower left-hand corner of the graph, which tells us that the deformation is zero when the load is zero. This certainly makes sense. The bar won't start to stretch until we apply a force to it. As we follow the curve up and to the right, we notice that the curve is almost perfectly straight from zero all the way up to about 36,000 pounds. The straight line means that the *deformation increases in direct proportion to the load*. For example, the deformation at 20,000 pounds is exactly twice as large as the deformation at 10,000 pounds. In this linear part of the load-deformation curve, the behavior of the steel bar is said to be *elastic*. **Elastic behavior** means that, if the load is removed, the deformation will also return to zero. When a member is elastic, it always returns to its original length after it is unloaded. This particular steel bar will remain elastic, as long as the load on it is kept below 36,000 pounds. When the load does reach 36,000 pounds, the deformation of the bar is just over 1/100″. The total length of the bar has increased from 10″ to 10.012″—a change of only about one tenth of one percent.

As the load is increased beyond 36,000 pounds, the behavior of the bar changes rather abruptly. There is suddenly a huge increase in deformation, with virtually no change in the load. The steel is beginning to fail. When a material undergoes large deformations with little change in load, it is said to be **yielding.** The point on the load-deformation curve where yielding begins is called the **yield point**, and the force at which yielding occurs is called the **yield strength**. Beyond the yield point, the steel stretches like taffy. And unlike the elastic behavior we observed earlier, any deformation that occurs beyond the yield point will not disappear after the load is removed. This permanent elongation of the member is called **plastic deformation**. Note that, as the plastic deformation increases, the bar eventually begins to carry more load. The load peaks at 58,000 pounds, which is called the **ultimate strength** of the member. After further plastic deformation, the specimen finally breaks into two pieces. This failure mode is called a **rupture**.

So what is the *tensile strength* of this steel member? Is it the yield strength or the ultimate strength? Since the tensile strength is the force at which the member fails, the answer to this question depends on how the structural engineer chooses to define "failure." For most practical structural applications, the engineer would probably want to ensure that the member does not yield. In such cases, "failure" would be defined as yielding, and the tensile strength would be 36,000 pounds—the yield strength. In some cases, however, the engineer might only want to ensure that the member does not rupture. In such cases, the tensile strength would be 58,000 pounds—the ultimate strength. This latter definition of failure might be appropriate, for example, when the engineer is designing for the effect of an extraordinary event like a major earthquake. In such cases, the engineer might be willing to accept some plastic deformation of the structure, as long as it does not collapse.

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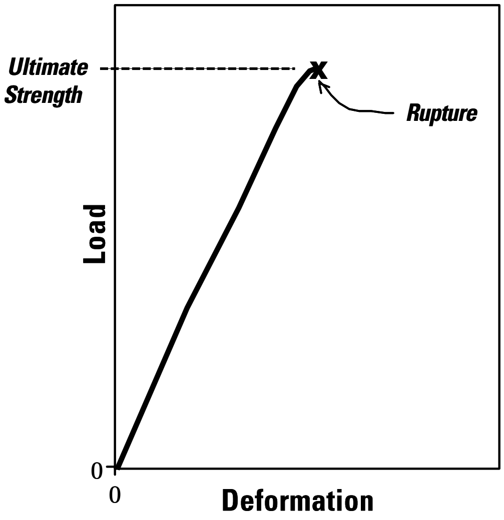
[2] The test machine measures the load applied to the bar. What we are really interested in, however, is the internal force in the bar. Fortunately, in this test, the magnitude of the load and the internal member force are exactly equal. Can you explain why?

This is an important and often misunderstood point—in structural engineering, there is often no single universally accepted definition of "failure." Rather, the engineer must exercise his or her professional judgment to determine the conditions under which a structure (or a component of a structure) no longer will function as intended.

One other characteristic of the load-deformation curve for the carbon steel bar is worth mentioning. Note that, at rupture, the bar has deformed two full inches—20% of its original length. This capacity to undergo very large plastic deformation after yielding is called **ductility**. Ductility is one of the most beneficial properties of steel, and it is one of the most important reasons why steel is so widely used in structures. When a ductile member begins to fail, its large plastic deformation provides an obvious warning that something is wrong with the structure. This warning provides an opportunity to evacuate people and make emergency repairs before the structure collapses. For this reason, ductility greatly enhances structural safety.

Not all structural materials are ductile. Materials that do not undergo large plastic deformation prior to failure are called **brittle** materials. A typical load-deformation curve for a brittle material is shown at right. Note that the material ruptures without yielding and thus without giving any warning that a failure is about to occur. For this reason, brittle materials are generally undesirable for structural members. Cast iron is a brittle material, which explains why cast iron has been entirely replaced by steel in modern structures. Concrete is a brittle material, which explains (in part) why concrete is always reinforced with steel bars when it is used as a structural material.



Load-deformation curve for a brittle material.

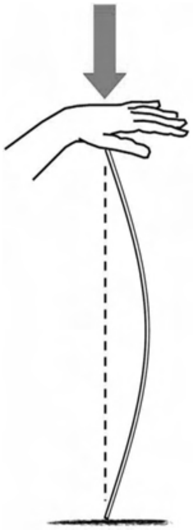
We have seen how one particular structural member made of one particular material can be tested to determine its tensile strength. If we were to repeat this test with many different members—different sizes, different cross-sections, and different materials—some patterns would begin to emerge. A careful analysis of these patterns would reveal the following facts about the tensile strength of structural members:

- ***Tensile strength depends on the cross-sectional area of a member.*** As the cross-sectional area increases, the tensile strength increases in direct proportion to the area. If the cross-section of our carbon steel bar were changed from 1″ x 1″ to 2″ x 2″, the cross sectional area would increase from 1 square inch to 4 square inches, and the yield strength would increase from 36,000 pounds to about 144,000 pounds—four times greater.
- ***Tensile strength depends on the type of material the member is made of***. Every material has its own characteristic strength, measured in units of *force per area (*for example, pounds per square inch or newtons per square meter). The yield strength of carbon steel is 36,000 pounds per square inch. Other types of steel with yield strengths of 50,000 pounds per square inch and higher are common. The tensile strength of a *member* can be calculated by multiplying the tensile strength of the *material* by the cross-sectional area of the member.
- ***Tensile strength does not depend on the length of a member.*** If we used the same 1″ x 1″ cross-section but changed the length of our specimen from 10″ to 20″, the tensile strength would remain exactly the same.
- ***Tensile strength does not depend on the shape of the cross-section.*** If we tested a hollow tube or circular rod with a cross-sectional area of 1 square inch, we would find that its tensile strength is exactly the same as the 1″ x 1″ square steel bar.

These observations will guide the design of our own testing program, later in this learning activity.

3. Compressive Strength

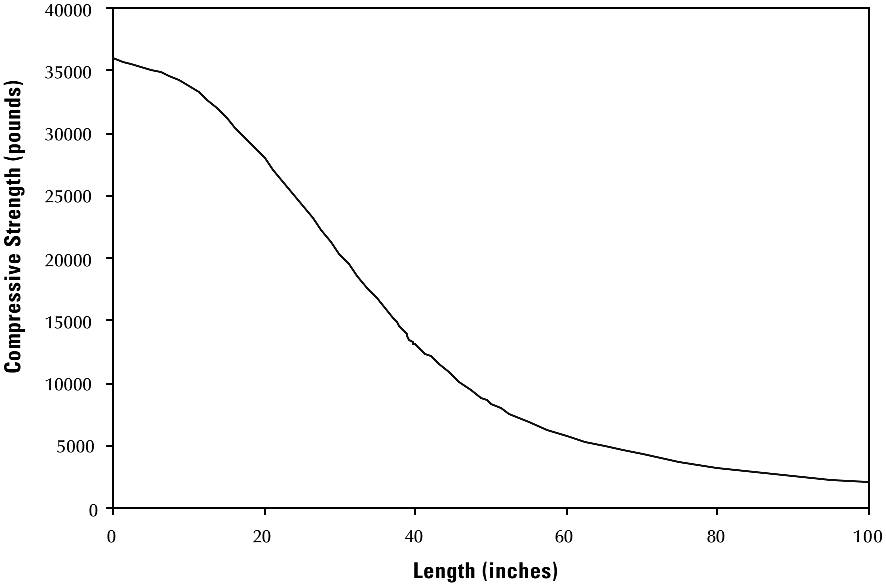
Compressive strength is the maximum compression force a member can carry before it fails. We can determine the compressive strength of a structural member by loading it in compression until it fails, then measuring the amount of force required to cause the failure.



To understand how a structural member fails in compression, try the following simple experiment. Hold a yardstick or meter stick vertically, with its bottom end on the floor. Now put the stick in compression by pushing downward on its top end. Gradually increase the compression force. At some point, the stick will suddenly bend sideways—in engineering terms, it will *buckle*. **Buckling** is a failure that occurs when compression causes a member to suddenly bend sideways, perpendicular to the direction of the applied load. Buckling is the most common failure mode for structural members in compression. When a member fails by buckling, its *compressive strength* is the internal force at which buckling occurs.

Try repeating the same experiment with a 12-inch wooden ruler. Unless you're really strong, you'll probably find that you can't push hard enough on the 12-inch ruler to make it buckle. This observation suggests an important characteristic of buckling failures: *Shorter members have greater compressive strength than longer ones.*

Now suppose we wanted to test the compressive strength of a 1″ x 1″ carbon steel bar. Again, we would need to use our hydraulic testing machine to load the bar. It would take about 34,000 pounds of compressive force to cause the 10″ long bar to buckle. Recognizing that the compressive strength of the bar depends on its length, we would need to test a series of 1″ x 1″ square bars, each with a different length. We would find that increasing the length of the bar to 20″ would cause a substantial reduction in its strength—to about 28,000 pounds. A 40″ bar would fail at around 13,000 pounds. If we tested members with lengths up to 100″ and plotted the results on a graph of compressive strength vs. length, the result would look like this:

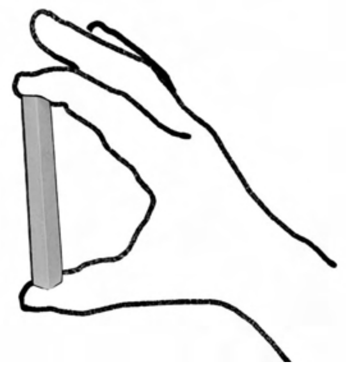


Compressive strength vs. length for a 1″x1″ carbon steel bar.

This graph vividly illustrates the effect of member length on compressive strength. Note that an 80″ member is less than one-tenth as strong as a 10″ member, even though their cross-sections are identical.

How do the size and shape of the cross-section affect compressive strength? In Learning Activity #1, we observed that hollow tubes seem to be more effective than solid bars at carrying compression. Let's test that observation now with another simple experiment. Using the same file-folder cardboard you used to build the Grant Road Bridge, cut out two identical rectangles measuring 5 centimeters wide and 10 centimeters long. Fold one of the two rectangles into a square tube measuring 1cm x 1cm. Glue the edges together as we did when we prefabricated the square tubes in Learning Activity #1. The second rectangle should remain unfolded—a 5cm-wide "bar" with a length of 10cm. We now have two structural members—a bar and a tube. Both are the same length, and both use *exactly* the same amount of material. Place each one with its ends between your thumb and forefinger, and squeeze. You'll find that the flat rectangular "bar" buckles with only the slightest compressive force. On the other hand, the tube is amazingly strong—almost impossible to buckle with one hand. This simple test clearly demonstrates another important characteristic of buckling failures: A *hollow tube has significantly higher compressive strength than a solid bar using the same amount of material.*

A cardboard bar buckles with only a slight load.

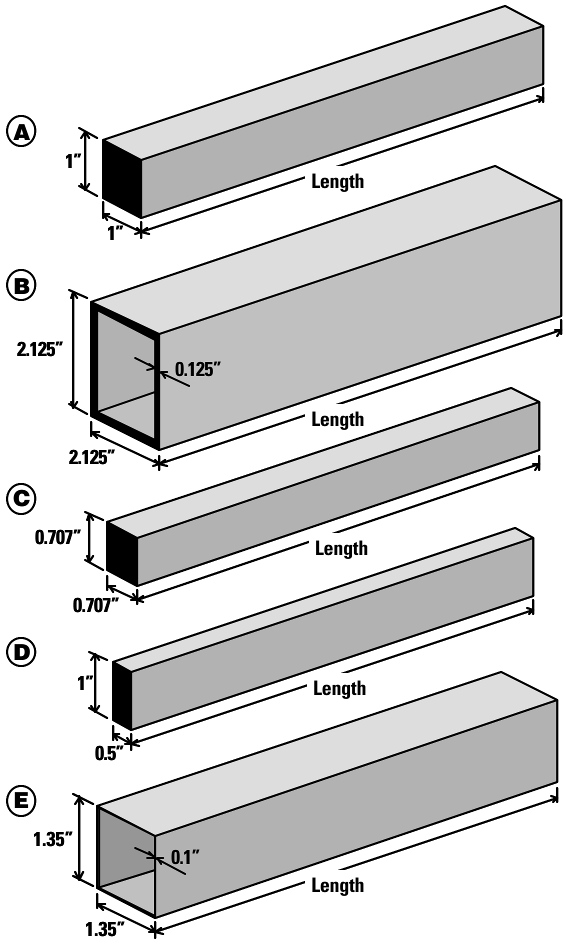


A cardboard tube is amazingly strong in compression

Now let's do the same sort of experiment with actual steel structural members. Our goal is to investigate how the cross-sectional area and cross-section shape affect the compressive strength. We can accomplish this goal by testing a series of steel specimens with various areas and shapes, then comparing the results. Since we will be studying the effects of two different variables—cross-sectional area and shape—we will need to design the experiments carefully. It is important that successive tests change only one variable at a time, so we can logically compare the results and determine the effect of each variable.

For example, consider the following five carbon steel test specimens:

- **Specimen A** is the same 1″ x 1″ solid square bar we have been using throughout this learning activity. The test results for this bar will provide a basis for comparison with the other specimens.
- **Specimen B** is a hollow tube measuring 2.125″ x 2.125″. The thickness of the tube walls is 0.125″ (1/8″). These dimensions result in a cross-sectional area of exactly 1 square inch—the same as Specimen A. Because their cross-sectional areas are equal, we can compare the test results from Specimens A and B to determine the effect of cross-section shape on compressive strength.
- **Specimen C** is a 0.707″ x 0.707″ solid bar. These dimensions result in a cross-sectional area of 0.5 square inches—exactly half the area of Specimen A. Since Specimens A and C have the same shape, we can compare the two sets of test results to determine the effect of cross-sectional area on strength.
- **Specimen D** is a 1″ x 0.5″ solid bar. Like Specimen C, this bar has a cross-sectional area of 0.5 square inches; however, the cross-section of Specimen D is rectangular, rather than square. Thus we can compare the test results from Specimens C and D to further examine the effect of cross-section shape on strength.
- **Specimen E** is a 1.35″ x 1.35″ hollow tube with walls 0.1″ thick. Its cross-sectional area is exactly 0.5 square inch. We can compare the test results from Specimens C and E to determine the effect of cross-section shape. We can compare the results from Specimens A and E to determine the effect of cross-sectional area.



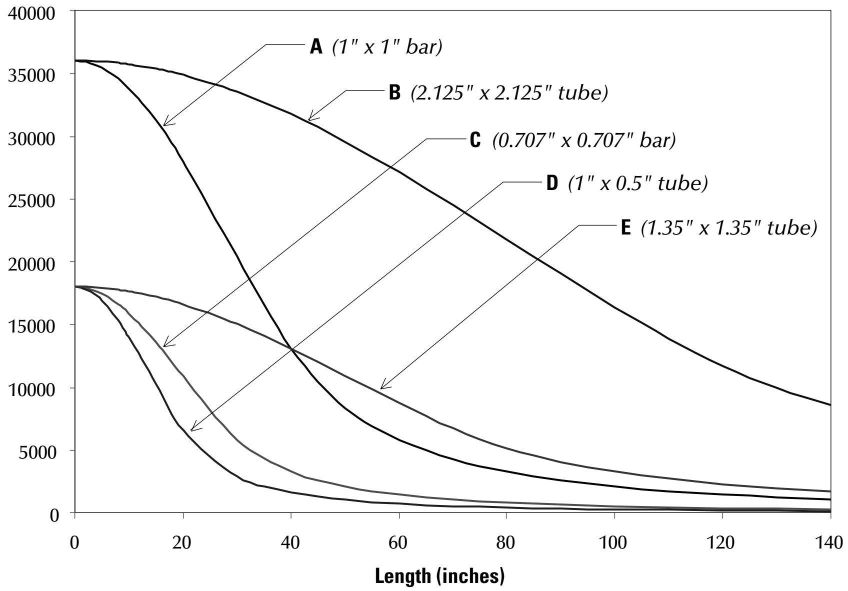
Carbon steel test specimens used to determine how cross-sectional area and shape affect compressive strength.

Q 1

Can you calculate the cross-sectional areas of these members?

Show that the cross-sectional area is exactly 1 square inch for Specimens A and B and exactly 0.5 square inch for Specimens C, D, and E.

If we do a series of compression strength tests on each of these five specimens using a variety of different lengths, the results will look like this:



A careful comparison of these five strength vs. length curves will confirm several of our earlier observations about compressive strength and will lead us to several new ones as well:

- ***Compressive strength depends on the length of the member.*** For all five specimens, increasing the length causes a substantial reduction in the compressive strength.
- ***Compressive strength depends on the shape of the cross-section.*** A comparison of the results for Specimens A and B clearly shows that a hollow tube has a substantially higher compressive strength than a solid bar, for a given length and cross-sectional area. The results for Specimens C and E confirm this observation. The results for Specimens C and D indicate that a square bar has somewhat higher strength than a rectangular bar with the same area.
- ***Compressive strength depends on the cross-sectional area of the member.*** The results for Specimens A and C indicate that increasing the cross-sectional area of a member increases its compressive strength. The results for B and E indicate that this same conclusion is valid for tubes. Note that, unlike tensile strength, the compressive strength is **not** proportional to the cross-sectional area. (For example, doubling the area generally will not double the strength.)
- ***Compressive strength depends on the material the member is made of.*** This series of tests uses only carbon steel; however, if we repeated these experiments with specimens made of other materials, we would find that stronger, stiffer materials have higher compressive strength than weaker, more flexible ones.

Q2

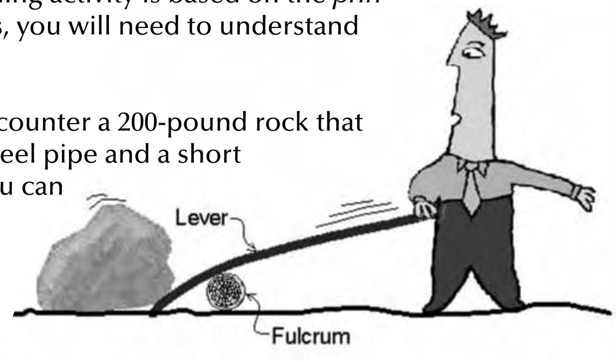
What can we learn from a comparison of Specimens A and E?

Compare the compressive strength vs. length curves for Specimens A and E on the graph above. What conclusion can you draw from this comparison?

4. The Principle of the Lever

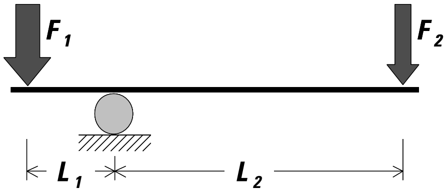
When actual structural members are tested in a laboratory, powerful hydraulic machines are used to perform the tests. We don't have hydraulic power available for this project, but we do have the power of the *lever* to help us apply a large, controlled, measurable tension or compression force to a cardboard structural member. The simple testing machine we will use in this learning activity is based on the *principle of the lever;* thus, to understand how the machine works, you will need to understand how a lever works.

Suppose you are doing a landscaping project, and you encounter a 200-pound rock that must be moved. The only "tools" available are a 6-foot long steel pipe and a short log. How can you move the rock? As the picture suggests, you can move the rock quite easily by using the steel pipe as a *lever* and the log as a *fulcrum*. A **lever** is a simple machine, consisting of a bar or rod that rotates on a pivot. The pivot is called a **fulcrum**. When you apply a downward force to one end of the lever, the lever pivots on the fulcrum and applies an upward force to the rock at the other end.



Using a lever to move a heavy rock.

The diagram below shows the forces acting on the lever. F_2 represents the downward force you are applying to one end of the lever, and F_1 represents the weight of the rock pushing down on the other end. Since the lever is lifting the rock, F_1 also represents the *upward force applied by the lever to the rock*. L_1 is the distance from the force F_1 to the fulcrum. L_2 is the distance from the force F_2 to the fulcrum.



The *principle of the lever* states that:

$$F_1L_1 = F_2L_2$$

In our example, we know that the weight of the rock, F_1, is 200 pounds. Let's place the log (the fulcrum) one foot away from the rock. Since the steel pipe (the lever) is six feet long, then L_1 is 1′ and L_2 is 5′. What force do you need to apply to the long end of the lever to lift the rock? If you substitute the known values of L_1, L_2, and F_1 into the equation above, and solve for F_2, you will find that *you can lift the 200-pound rock with a force of only 40 pounds.*

Q3

Using this lever, how much weight could you lift with 50 pounds?

Suppose you need to move another rock with this lever. Using the same bar and fulcrum location, you push downward with a force of 50 pounds to lift the rock. How much does the rock weigh?

This simple example shows how a lever can be used to significantly increase the amount of force that can be applied to an object. It also shows how we can use the principle of the lever to precisely calculate the amount of force applied to an object. The testing machine we will be using to conduct strength tests is a direct application of this principle.

5. Converting Mass to Weight

Weight is a *force*; thus, we express the weight of an object in units of force. In the lever example above, the weight of the rock and the forces applied to the lever are expressed in pounds—the standard measure of force in the U.S. Customary system of units.

The experiments conducted in this learning activity use metric units, also called SI units. (SI stands for *Systeme International*.) Determining the weight of an object in SI units is a bit more complicated than doing it in U.S. units. When you "weigh" an object on a metric scale, the number you read from the scale is usually in grams or kilograms, which are units of *mass,* not force. Thus, when you "weigh" an object on a metric scale, you actually do not measure its weight. You measure its mass.

To determine the weight of this object, you must convert its mass to a force, using the equation

$$W = mg$$

In this equation, W is the weight of the object, m is its mass, and g is the acceleration of gravity. In SI units, $g = 9.81$ meters/sec^2. If you express the mass m in kilograms, then the weight W will be in newtons.

Q4

What is the weight of a 5-kilogram mass?

In Learning Activity #1, we used a 5-kilogram stack of books to load the Grant Road Bridge. How much did this stack of books weigh?

We will use this mass-to-weight conversion extensively when we analyze the experimental data collected as part of this learning activity.

The Learning Activity

The Problem

The Need

The Town Engineer of Hauptville, New York, has decided to conduct a structural evaluation of the Grant Road Bridge, to ensure that it can safely carry the required highway loads. Before he can begin analyzing the structure, he will need to obtain information about the strengths of the various structural members used in the main trusses. He decides to hire a materials testing laboratory to design and conduct an experimental testing program to provide the necessary information.

Your Job

Your materials testing company, Universal Structural Materials Assessment, Inc., has been hired by the Hauptville Town Engineer to provide experimental data in support of his structural evaluation of the Grant Road Bridge. Your job is to design and conduct a program of experimentation to determine the strengths of all structural members used in the main trusses of the bridge. As a technical specialist, you are responsible for providing your client with complete, accurate data and presenting that data in a manner that is both understandable and usable.

The Solution

The Plan

Our plan to provide the Hauptville Engineer with the information he needs is as follows:

- Familiarize with the testing machine that we will use for our experiments.
- Design a testing program.
- Make the test specimens.
- Conduct tension and compression strength tests.
- Analyze and graph the experimental data.

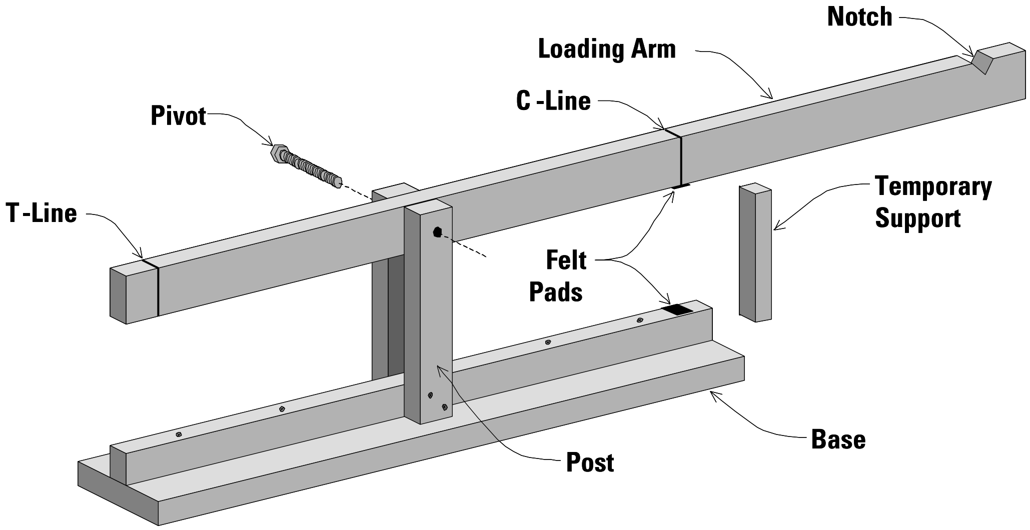
The product of our work will be a series of graphs that the Hauptville Engineer can use as the basis for his structural evaluation.

The Testing Machine

Description

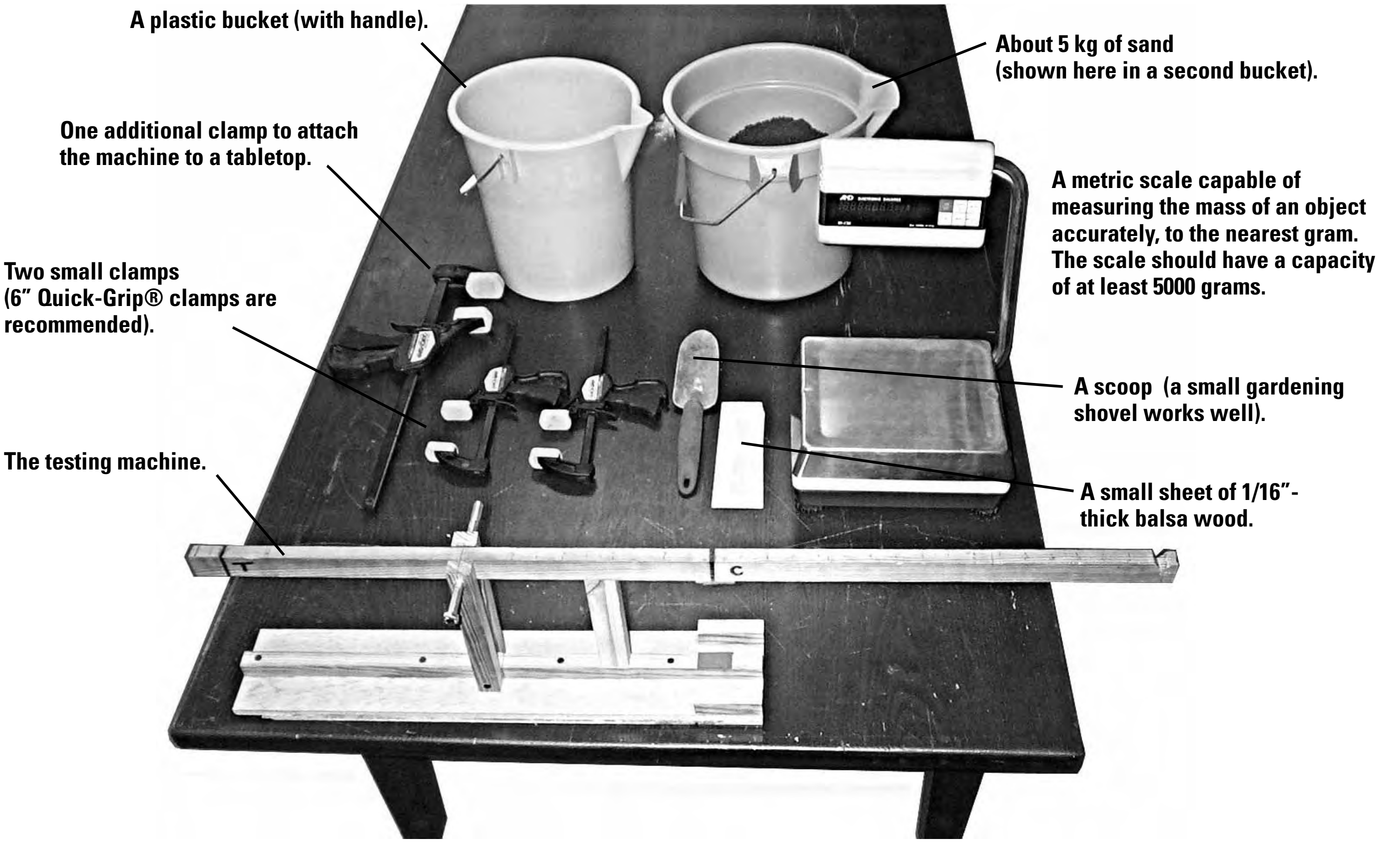
This simple lever-based testing machine will allow you to apply a controlled tension or compression force to a test specimen and measure that force with reasonable accuracy. Though you do not need to build the machine, you should understand how it works, in order to use it properly and to achieve accurate results.

The configuration and component parts of the testing machine are illustrated in the drawing below. The *loading arm* is fastened to the *posts* with a steel bolt, which serves as a pivot. The *T-Line* and *C-Line* are vertical marks on the loading arm, indicating the points where the tension and compression specimens will be fastened for testing. *Felt pads* are fastened to the underside of the loading arm and the top side of the base at the C-Line. These pads will ensure that compression test specimens are uniformly loaded. The *temporary support* is a wooden post that is used to support the loading arm while a tension specimen is being clamped into position.



Component parts of the testing machine.

The photo below shows everything you will need to conduct the tensile and compressive strength experiments.



How the Machine Works

When you test the tensile strength of a cardboard structural member, you will clamp the top of the test specimen to the loading arm at the T-Line. The bottom of the specimen will be clamped to the base. You will hang the plastic bucket from the notch at the end of the loading arm, then slowly fill it with sand until the specimen ruptures. After the failure, you will weigh the bucket and sand, and apply the *principle of the lever* to determine the internal force in the specimen at the instant of failure. The principle of the lever says that:

$$TL_1 = WL_2$$

In this equation, T is the internal force in the test specimen and W is the weight of the bucket and sand. Since L_1 and L_2 can be measured directly from your testing machine, and W is determined experimentally, we can solve this equation for the unknown internal force T. The result is:

$$T = \frac{WL_2}{L_1}$$

The procedure for testing compression members is the same, except that the specimen will be placed at the C-Line instead of the T-Line. When we apply the principle of the lever to find the unknown internal compression force C in the specimen, we get:

$$C = \frac{WL_2}{L_1}$$

Note that the principle of the lever applies even when both forces are on the same side of the fulcrum.

Design the Testing Program

Now that the testing machine is ready to go, you are probably anxious to start doing some experiments. But before we can start testing, we first need to design the testing program. The objectives of this planning process are to:

- Ensure that we get accurate data;
- Ensure that we get the right kinds of data to support the projects we will be doing later; and
- Ensure that we do not waste time or material by doing unnecessary tests.

To accomplish these objectives, we must apply some of the observations we made earlier about the tensile strength and compressive strength of structural members. Specifically, we need to look at each of the factors on which the tensile and compressive strength depend, and vary these factors systematically in our tests. As a minimum, the range of values for each factor must be adequate to analyze every member in the Grant Road Bridge.

The logical thought process leading to the design of our testing program is as follows:

- ***Tensile strength depends on the cross-sectional area of a member.*** Therefore, we must create test specimens with a variety of different cross-sectional areas. The cross-sectional area of a rectangular member is simply its width times its thickness. Since all of our specimens will have the same thickness (the thickness of the cardboard), we need to create test specimens with a variety of different widths.
- ***Tensile strength does not depend on the length of a member.*** Therefore, all of our tension test specimens can be the same length. We will use 20 centimeters, because this length fits the testing machine nicely.
- ***Tensile strength does not depend on the shape of the cross-section.*** Therefore, all of our tension test specimens can have the same type of cross-section. We will use a simple rectangular "bar."

- ***Compressive strength depends of the shape and size of the cross-section.*** Therefore, we must create compression test specimens for each of the different cross-sections we plan to use in our structure. We will test rectangular tubes with the same dimensions as the tubes used in the Grant Road Bridge model.
- ***Compressive strength depends of the length of the member.*** Therefore, we must create test specimens with the full range of different lengths we plan to use in our structure. We will use lengths from 5 to 16 centimeters.
- ***Tensile and compressive strength both depend on the material the member is made of.*** Therefore, to do a truly comprehensive testing program, we would need to create test specimens of various different materials. Since our projects will all use the same type of cardboard, however, we will only test this one material.

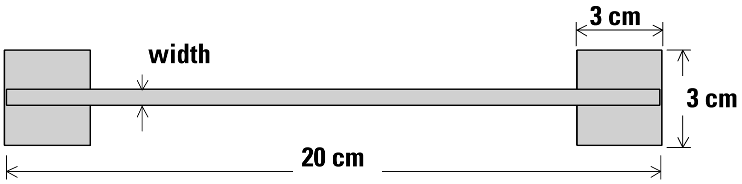
In designing the testing program, we must also consider the effects of experimental error and the natural variability of the properties we are attempting to measure. There are many possible sources of experimental error in our test setup. (We will discuss them in detail later.) Some of these can be controlled by conducting the tests very carefully; but no matter how careful we are, our experimental data will exhibit some natural variability. For this reason, we should repeat each of our experiments several times and average the results. Repeating each experiment several times is especially important for the compression tests, which are inherently more variable than the tension tests.

Taking all of these factors into account, our testing program will consist of the following experiments:

Test #	Cross-Section	Length	Number of Specimens
T1	4 mm-wide bar	20 cm	3
T2	6 mm-wide bar	20 cm	3
T3	8 mm-wide bar	20 cm	3
C1	10 mm x 10 mm tube	5 cm	3
C2	10 mm x 10 mm tube	10 cm	3
C3	10 mm x 10 mm tube	16 cm	3
C4	6 mm x 10 mm tube	5 cm	3
C5	6 mm x 10 mm tube	10 cm	3
C6	6 mm x 10 mm tube	16 cm	3

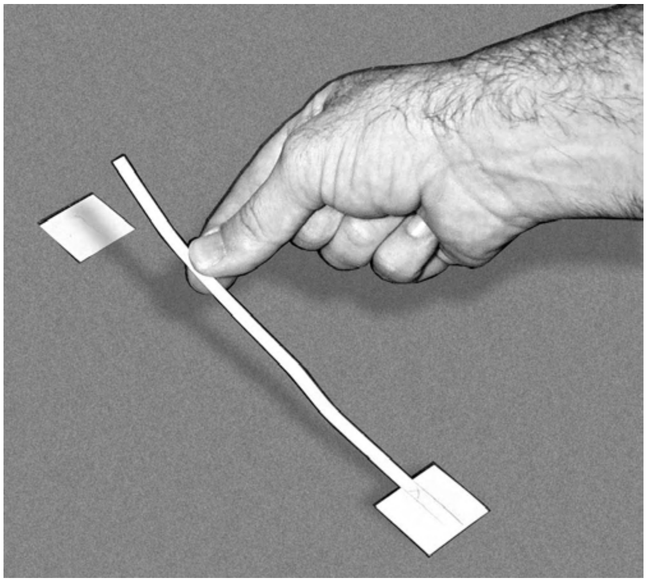
Make the Test Specimens

The configuration of a typical tension test specimen is shown below. The member itself is a strip of cardboard 20 centimeters long, sliced from a file folder just as we did in Learning Activity #1. Glued onto each end of the member is a 3-centimeter square of cardboard, which provides a surface for the clamps to grip when the specimen is placed in the testing machine.



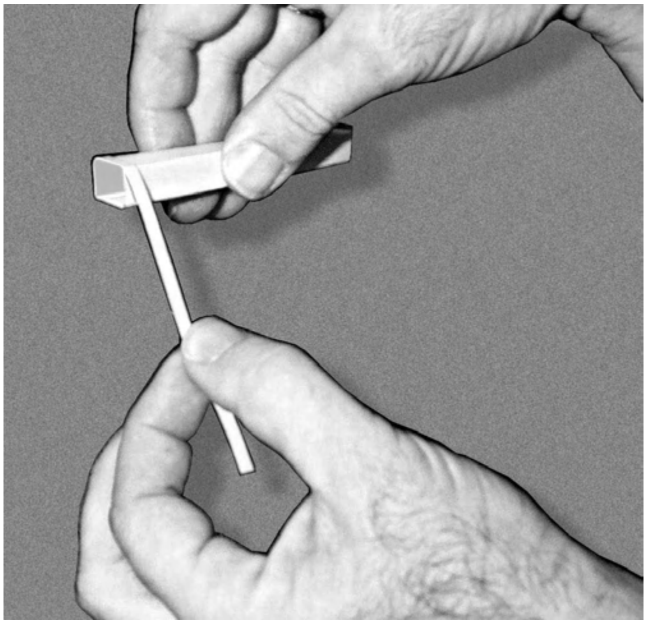
Tension test specimen.

To make a tension test specimen, apply glue to one of the cardboard squares. Place the member onto the glue, and hold it in position until the glue sets. Repeat for the opposite end of the member, as shown at right.



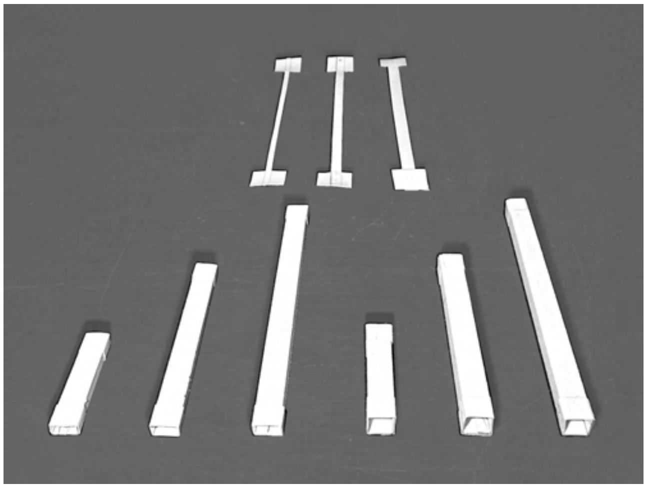
Making a tension test specimen.

To make the compression test specimens, start by laying out and fabricating cardboard tubes of the required sizes, exactly as we did in Learning Activity #1. Then reinforce the ends of each member by coating a 6mm-wide strip of cardboard with glue, and wrapping the strip around the entire perimeter of the member at each end. This procedure is illustrated in the photo at right. The purpose of this reinforcement is to ensure that the ends of the member do not crush when they are compressed by the testing machine.



Reinforcing the ends of a compression test specimen.

The picture at right shows one complete set of test specimens—three bars of different widths and two tubular cross-sections, each in three different lengths.



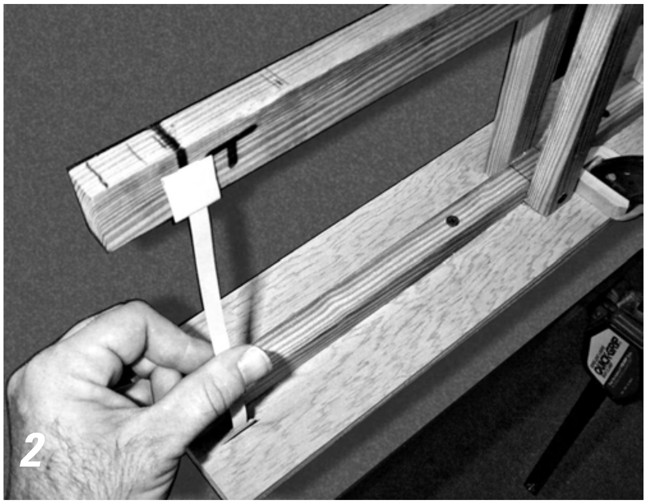
Conduct the Tension Tests

Use the following procedure to test each of your tension specimens:

1) To prepare for your first test, clamp the testing machine to the edge of a table, with the long end of the loading arm overhanging as shown. Place the temporary support under the loading arm, and hang a bucket from the notch at the end of the loading arm. Put a chair or stool below the bucket and, if necessary, stack books on the chair so that the space between the top of the stack and the bottom of the bucket is only about two inches. When a test specimen breaks, the bucket will fall; we don't want it to fall very far.

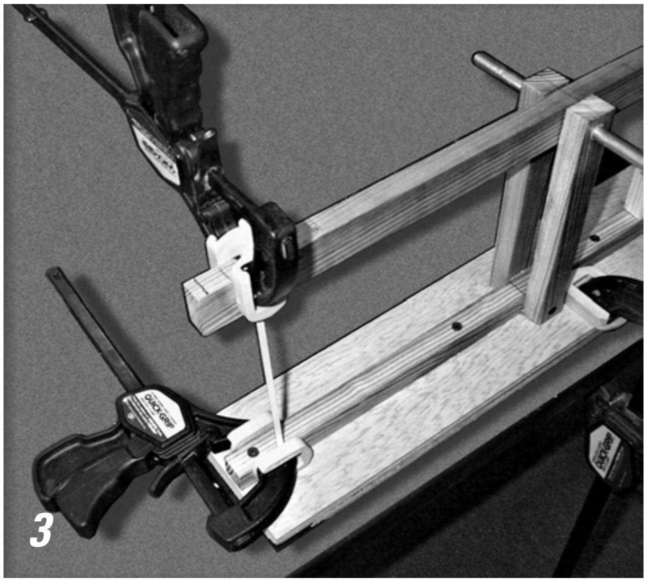
1

2) Place one of the 4mm tension specimens (Test T1) into position, centered on the T-Line.



2

3) Clamp the top of the specimen to the loading arm and the bottom of the specimen to the base. Ensure that the specimen remains straight and vertical. The clamps must be tight, so the specimen won't slip.

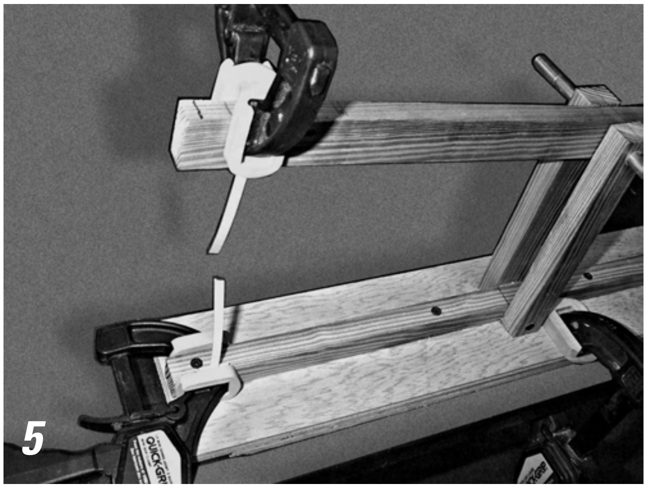


3

4) Remove the temporary support, and gently allow the weight of the bucket to pull the specimen tight. Now begin the loading process. Fill the scoop with sand, then slowly pour the sand from the scoop into the bucket. Wait 5 seconds, then add a second scoop of sand. Wait 5 seconds, and add a third scoop. Continue this process, always waiting 5 seconds between scoops, until the specimen breaks.


4

5) The failure of the specimen will happen suddenly, without warning—often during one of the 5-second pauses in the loading process. However, if the failure does occur while you are adding sand, stop immediately. The sand in the bucket should be the exact amount that caused the failure to occur.

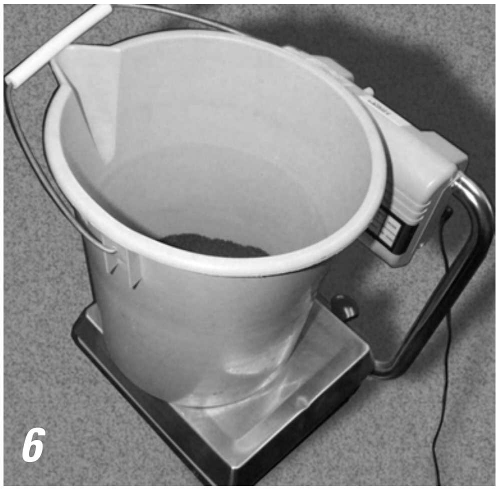

5

Q5

Is cardboard ductile or brittle?

Did your test specimen fail in a ductile or brittle manner? What does this characteristic tell you about the suitability of cardboard as a structural material?

6) Lift the bucket off of the testing machine, place it on the scale, and record the mass.


6

Now empty the bucket, and repeat the process for each of your tension specimens. For each test, keep careful records of the specimen size and the mass of the bucket and sand.

Q6

Why is it necessary for the loading arm to be balanced?

When the testing machine was built, the loading arm was balanced on the pivot (see Appendix C). If the loading arm had not been balanced, how would it affect your experimental results?

Analyze and Graph the Tension Data

To analyze our experimental data, we need to calculate the actual *tensile strength* that we measured in each test, then create a graph of tensile strength vs. member width. This graph will give us the capability to determine the tensile strength for *any* member width, not just the specific widths we tested. The analysis and graphing can be performed most easily and accurately by using a computer spreadsheet.

Calculate the Tensile Strength

For each tension test, you determined the mass of the bucket and sand that caused the specimen to fail. In order to determine the tensile strength, you must:

(1) Convert the *mass* to *weight*, using the equation:

$$W = mg$$

where g is 9.81 meters/second^2 and m must be expressed in kilograms.

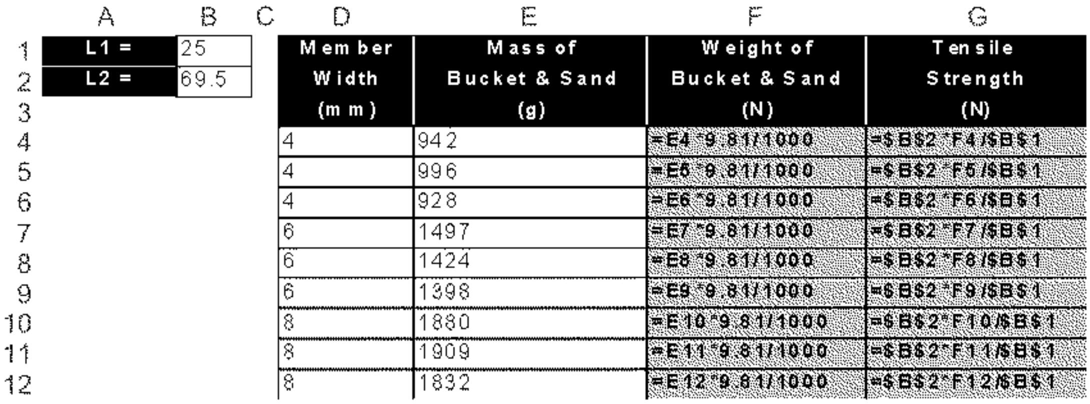
(2) Apply the principle of the lever to determine the force in the specimen at failure, using the equation:

$$T = \frac{WL_2}{L_1}$$

Because these calculations must be performed for each individual test, the analysis can be done very efficiently with a spreadsheet. The results should look something like this:

Test Number	Member Width (mm)	Mass of Bucket & Sand (g)	Weight of Bucket & Sand (N)	Tensile Strength (N)
T1	4	942	**9.2**	**25.7**
T1	4	996	**9.8**	**27.2**
T1	4	928	**9.1**	**25.3**
T2	6	1497	**14.7**	**40.8**
T2	6	1424	**14.0**	**38.8**
T2	6	1398	**13.7**	**38.1**
T3	8	1880	**18.4**	**51.3**
T3	8	1909	**18.7**	**52.1**
T3	8	1832	**18.0**	**50.0**

In this spreadsheet, the black cells are headings, the white cells are for entry of experimental data, and the gray cells are calculated automatically. The cell formulas required to create this spreadsheet with Microsoft Excel look like this:



	A	B	C	D	E	F	G
1	L1 =	25		Member Width (mm)	Mass of Bucket & Sand (g)	Weight of Bucket & Sand (N)	Tensile Strength (N)
2	L2 =	69.5					
3							
4				4	942	=E4*9.81/1000	=$B$2*F4/$B$1
5				4	996	=E5*9.81/1000	=$B$2*F5/$B$1
6				4	928	=E6*9.81/1000	=$B$2*F6/$B$1
7				6	1497	=E7*9.81/1000	=$B$2*F7/$B$1
8				6	1424	=E8*9.81/1000	=$B$2*F8/$B$1
9				6	1398	=E9*9.81/1000	=$B$2*F9/$B$1
10				8	1880	=E10*9.81/1000	=$B$2*F10/$B$1
11				8	1909	=E11*9.81/1000	=$B$2*F11/$B$1
12				8	1832	=E12*9.81/1000	=$B$2*F12/$B$1

Note that the spreadsheet is set up to allow *any* values of L_1 and L_2 to be entered. This is important, because the actual measurements L_1 and L_2 for your testing machine are likely to be different from these values. Note also that the mass was recorded in grams, so it had to be converted to kilograms (divided by 1000) as part of the weight calculation.

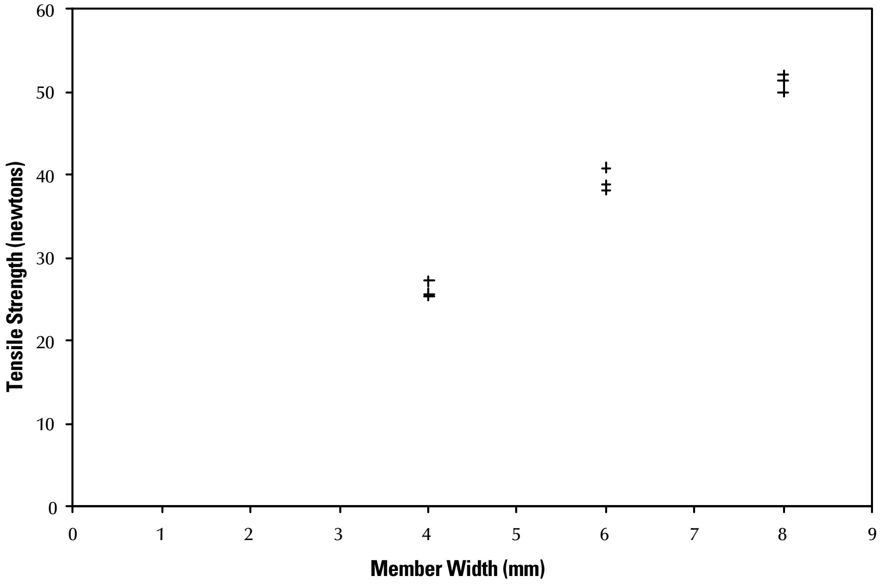
Q7

Can you verify these spreadsheet calculations?

Whenever you use a computer to solve a numerical problem, you should verify that the calculations are being done correctly. One way to verify computer results is to perform one complete set of calculations by hand and compare your answer with the one the computer calculated. Select any one of the tension test results above, calculate the tensile strength by hand, and compare your results to the computer solution. Are the computer results correct?

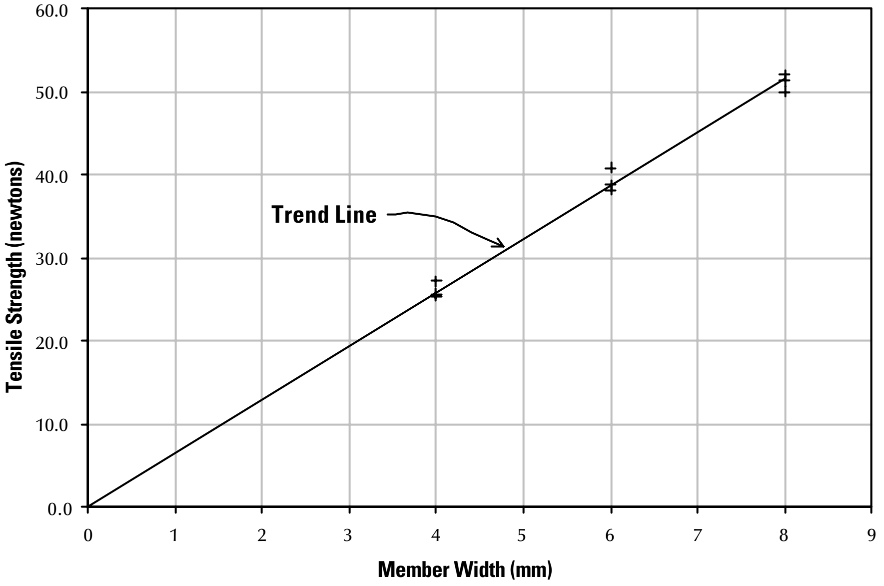
Create a Graph of Tensile Strength vs. Member Width

Once the spreadsheet is set up, it is a relatively simple task to create a graph of tensile strength vs. member width. The best type of graph for this type of data is an "x-y scatter plot," with no line connecting the data points. Use *member width* for the x-axis and *tensile strength* for the y-axis. The procedure for creating this graph depends of the spreadsheet software you are using. Check the program's "Help" menu for instructions. The result should look something like the graph on the following page . Each "+" symbol represents one test.



Tensile Strength vs. Member Length for cardboard members (experimental data).

As you might expect, there is some "scatter" in our experimental data—a clear indication of both experimental error and natural variability in the tensile strength of a material. Nonetheless, the data points all appear to lie along a straight line. This observation suggests that there is a linear relationship between member width and tensile strength. We can represent this linear relationship by drawing a "best fit" straight line directly on the x-y scatter plot. Many spreadsheet programs can do this automatically, using a function called "trend line" or "linear regression." Again, check the "Help" menu of your spreadsheet program to see if either of these functions is available. If not, you can print a copy of the graph you just created, then use a pencil and ruler to draw a straight line that best fits your data. Whether you draw the line yourself or use the spreadsheet to do it, you must ensure that the trend line passes through the point (0,0). We know that a member with zero width *must* have zero strength, so the point (0,0) *must* lie on the trend line. In a spreadsheet program, this is normally accomplished by setting the *y-intercept* of the trend line to zero. The result should look like this:



Tensile Strength vs. Member Length for cardboard members (experimental data with trend line).

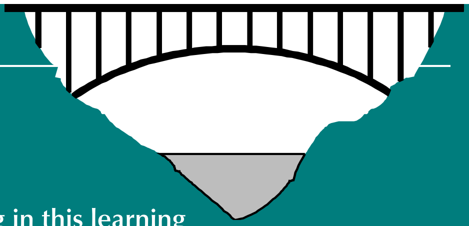
Notice that this chart can be used as a powerful design tool. Even though you only performed tests on specimens of three different widths, you can now easily determine the tensile strength of a member with *any* width between 0 and 8 mm. To use the correct mathematical term, you can *interpolate* any point on the graph. You could also guess at the strength of a member with a width larger than 8 mm; in other words, you could *extrapolate* from the graph. But be careful! Extrapolation can be dangerous. Just because our data appear to be highly linear in the range from 0 to 8 mm doesn't mean that the relationship between member width and tensile strength will continue to be linear for larger widths.

Q8

Can you use the graph to determine tensile strength?

What is the tensile strength of a cardboard member 5 millimeters wide? What is the strength of a double 4-millimeter bar, like the ones we used in the Grant Road Bridge model?

On an Actual Bridge Project

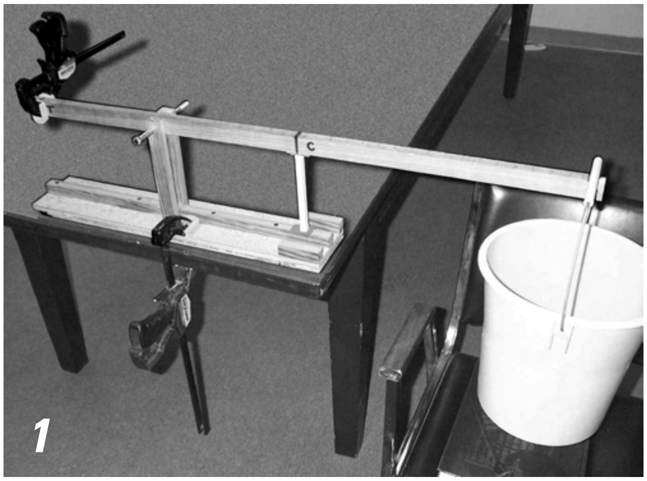


How do engineers obtain data from experimental strength testing?

The sort of strength testing we have been doing in this learning activity is generally not performed as part of the design process for individual structures. Rather, testing is done by researchers in universities, in government laboratories, and in industry. The results of these tests are incorporated into books called design codes, which are published by professional societies and made available to engineers. For example, the American Institute of Steel Construction publishes a design code for steel structures. The American Concrete Institute publishes a design code for concrete structures. These codes are updated frequently, so that practicing engineers can have access to the most current research results, without actually having to develop and conduct experiments for each project.

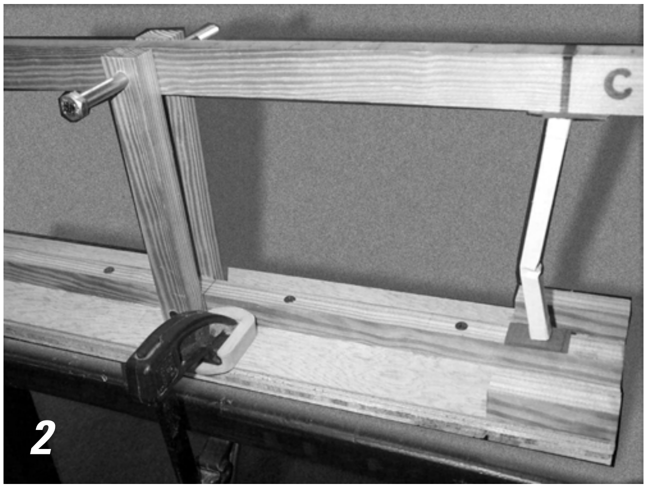
Test the Compression Specimens

1) To conduct the compression tests, set up the testing machine in the same way you did for the tension tests. Place one of the 16 cm-long specimens (Test C3) at the “C-Line,” between the loading arm and the base. Both the top and bottom of the specimen should be resting on the felt pads.


1

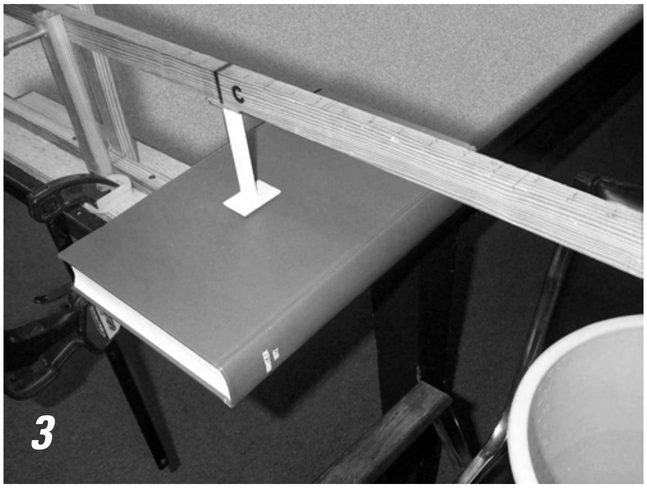
Once the specimen is in place, apply load just as you did for the tension tests. Add sand to the bucket one scoop at a time, pausing 5 seconds between scoops.

2) When the specimen buckles, remove the bucket, place it on the scale, and record the mass.


2

Empty the bucket, and repeat the process for each of your compression specimens. For each test, keep careful records of the specimen size and length and the mass of the bucket and sand.

3) To mount the 10cm-long and 5cm-long specimens, a minor adjustment to the testing machine will be required. It is *very important* that the loading arm be exactly horizontal at the start of each test; otherwise, the four sides of the tube will not be loaded equally. For the shorter specimens, it is necessary to block up the base of the member with books or pieces of wood until the loading arm is level. Use the 16 cm-long temporary support as a gage. Use shims made of 1/16″ balsa wood to make fine adjustments to the height of the specimen. Because balsa is soft, these shims will also perform the function of the lower felt pad, which is now covered up.


3

Q9

What are some possible sources of error in these experiments?

Now that you have completed the tension and compression tests, think about some of the factors that might cause your results to be inaccurate. List at least five possible sources of experimental error in these tests. Which sources of error do you think have the greatest effect on your results? How can you minimize them?

Analyze and Graph the Compression Data

To analyze our experimental data, we will first calculate the actual compressive strength measured in each test, then use these data to create a compressive strength vs. length graph for each cross-section. Again, these tasks are best performed with a computer spreadsheet.

Calculate the Compressive Strength

As long as the distance from the C-Line to the pivot of your testing machine is exactly equal to L_1, we can calculate the compressive strength C using the equation:

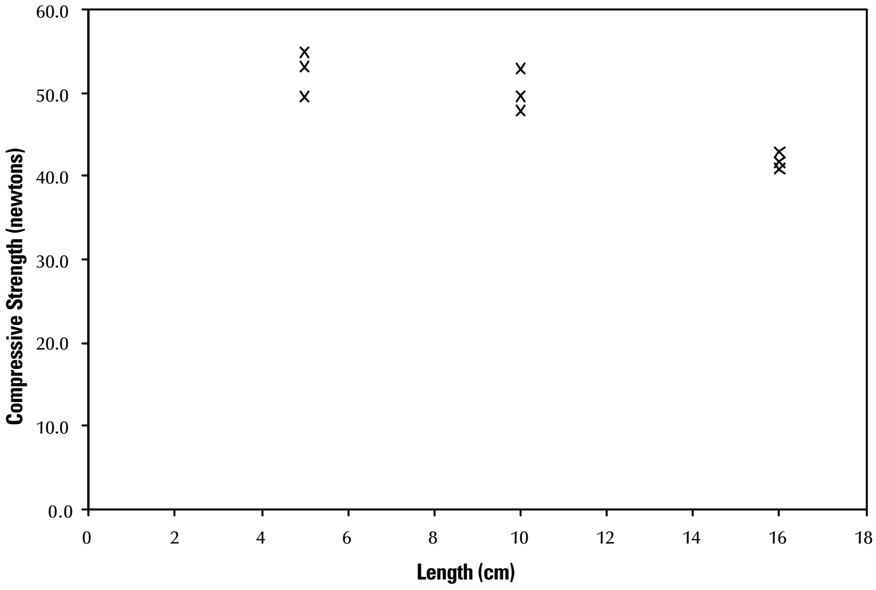
$$C = \frac{L_2 W}{L_1}$$

We can use a slightly modified version of our tension spreadsheet to do the data analysis. The result should look like this:

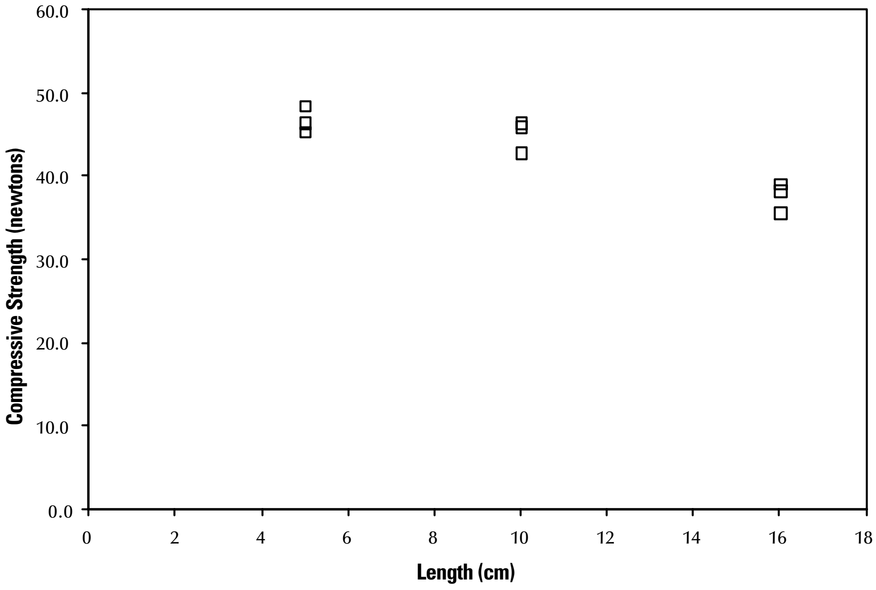
Test Number	Size (mm)	Length (cm)	Mass of Bucket & Sand (g)	Weight of Bucket & Sand (N)	Compressive Strength (N)
C1	**10 x 10**	5	1948	**19.1**	**53.1**
C1		5	2014	**19.8**	**54.9**
C1		5	1816	**17.8**	**49.5**
C2		10	1755	**17.2**	**47.9**
C2		10	1821	**17.9**	**49.7**
C2		10	1940	**19.0**	**52.9**
C3		16	1531	**15.0**	**41.8**
C3		16	1572	**15.4**	**42.9**
C3		16	1498	**14.7**	**40.9**
C4	**6 x 10**	5	1776	**17.4**	**48.4**
C4		5	1704	**16.7**	**46.5**
C4		5	1663	**16.3**	**45.4**
C5		10	1688	**16.6**	**46.0**
C5		10	1572	**15.4**	**42.9**
C5		10	1701	**16.7**	**46.4**
C6		16	1404	**13.8**	**38.3**
C6		16	1432	**14.0**	**39.1**
C6		16	1305	**12.8**	**35.6**

Create a Graph of Compressive Strength vs. Length

The compressive strength of a tube is affected by both the size of its cross-section and the length of the member. Thus we need to create a series of strength vs. length graphs, one for each different cross-section. Again, we will use the "x-y scatter plot," with *member length* as the x-axis and *compressive strength* as the y-axis. The resulting graphs for the 10mm x 10mm and 6mm x 10mm cross-sections should look like this:

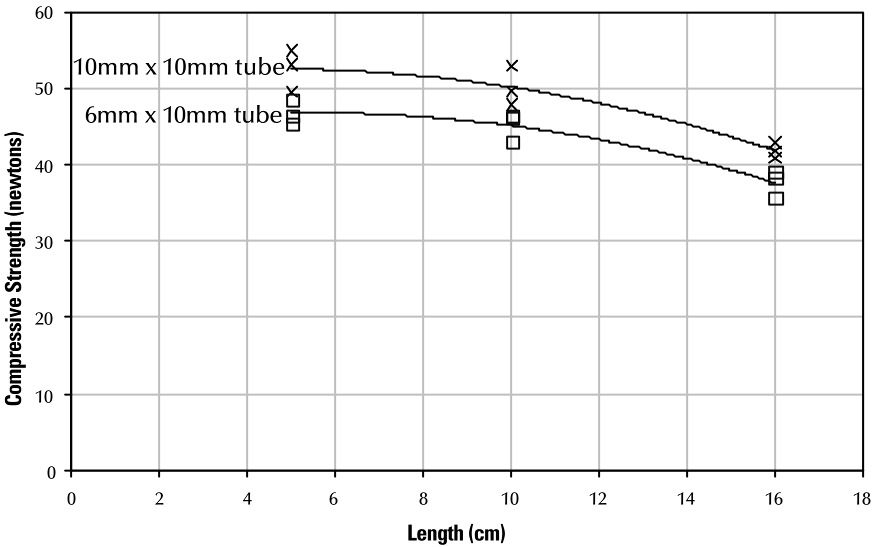


Compressive Strength vs. Length for a 10mm x 10mm cardboard tube (experimental data).



Compressive Strength vs. Length for a 6mm x 10 mm cardboard tube (experimental data).

Note also that the experimental data plotted on the compression strength graphs do not seem to be as strongly linear as the tension strength data were. Indeed, the relationship between compressive strength and length is *not* linear, so it would be incorrect to represent them with a linear trend line. Instead, we should use a "best fit" *curve* to represent the data. The graph below shows both sets of data, with polynomial trend lines added. Again, if you're spreadsheet does not have this capability, you can print the two graphs above, and sketch in the "best fit" curves by hand.



Compressive Strength vs. Length for cardboard tubes (experimental data with polynomial trend lines).

Like the tensile strength graph you developed earlier, this is an important design tool. Of course, this graph is only useful for two particular member sizes—the 6 mm x 10 mm tube and 10 mm x 10 mm tube. If you wanted to use a different size member in a design, you would need to run another series of tests and produce a corresponding chart for that cross-section.

Q 10

Can you use the graph to determine compressive strength?

What is the compressive strength of a 6mm x 10mm tube that is 12cm long? What is the compressive strength of an 8mm x 10mm tube that is 6cm long?

Conclusion

In doing this project, you had an opportunity to learn about structural members, how they fail, and how their various characteristics affect their strength. You learned about designing experiments, and you saw that the data obtained from well-designed experiments can be used to predict the strengths of structural members with reasonable accuracy. Most important, you produced a series of graphs that we will soon use to analyze and design model truss bridges.

Answers to the Questions

1) **Can you calculate the cross-sectional areas of these members?** In the case of solid square or rectangular members, the cross-sectional area is the width times the height of the member.

- For the 1″ x 1″ bar (Specimen A), the area is

$$Area = (1")(1") = \underline{\underline{1.0 \text{ square inch}}}$$

- For 0.707″ x 0.707″ bar (Specimen C), the area is

$$Area = (0.707")(0.707") = \underline{\underline{0.5 \text{ square inch}}}$$

- For the 1″ x 1/2″ bar (Specimen D), the area is

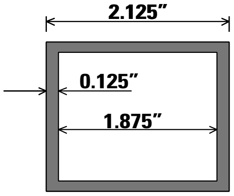
$$Area = (1")(0.5") = \underline{\underline{0.5 \text{ square inch}}}$$

In the case of hollow tubes, the easiest method is to calculate the cross-sectional area as if the bar were solid, then subtract the area of the hole.

- For the 2.125″ x 2.125″ tube (Specimen B), the area is

$$Area = [(2.125")(2.125")] - [(1.875")(1.875")] = \underline{\underline{1.0 \text{ sq. in.}}}$$

Note that the "hole" in the cross section measures 1.875″, which is determined by subtracting two times the wall thickness (0.125″) from 2.125″.



- For the 1.35″ x 1.35″ tube (Specimen E), the area is

$$Area = [(1.35")(1.35")] - [(1.15")(1.15")] = \underline{\underline{0.5 \text{ sq. in.}}}$$

2) **What can we learn from a comparison of Specimens A and E?** For any length greater than about 40″, the hollow tube E is stronger than the solid bar A—even though the tube uses only half as much steel. This observation clearly demonstrates that hollow shapes are generally much more economical than solid ones for structural members in compression.

3) **Using this lever, how much weight could you lift with 50 pounds?** The *principle of the lever* states that $F_1L_1 = F_2L_2$. In this case, we know F_2 is 50 pounds, L_1 is 1′ and L_2 is 5′. If we solve for F_1 and substitute these known values, we get

$$F_1 = \frac{F_2L_2}{L_1} = \frac{(50 \text{ pounds})(5')}{1'} = \underline{\underline{250 \text{ pounds}}}$$

Note that the principle of the lever can be used to determine the magnitude of *either* force acting on the lever.

4) **What is the weight of a 5-kilogram mass?** To find the weight of a 5-kilogram mass, use the equation

$$W = mg = (5\ \text{kilograms})\left(9.81\,\frac{\text{meters}}{\text{second}^2}\right) = \underline{\underline{49.05\ \text{newtons}}}$$

5) **Is cardboard ductile or brittle?** Cardboard fails suddenly, with no warning and with no evidence of yielding before rupture. Therefore, it is a brittle material and would not be appropriate for use in actual structures.

6) **Why was it necessary to balance the loading arm?** If the loading arm were not balanced, then the weight of the arm itself would affect the force in the specimen. For example, if the long side of the loading arm were heavier than the short end (even with the clamp attached), then the weight of the loading arm would add to the internal force in the specimen. But that force would not be accounted for in our analysis of experimental data, because the equation $T=WL_2/L_1$ assumes that W and T are the only forces acting on the loading arm. Thus an improperly balanced loading arm could be a significant source of experimental error.

7) **Can you verify these spreadsheet calculations?** The following calculations are a manual verification of the test result for the first specimen:

Given:

$L_1 = 25$ cm

$L_2 = 69.5$ cm

Mass of bucket and sand = 942 grams = 0.942 kilogram

Convert mass to weight:

$$W = mg = (0.942\ \text{kilograms})\left(9.81\,\frac{\text{meters}}{\text{second}^2}\right) = \underline{\underline{9.24\ \text{newtons}}}$$

Apply the principle of the lever:

$$T = \frac{WL_2}{L_1} = \frac{(9.24\ \text{newtons})(69.5\ \text{cm})}{25\ \text{cm}} = \underline{\underline{25.7\ \text{newtons}}}$$

This answer matches the spreadsheet result exactly.

8) **Can you use the graph to determine tensile strength?**

- A cardboard member 5mm wide should have a tensile strength of about 32 newtons.
- A single 4-mm bar should have a tensile strength of about 26 newtons. The doubled 4-mm bar has twice the cross-sectional area and therefore twice the strength—about 52 newtons.

9) **What are some possible sources of error in these experiments?** Some possible sources of error include:

- Inaccurate fabrication of the test specimens.
- Inaccurate positioning of the specimens in the testing machine.
- Inaccurate measurement of L_1 and L_2.
- Inaccurate measurement of the weight of sand in the bucket.
- Improperly calibrated scale.
- Loading arm not balanced.

- Poor experimental procedure, such as adding sand too rapidly, adding sand after the specimen has failed, or not ensuring that the loading arm is horizontal at the start of the test.
- Friction at the pivot of the loading arm.

Of these sources of error, inaccuracy in the fabrication and positioning of test specimens is probably most critical. If a 4 mm tension specimen is actually only 3 mm wide, the measured tensile strength will be too small by 25% percent. If a compression specimen is not placed exactly vertically in the testing machine, all four sides of the tube will not be loaded uniformly, the member will fail prematurely, and the measured compressive strength will be much too low. None of these sources of error can be eliminated, but they can be minimized by fabricating test specimens as precisely as possible and by careful attention to proper experimental procedures.

10) Can you use the graph to determine compressive strength?

- The compressive strength of a 6mm x 10mm tube with a length of 12cm is approximately 43 newtons.
- There is no strength vs. length graph for an 8mm x 10mm tube. Nonetheless, because this member size falls between the 6mm x 10mm tube and the 10mm x 10mm tube, we can reasonably interpolate between the two curves. The compressive strength of an 8mm x 10mm tube with a length of 6cm could be estimated at approximately 49 newtons.

Some Ideas for Enhancing this Learning Activity

Once students have completed the basic strength tests described above, have them develop and perform experiments to answer questions like these:

- Are different cross-section shapes—triangles, **I**-shapes, and circular tubes, for example—more or less efficient than square tubes in carrying compression?
- Do two 4 mm bars really have the same strength as one 8 mm bar?
- Can the tensile strength of a cardboard member be improved by coating it with glue?
- What is the tensile strength of a piece of licorice? The compressive strength of a pretzel stick?

Better yet, have the students formulate their own questions, then design experiments to find the answers.

After students have completed a series of tensile strength and compressive strength experiments, ask them to assess the design of the testing machine. Working in teams, the students should study and discuss the operation of the device, then recommend at least three improvements to its configuration that will improve the accuracy of the experimental results or improve the user-friendliness of the machine. This exercise will help to reinforce what the students have already learned about tensile strength, compressive strength, levers, and experimental error. It will also get them thinking about *design*—the subject of two future learning activities.

#3

Learning Activity #3:

Analyze and Evaluate a Truss

Overview of the Activity

In this learning activity, we will analyze and evaluate one of the main trusses from the Grant Road Bridge. We will create a mathematical model of the truss, then use this model as the basis for a structural analysis—a series of mathematical calculations to determine the internal force in every member of the truss. We will also use the experimental data from Learning Activity #2 to determine the strength of each truss member. Finally we will perform a structural evaluation—a comparison of the internal forces and strengths, to determine whether or not the truss can safely carry its prescribed loads.

Why?

Engineering design is an iterative process. To create an optimal design, the engineer must develop many different alternative solutions, evaluate each one, and then select the alternative that best satisfies the design requirements. But how are these alternative solutions evaluated? Engineers use many different criteria to evaluate a design; but in structural design, the most important of these criteria is the structure's ability to carry load safely. In most cases, an evaluation of structural safety can only be done mathematically. It would be impractical, uneconomical, and unsafe for the structural engineer to evaluate a bridge design by building a full-size prototype, then running heavy trucks across the structure to determine if it is strong enough. When a structure is built, it must be strong enough to carry its prescribed loads. The engineer must get it right the first time. For this reason, the structural engineer must be able to mathematically model, analyze, and evaluate the structure with a high degree of accuracy—and without the benefit of prototype testing. In this activity, you will learn how an engineer performs a structural evaluation. In Learning Activity #5, you will apply this process to design your own truss bridge.

Learning Objectives

As a result of this learning activity, you will be able to do the following:

- Calculate the components of a force vector.
- Add two force vectors together.
- Explain the following structural engineering concepts: *free body diagram, equilibrium, structural model, symmetry, static determinacy, stability, and factor of safety.*
- Use the Method of Joints to calculate the internal force in every member in a truss.
- Determine the strength of every member in a truss.
- Evaluate a truss, to determine if it can carry a given load safely.

Key Terms

To successfully complete this learning activity, you must understand the following key terms and concepts from Learning Activities #1 and #2:

truss **deck** **internal force** **tensile strength**
member **load** **tension** **compressive strength**
joint **reaction** **compression** **failure**

If you have forgotten any of these terms, it would be a good idea to review their definitions in the Glossary (Appendix D) before proceeding.

Information

Analysis

An **analysis** is an examination of a complex system, usually conducted by breaking the system down into its component parts. Once they are identified, the component parts and their relationships to the system as a whole can be studied in detail. For example, suppose your baseball team has been losing a lot of games, and you want to figure out why. Your team is a complex system. There are a lot of possible reasons why it might not be functioning as well as it could. To analyze the performance of the team, you'll need to break it down into its component parts. The obvious way to do this is to look at the team's individual members—nine players and a coach. But the team can also be broken down by its *functions*—hitting, pitching, fielding, and base running. To perform the analysis, you would look at each team member and each function in detail. You would examine batting, pitching, and fielding statistics, to determine whether poor performance in any of these areas might be responsible for the team's losing record. You might discover, for example, that the team's batting average against left-handed pitching has been particularly poor. This important analysis result might be used as the basis for designing a practice regimen to correct the problem.

Structural Analysis

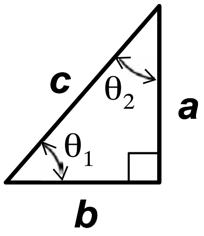
A **structural analysis** is a mathematical examination of a structure, conducted by breaking the structure down into its component parts, then studying how each part performs and how each part contributes to the performance of the structure as a whole. Usually, the products of a structural analysis are (1) reactions, (2) internal member forces, and (3) deflections—how much the structure bends or sways when it is loaded. Like the analysis of your baseball team, structural analysis is often used to determine if the system is performing as intended and, if it is not, to correct the problem.

There is an important difference between *structural analysis and structural design*. Structural analysis is concerned with examining *existing structures* to determine if they can carry load safely. Structural design is concerned with creating *new structures* to meet the needs of society. Though analysis and design are fundamentally different activities, they are closely interrelated—analysis is an integral part of the design process. We'll see how analysis and design fit together in Learning Activity #4.

To perform a structural analysis, we will apply a variety of mathematical tools from geometry, trigonometry, and algebra, as well as some basic concepts from physics. These concepts are reviewed in the following sections.

Some Basic Concepts from Trigonometry

A truss is a structure composed of members arranged in interconnected triangles. For this reason, the geometry of triangles is very important in structural analysis. To analyze a truss, we must be able to mathematically relate the angles of a triangle to the lengths of its sides. These relationships are part of a branch of mathematics called **trigonometry**. Here we will review some basic concepts from trigonometry that are essential tools for truss analysis.



This diagram shows a **right triangle**—a triangle with one of its three angles measuring exactly 90°. Sides ***a*** and ***b*** form the 90° angle. The other two angles, identified as θ_1 and θ_2, are always less than 90°. Side ***c***, the side opposite the 90° angle, is always the longest of the three sides. It is called the **hypotenuse** of the right triangle.

Thanks to an ancient Greek mathematician named Pythagoras, we can easily calculate the length of the hypotenuse of a right triangle. The **Pythagorean Theorem** tells us that

$$c = \sqrt{a^2 + b^2}$$

The Pythagorean Theorem shows how the lengths of the sides of a right triangle are related to each other. But how are the lengths of the sides related to the angles? Consider the definitions of two key terms from trigonometry—*sine* and *cosine*. Both definitions are based on the geometry of a right triangle, as shown above. The **sine** of an angle (abbreviated ***"sin"***) is defined as the length of the *opposite side* divided by the length of the *hypotenuse*. For example, the sine of the angle θ_1 would be calculated as

$$\sin\theta_1 = \frac{opposite}{hypotenuse} = \frac{a}{c}$$

In this case, side ***a*** is designated as the "opposite side," because it is farthest from the angle θ_1. For the angle θ_2, the opposite side is ***b***; thus, the sine of θ_2 is

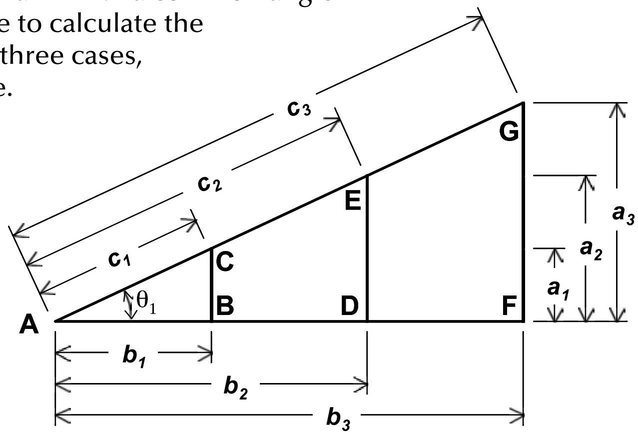
$$\sin\theta_2 = \frac{opposite}{hypotenuse} = \frac{b}{c}$$

The **cosine** of an angle (abbreviated ***"cos"***) is defined as the length of the *adjacent side* divided by the length of the *hypotenuse*. Applying this definition to our example, we have

$$\cos\theta_1 = \frac{adjacent}{hypotenuse} = \frac{b}{c}$$

$$\cos\theta_2 = \frac{adjacent}{hypotenuse} = \frac{a}{c}$$

It is important to recognize that the sine and cosine of an angle do not depend on the overall size of the triangle—only on the relative lengths of its sides. In the diagram at right, three different right triangles (ABC, ADE, and AFG) are drawn with a common angle θ_1. It doesn't matter which of the three triangles you use to calculate the sine and cosine of θ_1. You'll get the same answers in all three cases, because the relative lengths of the sides are all the same.



$$\sin\theta_1 = \frac{a_1}{c_1} = \frac{a_2}{c_2} = \frac{a_3}{c_3}$$

$$\cos\theta_1 = \frac{b_1}{c_1} = \frac{b_2}{c_2} = \frac{b_3}{c_3}$$

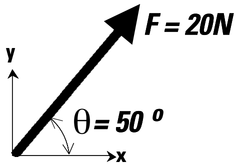
We'll see important applications of the sine and cosine when we analyze a truss, later in this learning activity.

Working with Vectors

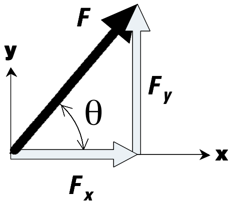
A force can be represented as a **vector**—a mathematical quantity that has both magnitude and direction. When we perform a structural analysis, we will calculate both the magnitude and direction of every force that acts on the structure. Thus, before when can analyze a structure, we need to learn how to work with vectors. Specifically, we need to learn two basic concepts from vector math—breaking a vector into its components and adding vectors together.

Breaking a Vector into its Components

When we analyze a truss, we will need to describe the *directions* of force vectors mathematically. To do this, we must first define a *coordinate axis system*. For a two-dimensional structure, we normally use an x-axis to represent the horizontal direction and a y-axis to represent the vertical. Once the coordinate axis system is established, we can represent the direction of any vector as an *angle* measured from either the x-axis or the y-axis. For example, the force vector at right has a magnitude *(**F**)* of 20 newtons and a direction *(*θ) of 50 degrees, measured counterclockwise from the x-axis.



This force can also be represented as *two equivalent forces*, one in the x-direction and one in the y-direction. Each of these forces is called a **component** of the vector ***F***. To determine the magnitudes of these two components, visualize a right triangle with the vector ***F*** as the hypotenuse and the other two sides parallel to the x-axis and y-axis. If ***F*** is the length of the hypotenuse, then the lengths of the two perpendicular sides are exactly equal to the x-component and y-component of ***F***. We use the symbol F_x to represent the x-component of ***F*** and the symbol F_y to represent the y-component.



From trigonometry, we can apply the definitions of the sine and the cosine to calculate the two components. Recall that

$$\sin\theta = \frac{opposite}{hypotenuse}$$

From the diagram on the previous page, we can see that $\boldsymbol{F_y}$ is the opposite side of the triangle, and ***F*** is the hypotenuse. Substituting, we get

$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{F_y}{F}$$

If we multiply both sides of this equation by ***F***, we get

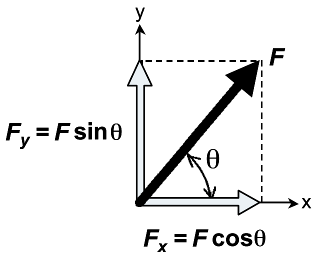
$$\underline{\underline{F_y = F\sin\theta}}$$

Similarly,

$$\cos\theta = \frac{adjacent}{hypotenuse} = \frac{F_x}{F}$$

$$\underline{\underline{F_x = F\cos\theta}}$$

Therefore, if we know the magnitude *(**F**)* and direction *(θ)* of a force, then we can use the equations above to calculate the two components of the force.



The diagram at right shows the correct way to represent the force ***F*** and its components—with all three vectors originating from the same point. The two dotted lines show that $\boldsymbol{F_x}$ and $\boldsymbol{F_y}$ are the same lengths as the sides of a right triangle with ***F*** as its hypotenuse.

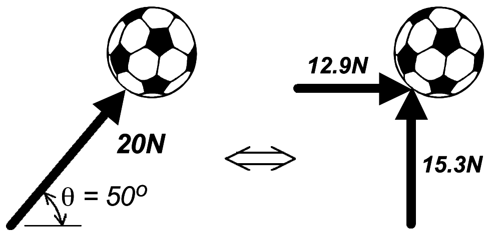
Returning to our example, if we substitute the actual numerical values F=*20N* and θ=50°, and use a calculator to determine the sine and cosine of the angle, we get the following results

$$F_y = F\sin\theta = 20\sin 50° = 20(0.766) = \underline{\underline{15.3N}} \uparrow$$

$$F_x = F\cos\theta = 20\cos 50° = 20(0.643) = \underline{\underline{12.9N}} \rightarrow$$

The small arrows to the right of the answers indicate the directions of the $\boldsymbol{F_y}$ and $\boldsymbol{F_x}$ vectors. When we write a vector quantity, we must always be careful to show *both* its magnitude and direction.

But what do these numbers really mean? Suppose you kick a soccer ball with a single 20-newton force at an angle of 50°. This force will cause the ball to move a particular direction and distance. Now suppose that two players kick the ball simultaneously—one with a 15.3-newton force in the y-direction and one with a 12.9-newton force in the x-direction. In this case, the ball will respond exactly as it did when you kicked it with the single 20-newton force. The ball will move the same direction and distance, because it "feels" exactly the same force. The two components of a force are exactly equivalent to that force and will produce exactly the same effect on an object.



The two components of a force are exactly equivalent to that force.

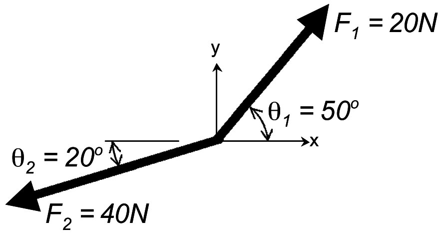
Adding Vectors Together

When two or more forces are applied to an object, it is often necessary to calculate the *total force* on the object. We calculate the total force by simply adding all of the individual force vectors together. To add vectors, however, we must follow an important rule:

The magnitudes of two or more vectors can be added together only if their directions are the same.

To add vectors whose directions are not the same, we must do the following:

- Break each vector into its equivalent x-component and y-component.
- Add all of the x-components together.
- Add all of the y-components together.



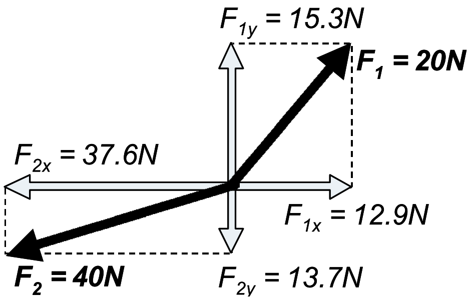
As an example, let's add the two forces $\mathbf{F_1}$ and $\mathbf{F_2}$, shown at right. We begin by calculating the components of the two vectors:

$$F_{1x} = F_1 \cos\theta_1 = 20 \cos 50^\circ = 20(0.643) = 12.9N \rightarrow$$

$$F_{1y} = F_1 \sin\theta_1 = 20 \sin 50^\circ = 20(0.766) = 15.3N \uparrow$$

$$F_{2x} = F_2 \cos\theta_2 = 40 \cos 20^\circ = 40(0.940) = 37.6N \leftarrow$$

$$F_{2y} = F_2 \sin\theta_2 = 40 \sin 20^\circ = 40(0.342) = 13.7N \downarrow$$



Again the direction of each vector component is indicated with an arrow. We must pay careful attention to these directions when we add components together. Note that F_{1x} and F_{2x} point in opposite directions. The directions of F_{1y} and F_{2y} are also opposite.

When we add the x-components, we will assume that the direction indicated by the x-axis is positive. Then the sum of the two x-components is

$$F_{TOTALx} = F_{1x} - F_{2x} = +12.9 - 37.6 = -24.7N$$

In this equation, F_{1x} is positive, because it points to the right—the same direction as the positive x-axis. F_{2x} is negative, because it points to the left—opposite the direction of the positive x-axis. The answer is negative, which means that the x-component of the total force is to the left. We write the final answer as

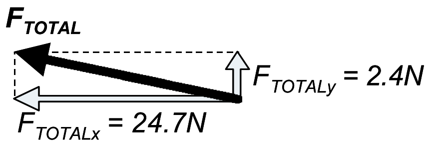
$$F_{TOTALx} = \underline{\underline{24.7N}} \leftarrow$$

Assuming that the direction of the positive y-axis (upward) is positive, the sum of the y-components is

$$F_{TOTALy} = F_{1y} - F_{2y} = +15.3 - 12.9 = +2.4N = \underline{\underline{2.4N}} \uparrow$$

In this case, the total is positive, so we conclude that the y-component of the total force is upward.

The total force and its two components are illustrated at right. If we needed to know the actual magnitude of F_{TOTAL} we could calculate it by using the Pythagorean Theorem; however, for this learning activity, we will only need to calculate the total x-component and the total y-component, as shown here.



Equilibrium

In Learning Activity #1, we defined *equilibrium* as a condition in which the total force acting on an object is zero. Now that we know how to actually calculate the total force on an object, we can apply the concept of equilibrium as a powerful problem-solving tool. Specifically, if we know that an object is in equilibrium—because it is not moving—then we know that the total force on that object is zero; and we can use this fact to calculate the magnitude and direction of unknown forces acting on the object.

Because we calculate total force by adding up the x-components and y-components separately, there are really two conditions that must be satisfied if an object is in equilibrium.*

First, the sum of the x-components of all forces acting on the structure must be zero. We write this condition as

$$\sum F_x = 0$$

where the symbol Σ means "the sum of," and the entire expression is read, "The sum of the forces in the x-direction equals zero."

The second equilibrium condition is that the sum of all forces in the y-direction must equal zero, which we write as

$$\sum F_y = 0$$

These two equations are commonly known as the **equations of equilibrium**. They are simple yet powerful mathematical tools, with many different applications in science and engineering. In this learning activity, the equations of equilibrium will enable us to calculate the reactions and internal member forces in a truss.

Creating a Structural Model

A **structural model** is a mathematical idealization of a structure—a series of simplifying assumptions about the structure's configuration and loading that allow us to predict its behavior mathematically.

When we model a two-dimensional truss, we typically make the following general assumptions:

- The truss members are perfectly straight.
- The joints that connect the truss members together are frictionless pins.
- Loads and reactions are applied only at the joints.

---

* Actually, there are three equilibrium conditions for a two-dimensional structure. In addition to the two described above, the sum of the *moments* about any point must also equal zero. The concept of a moment is a very important one; however, it is beyond the scope of this book. The problems used in this and subsequent learning activities have been chosen so that this third equilibrium condition is not required to obtain a correct solution.

Taken together, these assumptions imply that *the members of a truss do not bend*. Truss members are assumed to carry load either in pure *tension* or in pure *compression*. These assumptions allow us to use a simple type of structural analysis that ignores the effects of bending.

None of these assumptions is perfectly accurate, however. In an actual truss bridge, members are never perfectly straight, due to minor variations in the manufacturing and fabrication processes. Modern trusses use gusset plate connections, which do not behave like pins; and even in older bridges with pinned connections, the pins are certainly not frictionless. Furthermore, actual trusses can never be loaded entirely at the joints, if only because the weight of the members themselves is distributed throughout the structure. Fortunately, the inaccuracies in our assumptions generally produce only minor inaccuracies in our structural analysis results, and experienced engineers know how to compensate for these small errors to ensure the safety of their designs.

Having made these general assumptions about the structure, we must also make a number of specific decisions about how to represent the particular truss we are modeling. These decisions include:

- The geometric configuration of the truss, including the locations of all joints, the configuration of the members, and all relevant dimensions.
- The configuration of the supports.
- The magnitude and direction of the loads that will be applied to the structure.

Once we have decided how we will represent the structure, supports, and loads, we should always complete the modeling process by creating one or more drawings that clearly illustrate the structural model.

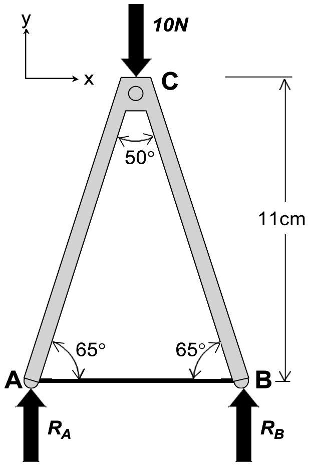
The Free Body Diagram

One of the most important tools in structural engineering is a simple sketch called the *free body diagram*. A **free body diagram** is a drawing of a "body"—a structure or a portion of a structure—showing all of the forces acting on it. Drawing a free body diagram is the essential first step in any structural analysis.

To draw a free body diagram:

- Draw the outline of the structure, completely isolated from its surroundings. Do not show any of the supports that connect the structure to its foundations.
- At the location of each support, draw and label the appropriate *reactions*.
- Draw and label all of the loads applied to the structure.
- Draw all relevant dimensions.
- Draw the x-y coordinate axis system.

In Learning Activity #1, we built a simple three-member truss by tying a short piece of string to the handles of a nutcracker. We then applied a 10-newton downward load to the top of the structure. The free body diagram for our nutcracker truss is shown here. Note that the downward 10-newton load is resisted by two upward reactions at the bottom ends of the handles. Because the magnitudes of these forces are unknown, they are labeled R_A and R_B. The three joints of the truss are labeled **A**, **B**, and **C**, for future reference.



Free Body Diagram of the nutcracker truss

Calculating Reactions

Reactions are forces developed at the supports of a structure, to keep the structure in equilibrium. Given that the reactions R_A and R_B on our nutcracker truss are in the y-direction, we can determine their magnitude using the equilibrium equation

$$\sum F_y = 0$$

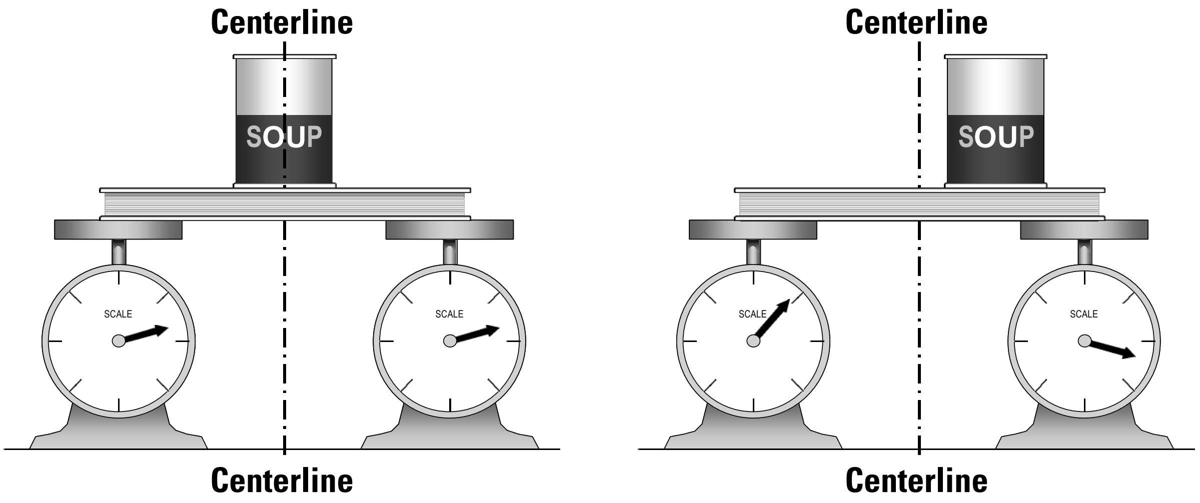
Assuming that the upward direction is positive, the sum of the forces in the y-direction is

$$\sum F_y = R_A + R_B - 10 = 0$$

or

$$R_A + R_B = 10$$

Because the structure, the load, and the supports are all symmetrical about the centerline of the nutcracker, the two reactions labeled R_A and R_B must be equal. To understand why this is true, try the simple experiment illustrated below. Set up two scales a few inches apart, and lay a book across them. Ensure that the book is centered between the two scales. Then place a relatively heavy object like a full can of soup on top of the book. Gradually slide the soup can from one end of the book to the other, and watch the readings on the two scales as you move the can. You will notice that, when the can is perfectly centered between the two scales, the readings on the scales are exactly equal. At any other position, the readings are unequal. In this experiment, the soup can is the *load*; the book is the *structure*, and the scales directly measure the two *reactions*. The experiment clearly demonstrates that the reactions are equal if the loads, the supports, and the structure itself are symmetrical about a vertical centerline.



When the load is centered, the readings on the two scales are equal. Otherwise, the readings are unequal.

If the two reactions of our nutcracker truss are equal, then

$$R_A = R_B$$

If we substitute this expression into the equilibrium equation on the previous page, we get

$$R_A + R_B = R_B + R_B = 2R_B = 10$$

$$R_B = +5N = \underline{\underline{5N}}\uparrow$$

And since $R_A = R_B$,

$$R_A = \underline{\underline{5N}}\uparrow$$

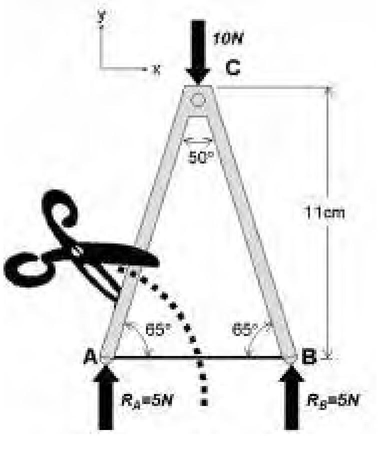
How do we calculate the reactions if the structure, the loads, or the supports are *not* symmetrical? This calculation requires the use of a third equilibrium condition—the condition that the sum of the "moments" about any point is zero. The concepts of "moments" and "moment equilibrium" are quite important but are beyond the scope of this book. All of the bridges we analyze and design here will be symmetrical.

Calculating Internal Member Forces

Once the reactions have been calculated, we can use a technique called the Method of Joints to calculate the internal member forces in a truss. To use the Method of Joints, we will use the following procedure:

1) Isolate one joint from the truss.
2) Draw a free body diagram of the joint.
3) Write and solve the equations of equilibrium to determine the member forces.
4) Repeat the process for the remaining joints.

Let's use our nutcracker truss to illustrate how the Method of Joints is used to analyze a structure.

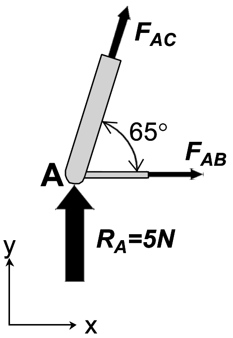


Step 1: Isolate one joint from the truss.

We'll begin our analysis with Joint A, at the lower left-hand corner of the truss. When you isolate a joint, imagine that you are physically cutting it out of the truss with a sharp scissors. You must cut through *all* of the members that connect the joint to the remainder of the structure. To isolate Joint A, you'll need to cut through Members AC (the handle) and AB (the string), as shown at right.

Step 2: Draw a free body diagram of the joint.

Now that we have cut Joint A out of the truss, we will draw a free body diagram of the joint itself. Like the diagram of the entire truss, this free body diagram must include any loads and reactions acting on the "body." Thus the upward reaction R_A is included, along with its known magnitude of 5 newtons.



In addition to the *external* loads and reactions, the free body diagram of Joint A must also include the *internal* member forces F_{AB} and F_{AC}. When we isolated the joint, we cut through Members AC and AB, thus "exposing" the internal forces in these members. We don't know the magnitudes of the two internal forces, so we simply label them with the variables F_{AB} and F_{AC}. We also don't know the directions of these forces. For now, we will simply assume that they are in *tension*. When a member is in tension, it pulls on the joint; thus, we indicate tension by showing the F_{AB} and F_{AC} vectors pointing *away from the joint,* along the centerlines of their respective members. Remember that we have only *assumed* F_{AB} and F_{AC} to be in tension. We will check this assumption when we solve the equations of equilibrium in Step 3.

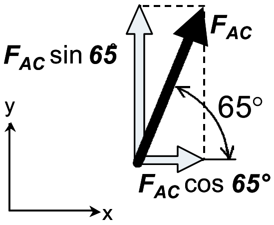
Step 3: Write and solve the equations of equilibrium.

When a structure is in equilibrium, every part of that structure must also be in equilibrium. We know that our nutcracker truss is in equilibrium, because it isn't moving; therefore, Joint A must be in equilibrium as well. Because Joint A is in equilibrium, we can write its two equilibrium equations. Let's start with the sum of forces acting in the y-direction. To write this equation, look at the free body diagram of Joint A, and identify every force that acts in the y-direction or has a component in the y-direction. Each of these forces must appear in the equilibrium equation. Assuming that the upward direction is positive,

$$\sum F_y = 0$$

$$+5 + F_{AC} \sin 65° = 0$$

To write this equation, it was necessary to break the force vector F_{AC} into its x-component and y-component, as shown at right. The y-component is $F_{AC} \sin 65°$, and because it points upward, it is positive. The x-component is $F_{AC} \cos 65°$, but this component does not appear in the ΣF_y equilibrium equation, because it does not act in the y-direction.



Since this equation has only one unknown variable, we can calculate $\sin 65°$ and solve for F_{AC} directly:

$$+5 + F_{AC}(0.906) = 0$$

$$F_{AC}(0.906) = -5$$

$$F_{AC} = \frac{-5}{0.906} = -5.52N$$

Because the answer is negative, our initial assumption about the direction of F_{AC} must have been incorrect. We assumed that the force F_{AC} is in tension; the negative answer tells us it is in compression. We can now write the final answer as

$$F_{AC} = \underline{\underline{5.52N \text{ (compression)}}}$$

Note that, for internal forces, we do not show the direction of the force vector with an arrow; rather we simply label the force as either tension or compression.

Now we can write the second equilibrium equation—the sum of the forces in the x-direction—for Joint A. Again look at the free body diagram of the joint, and identify every force that acts in the x-direction or has a component in the x-direction. Include each of these forces in the equilibrium equation:

$$\sum F_x = 0$$

$$+F_{AB} + F_{AC} \cos 65° = 0$$

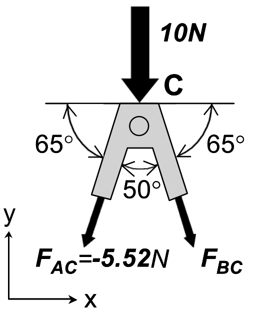
Note that the x-component of the force F_{AC} appears in this equation, while the y-component does not. To solve the equation, we must calculate $\cos 65°$, substitute the value of F_{AC} we calculated above, and then solve for F_{AB}.

$$+F_{AB} + (-5.52)(0.423) = 0$$

$$F_{AB} = +2.33N = \underline{\underline{2.33N \text{ (tension)}}}$$

Step 4: Repeat the process for the remaining joints.

Next we will apply the same solution process to Joint C. We begin by isolating the joint—cutting through the two handles, AC and BC. When we cut through these two members, we expose their internal forces F_{AC} and F_{BC}. Thus these two forces must be included on the free body diagram, along with the 10-newton load.



Note that both internal forces are again shown pointing away from the joint, indicating tension. This might appear to be incorrect, since we already know that F_{AC} is in compression. In fact, it is not an error but a technique to help prevent errors. To ensure that there is mathematical consistency when we move from joint to joint, it is best to *always* show internal member forces acting in tension. When our calculations show that a force is actually in compression, we write its magnitude as a negative number. The minus sign ensures that it is mathematically represented as a compression force in the equilibrium calculation. (We'll see how this works shortly.)

On the free body diagram, we can see that the two handles are connected together at an angle of 50°. It is also important to recognize that each handle forms an angle of 65° measured from horizontal. If you have studied plane geometry, you should be able to prove that this is true.

Again, once we have carefully drawn the free body diagram of the joint, we can write an equilibrium equation to determine the unknown member force F_{BC}. In this case, either equation will do the job. Let's use the sum of the forces in the y-direction:

$$\sum F_y = 0$$

$$-10 - F_{AC}\sin 65^\circ - F_{BC}\sin 65^\circ = 0$$

Now we can substitute the calculated value of F_{AC} and solve for F_{BC}. Remember that F_{AC} is in compression, so the value you substitute must have a minus sign.

$$-10 - (-5.52)(0.906) - F_{BC}(0.906) = 0$$

$$-10 + 5 - F_{BC}(0.906) = 0$$

$$-F_{BC}(0.906) = 5$$

$$F_{BC} = \frac{-5}{0.906} = -5.52N = \underline{\underline{5.52N\ (compression)}}$$

We shouldn't be surprised that F_{BC} turns out to be exactly the same as F_{AC}. Given that the structure, loads, and reactions are all symmetrical, it certainly makes sense that the compression forces in the two handles are also the same. Nonetheless, it is reassuring that our mathematical equations and our common sense both produce the same result. For further reassurance, try writing the second equilibrium equation, $\Sigma F_x=0$, for Joint C. This calculation will also show that $F_{BC}=-5.52$ newtons. If you write the equilibrium equations for Joint B, you will again find that $F_{BC}=5.52$ newtons (compression) and $F_{AB}=2.33$ newtons (tension).

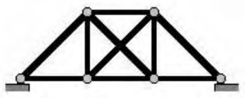
Static Determinacy and Stability

The Method of Joints is a simple, powerful tool for calculating the forces in truss members. Unfortunately, the method does not work for all trusses. If a structure has more unknown member forces and reactions than the number of available equilibrium equations, then the Method of Joints is not sufficient to perform the structural analysis. A structure that cannot be analyzed using the equations of equilibrium alone is called **statically indeterminate**. A structure that *can* be analyzed using the equations of equilibrium alone is called **statically determinate**. Only statically determinate trusses can be analyzed with the Method of Joints.

A statically determinate truss with two reactions must satisfy the mathematical equation

$$2j = m + 3$$

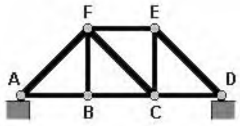
where ***j*** is the number of joints and ***m*** is the number of members. For example, our nutcracker truss has 3 members and 3 joints. Substituting these numbers into the equation above, we find that ***2j*** and ***m+3*** are both equal to 6, so the mathematical condition for static determinacy is satisfied. If ***2j is less than m+3***, then the truss is *statically indeterminate*. For example, the truss at right has 6 joints and 10 members. Thus ***2j*** is 12, and ***m+3*** is 13. Since ***2j*** is less than ***m+3***, the structure is indeterminate. Such a structure cannot be analyzed using the equations of equilibrium alone. If you tried to use the Method of Joints to analyze this truss, you would find that you have more unknown forces than you have equations available to solve for them. It is possible to analyze a statically indeterminate structure, but the solution process requires advanced engineering concepts that are beyond the scope of this book.



A statically indeterminate truss.

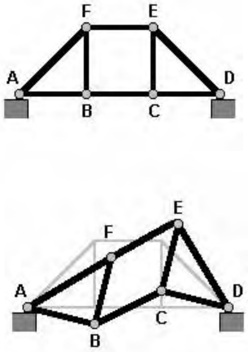
If ***2j is greater than m+3***, then the truss is **unstable**. An unstable truss does not have enough members to form a rigid framework. Such a structure cannot carry any load.

In general, a truss is stable if all of its members are arranged in a network of interconnected triangles. For example, the simple truss at right is composed of 6 joints and 9 members, which together form four interconnected triangles (ABF, BCF, CEF, and CDE). This truss also satisfies the mathematical condition for determinacy, since both ***2j*** and ***m+3*** are equal to 12.



A statically determinate truss.

If member CF is removed, however, the truss becomes unstable. Without its diagonal member, the center panel of the truss now consists of a rectangle (BCEF) formed by four members, rather than two triangles (BCF and CEF). This configuration is unstable because there is nothing to prevent the rectangle BCEF from distorting into a parallelogram, as shown below. (Remember that we assume all truss joints to be frictionless pins.) For this truss, ***2j*** is still 12, while ***m+3*** is only 11. Since *2j* is greater than *m+3*, the mathematical test confirms our observation that the truss is unstable.



An unstable truss.

As you might expect from this example, an unstable truss can generally be made stable by simply adding members until an appropriate arrangement of interconnected triangles is achieved.

Factor of Safety

When an engineer designs a structure, he or she must consider many different forms of *uncertainty*. There are three major types of uncertainty that affect a structural design:

- There is always substantial uncertainty in predicting the loads a structure might experience at some time in the future. Wind, snow, and earthquake loads are highly unpredictable. The engineer can never be certain of the maximum number of people that might occupy an apartment building or the weight of the heaviest truck that might cross a bridge. Truck weights are regulated by law in the United States, but illegally heavy trucks occasionally do drive our highways, and it only takes one of them to collapse a bridge. You can post a 20-ton Load Limit sign on a bridge, but that doesn't mean the driver of a 30-ton truck won't try to cross it anyway.

The loads applied to structures are highly unpredictable.

- The strengths of the materials that are used to build actual bridges are also uncertain. Manufacturers of construction materials generally pay careful attention to the quality of their products; nonetheless, it is always possible for a batch of substandard steel or concrete to be used in a structure. Even the most conscientious construction contractors occasionally make mistakes on a project, and some construction errors can reduce the ability of a structure to carry load.
- The mathematical models we use for structural analysis and design are never 100% accurate. We have already seen this in our discussion of structural models—actual trusses do not have perfectly straight members or frictionless pinned connections. Yet we must make these sorts of simplifying assumptions, or the truss analysis simply cannot be performed.

The engineer accounts for all forms of uncertainty by making the structure somewhat stronger than it really needs to be—by using a **factor of safety** in all analysis and design calculations. In general, when it is used in the analysis of an existing structure, the factor of safety is a defined as

$$\text{Factor of Safety} = \frac{\text{Failure Level}}{\text{Actual Level}}$$

In a truss, the *actual* force in a member is called the *internal member force*, and the force at which *failure* occurs is called the *strength*. Thus we can rewrite the definition of the factor of safety as

$$\text{Factor of Safety} = \frac{\text{Strength}}{\text{Internal Member Force}}$$

For example, if a structural member has an internal force of 5000 pounds and a strength of 7500 pounds, then its factor of safety, ***FS***, is

$$FS = \frac{7500}{5000} = 1.5$$

If the factor of safety is less than 1, then the member or structure is clearly unsafe and will probably fail. If the factor of safety is 1 or only slightly greater than 1, then the member or structure is nominally safe but has very little margin for error—for variability in loads, unanticipated low member strengths, or inaccurate analysis results. Most structural design codes specify a factor of safety of 1.6 or larger (sometimes considerably larger) for structural members and connections.

In Learning Activity #5, we'll see how the factor of safety is applied in the design process.

On an Actual Bridge Project

Load and Resistance Factor Design

The factor of safety has been used in structural engineering for over a century. In recent years, however, a new design philosophy called load and resistance factor design (LRFD) has become increasing popular. LRFD is based on the idea that the largest loading a structural member experiences in its lifetime must be less than the smallest possible strength of that member. In an LRFD-based design, the engineer estimates this "largest loading" by adjusting the loads used in the structural analysis. All loads are multiplied by a code-specified load factor—a number that is always greater than 1. The actual magnitude of the load factor depends on how uncertain the loads are. The self-weight of a structure can be predicted accurately, so it has a relatively low load factor (normally 1.2 to 1.4). Wind, traffic, and earthquake loads are much more unpredictable, so their load factors are usually much higher. To estimate the smallest possible strength of a member, the engineer multiplies the nominal member strength by a code-specified resistance factor—a number that is always less than 1. The resistance factor accounts for the possibility of understrength materials, fabrication errors, and other uncertainties that may cause a member to be weaker than the engineer intended. Ultimately load factors and resistance factors serve the same function as the factor of safety—they ensure the safety of a structure by providing a margin for error. Many experts view the LRFD as a superior design philosophy, because it more accurately represents the sources of uncertainty in structural design.

The Learning Activity

The Problem

The Need

One year after the completion of the new Grant Road Bridge, the Hauptville Town Engineer inspects the structure and finds that it is performing well. Though the bridge has been carrying a lot of traffic, its structural members show no signs of distress or deterioration. Nonetheless, the Town Engineer is still somewhat concerned about the bridge. Because of a major construction project nearby, many heavily loaded dump trucks have been using Grant Road recently. What if one of these trucks is heavier than the legal weight limit? How much of an overload would cause the structure to collapse? The Town Engineer decides to perform a complete structural evaluation to determine the overall level of safety of the Grant Road Bridge. He begins by hiring Universal Structural Materials Assessment, Inc. to test the strength of the structural members used in the bridge. (We did this part of the structural evaluation in Learning Activity #2.) Once the Engineer has received the test results from Universal, he is ready to begin his analysis.

Your Job

You are the Town Engineer of Hauptville. Your job is to analyze the Grant Road Bridge and evaluate its overall level of safety. Specifically, you must calculate the factor of safety for every member in one of the main trusses, then determine the overall safety factor for the structure.

As the Town Engineer, you have the professional responsibility to protect the health and safety of the people who use this bridge. You fulfill this responsibility by performing the structural evaluation conscientiously—by using good judgment, by performing calculations carefully and accurately, and by asking a colleague to check your work.

The Solution

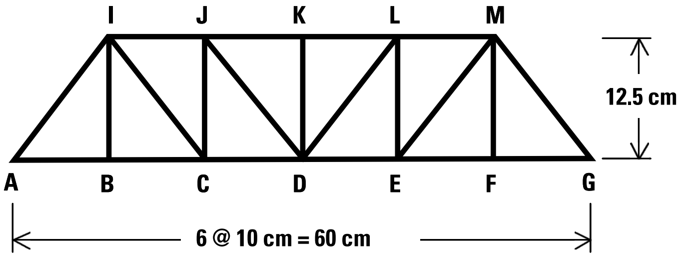
The Plan

Our plan to conduct the structural analysis and evaluation of the Grant Road Bridge consists of the following tasks:

- Create the structural model.
- Check the structural model for static determinacy and stability.
- Calculate the reactions.
- Calculate the internal member forces.
- Determine the strengths of the members
- Calculate the factor of safety for every member in the structure
- Evaluate the safety of the structure.
- Check our assumptions.

Create the Structural Model

To model the Grant Road Bridge, we must define (1) the geometry of the structure, (2) the loads, and (3) the supports and reactions. We begin by idealizing the three-dimensional bridge structure as a pair of two-dimensional Pratt trusses. Since these two trusses are identical, we only need to analyze one of them. The geometry of the truss is shown below. The dimensions indicate the locations of the *centerlines* of the members. Joints are identified with letters—the same letter designations that were used on the bridge plans provided in Learning Activity #1. To facilitate the analysis, we will assume that the truss members are perfectly straight, the joints are frictionless pins, and the loads are applied only at the joints. We will also assume that the weight of the truss itself is zero.

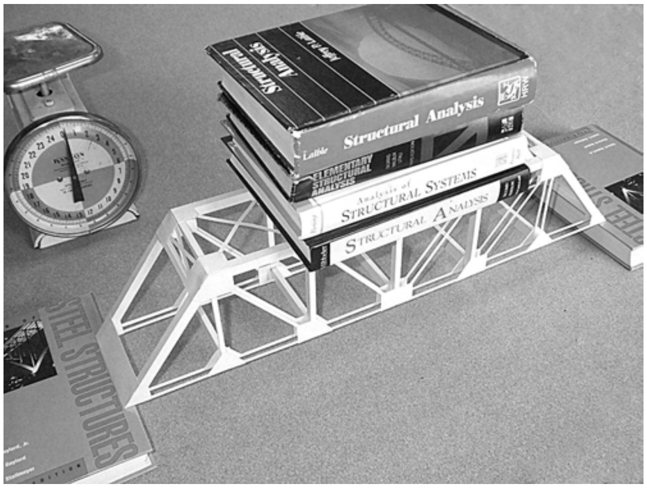


Q1

Why did we assume that the weight of the truss is zero?

Obviously, the actual weight of the truss is not zero. Why did we make this assumption, when we know it is not true? How do you think it will affect the accuracy of our structural analysis?

When we load-tested the Grant Road Bridge in Learning Activity #1, we applied the load in two different ways—with a stack of books placed on the top chord and with a bucket of sand suspended from the floor beams. Before we can define the loads for our structural model, we need to decide which of these two loading configurations to use. As a general rule, a structural evaluation should be based on the most severe loading condition—the one that produces the highest member forces. If the analysis shows that the truss is safe for the most severe loading, then the structure will certainly be safe for less severe ones. Unfortunately, in this case, it is not immediately obvious which of the two loading configurations is more severe. The best we can do is to make an assumption and check it later. For now, we will assume that the top-chord loading, shown here, is more severe.



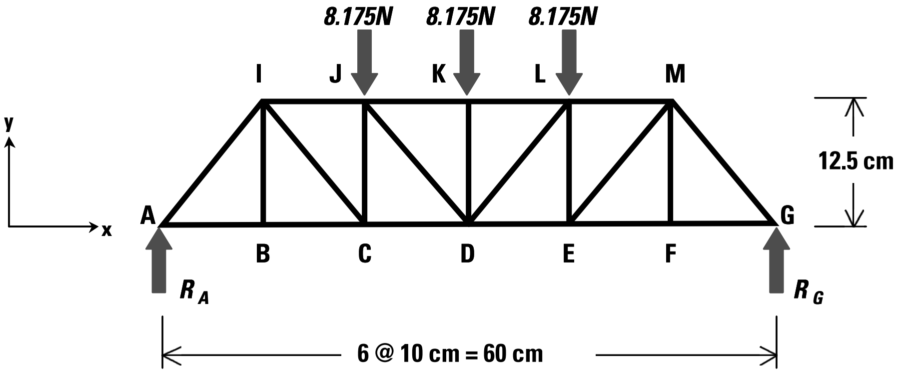
We will assume that the top chord loading is more severe, then check the bottom-chord loading later.

Having decided on the location of the load, we must now determine its magnitude. In Learning Activity #2, we applied the equation ***W=mg*** to determine that the weight of a 5-kilogram mass is 49.05 newtons. When we placed the stack of books onto the top chord of the truss, the weight of the stack was supported on joints J, J′, K, K′, L, and L′. We can reasonably assume that the weight of the books is distributed equally to these six joints. Therefore, the downward force applied to each joint is

$$\text{Load per Joint} = \frac{\text{Total Load}}{\text{Number of Joints}} = \frac{\mathbf{49.05N}}{\mathbf{6}} = \mathbf{8.175}N$$

Since there are two main trusses, three ***8.175N*** loads will be applied to each truss. Since all of the loads are downward, and the bridge is supported only at its ends, we will add upward reactions R_A and R_G at joints A and G.

A complete free body diagram of the truss looks like this:



Check Static Determinacy and Stability

Before we can use the equations of equilibrium to analyze this truss, we must first verify that it is statically determinate and stable. The mathematical condition for static determinacy and stability is

$$2j = m + 3$$

where ***j*** is the number of joints and ***m*** is the number of members. Our truss from the Grant Road Bridge has 12 joints and 21 members. Substituting these numbers into the equation above, we find that ***2j*** and ***m+3*** are both equal to 24, so the mathematical condition for static determinacy and stability is satisfied. Furthermore, we note that the truss is composed entirely of interconnected triangles, which confirms our conclusion that the structure is stable.

Calculate Reactions

On the free body diagram above, the forces R_A and R_G are the unknown reactions at Joints A and G. We know that the truss is in equilibrium; therefore, the sum of all forces acting on the structure must be zero. Since all of the forces—loads and reactions—are acting in the y-direction, only one of our two equilibrium equations is relevant to the calculation of reactions:

$$\sum F_y = 0$$

$$R_A + R_G - 8.175 - 8.175 - 8.175 = 0$$

Since the structure, the loads, and the reactions are all symmetrical about the centerline of the truss, the two reactions R_A and R_G must be equal. (The centerline of the truss is a vertical line passing through Member DK). Substituting $R_A = R_G$ into the equilibrium equation above, we get

$$R_A + R_A - 24.525 = 0$$

$$2R_A = 24.525$$

$$R_A = 12.26N \uparrow$$

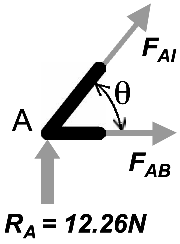
And since $R_A = R_G$, then

$$R_G = 12.26N \uparrow$$

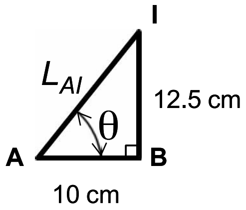
Calculate Internal Member Forces

We will use the Method of Joints to calculate the internal force in each member of the truss. To apply this method, we will isolate a joint from of the structure, cutting through the attached members and exposing their internal member forces. We will draw a free body diagram of the joint, then use the equations of equilibrium to determine the unknown member forces. We will repeat the process for successive joints, until we have calculated all of the internal member forces in the structure.

Joint A



We'll start by isolating Joint A and drawing a free body diagram of it. The free body diagram must show *all* forces acting on the joint. Thus the reaction R_A is shown, along with its known magnitude of 12.26N. The member forces F_{AI} and F_{AB} are also included on the diagram. Because we do not know the magnitudes or directions of these forces, we simply show them in variable form, and we assume their directions to be in tension. To indicate that a member force is in tension, we draw the force vector pointing away from the joint, along the centerline of the member.



Before we can write the equilibrium equations for this joint, we need to figure out what the angle θ is. Actually, we don't really need to know the angle itself; rather, we only really need to know the sine and cosine of the angle—**sin**θ and **cos**θ. We can determine the sine and cosine directly from the geometry of the truss. Note that Members AB, AI, and BI form a right triangle, with Member AI as the hypotenuse. We can apply the Pythagorean Theorem to calculate the length, L_{AI}, as follows:

$$L_{AI} = \sqrt{10^2 + 12.5^2} = 16.01cm$$

Now we can apply the basic definitions of the sine and cosine to find **sin**θ and **cos**θ:

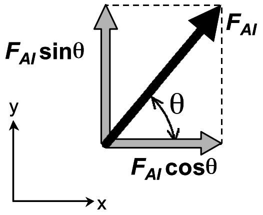
$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{12.5}{16.01} = 0.7809$$

$$\cos\theta = \frac{adjacent}{hypotenuse} = \frac{10}{16.01} = 0.6247$$

We are finally ready to write the equilibrium equations for Joint A. We will start with the equation for the sum of forces in the y-direction. Assuming that the upward direction is positive,

$$\sum F_y = 0$$

$$12.26 + F_{AI}\sin\theta = 0$$



To write this equation, we had to represent the force F_{AI} in terms of its x-component and y-component. The y-component is $F_{AI}\sin\theta$, and its direction is upward, so it is positive in the equilibrium equation. The x-component is $F_{AI}\cos\theta$; however, this component is not included in the ΣF_y equilibrium equation, because it does not act in the y-direction.

We can now substitute the known value of **sin**θ into the equilibrium equation, and solve for the unknown force F_{AI}.

$$12.26 + F_{AI}(0.7809) = 0$$

$$F_{AI}(0.7809) = -12.26$$

$$F_{AI} = -\frac{12.26}{0.7809} = -15.70N$$

Because the answer is negative, our initial assumption about the direction of F_{AI} was incorrect. We assumed that F_{AI} is in tension. The negative member force indicates that it is in compression. Thus our final answer is

$$F_{AI} = \underline{\underline{15.70N\ (compression)}}$$

Now we can write the equilibrium equation for forces in the x-direction. Assuming that the positive direction is to the right,

$$\sum F_x = 0$$

$$F_{AB} + F_{AI}\cos\theta = 0$$

We know **cos**θ, and we have just solved for F_{AI}. We can substitute these values into the equilibrium equation and solve for F_{AB}. But be careful! When you substitute F_{AI}, don't forget the minus sign.

$$F_{AB} + (-15.70)(0.6247) = 0$$

$$F_{AB} = +9.81N$$

Because the answer is positive, our assumption about the direction of F_{AB} was correct.
The final answer is

$$F_{AB} = \underline{\underline{9.81N\ (tension)}}$$

Q2

Why did we choose to do Joint A first?

Is there a reason why Joint A was a good place to start this analysis? What would have happened if we had started with a different joint?

Q3

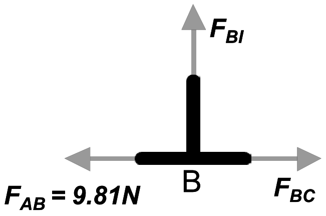
Why did we choose to solve the y-direction equilibrium equation first?

Is there a reason why it was a good idea to solve $\Sigma F_y = 0$ before solving $\Sigma F_x = 0$?

Joint B

At this point, we should analyze Joint B. It has only three connected members, and we already know the internal force in one of the three (Member AB). Thus there are only two unknown forces, and we will be able to solve for them with the two available equilibrium equations.

Again we draw a free body diagram of the joint, with all member forces assumed to be in tension—pointing away from the joint. The known magnitude of the force F_{AB} is included on the diagram. The equilibrium equation for forces in the x-direction is



$$\sum F_x = 0$$

$$-9.81 + F_{BC} = 0$$

$$F_{BC} = +9.81N = \underline{\underline{9.81N\ (tension)}}$$

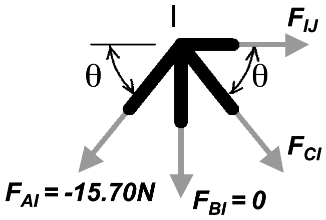
The equilibrium equation for forces in the y-direction produces an interesting result:

$$\sum F_y = 0$$

$$F_{BI} = 0$$

It should come as no surprise that this member has zero internal force. When you load-tested the Grant Road Bridge in Learning Activity #1, you should have noticed that this member—the hip vertical—was slack. It appeared to have no internal force at all. Now we have verified our observation using the Method of Joints!

Joint I



It takes some careful thought to recognize that Joint I should be the next joint we analyze. As the free body diagram indicates, this joint has four connected members and, therefore, it also has four internal member forces. Note, however, that we have already calculated two of these—F_{AI} and F_{BI}. Thus there are only two unknown forces, which we can calculate with our two equilibrium equations.

Note that all four of the force vectors are pointing away from the joint, even though we already know that one of them, F_{AI}, is in compression. The negative magnitude of F_{AI} ensures that it is mathematically represented as a compression force.

It is important to note that both angles labeled as θ on the free body diagram are exactly the same as the angle θ on the diagram of Joint A. (If you can't see that these angles are all equal, prove it to yourself by drawing the corresponding right triangles, just as we did for Joint A.) Thus the values we calculated for **sin**θ and **cos**θ for Joint A are still valid here.

If we begin with the equilibrium equation in the y-direction, we will be able to solve for F_{CI} directly:

$$\sum F_y = 0$$

$$-F_{AI}\sin\theta - F_{BI} - F_{CI}\sin\theta = 0$$

$$-(-15.70)(0.7809) - 0 - F_{CI}(0.7809) = 0$$

$$F_{CI} = +15.70 = \underline{\underline{15.70N\ (tension)}}$$

Now we can use the second equilibrium equation to solve for F_{IJ}:

$$\sum F_x = 0$$

$$-F_{AI}\cos\theta + F_{CI}\cos\theta + F_{IJ} = 0$$

$$-(-15.70)(0.6247) + 15.70(0.6247) + F_{IJ} = 0$$

$$F_{IJ} = -19.62N = \underline{\underline{19.62N\ (compression)}}$$

Joint C

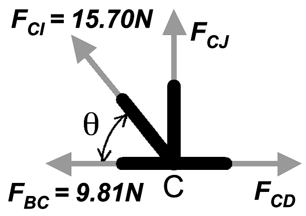
Next we will calculate the unknown member forces at Joint C.

Q 4

Why Joint C?

Why was Joint C the best joint to analyze at this point in the solution process?

Based on the free body diagram of the joint, we can write the two equilibrium equations and solve for the two unknown member forces as follows:



$$\sum F_x = 0$$

$$-F_{BC} - F_{CI}\cos\theta + F_{CD} = 0$$

$$-9.81 - (15.70)(0.6247) + F_{CD} = 0$$

$$F_{CD} = +19.62N = \underline{\underline{19.62N\ (tension)}}$$

$$\sum F_y = 0$$

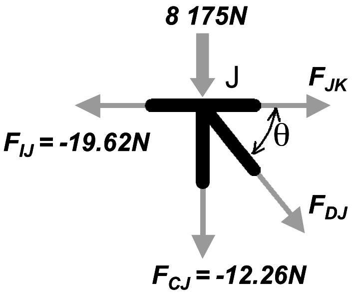
$$F_{CI}\sin\theta + F_{CJ} = 0$$

$$(15.70)(0.7809) + F_{CJ} = 0$$

$$F_{CJ} = -12.26N = \underline{\underline{12.26N\ (compression)}}$$

Joint J

The free body diagram of Joint J is shown at right. Note that the 8.175N load at Joint J *must* be included on the diagram. (Failure to put loads on the free body diagram is one of the most common errors in truss analysis.) We can write the two equilibrium equations and solve for the two unknown member forces as follows:



$$\sum F_y = 0$$

$$-8.175 - F_{CJ} - F_{DJ}\sin\theta = 0$$

$$-8.175 - (-12.26) - F_{DJ}(0.7809) = 0$$

$$F_{DJ}(0.7809) = 4.085$$

$$F_{DJ} = +5.23N = \underline{\underline{5.23N\ (tension)}}$$

$$\sum F_x = 0$$

$$-F_{IJ} + F_{DJ}\cos\theta + F_{JK} = 0$$

$$-(-19.62) + (5.23)(0.6247) + F_{JK} = 0$$

$$F_{JK} = -22.89N = \underline{\underline{22.89N\ (compression)}}$$

Q5

Can you apply the Method of Joints to calculate a member force?

Which joint should you analyze to determine the member force F_{DK}? Solve the appropriate equilibrium equations to show that $F_{DK}=8.175$N *(compression)*.

Summary of Structural Analysis Results

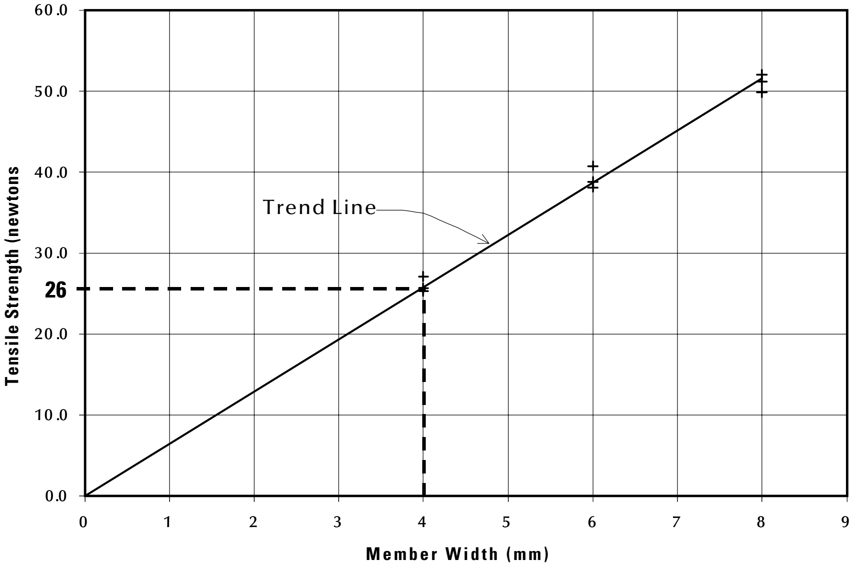
At this point, we have only analyzed half of the truss. However, if we take advantage of symmetry, we can determine the internal forces in all remaining members without doing any further calculations. When we determined the reactions R_A and R_G, we noted that these two forces must be equal because the structure, its loads, and its reactions are all symmetrical about the centerline of the truss. The same principle holds true for internal member forces. Because the structure, loads, and reactions are all symmetrical, the member forces must also be symmetrical about the centerline. Members that are "mirror images" of each other have equal internal forces. F_{GM} and F_{AI} must be equal; F_{LM} and F_{IJ} must be equal; F_{DJ} and F_{DL} must be equal, and so forth.

So we're done! The results of the analysis are summarized in the table below.

Members	Force		Members	Force	
AB, FG	9.81 N	(tension)	BI, FM	0 N	
BC, EF	9.81 N	(tension)	CI, EM	15.70 N	(tension)
CD, DE	19.62 N	(tension)	CJ, EL	12.26 N	(compression)
IJ, LM	19.62 N	compression)	DJ, DL	5.23 N	(tension)
JK, KL	22.89 N	(compression)	DK	8.175 N	(compression)
AI, GM	15.70 N	(compression)			

Determine the Strengths of the Members

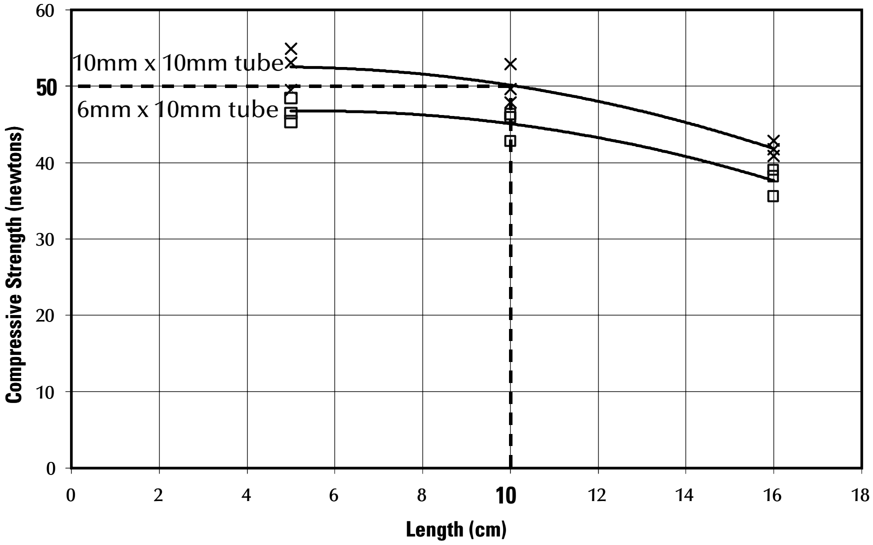
Now that we have calculated the force in each member, we must determine the corresponding strength of each member. To do this, we will use the graphs we developed in Learning Activity #2. We'll start with the *bars*—the bottom chords, diagonals, and hip verticals. The table above tells us what we have already observed in our Grant Road Bridge model—that all of the bars are in tension (except the hip verticals, which have zero internal force). Thus we need to determine the *tensile strength* of the bars. All of the bars used in the Grant Road Bridge are 4mm wide. Using the tensile strength vs. member width graph we developed in Learning Activity #2, we find the tensile strength of a 4mm bar to be 26 newtons, as shown below.



Determining the tensile strength of a 4mm bar

Note, however, that all of the bottom chords, diagonals, and hip verticals are actually *doubled* 4mm bars. Thus the tensile strength of these members is exactly twice that of a single 4mm bar, or 52 newtons.

Our structural analysis shows that all of the tubes—the top chords, the end posts, and the interior verticals—are in compression. Thus we must determine the compressive strength of these members, using the strength vs. length graph we developed in Learning Activity #2. The top chord members are all 10mm x 10mm tubes, and each has a length of 10 centimeters. The strength of these members is approximately 50 newtons, as indicated below.



Determining the compressive strength of a 10mm x 10mm tube that is 10 centimeters long

Q6

Can you determine the strength of a member?

What is the compressive strength of the vertical tube members (CJ, DK, and EL) and the end posts (AI and GM)?

Calculate the Factor of Safety

Once we know the strength of a member and the internal force it is actually experiencing, we can calculate its factor of safety. For the bottom chord member CD, the factor of safety is:

$$FS_{CD} = \frac{\textit{Strength}}{\textit{Internal Member Force}} = \frac{52}{19.62} = 2.7$$

For the top chord member JK, the factor of safety is

$$FS_{JK} = \frac{\textit{Strength}}{\textit{Internal Member Force}} = \frac{50}{22.89} = 2.2$$

Q7

Can you calculate the factor of safety for a member?

Calculate the factor of safety for all remaining members in the truss, and add them to the summary table below (along with the member strengths not already recorded in the table).

Members	Force		Strength	FS
AB, FG	9.81 N	(tension)	52	
BC, EF	9.81 N	(tension)	52	
CD, DE	19.62 N	(tension)	52	2.7
IJ, LM	19.62 N	(compression)	50	
JK, KL	22.89 N	(compression)	50	2.2
AI, GM	15.70 N	(compression)		
BI, FM	0 N	--	52	--
CI, EM	15.70 N	(tension)	52	
CJ, EL	12.26 N	(compression)		
DJ, DL	5.23 N	(tension)	52	
DK	8.175 N	(compression)		

Evaluate the Structure

As the Town Engineer of Hauptville, you have finished what you set out to do—a complete structural evaluation of the main trusses of the Grant Road Bridge. Yet the results of these calculations are just numbers. They are of little use, until you study them, think critically about them, and draw meaningful conclusions from them.

Once you have completed the summary table on the previous page, you should be able to make the following observations:

- Members JK and KL have a factor of safety of 2.2—the smallest of any member in the truss. Since the failure of Member JK or Member KL would cause the entire structure to collapse, we can say that the factor of safety of the *entire structure* is 2.2.
- Since 2.2 is obviously larger than 1, our analysis tells us that the structure will not collapse when the 5 kg mass is placed on the top chord. Because the factor of safety is considerably larger than 1, we can have a high degree of confidence that the structure will not fail, even if we made some minor errors in construction or if the actual load is significantly larger than 5 kg.
- Theoretically, the bridge would collapse if the mass of the stack of books at mid-span were increased to ***(5 kg)(2.2)=11.0 kg***.
- Many members of the truss have safety factors that are substantially larger than 2.2. These members are actually much stronger than they need to be.

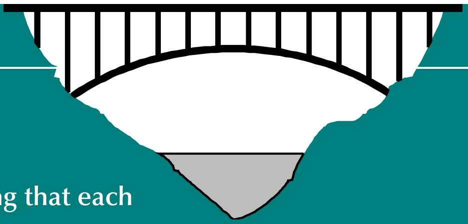
Q8

Why are some truss members stronger than they need to be?

For example, Member CI is a doubled 4mm bar with a safety factor of 3.3. Had the structural engineer chosen to use a doubled 3mm bar for this member, the safety factor would still be 2.5. The member would use less material; and because its safety factor would still be greater than 2.2, the overall safety of the structure would not be adversely affected. Why did the structural engineer choose not to use a smaller member size?

It is important to note that this structural evaluation is valid only for one particular loading. If we change either the magnitude or the position of the load, the member forces and factors of safety will also change.

On an Actual Bridge Project

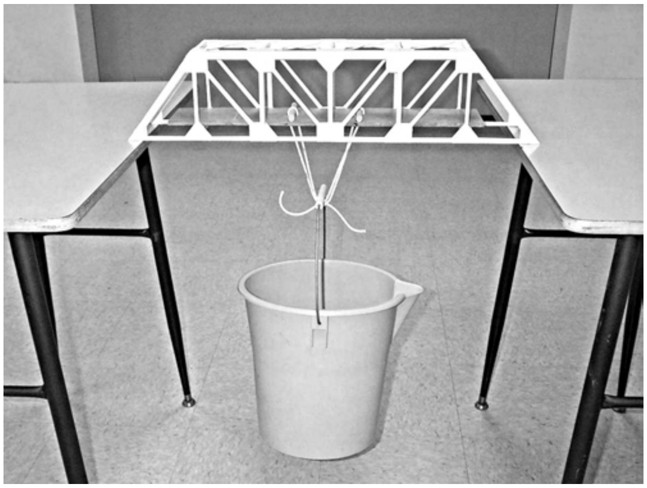


Deflections are important too.

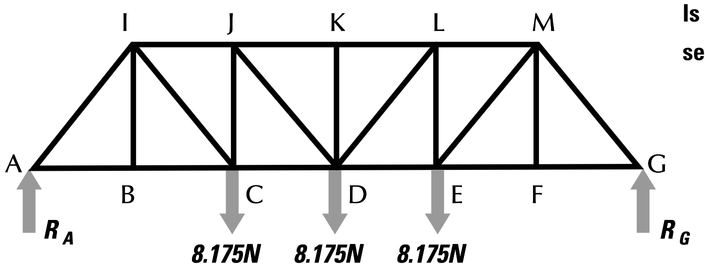
We evaluated the Grant Road Bridge by checking that each member in the structure can carry load safely. On an actual bridge project, the engineer would also check to ensure that the *deflections* are acceptably small. A **deflection** is the distance a structure moves when it is loaded. A bridge *always* deflects when it is loaded. On a well-designed bridge, these deflections are so small that they are imperceptible to drivers and pedestrians crossing the span. If a bridge is too flexible, then drivers and pedestrians will feel its movement and will perceive the structure to be unsafe—even if its strength entirely adequate. The engineer is responsible for ensuring not only that the bridge *is safe*, but also that the public *perceives it to be safe*. Thus the engineer carefully computes the structure's deflections under various loading conditions and ensures that these computed deflections comply with the appropriate design codes.

Bottom-Chord Loading

The analysis above was based on a number of assumptions. Perhaps the most important of these was our assumption that placing the load on the top chord of the truss is more severe than suspending the load from the floor beams. We now have the analytical tools to check this assumption. Since the floor beams are attached to the bottom chords, loading the floor beams is essentially the same as loading the bottom chord joints of the truss. Thus our revised structural model should have the three 8.175 N loads applied at the bottom chord, as shown below:



Is bottom-chord loading more or less severe than top-chord loading?



If we repeat the structural analysis using this new loading condition, we get the following results:

Members	Force		Members	Force	
AB, FG	9.81 N	(tension)	BI, FM	0 N	
BC, EF	9.81 N	(tension)	CI, EM	15.70 N	(tension)
CD, DE	19.62 N	(tension)	CJ, EL	4.09 N	(compression)
IJ, LM	19.62 N	compression)	DJ, DL	5.23 N	(tension)
JK, KL	22.89 N	(compression)	DK	0 N	(compression)
AI, GM	15.70 N	(compression)			

Q9

Can you apply the Method of Joints to analyze a truss?

Use the Method of Joints to analyze the Pratt truss with bottom-chord loading. Prove that the results in the table above are correct.

Note that moving the loads from the top chord to the bottom chord caused the internal forces to change *only* in Members CJ, EL, and DK. In all three cases, the forces got smaller. The forces in the most heavily loaded members—the top and bottom chords and the end posts—remained unchanged; thus the overall factor of safety of the structure remains unchanged. We can conclude that using the top-chord loading in Learning Activity #1 (and in our initial structural analysis) was entirely appropriate. The bottom-chord loading is more realistic, because real bridges carry traffic loads on their floor beams; however, the top-chord loading is much easier to do and produces nearly identical structural analysis results.

Q10

Can members with zero force be removed?

In the analysis above, Members BI, FM, and DK have zero internal force. It would seem that the truss does not need these members to carry load, and we might simply remove them from the structure to save material. Do you think these zero-force members can safely be removed from the truss?

Q11

Can you analyze a different truss?

Select any truss from the Gallery of Structural Analysis Results (Appendix B), and calculate the internal forces in all of its members.

Conclusion

In this learning activity, we applied concepts from geometry, trigonometry, algebra, and physics to calculate the internal forces in every member of a truss. In doing so, you saw how math and science are applied to solve an important, real-world problem. You also saw how data from laboratory experiments—the strength tests we did in Learning Activity #2—can be integrated into the solution of an engineering problem. Most important, you learned to use an important analysis tool called the Method of Joints. This technique isn't easy! It requires you to construct and solve equilibrium equations—lots of them—while paying careful attention to the magnitudes and directions of forces. Wouldn't it be great if we could use the computer to do this work for us? Well, we can. In Learning Activity #4, we will use the West Point Bridge Designer software to analyze and evaluate a truss bridge.

Answers to the Questions

1) **Why did we assume that the weight of the truss is zero?** We *could* include the self-weight of the bridge in our structural analysis, but doing so would *greatly* complicate the analysis. In Learning Activity #1, we found that the mass of our Grant Road Bridge model is about 55 grams. This mass is very small in comparison with the 5 kilogram mass the bridge is designed to carry. Including the self-weight in our analysis would change our calculated member forces by only about one percent. For the sake of simplicity, we can ignore self-weight, and recognize that this assumption has a very small effect on the accuracy of the structural analysis.

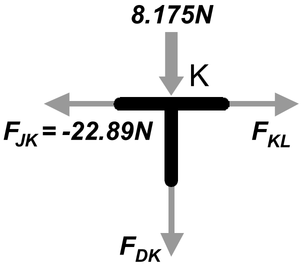
2) **Why did we choose to do Joint A first?** At any given joint, we can write two equilibrium equations—$\Sigma F_x=0$ and $\Sigma F_y=0$. With only two available equations, we can solve for only two unknown member forces at each joint. At the start of the solution process, Joint A has just two unknown member forces—F_{AB} and F_{AI}. Joint G is the only other joint in the entire truss with only two unknown forces. All of the others have three or more. Thus the solution process should start at either Joint A or Joint G.

Note that the reaction force R_A is also applied at Joint A. Had this force also been unknown, it would have been impossible to solve the equilibrium equations at this joint—there would have been three unknowns and only two equations. That's why it is generally necessary to solve for reactions before we use the Method of Joints to calculate member forces.

3) **At Joint A, why did we choose to solve the y-direction equilibrium equation first?** At Joint A, the x-direction equilibrium equation has two unknown member forces, F_{AB} and F_{AI}. Had we written this equation first, we would have been unable to solve for either of the unknowns immediately. The y-direction equation includes only one unknown force, F_{AI}. By solving $\Sigma F_y=0$ first, we were able to solve for F_{AI} directly. Then, when we wrote $\Sigma F_x=0$, we could substitute the known value of F_{AI} and solve for F_{AB}. At any given joint, we can often (but not always) avoid the chore of solving two equations simultaneously by identifying an equilibrium equation that has only one unknown member force—and solving it *first*.

4) **After solving for the unknown member forces at Joints A, B, and I, why did we choose Joint C?** At Joints A, B, and I we calculated the member forces F_{AB}, F_{AI}, F_{BC}, F_{BI}, F_{CI}, and F_{IJ}. At this point in the solution process, we already knew two of the four member forces at Joint C—F_{BC} and F_{CI}. Only F_{CD} and F_{CJ} were unknown. So we selected Joint C because it had only two unknown member forces, which could be solved with our two equations of equilibrium. As a general rule, when using the Method of Joints to analyze a truss, always select a joint with only two unknown member forces as the next step in the analysis.

5) **Can you apply the Method of Joints to calculate the force in Member DK?** The best joint to analyze in order to determine the member force F_{DK} is Joint K. The free body diagram of this joint is shown at right. From this diagram, we can calculate F_{DK} directly from the y-direction equilibrium equation

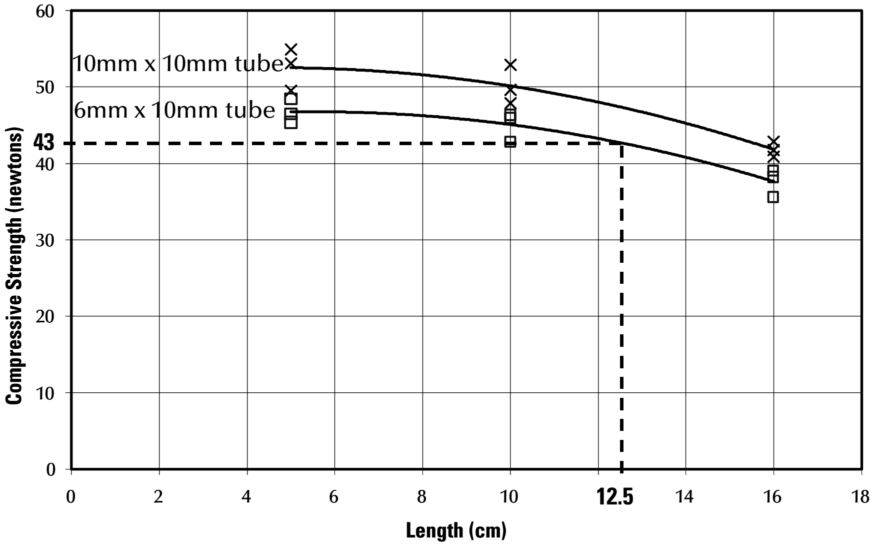


$$\sum F_y = 0$$

$$-8.175 - F_{DK} = 0$$

$$F_{DK} = -8.175N = \underline{\underline{8.175N\ (compression)}}$$

6) **Can you determine the strength of the verticals and end posts?** To determine the compressive strength of the verticals and end posts, we will use the strength vs. length graph we developed in Learning Activity #2. The verticals are 10mm x 6mm tubes, and each has a length of 12.5 centimeters. Thus the compressive strength of the vertical tubes is 43 newtons, as indicated below:

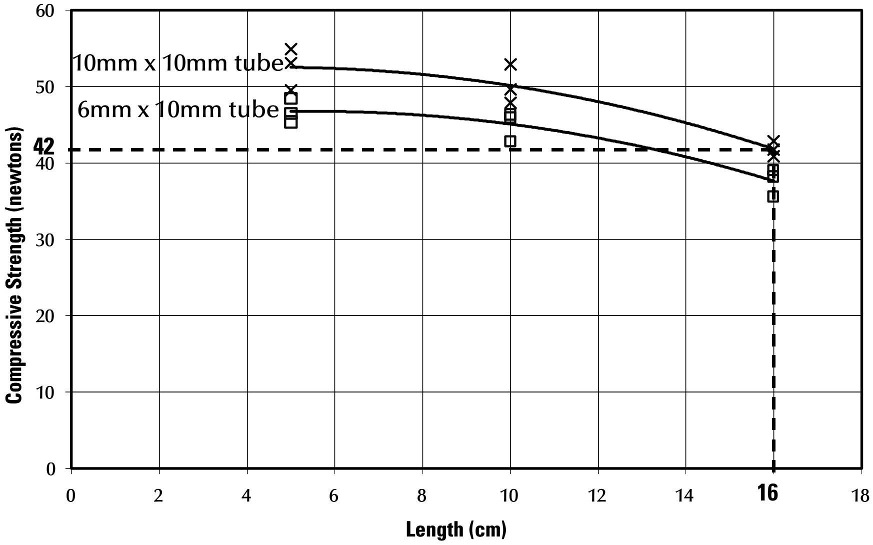


Determining the compressive strength of a 6mm x 10mm tube that is 12.5cm long

The end posts are 10mm x 10mm tubes. Since they are diagonal members, we must calculate their length using the Pythagorean Theorem:

$$L_{AI} = \sqrt{(10.0cm)^2 + (12.5cm)^2} = 16.0cm$$

As the graph below indicates, a 10mm x 10mm tube with a length of 16 centimeters has a compressive strength of 42 newtons.



Determining the compressive strength of a 10mm x 10mm tube that is 16cm long

7) **Can you calculate the factor of safety for all of the remaining truss members?** Once the internal member force and strength are known for each member in the truss, the corresponding factor of safety can be calculated using the equation

$$FS = \frac{\text{Strength}}{\text{Internal Member Force}}$$

The results are summarized in the table below.

Members	Force		Strength	FS
AB, FG	9.81 N	(tension)	52	5.3
BC, EF	9.81 N	(tension)	52	5.3
CD, DE	19.62 N	(tension)	52	2.7
IJ, LM	19.62 N	(compression)	50	2.5
JK, KL	22.89 N	(compression)	50	2.2
AI, GM	15.70 N	(compression)	42	2.7
BI, FM	0 N	--	52	--
CI, EM	15.70 N	(tension)	52	3.3
CJ, EL	12.26 N	(compression)	43	3.5
DJ, DL	5.23 N	(tension)	52	9.9
DK	8.175 N	(compression)	43	5.3

8) **Why are some truss members stronger than they need to be?** For example, why did the structural engineer not use a doubled 3mm bar for Member CI, rather than the doubled 4mm bar she specified in the design? Making this change would not adversely affect the overall safety of the structure; yet reducing the member size would clearly use less material. And using less material ought to lower the cost of the structure, right?

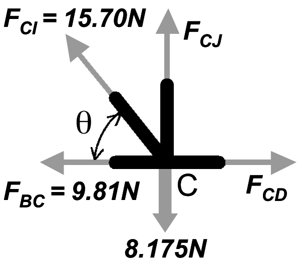
Member CI and a number of other members in the Grant Road Bridge are “too strong” because the structural engineer chose to use only a limited number of different member sizes in her design. In this case, she used only three—the two tubes and the doubled 4mm bar. She chose the 4mm bar so that the tension members with the largest internal force—CD and DE—would have a factor of safety greater than 2. Then she simply specified the same 4mm bar for all of the other tension members, knowing that this size would be more than adequate for members whose internal forces were lower.

To understand why the engineer chose to use a limited number of member sizes, just think about your own experience building the Grant Road Bridge. Suppose the engineer had designed every member with a safety factor of exactly 2. Most likely, each main truss would have required five different bar sizes and five different tube sizes. With so many different sizes, it would have taken you much longer to measure, cut out, and assemble the members. The connections would also have been much more complicated, and you would have been more likely to make construction errors—to put a 3mm bar where a 4mm bar is supposed to be used, for example. You would have saved some material, because every member would only be as strong at it absolutely needs to be. But this small reduction in material cost probably would not have been worth all of the extra work.

The same is true for the construction of a real structure—there can be substantial cost saving in using a few standard member sizes, because doing so can greatly simplify the fabrication and construction processes.

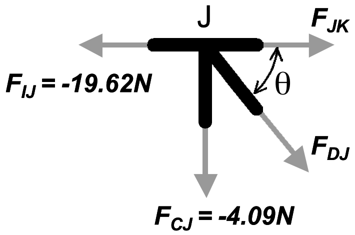
9) Can you apply the Method of Joints to analyze a truss? To analyze the truss with bottom-chord loading, follow exactly the same procedure as you did for the truss with top-chord loading. You will find that the calculation of reactions, and the analyses of Joints A, B, and I are *exactly* the same as for the top-chord loading. Starting at Joint C, however, the two solutions differ. The truss with bottom-chord loading has an 8.175N load at Joint C, as shown in the free body diagram. As a result, the x-direction equilibrium equation remains the same, but the y-direction equilibrium equation changes:

$$\sum F_x = 0$$
$$-F_{BC} - F_{CI}\cos\theta + F_{CD} = 0$$
$$-9.81 - (15.70)(0.6247) + F_{CD} = 0$$
$$F_{CD} = +19.62N = \underline{\underline{19.62N \text{ (tension)}}}$$



$$\sum F_y = 0$$
$$F_{CI}\sin\theta + F_{CJ} - 8.175 = 0$$
$$(15.70)(0.7809) + F_{CJ} - 8.175 = 0$$
$$F_{CJ} = -4.09N = \underline{\underline{4.09N \text{ (compression)}}}$$

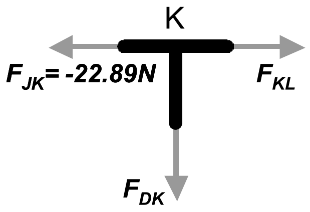
Next we analyze Joint J. In this case, the absence of an external load at the joint causes the y-direction equilibrium equation to change.



$$\sum F_y = 0$$
$$-F_{CJ} - F_{DJ}\sin\theta = 0$$
$$-(-4.09) - F_{DJ}(0.7809) = 0$$
$$F_{DJ}(0.7809) = 4.09 \quad F_{DJ} = +5.23N = \underline{\underline{5.23N \text{ (tension)}}}$$

$$\sum F_x = 0$$
$$-F_{IJ} + F_{DJ}\cos\theta + F_{JK} = 0$$
$$-(-19.62) + (5.23)(0.6247) + F_{JK} = 0$$
$$F_{JK} = -22.89N = \underline{\underline{22.89N \text{ (compression)}}}$$

Finally we analyze Joint K to determine F_{DK}.



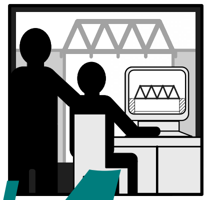
$$\sum F_y = 0$$
$$-F_{DK} = 0$$
$$F_{DK} = \underline{\underline{0N}}$$

10) Can members with zero force be removed? Members BI and FM could be removed from our model of the Grant Road Bridge with no adverse consequences; however, they could not be safely removed from the actual highway bridge. In the actual bridge, Members BI and FM serve an important structural function—they support the floor beams attached to the truss at Joints B and F. These floor beams help to support the bridge deck and transmit vehicular loads from the deck to the main trusses. Thus, in the actual bridge, Members BI and FM are in tension and are essential to the structure's load-carrying ability.

Member DK could not be safely removed from either the model or the actual bridge. Without Member DK, we would have just one continuous member from Joint J to Joint L—there would be no reason for a joint at K. This new member—let's call it Member JL—would be twice as long as Member JK or Member KL. As a result, Member JL would be much weaker in compression than JK or KL. (Recall from Learning Activity #2 that compression strength decreases substantially with increasing length.) To keep Member JL from buckling, a considerably larger tube would be required. Thus, even though Member DK has no internal force, it effectively strengthens the structure by dividing Member JL into two shorter, stronger compression members.

It is also worth noting that, in an actual bridge, the internal force in Member DK would not be zero. It would actually carry part of the self-weight of Members JK and KL, resulting in a small compressive internal force. In our analysis, the internal force in Member DK is zero only because we assumed the self-weight to be zero.

11) Can you analyze a more complex truss? The Gallery of Structural Analysis Results (Appendix B) provides the calculated internal member forces for a wide variety of common truss configurations.



#4

Learning Activity #4:

Design a Truss Bridge with a Computer

Overview of the Activity

In this learning activity, we will use a specially developed software package called the West Point Bridge Designer to design a truss bridge. We will use the software to create a structural model, then run a simulated load test to evaluate the design. The load test will help us to identify members with inadequate strength. We will strengthen these members by increasing their size. Finally we will optimize the design by minimizing its cost.

Why?

Design is the essence of engineering. To learn about engineering, you must learn about design. And the only way to fully appreciate the challenges and rewards of design is to *do it*. In this learning activity, we will design a bridge, with the aid of a computer and some special software. Then in Learning Activity #5, we will do the same sort of design by hand.

Why use the computer first? The West Point Bridge Designer software allows you to learn about the *design process,* without having to worry about the mathematical calculations that would normally be required in certain phases of the process. All of the quantities you measured or calculated manually in Learning Activities #2 and #3—tensile strength, compressive strength, loads, reactions, and member forces—are computed automatically by the West Point Bridge Designer. By doing all these computations for you, the software allows you to focus on the creative part of the design process—structural modeling—and to explore many more design alternatives than you could do otherwise. Modern computer-aided design software provides essentially the same benefits to practicing engineers. Thus when you use the West Point Bridge Designer you are also learning how engineers use the computer as a design tool.

Learning Objectives

As a result of this learning activity, you will be able to do the following:

- Describe the *problem-solving process*.
- Describe the *engineering design process*.
- Explain how the engineering design process is applied to the design of a highway bridge.
- Explain how engineers use computers to enhance the engineering design process.
- Design a truss bridge, using the West Point Bridge Designer software.

Key Terms

To successfully complete this learning activity, you must understand the following key terms and concepts from previous learning activities:

truss
member
top chord
bottom chord
diagonal
vertical
deck
floor beam
foundation
abutment
pier
joint
load
internal force
tension
compression
strength
failure
Owner
Design Professional
Constructor
plans & specifications
quality control
shop drawings

If you need to refresh your memory on any of these terms, see the Glossary in Appendix D.

Information

Problem-Solving

Engineering design is really just a specialized form of problem-solving. All engineers are problem-solvers, but you certainly don't need to be an engineer to solve problems effectively.

We are all confronted with problems every day. Sometimes they are large and complicated, like deciding which college to attend or figuring out how to program your VCR. Other times they are small and simple, like deciding which movie to watch tonight. But no matter what problem you're facing, you'll solve it more effectively and more efficiently if you use a methodical process to achieve a solution. Consider this example:

A teacher has just received a new set of reference books for his classroom but has no place to put them. He asks Rush, one of his students, to put up some bookshelves as a service project. Rush is in a hurry. He has basketball practice right after class and wants to go to a movie after practice. He decides to do the bookshelf project quickly and with as little effort as possible. He goes to the local hardware store and buys a four-foot length of wooden shelving, two metal shelf brackets, and four small woodscrews. He returns to the classroom, drills four holes in the wall, attaches the brackets to the wall with the four screws, and places the shelf on top of the bracket. Then he heads off to basketball practice. When the teacher returns to the classroom a few minutes later, he is pleasantly surprised to find that Rush has already finished the job. The teacher is immediately disappointed, however, when he notices that the shelf is too short. The reference books require six feet of shelf space, not four. But Rush never bothered to ask how large the shelf needed to be. Nonetheless, the teacher begins placing the books on the shelf. After only five or six books are in place, the screws suddenly pull out of the wall, and the shelf crashes to the floor.

The next day, the teacher asks Anne to do the job. Anne is also in a hurry, but she realizes that the best way to get a job done quickly is to do it right the first time. Before doing anything, she asks the teacher a series of questions. How many books are there? How much do they weigh? Where is the best place in the classroom for the books to be located? What color would best match the classroom décor? With the answers to these questions, she sits down and sketches three alternative solutions—a single six-foot shelf; two three-foot shelves, one above the other; and a simple three-shelf bookcase that would stand on the floor. She shows the sketches to the teacher and explains the advantages and disadvantages of each alternative. The single six-foot shelf will be least expensive but will use a large amount of wall space. The two-shelf arrangement will use less wall space, but the upper shelf will be harder to reach. The bookcase will be movable, but it will also be most expensive. The teacher decides on the two-shelf arrangement. Anne takes careful measurements and makes a final sketch of her design. She takes the sketch to the hardware store and gets the manager's recommendation on the number and types of brackets and screws required for the job. Then she brings all of the materials back to the classroom, carefully plans how she will assemble the shelves, and starts the job. She ensures that the mounting screws are driven into the wooden wall studs, so they don't pull out. Finally, when the job is complete, she carefully places all of the reference books on the shelves to ensure that they are strong enough. She adds a few extra books, just to be sure. Anne gets an A for the project. More important, she gets the satisfaction of knowing that she did the job well and, in the process, made a positive contribution to her class.

Obviously, Anne's solution to the bookshelf problem is a lot more effective than Rush's. Anne succeeds because she follows a methodical **problem-solving process**, consisting of the following seven steps:

1) **Identify the problem** – The teacher asks Anne to put up a new bookshelf.

2) **Define the problem** – Anne asks questions until she understands exactly what the teacher wants.

3) **Develop alternative solutions** – She sketches three different bookshelf configurations.

4) **Analyze and compare alternative solutions** – She determines the advantages and disadvantages of each configuration.

5) **Select the best alternative** – She presents her three alternatives to the teacher, and the teacher chooses the one that best meets his needs.

6) **Implement the solution** – Anne makes a final sketch, buys the necessary materials, obtains the advice of an expert, plans the job, and puts up the shelves.

7) **Evaluate the results** – Anne loads the bookshelves to ensure that they are strong enough.

Rush's problem-solving process can be summed up in three words: *Just do it!* This approach might work for selling sneakers but usually doesn't work very well for solving problems—especially large, complicated ones. Rush fails because he tries to solve a problem he doesn't really understand; he doesn't consider a range of alternative solutions; he doesn't acquire the necessary technical expertise to do the job; and he doesn't develop a plan before starting to work. In Rush's case, the consequences of the failure aren't too severe—a few holes in the wall and some wasted lumber. But what if Rush had been a structural engineer and the bookshelf had been a bridge? Then the consequences of his haphazard problem-solving process could have been tragic.

If Rush ever has to put up another set of bookshelves, he probably won't make the same mistakes as he did on his first attempt. But what if he needs to change the oil in his dad's car or build a deck or apply to college? Will he make the same sorts of careless errors every time he is confronted with a new and unfamiliar problem? He won't, if he learns that the problem-solving process can be applied to *any kind of problem* and can substantially improve his chances of achieving a successful solution on the *first* try.

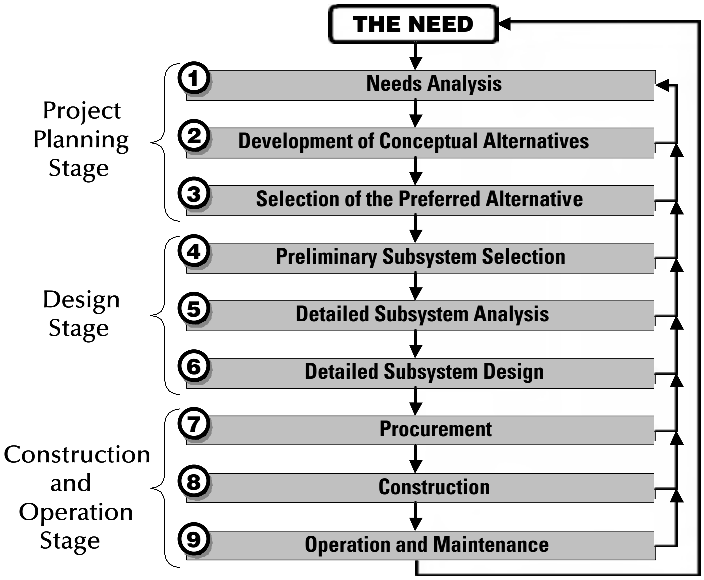
The Engineering Design Process

Engineering design is a specialized form of problem-solving—the application of math and science to create something that meets a human need. What are the "human needs" that engineers fulfill? The products of engineering are all around us. Every building, every bridge, every highway, every car, every airplane, every electrical appliance, and every computer you have ever used was designed by a team of engineers. The many forms of technology that form the fabric of modern civilization are all products of engineering.

Yet engineering design is much more than just math, science, and technology. It is also a creative process—one requiring the ability to conceive of problem solutions that no one has previously imagined. Theodore von Kármán, an eminent scientist and engineer, expressed this sentiment perfectly when he said, "The scientist describes what is; the engineer creates what never was." [1]

Some of the finest examples of creative engineering design are the world's great suspension bridges. With their graceful cables, monumental towers, and grand scale, these structures are both beautiful and awe-inspiring. Many have become symbols of the cities in which they reside. Can you even imagine San Francisco's Golden Gate without the Golden Gate Bridge; or New York City's East River without the Brooklyn Bridge? Yet there was, in fact, a time when each of these great bridges existed only in the mind of an engineer. Indeed, the real creativity in the design of the Brooklyn Bridge lies not in its physical appearance, but in the ability of its designer, John Roebling, to envision this spectacular structure, even though nothing like it had ever been attempted before.

How does a great bridge come to be? How is the engineer's dream translated into a steel and concrete structure carrying tens of thousands of vehicles per day? The answer lies in the *engineering design process*—a systematic approach that engineers use to create technological solutions to problems.



As it is depicted here, the engineering design process consists of nine distinct phases, grouped into three major stages—*project planning, design,* and *construction and operation*. This diagram is intended to illustrate the process in a general way. It applies equally well to the design of a skyscraper, a car, or a stereo system. But to see what actually happens in each phase, let's apply this process to a specific example—the planning, design, and construction of a major highway bridge.

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[1] Quoted in A. L. Mackay, Dictionary of Scientific Quotations (London: Adam Hilger), 1991.

Bridge Design

The Project Planning Stage

Let's imagine that a state Department of Transportation (DOT) decides to route a new highway across a major river. The DOT has initiated the engineering design process by identifying a **need** for a new highway bridge. To meet this need, the department hires an engineering firm to design the structure. The DOT is acting as the Owner for the project. The Design Professional is a senior structural engineer from the engineering firm—a woman named Anne, who first became interested in engineering when her high-school teacher asked her to put up a set of bookshelves in the classroom many years ago. Anne assembles the Design Team, whose members include structural, transportation, geotechnical, hydraulic, and environmental engineers from the firm, as well as other technical specialists hired as consultants for this particular project.

The project planning stage begins with a **needs analysis** (Phase 1 on the diagram on the previous page), in which the Design Team and the Owner work together to define the project requirements and constraints as fully as possible. The needs analysis includes a determination of:

- the proposed location of the bridge,
- the amount of vehicle traffic it can be expected to carry,
- the number of traffic lanes required,
- aesthetic requirements for the completed structure,
- the laws and regulations that will affect the design, and
- the project budget.

As part of the needs analysis, the Design Team also conducts a thorough site investigation, aimed at determining such factors as:

- the width and depth of the river,
- the potential for ice buildup in winter,
- the height and slopes of the river banks,
- the type of soil and the depth to good foundation material—like strong rock,
- the conditions at the proposed location of the **approaches**—the roadways leading up to the ends of the bridge,
- the amount of boat traffic in the river,
- the required width and height of the navigation channel in the river, and
- the potential environmental impact of the structure.

Once the **needs analysis** is complete, the **development of conceptual alternatives** (Phase 2) begins. During this phase, the Design Team develops a range of distinctly different alternative bridge types and configurations. These alternatives are analyzed and compared, with the ultimate objective of selecting the bridge type that is best suited to the needs of the Owner and the characteristics of the site.

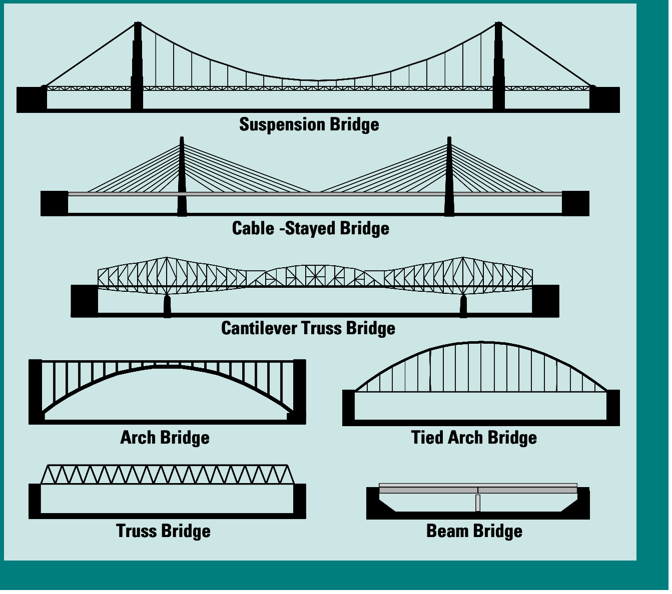
The development of conceptual alternatives is heavily influenced by the needs analysis. For example:

- If the river is very deep at the crossing point, it will be difficult and expensive to build piers in the water. In this case, a long-span structure with fewer piers is called for.
- Even if it is feasible to put piers in the river, their locations may be dictated by government regulations governing the width of the navigation channel for boats.
- Such constraints on the locations of the piers will often determine the required span length of the bridge. And the required span length often has a major influence on which bridge configuration is most economical. (See the sidebar at right.)
- The height and steepness of the riverbanks can also influence the selection of the bridge type. High, steep banks might favor the use of an arch bridge. Low-lying banks might require the construction of ramps on either end of the bridge, in order to raise the structure to an adequate height over the water. But the feasibility of building long ramps will depend on how much development—buildings, streets, factories, and so forth—exists along the shoreline at the proposed bridge location.
- Often the Owner will want the bridge to cost as little as possible. Even if minimizing cost is *not* an objective, however, keeping the construction cost within the project budget is essential. Thus the financial constraints identified during the needs analysis are critical to the development of conceptual alternatives.
- Sometimes the Owner places a high value on aesthetics and is willing to pay extra for a visually appealing structure. This might be the case, for example, if the bridge is to be located in a downtown waterfront area, where improving residents' quality of life and promoting tourism would be important design requirements.
- Sometimes aesthetic considerations will *require* the use of a certain bridge type, even if it is not as efficient or economical as other alternatives. For example, sometimes an older bridge becomes functionally inadequate (even if it is still structurally safe) because its roadway does not have enough lanes to handle modern traffic demands.   In such cases, a new bridge is often built immediately adjacent to the older one, so that each bridge carries several lanes of traffic. When a new bridge is built adjacent to an older one, the new one is often designed to look identical to the older one—even if a different configuration might be more efficient.

On an Actual Bridge Project

Which bridge type is most economical?

Different types of structures tend to be most economical for different span lengths. For very long spans of 3,000 feet or more, suspension bridges are generally used. For spans of 1,500 to 3,000 feet, cable-stayed bridges are becoming increasingly popular. Various types of arch bridges and cantilever trusses are often most economical for spans in the range of 1,000 to 2,000 feet, while beam bridges and truss bridges are most common for spans under 1,000 feet. For excellent descriptions of these bridge types and how they work, see David Macaulay's beautifully illustrated book Building Big (Houghton Mifflin, 2000).



These examples demonstrate that developing conceptual alternatives to meet a particular set of project requirements is not an easy task. During this phase, the Design Team doesn't have the time or resources to do detailed design work. Conceptual alternatives are only developed in enough detail (1) to ensure that the completed structure will meet the Owner's needs and (2) to perform a reasonably accurate cost estimate. In this phase, the designer's experience may be more important than any other factor in determining which project alternatives are feasible and which are not.

At the conclusion of this phase, the Design Team often produces a report called a **type study**. The type study describes the alternative bridge configurations that were considered by the designers. It explains the advantages and disadvantages of each and provides the Design Professional's recommended alternative. This recommendation is based on thorough consideration of many criteria, including:

- How well the design satisfies the Owner's requirements,
- construction cost,
- constructability,
- expected duration of the construction project,
- environmental impact,
- community impact,
- requirements for obtaining land and legal rights-of-way,
- traffic safety, and
- aesthetics.

The type study is presented to the Owner, who may accept the designer's recommendation or opt for one of the other alternative configurations. The Owner's **selection of a preferred alternative** (Phase 3) is the final phase of the Project Planning Stage. The product of this phase—the selected alternative—is called the **conceptual design**.

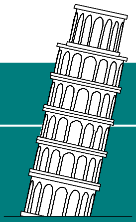
In our example project, the width of the river at the proposed bridge site is 2000 feet. The river is quite deep, so all piers and abutments must be placed on the shore. The riverbanks are low and flat. Based on these considerations and the Owner's requirements, the Design Team develops three alternative bridge configurations—a truss, a tied arch, and a cable-stayed bridge. Each will have a main span of 2000 feet, and each will carry six lanes of traffic. The Design Team's preliminary cost analysis indicates that the cable-stayed bridge will be least expensive. The Owner has also expressed a preference for the appearance of the cable-stayed structure. In considering the other selection criteria, the designers find that none of the three conceptual alternatives has a clear advantage. Thus the Design Professional's type study recommends the cable-stayed bridge, and the Owner accepts her recommendation.

The Design Stage

With the conceptual design complete, the Design Stage begins. A major highway bridge is an enormously complex system. The Design Team deals with this complexity by breaking the large system into a series of smaller, simpler components or **subsystems**. For our cable-stayed bridge, these subsystems might include the towers, the foundations, the deck, the main beams supporting the deck, the cables, the highway approaches, the electrical lighting system, the toll plaza, and the landscaping of the site—to name only a few. Responsibility for each subsystem is assigned to technical specialists with expertise in that area. For example, structural engineers will work on the towers, deck, beams, and other structural elements; geotechnical engineers will have responsibility for the foundations; transportation engineers will be assigned to handle the highway approaches and the geometric layout of the toll plaza.

The Design Stage begins with the **preliminary selection of subsystems** (Phase 4). Here the technical specialists decide what types of subsystems best meet the needs of the project. For example, the structural engineer decides whether to use steel or reinforced concrete for the towers. The geotechnical engineer decides whether to use **spread footings** or **piles** for the foundations. (See the sidebar at right.) At this point, the subsystem selections are only tentative. They might be changed later, depending on the results of the two succeeding phases.

The remainder of the Design Stage consists of the **analysis** and **design** of these subsystems (Phases 4 and 5). Analysis and design are *not* distinct, sequential phases. Rather, they form a cycle, which usually must be repeated numerous times before a satisfactory design solution is reached. Each repetition of the analysis-design cycle is called an **iteration**.

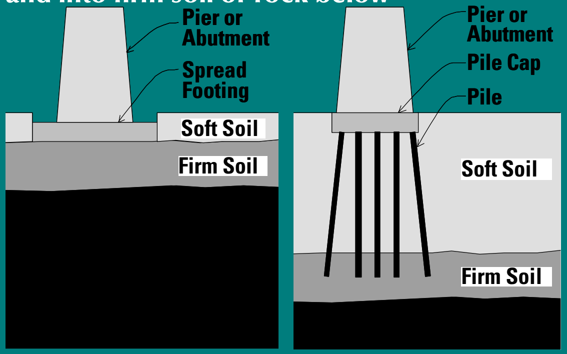


On an Actual Bridge Project

Spread footings or piles?

The purpose of a foundation is to distribute the weight of a structure (and all the loads acting on it) to the soil on which the structure rests. Without a well-designed foundation, the structure will settle excessively and might even collapse.

The type of foundation used for a particular structure depends primarily on the quality of the soil below the surface of the ground. If there is good firm soil or solid rock relatively close to the surface, the geotechnical engineer will typically choose a spread footing—a flat slab of concrete placed directly on the firm soil or rock. However, if the soil near the surface is soft, the bridge will probably need to be supported on piles—long steel or concrete shafts that are driven down through the soft soil layers and into firm soil or rock below



In our example, Anne, the structural engineer, has decided to use concrete towers, similar to the ones used on the Sunshine Skyway Bridge pictured below. On the first iteration, she models the structure, then she *analyzes* it to determine how much compressive force each tower will be required to carry. Based on these analysis results, she *designs* the tower. She determines the overall dimensions, the type of concrete, and the amount of steel reinforcement required to safely carry the load. Yet part of the load that the tower must carry is its own weight, which can't be calculated accurately until the exact dimensions of the tower have been determined. Since these dimensions weren't known when the initial structural analysis was conducted, the loads Anne used for this first iteration could only have been a rough estimate. Once she completes the first iteration, she determines the dimensions, and then re-calculates the loads with greater accuracy. But since the loads have now changed, a new structural analysis is necessary. The analysis results from this second iteration will be somewhat different from the previous results; thus, in the second design iteration, Anne might need to change the dimensions of the tower again. New dimensions will cause the loads to change again, and yet another analysis will be necessary. If it seems that these cycles of analysis and design will go on endlessly, don't worry! With each new iteration, the differences between the new loads and the previous ones get smaller and smaller. After several cycles, the differences become insignificant, and no further iterations are necessary.

Throughout the Design Stage, analysis-design cycles are conducted for each of the bridge subsystems. Because most of these subsystems are interconnected, changes in one often affect the design of others. For example, as the structural engineer's estimate of the weight of the towers changes, the geotechnical engineer will need to modify the design of the tower foundations accordingly. Even though the Design Team members are all working on different parts of the project, they must work closely together to ensure that their subsystem designs are well integrated.

As the upward arrows on the design process diagram suggest, iterations can occur at *any* phase of the design process, not just between Phases 5 and 6. During the development of conceptual alternatives, the Design Team may identify shortcomings in the needs analysis that must be rectified. When presented with the completed type study, the Owner may decide that none of the conceptual alternatives are acceptable and send the designers back to drawing board to develop new ideas. Sometimes, analysis reveals that a previously selected subsystem won't work or that an alternative subsystem will work more efficiently. In each of these situations, the engineering design process takes one or more steps *backward*, then resumes with a new design requirement, a new alternative, or a new subsystem. But these apparent setbacks should not be regarded as failures. They are inevitable and, indeed, they are desirable, because they often result in a better solution to the problem at hand.

As the Design Stage progresses, the Owner often uses formal **design reviews** to monitor the conduct of the design process. Through this management technique, the Owner ensures that the design is progressing on schedule and the project requirements are being addressed. To conduct a design review, the Owner requires the Design Professional to submit a complete draft of the design—drawings, reports, and draft specifications—at certain specified levels of completion. For example, the Owner might require design submittals when the work is 30%, 60% and 90% complete. The Owner—in this case, the Department of Transportation—reviews each submittal and provides written comments back to the Design Professional. Members of the Design Team are expected to address each of these comments in subsequent design submittals, to ensure that the DOT is 100% satisfied with the completed design. The Design Stage concludes with the delivery of a complete set of **plans and specifications** to the Owner.

The Construction and Operation Stage

During this final stage of the engineering design process, the Owner selects a Constructor, the Constructor builds the bridge, and the completed structure is placed into service. The **procurement** phase (Phase 7) begins with the selection of a Constructor. In the United States, this selection is normally done by competitive bidding—a process called **design-bid-build project delivery**. Design-bid-build project delivery generally works like this:

- The Owner advertises the project through public notices and ads in industry publications.
- Construction contractors obtain copies of the plans, specifications, and other bidding documents from the Owner.
- Construction contractors prepare and submit their bids. A **bid** is the contractor's estimate of how much it will cost to construct the bridge. In submitting a bid, the contractor is saying, "I can build this structure for x dollars." To ensure the fairness of the process, bids are always submitted in sealed envelopes.
- On a designated day, the Owner conducts a **bid opening**. In a public meeting, all bids are opened and read aloud. Each bid is checked to ensure that it includes all required information and that the bid amount is not unreasonably low. A bid that meets these conditions is called *responsive* and *responsible*.
- The Owner awards the construction contract to the lowest responsive, responsible bidder. This contractor becomes the Constructor for the project.

It is important to note that design-bid-build is not the *only* form of project delivery available to Owners. An alternative called **design-build project delivery** is becoming increasingly popular in the U.S. and is, in fact, the norm outside of the U.S. Design-build project delivery is described in Learning Activity #5.

Once the contract is awarded, the Owner issues a **notice to proceed**—an official authorization to start work on the project. But even after the notice is issued, the Constructor has a lot of things to do before construction can actually begin. These include hiring sub-contractors, preparing a construction plan and a project schedule, setting up the construction site, establishing jobsite safety and quality control procedures, ordering materials, and preparing shop drawings.

Once **construction** (Phase 8) actually gets underway, the contractor builds the bridge—much as you did in Learning Activity #1—along with all associated highways and facilities. The overall objective of the construction phase is to complete the project on time, within the Owner's budget, and to the level of quality required by the plans and specifications. The Design Team typically has only minimal involvement in this phase. The design engineers usually review the Constructor's shop drawings and are often called upon to respond to the Constructor's requests for information, as unforeseen circumstances are encountered on the construction site.

Finally, after many months of design and construction, it is Opening Day for the new bridge. A brass band plays, and the Commissioner of Transportation makes an inspirational speech. The Governor cuts the ribbon, and a wave of traffic surges forward across the span. The crowd cheers. The bridge is complete, and every member of the Project Team takes a moment to revel in their accomplishment. But only a moment! Back at the office, a new project—a new challenge and a new opportunity to serve society—is waiting.

Ideally, the nine-phase engineering design process concludes with many years of conscientious **operation and maintenance** (Phase 9) by the Department of Transportation. Trained technicians carefully inspect the entire structure every two or three years, looking for signs of deterioration and identifying needed repairs. Maintenance crews regularly clean the storm drains and expansion joints, repaint weathered steel, and repair cracked asphalt. As a result of this modest investment in routine maintenance and repair, the bridge might serve its purpose safely and effectively for a century or more. Ultimately, of course, the structure will become obsolete, or the cost of maintaining it will exceed the cost of building a new bridge. At this point, there is a **need** for a replacement bridge, and the design process begins all over again with Phase 1.

Unfortunately, bridges are often not effectively maintained. Sometimes bridges are placed into service then simply forgotten. More often, state and local governments have well-designed inspection and maintenance programs in place but don't receive enough funding to perform needed repairs. But trying to save money by cutting back on bridge maintenance is "penny wise and pound foolish." Bridges that are not maintained deteriorate rapidly, resulting in the need for expensive rehabilitation or replacement projects. Such projects ultimately cost far more than routine maintenance programs, especially considering the hidden costs associated with closing down major highways for months at a time, while bridges are rehabilitated or replaced.

Computer-Aided Design

Few things have revolutionized the engineering design process more that the widespread availability of computers and various forms of computer-aided design (CAD) software. CAD software provides designers with incredibly powerful tools for drawing, modeling, analyzing, and evaluating engineered systems. Specifically, modern computer-aided design programs enhance the design of bridges in the following ways:

- Using CAD software, an engineer can create accurate two-dimensional and three-dimensional drawings of a bridge. As the design evolves, CAD drawings can be easily updated. Hand-drawn plans must be redone from scratch every time the design changes.
- CAD provides the capability to accurately visualize a completed structure, long before the first shovel of soil is turned or the first batch of concrete is poured. Thus CAD can be used effectively in the development of conceptual alternatives (Phase 2) and in the presentation of alternatives to the Owner (Phase 3).
- Some CAD software provides the capability to create a highly accurate structural model, then analyze the structure to determine its internal forces, then automatically select steel or concrete members strong enough to carry these forces. As such, CAD can substantially increase the efficiency of Phases 5 and 6 in the design process. By expediting the analysis-design cycle, CAD makes it possible for the engineer to explore a wider variety of design alternatives and thus achieve greater efficiency and lower cost.

- Because computer-generated designs can be saved in electronic form, CAD gives teams of engineers unprecedented capability to share design information via computer networks and the Internet. This capability allows the members of the Design Team to coordinate their efforts across the office or across the globe.

With their ease of use, realistic graphics, and incredible computational capability, CAD programs sometimes make us think that human engineers have become obsolete. But nothing could be farther from the truth! Like the electronic calculator and the slide rule before it, like the T-square and the draftsman's scale, CAD software is only a tool. Like any tool, it can improve human efficiency, but it can never substitute for human creativity and good judgment. And like any tool, CAD can be misused. Some aspects of structural modeling and design simply cannot be automated. They require in-depth understanding of engineering principles, an appreciation for constructability, and good old-fashioned common sense. Designers who use CAD software as a "black box"—a problem-solving tool whose inner workings they do not fully understand—are toying with disaster, because they have no basis for assessing whether or not the computer's answers make sense. More than one experienced engineer has said, "Never use the computer for a task that you can't already do by hand."

The West Point Bridge Designer

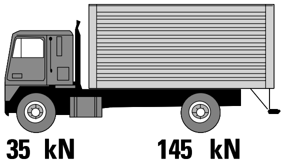
The **West Point Bridge Designer** is a computer-aided design program developed to introduce you to the engineering design process and to demonstrate how engineers use the computer as a problem-solving tool. The software will enable you to create a structural model of a truss, then run a simulated load test to determine if the structure can safely carry its applied loads—its own self-weight and the weight of a standard highway truck loading. In effect, the West Point Bridge Designer does automatically what we did manually in Learning Activity #3. The only difference is that the projects provided in the West Point Bridge Designer are full-sized bridges made of steel, while the project provided in Learning Activity #3 is a model bridge made of cardboard.

The West Point Bridge Designer attempts to mimic the "look and feel" of an industry-standard CAD program, but with a simpler user interface that allows fewer opportunities for errors. Ease of use is achieved primarily by using seven built-in design projects. Once you have selected a project, the software automatically establishes the scale of the drawing, the bridge supports, and the drawing grid. You can begin creating your structural model immediately, using a few simple drawing and editing tools. With a standard CAD program, you would have to define the scale, supports, and grid by yourself, and you would be confronted by an overwhelming array of drawing and editing tools. The very characteristics that make standard CAD software so powerful and flexible also make it seem complex and intimidating to new users.

The Bridge Designer helps prevent errors by allowing you to place joints only at pre-defined grid points, by requiring you to draw members from joint to joint, by allowing member properties to be selected from drop-down lists, and by providing truss templates to guide your creation of a stable structural model.

The most important feature of the West Point Bridge Designer is its simulated load test. Once you have created a complete, stable structural model, you can run the load test with the click of a single button. When you initiate the load test, the software will perform the following actions behind the scenes:

- Create supports at the appropriate locations in your structural model.
- Calculate the weight of all members, and apply these forces to the structure as loads.
- Calculate the weight of the concrete bridge deck, asphalt road surface, and floor beams, then apply the corresponding loads to the structure.
- Apply a standard AASHTO H20-44 truck loading to the structure at multiple positions, representing the movement of the truck across the bridge. AASHTO is the American Association of State Highway and Transportation Officials, an organization that develops design codes and specifications for highway bridges in the United States. The H20-44 truck is a vehicle with two axles spaced 4.0 meters apart. The front axle weighs 35 kilonewtons (kN), and the rear axle weighs 145 kilonewtons (kN). These axle weights are further increased by a *dynamic load allowance* of 33%, to account for the effects of the moving load. This means that a moving truck causes about 33% higher internal forces in the truss members than a stationary truck would cause.



- Check the structural model for stability. If the structural model is unstable, the West Point Bridge Designer will stop the load test, inform you of the problem, and provide some suggestions for fixing it.
- Perform a structural analysis, considering the combined effects of the bridge self-weight and truck loading. For each truck position, the software calculates the displacement of each joint and the member force for each member in the structural model.
- For each member, compare the calculated member forces for all truck positions, and determine the absolute maximum tension force and the absolute maximum compression force. These are the critical forces that determine whether a given member is safe or unsafe.
- Calculate the tensile strength and compressive strength of each member, using standard AASHTO strength equations.
- For each member, compare the absolute maximum tension force with the tensile strength, and compare the absolute maximum compression force with the compressive strength. If the force exceeds the strength in either case, the member is unsafe; if not, the member is safe.
- Display the load test animation.

The results of the load test are provided in a variety of different forms, both numerical and graphical. As you will see in this learning activity, the load test results will help you to strengthen any unsafe members and to optimize your design to minimize its cost.

Q1

What portion of the engineering design process does WPBD address?

When you use the West Point Bridge Designer to design a truss, what phases of the nine-phase engineering design process are you doing?

The Learning Activity

The Problem

The Need

With the success of the Grant Road Bridge project, the Town Engineer of Hauptville has decided to replace several other obsolete bridges in the town. The first of these is an old concrete beam bridge that carries Lee Road over Union Creek, a short distance from the Grant Road Bridge. The old bridge will be demolished and replaced with a more modern structure. The Town Engineer is very satisfied with the work done by Thayer Associates on the Grant Road Bridge, so he hires this firm to design the new Lee Road Bridge as well.

Design Requirements

The Hauptville Town Engineer works closely with civil engineers from Thayer Associates to develop the following design requirements for the bridge:

- The new bridge will be constructed on the abutments from the old structure. These existing supports are 24 meters apart.
- The bridge must carry two lanes of traffic.
- The bridge must meet the structural safety requirements of the AASHTO bridge design code.
- For consistency with the nearby Grant Road Bridge, the new structure should be a truss. It is not necessary for it to be a Pratt Through Truss, however.
- The bridge will be made of steel.

Because of the limited project budget, it is essential that the new bridge cost as little as possible.

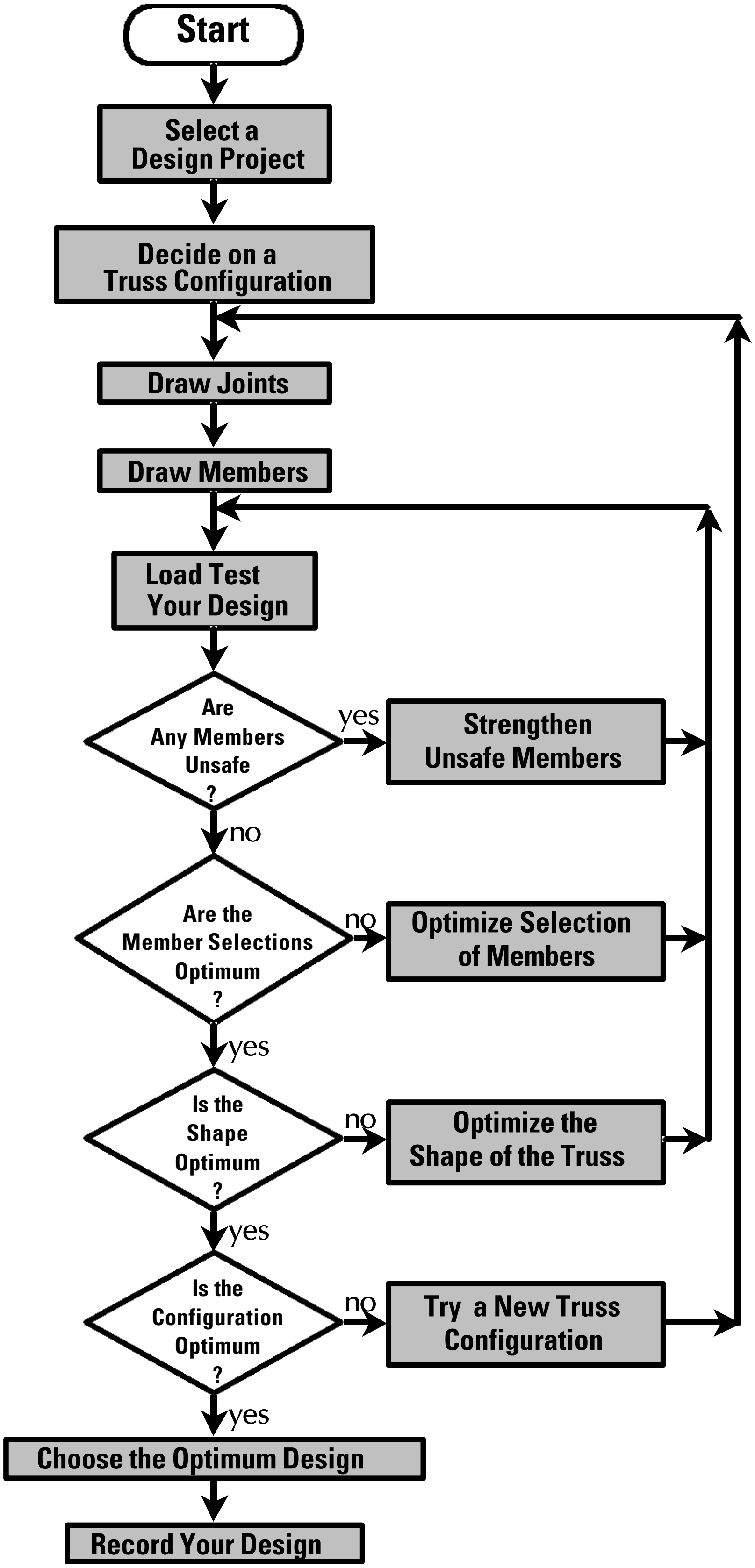
Your Job

You are a structural engineer employed by Thayer Associates. You are assigned to design the main trusses for the Lee Road Bridge. Your responsibility is to design trusses that are safe, that satisfy all of the other design requirements, and that cost as little as possible.

The Solution

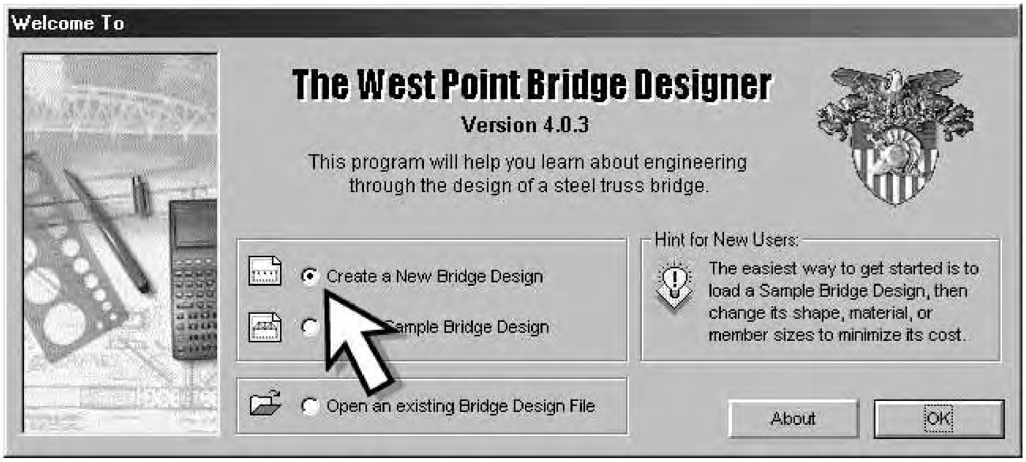
The Plan

Our plan to design the Lee Road Bridge is depicted in the flowchart on the following page. Each rectangle in the chart represents one step in the design process. The arrows indicate the order in which the steps should be performed. The diamonds represent decision points. When you reach one of these, your next step will depend on the answer to the question.


Start
Select a Design Project
Decide on a Truss Configuration
Draw Joints
Draw Members
Load Test Your Design
Are Any Members Unsafe ?
yes
Strengthen Unsafe Members
no
Are the Member Selections Optimum ?
no
Optimize Selection of Members
yes
Is the Shape Optimum ?
no
Optimize the Shape of the Truss
yes
Is the Configuration Optimum ?
no
Try a New Truss Configuration
yes
Choose the Optimum Design
Record Your Design

Start the West Point Bridge Designer

To run the West Point Bridge Designer, click the Windows **Start** button, then select **Programs**, then **West Point Bridge Designer**, and finally **WPBD4.exe**. Read the Tip of the Day, then click the **OK** button to close it. At this point, you'll see the Welcome screen. Ensure that the **Create a New Bridge Design** option is selected, and click **OK**.



Welcome To

The West Point Bridge Designer

Version 4.0.3

This program will help you learn about engineering through the design of a steel truss bridge.

● Create a New Bridge Design

○ ample Bridge Design

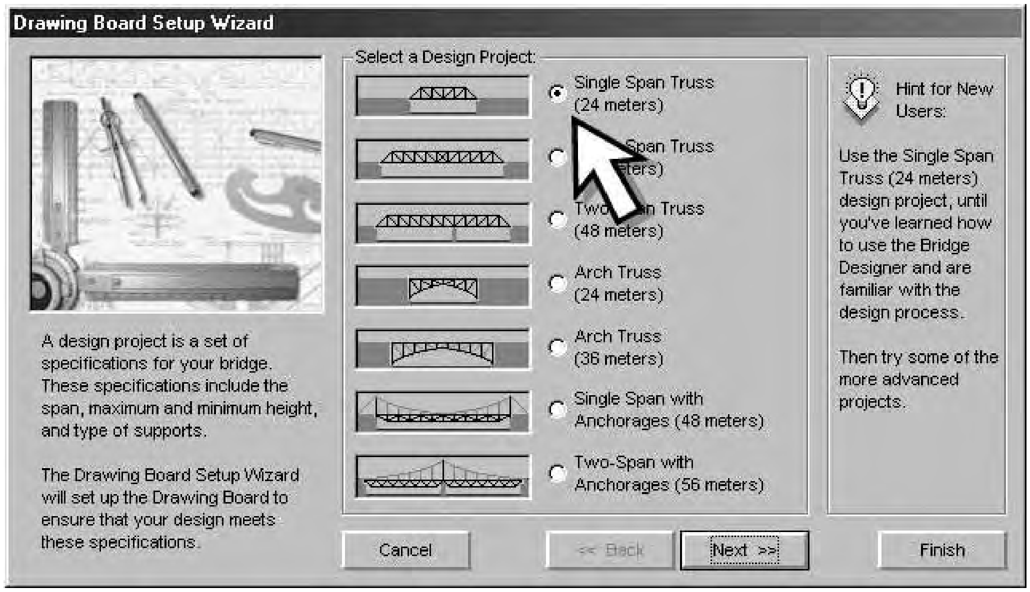
○ Open an existing Bridge Design File

Hint for New Users:

The easiest way to get started is to load a Sample Bridge Design, then change its shape, material, or member sizes to minimize its cost.

Select a Design Project

The Drawing Board Setup Wizard is now displayed, and you are prompted to select a design project. The Lee Road Bridge has a span of 24 meters, so select the **Single Span Truss (24 meters)** option, and click the **Next** button.



Drawing Board Setup Wizard

Select a Design Project:

● Single Span Truss (24 meters)

○ Span Truss eters)

○ Two n Truss (48 meters)

○ Arch Truss (24 meters)

○ Arch Truss (36 meters)

○ Single Span with Anchorages (48 meters)

○ Two-Span with Anchorages (56 meters)

A design project is a set of specifications for your bridge. These specifications include the span, maximum and minimum height, and type of supports.

The Drawing Board Setup Wizard will set up the Drawing Board to ensure that your design meets these specifications.

Hint for New Users:

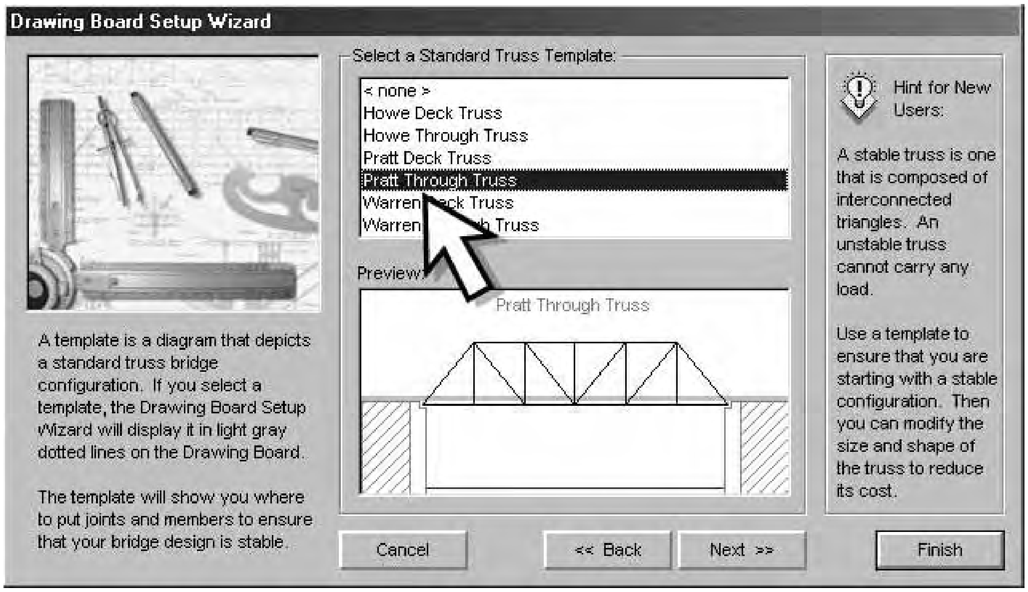
Use the Single Span Truss (24 meters) design project, until you've learned how to use the Bridge Designer and are familiar with the design process.

Then try some of the more advanced projects.

Decide on a Truss Configuration

The West Point Bridge Designer will allow you to create virtually any truss configuration (including statically indeterminate ones), as long as the resulting structural model is stable. But determining whether or not a configuration is stable can sometimes be tricky. Until you've gained some experience, it's best to start with a simple, standard configuration—like the Pratt, Howe, or Warren truss. The West Point Bridge Designer provides *templates* for a variety of different standard truss configurations. If you use a template, the locations of all joints and members for the standard truss you selected will be displayed with light gray lines on the Drawing Board.

The Drawing Board Setup Wizard now prompts you to select a template. Choose the **Pratt Through Truss**, and click **Next**.


Drawing Board Setup Wizard

A template is a diagram that depicts a standard truss bridge configuration. If you select a template, the Drawing Board Setup Wizard will display it in light gray dotted lines on the Drawing Board.

The template will show you where to put joints and members to ensure that your bridge design is stable.

Select a Standard Truss Template:

- < none >
- Howe Deck Truss
- Howe Through Truss
- Pratt Deck Truss
- Pratt Through Truss
- Warren Deck Truss
- Warren Through Truss

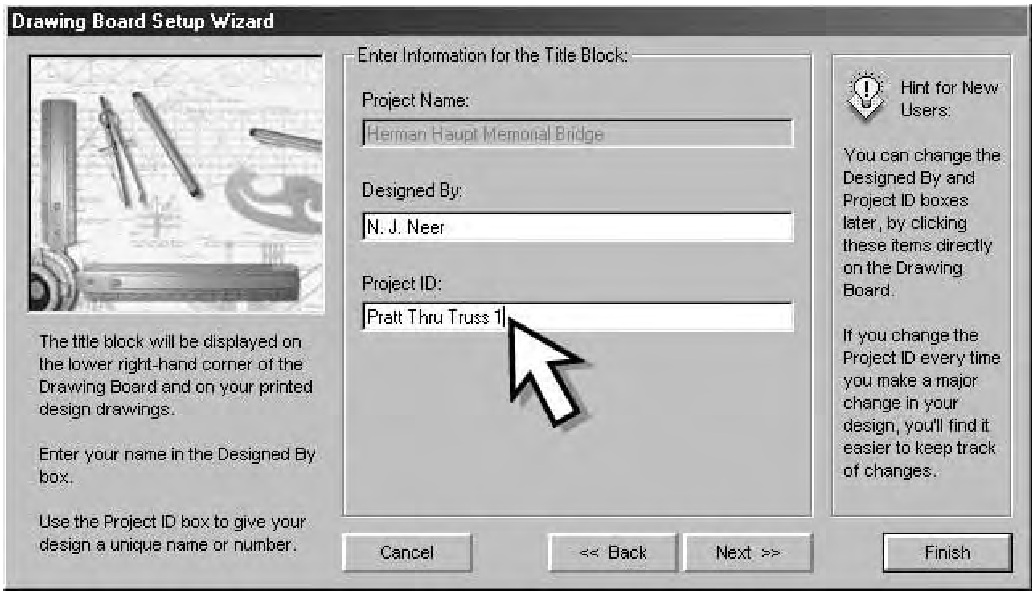
Preview:

Hint for New Users:

A stable truss is one that is composed of interconnected triangles. An unstable truss cannot carry any load.

Use a template to ensure that you are starting with a stable configuration. Then you can modify the size and shape of the truss to reduce its cost.

Now enter your name in the **Designed By** box and, if you like, add a **Project ID**. The Project ID is a name or number you assign to your design for future reference. If you are planning to try a variety of different design alternatives, the Project ID is a good way to keep track of them. Both your name and the Project ID will appear in the Title Block on the Drawing Board and are also included on the hard-copy printout of your design.


Drawing Board Setup Wizard

The title block will be displayed on the lower right-hand corner of the Drawing Board and on your printed design drawings.

Enter your name in the Designed By box.

Use the Project ID box to give your design a unique name or number.

Enter Information for the Title Block:

Project Name: Herman Haupt Memorial Bridge

Designed By: N. J. Neer

Project ID: Pratt Thru Truss 1

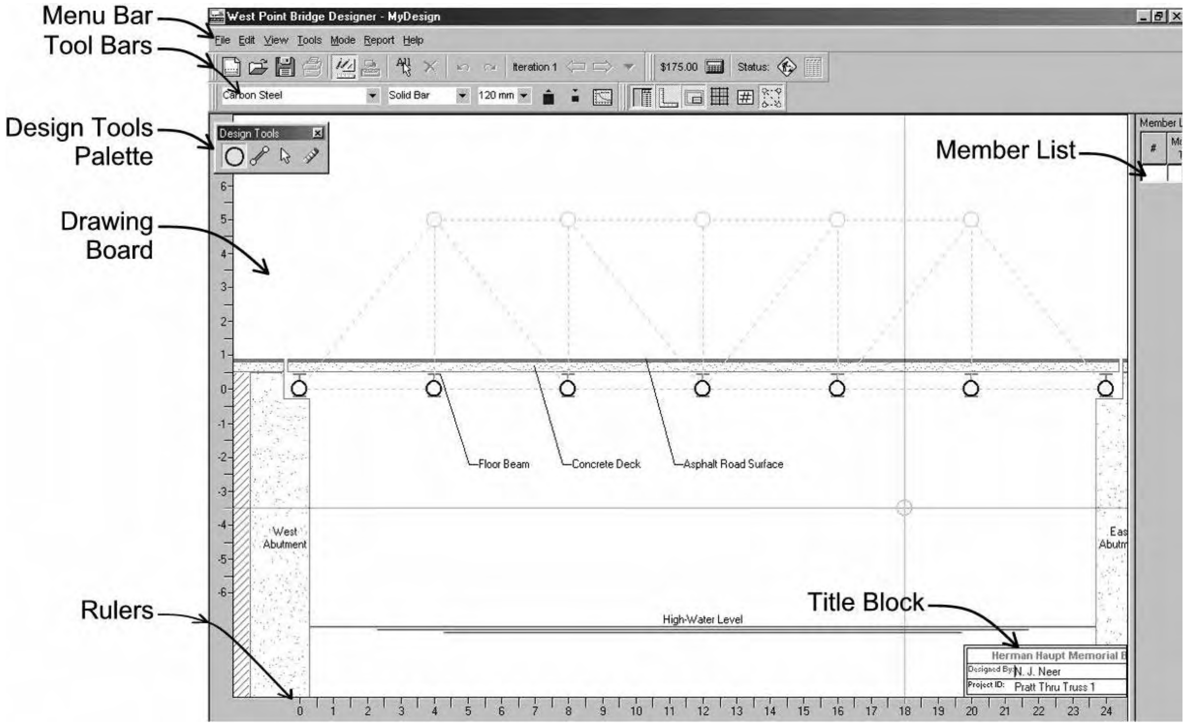
Hint for New Users:

You can change the Designed By and Project ID boxes later, by clicking these items directly on the Drawing Board.

If you change the Project ID every time you make a major change in your design, you'll find it easier to keep track of changes.

Click **Next**, then **Finish**. The Drawing Board Setup Wizard disappears, revealing the Bridge Design Window—the graphical environment in which you will create, test, and optimize your design. The Bridge Design Window includes the following major elements:

- Menu bar and Toolbars – Commands for creating, modifying, testing, recording, and reporting a bridge design.
- Drawing Board – The portion of the screen on which you will draw joints and members to create a structural model.
- Design Tools palette – Special toolbar containing tools that are used to create and modify a structural model on the Drawing Board.
- Rulers – Guides that show the vertical and horizontal dimensions of the structural model.
- Title Block – Portion of the Drawing Board displaying the designer's name and Project ID.
- Member List – List of all members in the current structural model, normally hidden on the right-hand side of the Drawing Board. (The Member List can be displayed by dragging it to the left with your mouse).



The Drawing Board already shows those portions of the bridge that you *will not* be designing—the abutments, the floor beams, the concrete deck, and the road surface. Note also that the first seven joints in the structural model have been created automatically. They are located at the points where the floor beams are attached to the main truss. Since the positions of the floor beams are fixed, these seven joints cannot be moved or deleted. The Drawing Board also displays the configuration of a Pratt Through Truss, marked with light gray lines. This is the template you selected earlier.

Note that the Design Tools palette has four available tools—the **Joint Tool**, the **Member Tool**, the **Select Tool**, and the **Eraser Tool**. The **Joint Tool** should already be selected.

Draw Joints

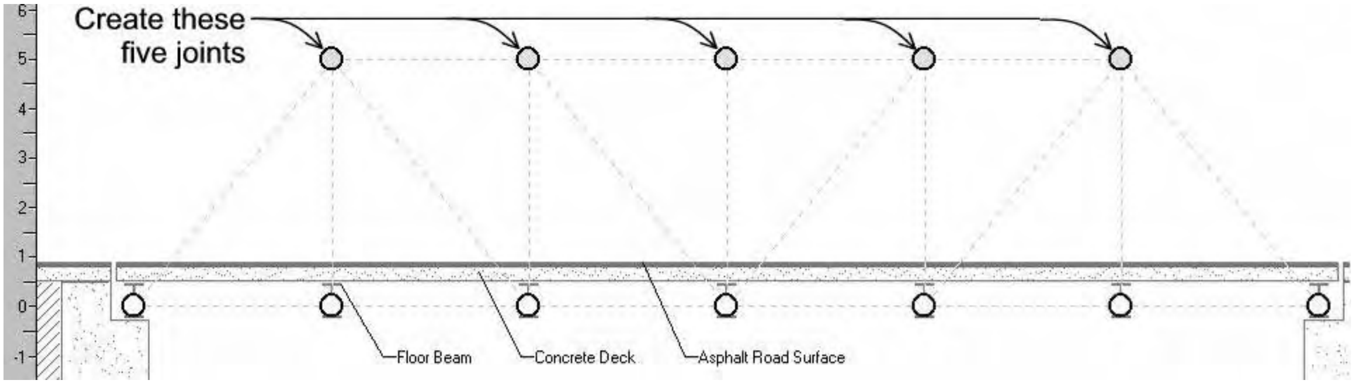
With the Drawing Board setup completed, we can begin creating our structural model by drawing **joints** and **members**. We'll begin with the joints. The Drawing Board Setup Wizard has already created the first seven joints automatically. You must create five more—the five joints that lie along the top chord of the truss. The light gray circles in the template show where to put them.

To draw a joint, use your mouse to position the red cross-hairs at the desired location, then click the left mouse button. Repeat this process for all five joints.

TIP

The red cross-hairs allow you to position joints accurately, using the Rulers along the bottom and left side of the Drawing Board.

When you're done, the structural model should look like this:

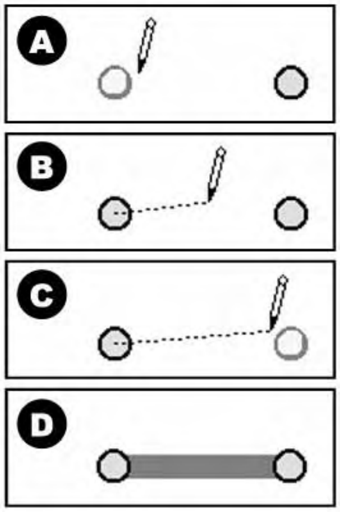


TIP

If you make a mistake while creating or modifying your structural model, just click the Undo button.

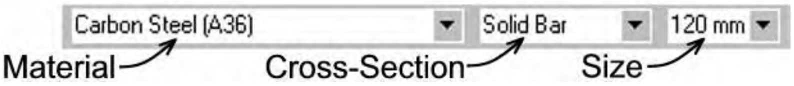
Draw Members

To draw a member, first select the **Member Tool** on the Design Tools palette. The mouse pointer will change to a pencil.



Members are always drawn from joint to joint. We'll begin by drawing the bottom-chord member on the left end of the truss. Move the mouse pointer close to the first joint—the one immediately above the West Abutment. You'll know when you are close enough, because the joint will "light up"—its color will brighten, and it will take on a three-dimensional appearance **(A)**. Press the left mouse button and hold it down, then drag the mouse pointer toward the second joint **(B)**. As you drag the mouse, you'll see a "rubber band"—a dotted line connecting the first joint and the mouse pointer. When the mouse pointer gets close to the second joint, this joint will also light up, to let you know that you are close enough to complete the member **(C)**. As soon as the second joint lights up, you can release the left mouse button. A solid member will be drawn between the two joints **(D)**.

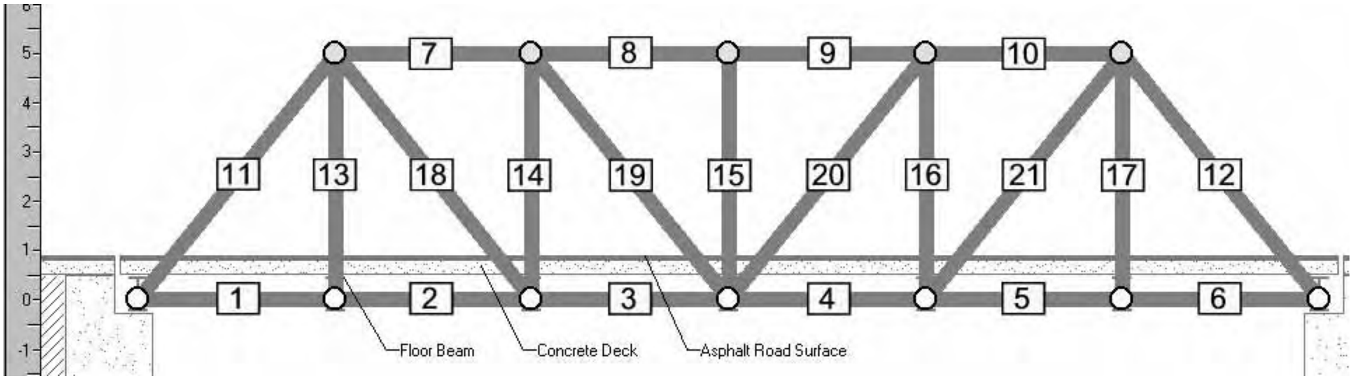
The member you just created is made of carbon steel; it has a solid square cross-section; and its dimensions are 120mm x 120mm. We know this because these three properties are currently displayed in the three Member Properties Lists on the toolbar. These particular properties—carbon steel, solid bar, and 120 mm—are default values. We'll change them later, as we optimize the design.


Carbon Steel (A36)
Solid Bar
120 mm

Using the procedure described above and following the dotted lines on the template, draw all 21 members in the structural model. In general, you can create members in any order you like; however, for this learning activity, you should create them in the following sequence (also shown in the picture below):

- Members 1 through 6 – bottom chord, from left to right
- Members 7 through 10 – top chord from left to right
- Members 11 and 12 – left and right end posts
- Members 13 through 17 – verticals from left to right
- Members 18 through 21 – diagonals from left to right

As we test and optimize the design, we will refer to individual members by their member numbers. If you create your members in a different sequence than the one shown here, your member numbers will be different, and you won't be able to follow along.



TIP

To display the member numbers on your structural model, click the Member Numbers button.

Load Test Your Design

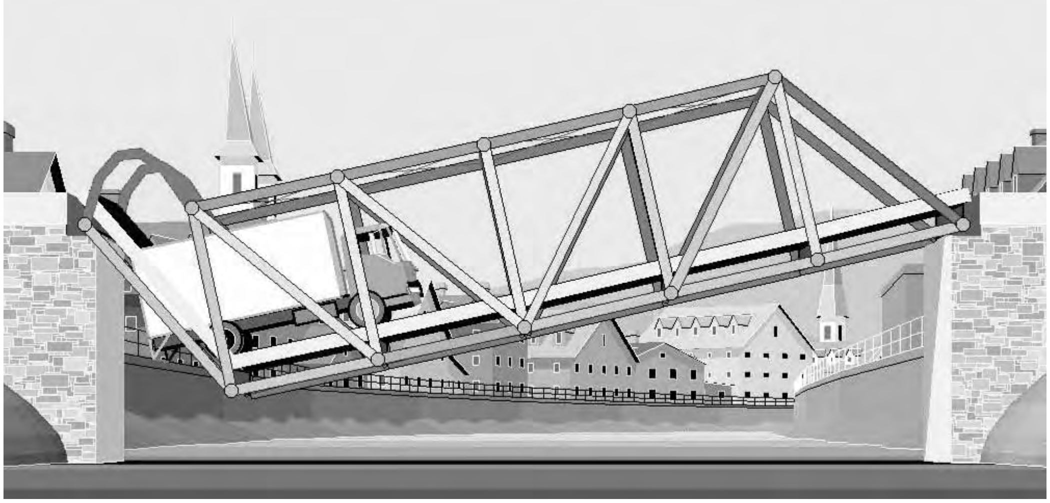
The structural model is now complete. Of course, we won't know if the truss is strong enough to carry its specified loads until we perform a structural analysis and evaluate the safety of each member. We could do this by hand, just as we did in Learning Activity #3. But that would take a lot of time, leaving us with much less time to optimize our design and explore alternative design concepts. So, instead, let's use the computer to perform our structural analysis and evaluation for us.

To evaluate your design, click the **Load Test Mode button** on the main toolbar. After a few seconds, the Drawing Board disappears, and in its place, a full-color three-dimensional rendering of your bridge (with the town of Hauptville in the background) appears. Your bridge is first subjected to its own weight and the weight of the concrete deck and asphalt road surface. Then a standard AASHTO H20-44 truck crosses the bridge to validate the structure's ability to carry a vehicular load safely. If the truck crosses the bridge, your design is successful—it is strong enough to carry the specified loads safely.

TIP

As the Load Test runs, members in tension turn blue, and members in compression turn red. The intensity of the color depends on the force-to-strength ratio. If the color is pale blue or red, it means that the internal force in that member is much less than its strength. If the color is bright blue or red, it means that the internal force is nearly equal to the strength (adjusted by an appropriate factor of safety).

But our design is definitely not successful. It does carry its own weight, but as soon as the truck moves onto the bridge, the structure collapses.



There's an old expression that engineers use when an idea doesn't work out exactly as intended: "Back to the drawing board!" In many ways, this expression captures the optimistic spirit of engineering. If one idea doesn't work, then the engineer keeps trying until he or she finds one that does. No design is ever accomplished on the first attempt. The design process is inherently iterative, and most successful designs are the result of many trials and many alternative design concepts. And so we, too, must go back to the drawing board—to find out why the bridge collapsed and to correct the problem.

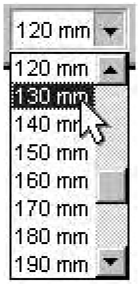
Strengthen Unsafe Members

Click the **Drawing Board Mode button** to go back to the Drawing Board. When the Drawing Board display is restored, you will notice that the two end posts, Members 11 and 12, are both highlighted in bright red. The red highlighting indicates that these two members are unsafe in compression. The internal force in each member is greater than the corresponding compressive strength. Both of these members will need to be strengthened before the design can be judged successful.

There are several possible ways to strengthen a member. We can use a stronger material; we can use a larger member size; or, if the member is in compression, we can make it shorter. Of these three, it is generally best to begin by increasing the member size. We will experiment with changing materials and member length later in the design process.

Before you can make any changes to member properties, you must select the member you want to change. Note that the **Select Tool** has automatically been chosen from the Design Tools palette. Move the mouse pointer over Member 11, and click the left button. The member will turn light blue, indicating that it has been selected.

Now click the drop-down button next to the Member Size list on the toolbar. From the list of available member sizes, choose the next larger size—**130mm**. The size of Member 11 is now increased to a 130mm by 130mm square bar. To de-select the selected member, just click anywhere on the Drawing Board other than on a joint or member.



Repeat this process for Member 12.

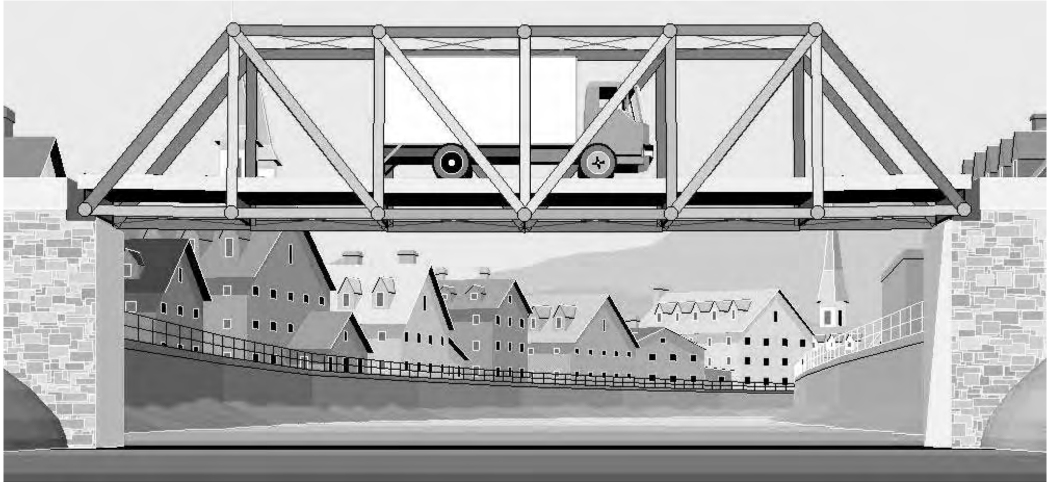
The two unsafe members have now been strengthened. But are they strong enough to carry load safely? The only way to tell is to run the load test again. Click the **Load Test Mode button**, and observe the load test animation. The bridge still collapses! Upon returning to the Drawing Board this time, however, note that only Member 11 is highlighted in red. Increasing the member size to 130mm was good enough for Member 12, but not for Member 11.

Select Member 11, and increase its size to 140mm.

TIP

To increase the size of a member to the next larger available size, select the member, then click the Increase Member Size button. It's faster than using the drop-down list.

Now run he load test once more. This time, the truck crosses the bridge safely.



Optimizing the Design

Congratulations! You now have a successful design for the Lee Road Bridge. The design is successful because it spans the required distance and carries all of the specified loads safely. But don't relax yet! Recall that our design objective is to minimize the cost of the bridge. We don't yet know if we have met this objective.

The cost of one main truss is displayed on the main toolbar and automatically updated as you create and modify your structural model. The current cost is **$5672.55**. Our challenge is to determine if this cost can be reduced, without compromising the safety of the structure. This process is called **optimizing the design**.

We will optimize our design for the Lee Road Bridge in three distinct phases. First, we will optimize the selection of member properties—material, cross-section, and size—for our Pratt Through Truss. Then we will optimize the shape of the truss—by exploring a range of different shapes and selecting the one that costs the least. Finally we will optimize the configuration itself—by exploring a range of alternative configurations and picking the least expensive one.

There are many millions of possible alternative designs for the Lee Road Bridge. We haven't the space to explore even a tiny fraction of them in this book. Rather we will look at a few representative examples, as a means of illustrating the *optimization process*. Once you have mastered this process, your ability to consider alternatives—and ultimately to develop a truly optimal design—will be limited only by your own time and motivation.

Optimize Member Properties

To optimize the member properties for our design, we must select a material, cross-section, and size for each member, such that the total cost of the truss is minimized.

While the load test animation is still running, take a careful look at the colors of the members. Recall that the *intensity of color* is proportional to the *internal force-to-strength ratio*. If the force-to-strength ratio of a member is greater than one, the member is unsafe. An optimally designed member should have a force-to-strength ratio slightly less than one. Such a member would be strong enough to safely carry its internal force, yet would use no more material than absolutely necessary. In the load test animation, such a member would be bright blue or bright red, depending on whether it is in tension or compression.

The load test animation shows us that few, if any, of the members in our current design are optimally designed. The pale blue bottom chords and diagonals are clearly much stronger than they really need to be. The light pink verticals are similarly over-designed. The top chords and end posts are bright red, but they aren't optimally designed either! Remember the important observation we made in Learning Activity #2—that hollow tubes are considerably more efficient than solid bars for carrying compression. The top chords and end posts in our current structural model are solid bars, so we should be able to reduce their cost substantially by changing them to hollow tubes.

It's time to go "back to the drawing board" once more—this time to optimize our design.

Let's begin the optimization process by changing the two end posts and the four top chord members from bars to tubes. To save time, let's make this change on all six members simultaneously.

TIP

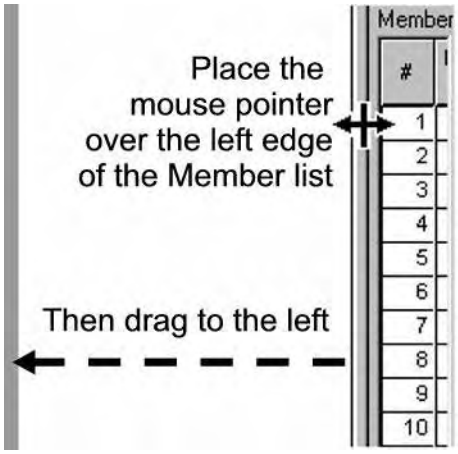
You can change the properties of two or more members simultaneously by using multiple selection. Hold down the Ctrl key on your keyboard, then click on each member you want to select. Any changes you make to member properties will affect all of the selected members.

Use multiple selection to choose Members 7, 8, 9, 10, 11, and 12 simultaneously. Now click the drop-down button next to the **Cross-Section list** on the toolbar, and choose **Hollow Tube**. Members 7 through 12 have now been changed from solid bars to hollow tubes, though their sizes remain unchanged.

Anytime we make a change to the design, we must validate it by running the load test. In this case, you will find that all of the members we just changed to tubes are now unsafe. We must work through a methodical process to determine *the smallest possible member size that passes the load test* for each of these six members. Here's how to do it:

- Select Members 7 through 12, then click the **Increase Member Size button** one time. This will change each member to the next larger available size. Members 7, 8, 9, and 10 will all be 130mm; Member 11 will be 150mm; and Member 12 will be 140mm.
- Run the load test. Again all six members fail, so all six need to be strengthened again.
- You will need to repeat this process—increase all six members to the next available size, and run the load test—four more times before any of the members are strong enough to pass the load test. On this design iteration, the size of Members 7, 8, 9, and 10 is 170mm. Member 11 is 190mm, and Member 12 is 180mm. On this iteration, Members 8 and 9 still fail the load test, but the other four pass.
- On the next iteration, increase only Members 8 and 9 to 180mm, and run the load test once more. At this point all members are found to be safe, and the design is again successful.

The cost of the truss is now **$5092.30**—a reduction of about 10% from our original cost. Clearly changing compression members from solid bars to hollow tubes does produce a cost saving. Thus we ought to do the same for any other members that carry load in compression. But how can we be sure that we have identified all compression members in the structural model? The load test animation is helpful, but in cases where the compressive member force is small, the color change in the animation may not be noticeable. The best way to identify compression members with certainty is to use the Member List.

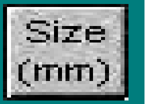


When you started up the West Point Bridge Designer, you probably noticed the Member List on the right-hand side of the Drawing Board. The list is not particularly useful for creating a structural model, so it is hidden from view at startup. When you begin editing your model, however, you'll find the Member List to be a very powerful tool. To view the entire Member List, place the mouse pointer over the left edge of the list. When the pointer changes to a double arrow, press the left mouse button and drag to the left.

The Member List shows the properties of each member—material, cross-section, size, and length—as well as the results of the most recent load test. Load test results are shown as a ratio of force to strength, for both tension and compression.

TIP

You can sort the Member List by clicking any one of the column headings. For example, clicking the Size heading will sort the list from the largest member size to the smallest. Clicking the same heading a second time will sort the list from smallest to largest.



Using the Member List, we can easily identify the members that carry load in compression. Any member for which the number in the "Compression Force/Strength" column is *not zero* carries some compressive force. If you click the **Compression Force/Strength** column heading, all of the non-zero entries in this column will be grouped at the top of the list. In our current structural model, these members include the top chords and end posts (which we have already changed to tubes), the three interior verticals (Members 14, 15, and 16), and the two interior diagonals (Members 19 and 20). All of these members should be designed as hollow tubes.

Note that three of these members—16, 19, and 20—carry *both* compression and tension. This condition occurs because of the moving load. As the AASHTO H20-44 truck crosses the bridge, certain load positions cause these members to be in tension, and certain positions cause them to be in compression. In most cases, you'll find that a member with *any compression force whatsoever* will be more economical when it is designed as a hollow tube.

When we change Members 14, 15, 16, 19, and 20 to hollow tubes (but leave their size unchanged at 120mm), the structure still passes the load test. Thus, in this case, we must find the optimum by systematically *reducing* the size of these members until they fail the load test. Here's how:

- Reduce the size of all five members from 120mm to 110mm—the next lower available size. Now both Members 14 and 16 fail the load test, which tells us that 120mm is the optimum size for these two members.
- On the next iteration, change Members 14 and 16 back to 120mm, and reduce the other three to 100mm. All three still pass the load test.

- On the next iteration, reduce Members 15, 19, and 20 to 90mm. Here Members 19 and 20 fail; thus their optimum size is 100mm.
- Change Members 19 and 20 back to 100mm, and continue reducing the size of Member 15 until it finally fails at 30mm. Its optimum size is 35mm.

  The cost of the truss is now $4334.32—another substantial cost reduction.

Now we can optimize the members that carry only tension—Members 1 through 6, 13, 17, 18, and 21. Again, we can systematically reduce the size of these members, running the load test after every change. When a member fails, we know that the next larger size is the optimum. At the conclusion of this process, our design costs **$2838.83**, and the member sizes are as indicated below:

#	Material	Sec.	Size (mm)	#	Material	Sec.	Size (mm)	#	Material	Sec.	Size (mm)
1	Carbon Steel	Bar	50	**8**	Carbon Steel	Tube	180	**15**	Carbon Steel	Tube	35
2	Carbon Steel	Bar	50	**9**	Carbon Steel	Tube	180	**16**	Carbon Steel	Tube	120
3	Carbon Steel	Bar	65	**10**	Carbon Steel	Tube	170	**17**	Carbon Steel	Bar	45
4	Carbon Steel	Bar	65	**11**	Carbon Steel	Tube	190	**18**	Carbon Steel	Bar	55
5	Carbon Steel	Bar	50	**12**	Carbon Steel	Tube	180	**19**	Carbon Steel	Tube	100
6	Carbon Steel	Bar	50	**13**	Carbon Steel	Bar	45	**20**	Carbon Steel	Tube	100
7	Carbon Steel	Tube	170	**14**	Carbon Steel	Tube	120	**21**	Carbon Steel	Bar	50

Q2

Why are these member selections not symmetrical?

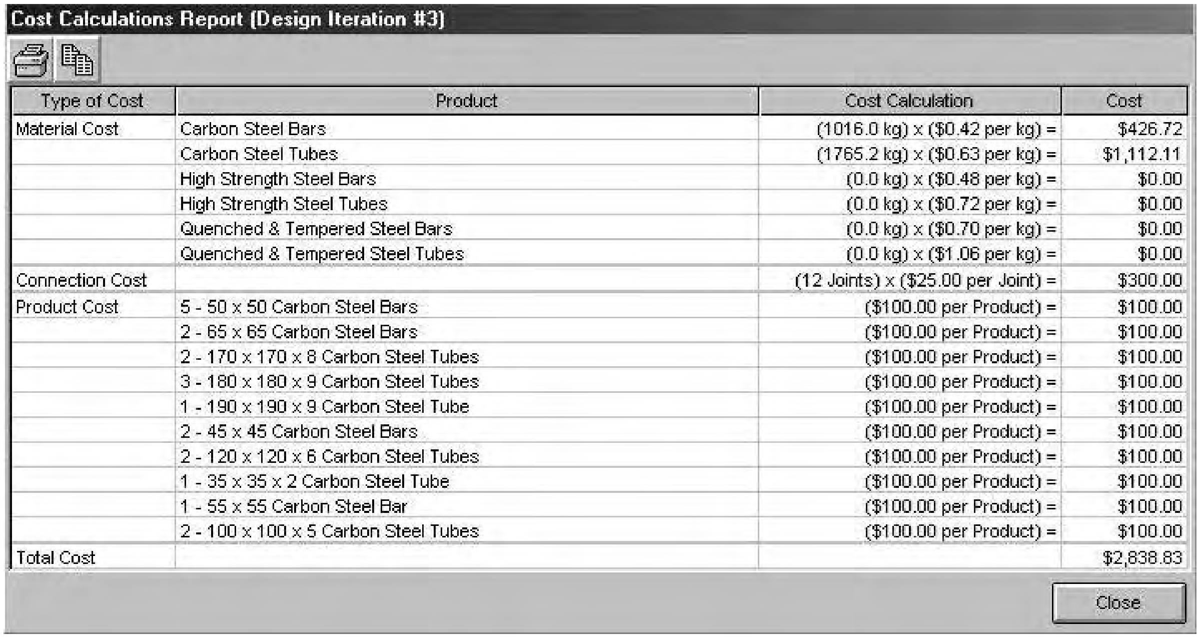
Given that the shape of this truss is perfectly symmetrical, we might expect that the optimal member selections would be symmetrical as well. Yet they are not. The two end posts—Members 11 and 12—are different sizes, as are the diagonals 18 and 21. Why are the optimum member selections not symmetrical, even though the truss itself is symmetrical?

For a truss made entirely of carbon steel, these members are as small as they can possibly be without failing. (Try running the load test now; you'll see that all of the members are either bright blue or bright red.) Thus our truss is as light as it can be and, since steel is priced by weight, we have effectively minimized the *material cost* of this carbon steel truss.

Note, however, that this 21-member structure is composed of ten *different* member sizes. From a practical perspective, this would be a very difficult truss to build. Ordering, fabricating, and assembling all of these different sized members (and their connections) would be both challenging and expensive for the Constructor. Thus, if we minimize the material cost of this truss, we cause its fabrication and construction costs to be quite high. Clearly there is an economic benefit in *standardization*. Using fewer member sizes results in lower fabrication and construction costs. Yet standardization also causes material costs to increase, so too much standardization might be just as uneconomical as too little. In structural design, there is a clear tradeoff between light weight and standardization. The two are almost always competing interests. The engineer's challenge is to find the right balance between them.

The West Point Bridge Designer accounts for this tradeoff when it calculates the cost of your design. Click the **Report Cost Calculations button** on the toolbar to see how the calculation is done. Note that the *total cost* of the truss is broken down into three components—*material cost*, *connection cost*, and *product cost*:

- The **Material cost** is calculated by (1) determining the total mass of the three available materials—carbon steel, high-strength steel, and quenched and tempered steel—in your structural model, (2) multiplying the mass of each material type by the corresponding unit cost, in dollars per kilogram, and (3) adding these together to get the total material cost. Each of the three different types of steel has a different unit cost. Carbon steel is least expensive; quenched and tempered steel is most expensive.
- The **connection cost** is calculated as $25 per joint in your structural model.
- The **product cost** is $100 for each product in your structural model. A *product* is defined as any unique combination of material, cross-section, and size. For example, a 120mm carbon steel bar, a 120mm high-strength steel bar, a 130mm carbon steel bar, and a 120mm carbon steel tube are four different products.



Type of Cost	Product	Cost Calculation	Cost
Material Cost	Carbon Steel Bars	(1016.0 kg) x ($0.42 per kg) =	$426.72
	Carbon Steel Tubes	(1765.2 kg) x ($0.63 per kg) =	$1,112.11
	High Strength Steel Bars	(0.0 kg) x ($0.48 per kg) =	$0.00
	High Strength Steel Tubes	(0.0 kg) x ($0.72 per kg) =	$0.00
	Quenched & Tempered Steel Bars	(0.0 kg) x ($0.70 per kg) =	$0.00
	Quenched & Tempered Steel Tubes	(0.0 kg) x ($1.06 per kg) =	$0.00
Connection Cost		(12 Joints) x ($25.00 per Joint) =	$300.00
Product Cost	5 - 50 x 50 Carbon Steel Bars	($100.00 per Product) =	$100.00
	2 - 65 x 65 Carbon Steel Bars	($100.00 per Product) =	$100.00
	2 - 170 x 170 x 8 Carbon Steel Tubes	($100.00 per Product) =	$100.00
	3 - 180 x 180 x 9 Carbon Steel Tubes	($100.00 per Product) =	$100.00
	1 - 190 x 190 x 9 Carbon Steel Tube	($100.00 per Product) =	$100.00
	2 - 45 x 45 Carbon Steel Bars	($100.00 per Product) =	$100.00
	2 - 120 x 120 x 6 Carbon Steel Tubes	($100.00 per Product) =	$100.00
	1 - 35 x 35 x 2 Carbon Steel Tube	($100.00 per Product) =	$100.00
	1 - 55 x 55 Carbon Steel Bar	($100.00 per Product) =	$100.00
	2 - 100 x 100 x 5 Carbon Steel Tubes	($100.00 per Product) =	$100.00
Total Cost			$2,838.83

Note that the $2838.83 total cost of our truss includes $1538.83 in material cost, $300.00 in connection cost, and $1000.00 in product cost. The material cost is already minimized (for carbon steel), and we can't change the connection cost without changing the configuration of the truss. Thus the only way we might reduce the total cost of the design is to reduce the product cost. We know that each reduction in product cost will cause a corresponding increase in material cost, so we must be careful to make only those changes that cause the total cost to go down. The following sequence of modifications to the structural model illustrates how to optimize the product cost of the design:

- In the current design, there are two 45mm bars and five 50mm bars. If we increase the two 45mm bars to 50mm, we eliminate the $100 product cost associated with the 45mm bars, and we only increase the material cost slightly. This change reduces the total cost to **$2754.49**.
- Similarly, changing the single 55mm bar to 65mm eliminates another product and reduces the total cost to **$2679.82**.
- Change the single 35mm tube to a 50mm bar, and the total cost is further reduced to **$2614.51**.
- Change the two 100mm tubes to 120mm tubes. The cost is now **$2567.45**.
- Change the two 170mm tubes to 180mm, to reduce the cost to **$2505.91**.

- We have now successfully reduced the number of products to five. Can we go further? One possibility might be to increase the nine 50mm bars to 65 mm, but this change causes the total cost to rise. Making nine members substantially larger causes the material cost to increase much more than $100; thus the cost of eliminating this product is greater than the benefit. We certainly don't want to incorporate this change into our design, so use the **Undo button** to eliminate it.
- Changing four 120mm tubes to 180mm produces the same result. The total cost increases, so the change is not justified.

By now, you should be able to see the logic behind this phase of the optimization process. In every case, we achieve greater standardization by making members *larger*. Because larger members are also *stronger*, there is little risk that the changes will cause the design to become structurally unsafe. A given change proves to be worthwhile if the increased material cost is less than the $100 saving we gain by eliminating a product.

Now, however, we will make one more change that seems illogical. Our current structural model has a single 190mm tube—Member 11, the end post on the left side. If we reduce the size of that member to a 180mm tube, we can eliminate a product *and* reduce the material cost, resulting in a total cost of **$2394.51**. Since we made Member 11 smaller, we would logically expect it to fail the load test. But when we run the load test, the structure carries its load successfully!

Q3

Why does Member 11 pass the load test?

When we first began the process of optimizing the design, Member 11 required a 190mm tube to pass the load test. At that time, reducing the member size to 180mm caused the structure to fail. Yet now the 180mm tube passes the load test. What happened?

We have come a long way since our $5672.55 truss passed the load test for the first time. By methodically optimizing first the material cost, then the product cost, we have reduced the total cost of the design by more than half. Yet there may be room for even more improvement. So far, we have used only carbon steel. We won't know if our design is truly optimal until we try the other two types of steel. Again, the way to do this successfully is to do it methodically—changing one particular group of members and carefully observing the result; then using that result to guide subsequent changes.

- Let's try changing the eight 50mm carbon steel bars to high-strength low-alloy (HSLA) steel. Note that, by changing all eight bars, we won't add to the total number of products in the structural model. We just substitute a new HSLA steel product for an old carbon steel one. Of course, HSLA steel is more expensive than carbon steel, so the immediate effect of the change is to cause the total cost to rise. But HSLA steel is stronger than carbon steel, so we should be able to make these members smaller without compromising the safety of the structure. In fact, a bit of trial and error will show that these eight bars can be reduced to 45mm and still pass the load test. The total cost drops to **$2371.61** as a result.
- Since changing to HSLA steel provided some cost benefit, let's try changing these same eight members to the strongest of the three materials—quenched and tempered steel. In this case, the substantial increase in material strength allows us to safely reduce the size of these members to 35mm, and the total cost is further reduced to **$2337.99**.
- Since quenched and tempered steel worked well for our first batch of tension members, let's try it for the second as well. If we change the three 65mm bars to quenched and tempered steel, we can reduce their size to 45mm, and the cost of the truss drops to **$2297.62**.

- Clearly the higher strength steels reduced the cost of tension members in our design. Will they do the same for the compression members? Let’s try changing the six 180mm tubes from carbon steel to HSLA steel. Unfortunately, in this case, using the stronger steel does not allow us to use a smaller member size. When we change these members to 170mm tubes, the structure fails the load test. Using quenched and tempered steel for these members produces exactly the same result.
- Similarly, using the higher strength steels for the four 120mm tubes produces no benefit.

Q4

Why did high-strength steel produce no benefit for compression members?

For tension members, using high-strength low-alloy steel and quenched and tempered steel produced significant cost benefits. For compression members, using these steels produced no benefit at all. Why not?

The optimization of member properties for our Pratt Through Truss is complete. The total cost is **$2297.62**, and further cost reductions are not possible without changing the configuration of the structural model or compromising the safety of the design. The final selections for materials, cross-sections, and member sizes are as follows:

#	Material	Sec.	Size (mm)	#	Material	Sec.	Size (mm)	#	Material	Sec.	Size (mm)
1	Q&T Steel	Bar	35	**8**	Carbon Steel	Tube	180	**15**	Carbon Steel	Tube	35
2	Q&T Steel	Bar	35	**9**	Carbon Steel	Tube	180	**16**	Carbon Steel	Tube	120
3	Q&T Steel	Bar	45	**10**	Carbon Steel	Tube	180	**17**	Q&T Steel	Bar	35
4	Q&T Steel	Bar	45	**11**	Carbon Steel	Tube	180	**18**	Q&T Steel	Bar	45
5	Q&T Steel	Bar	35	**12**	Carbon Steel	Tube	180	**19**	Carbon Steel	Tube	120
6	Q&T Steel	Bar	35	**13**	Q&T Steel	Bar	35	**20**	Carbon Steel	Tube	120
7	Carbon Steel	Tube	180	**14**	Carbon Steel	Tube	120	**21**	Q&T Steel	Bar	45

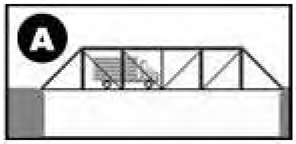
Optimize the Shape of the Truss

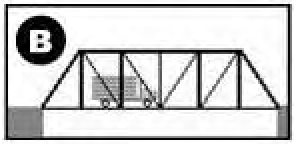
Our current design for the Lee Road Bridge is both safe and reasonably efficient. We could easily end the design process here and be entirely satisfied with our product. However, at this point in the process, we have only considered a single design alternative—a standard Pratt Through Truss configuration with a height of 5 meters. We have no way of knowing whether or not this configuration is most economical without considering other alternatives. To achieve a truly optimal design, we must fully consider a broad range of alternative configurations, then select the best one.

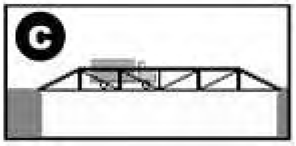
Before trying a totally new truss configuration, we should first optimize the shape of the current structural model. The West Point Bridge Designer allows you to change the shape of the truss by moving joints—a simple modification that can produce significant reductions in the cost of your design.

TIP

To move a joint in your structural model, first choose the **Select Tool** from the Design Tools palette. Then move the mouse pointer over the joint you want to move. Press the left mouse button and, while holding it down, drag the joint to its new location. When you release the mouse button, the joint and all attached members will be re-drawn in the new location.







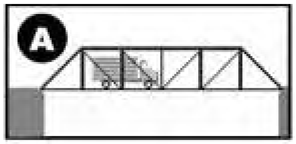
One way to optimize the shape of the structural model is to vary its *height*. The existing truss **(A)** has a height of 5 meters. We might try increasing the height, say to 6 or 7 meters **(B)**, or we might reduce the height to 3 or 4 meters **(C)**. In either case, you can make the change simply by dragging the five top-chord joints up or down. The basic Pratt truss configuration is not changed.

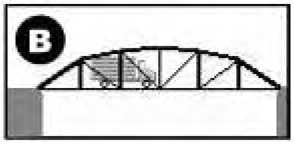
Once you have adjusted the height of the truss, you'll need to evaluate and optimize the new configuration—run the load test; identify and strengthen all unsafe members; and optimize the member properties. Only then can you determine whether or not the change was effective in reducing the cost of your design.

Q5

How would the cost of the design change if we change the height of the truss?

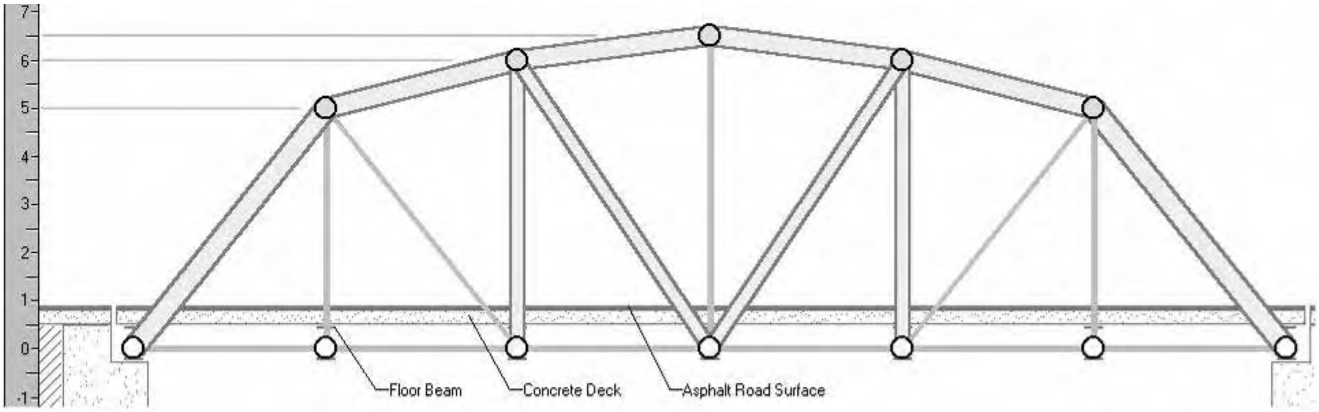
If we increase the height of the truss, do you think the cost of the design will increase or decrease? What if we decrease the height? Explain your answer.





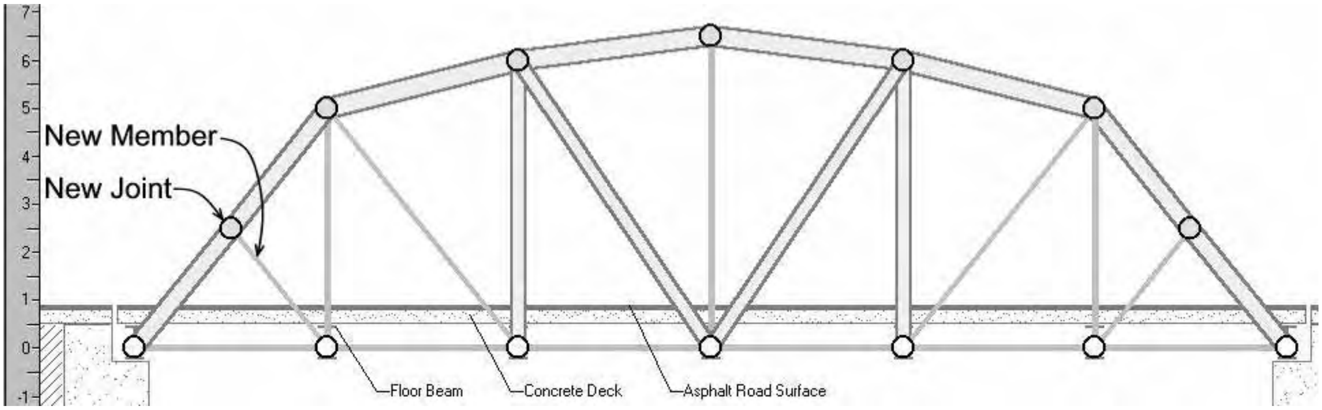
Another way to optimize the structural model is to change its overall shape. For example, we might change the basic Pratt Through Truss **(A)** to a Parker Truss **(B)**, by giving the top chord a more rounded shape. Once again, you can make this change simply by dragging the top-chord joints up or down. Often this minor adjustment can reduce the cost of a design significantly. The Gallery of Truss Bridges (Appendix A) shows a number of actual truss bridges that use this rounded configuration—an indication that it might provide some added structural efficiency.

Let's give it a try. Starting with the current $2297.62 Pratt Through Truss design, change it to a Parker Through Truss by (1) moving the center top-chord joint up 1.5 meters and (2) moving the two adjacent joints up 1.0 meter. The resulting truss is 6.5 meters high, as shown below:



This structure still passes the load test, but the increased member lengths cause the cost to increase to $2366.04. Increasing the height of a truss generally causes the member forces in the top and bottom chords to decrease. Thus we would expect to be able to reduce the sizes of these members. And indeed we can. The top chord—Members 7 through 10—can be reduced from 180mm to 160mm. The bottom chord members 3 and 4 can be reduced from 45mm solid bars to 40mm, and the diagonals 18 and 21 can be reduced to 35mm. Unfortunately, these changes only bring the total cost down to $2315.00—higher than the basic Pratt Through Truss we started with.

One problem with our Parker Truss is that the two end posts—Members 11 and 12—still require the use of 180mm tubes. When we reduced the size of the top chord members from 180mm to 160mm tubes, we added an additional product to the design. The added $100 product cost was greater than the material cost reduction we gained from using smaller members. We could make a substantial reduction in the total cost, if we could figure out a way to use 160mm tubes for the end posts. And we can! Recall that the strength of a member in compression is strongly influenced by its length. Compressive strength decreases sharply with increasing length. Thus if we can make the end posts shorter, we can use a smaller tube size to carry the same internal force. We can make the end posts shorter by breaking them in half—by adding a new joint in the middle of each one—then adding a new member to preserve the stability of the structural model. The result is shown below.



To achieve this new configuration, you'll need to:

Delete Members 11 and 12. To do this, select the two members, then click the **Delete button** on the toolbar.

Select the **Joint Tool** from the Design Tools palette, and draw the two new joints—at a vertical position of 2.5 meters and horizontal positions of 2.0 meters and 22.0 meters.

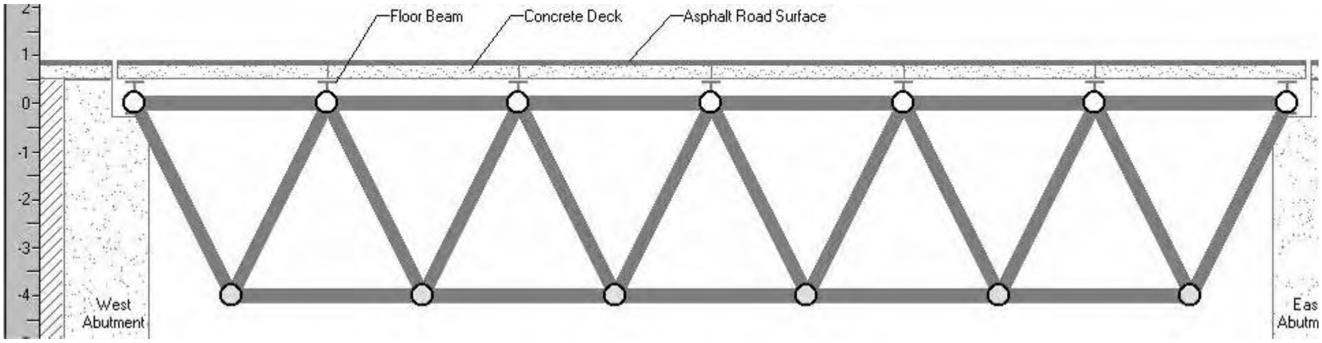
Select the **Member Tool** from the Design Tools palette. Change the default values in the Member Properties Lists to **Carbon Steel**, **Hollow Tube**, and **160mm**. Then draw the four new end post members. Finally, change the default values in the Member Properties Lists to **Quenched & Tempered Steel**, **Solid Bar**, and **35mm**, and draw the two new diagonals.

This optimized Parker Through Truss has a total cost of **$2226.28**.

Try a New Configuration

Recognizing that we were able to make a modest improvement in the cost of our structural model just by changing its shape, it is clear that we might do even better by starting with an entirely different truss configuration. The more configurations we explore, the more likely we will find one that is even more efficient than our current design.

Let's try the standard Warren Deck Truss configuration. If you follow the template provided by the West Point Bridge Designer, your initial structural model will look like this:



If you optimize the member properties for this truss (following the same procedure we used for the Pratt Through Truss above), the lowest total cost you will be able to achieve is about **$2460**.

Q6

Can you optimize member properties for a truss?

Starting with the standard Warren Deck Truss configuration shown above, optimize the member properties—material, cross-section, and size—such that the total cost of the design is less than $2500. Do not change the shape of the truss.

At $2460, the standard Warren Deck Truss is significantly more expensive than the Parker Through Truss we designed earlier. However, if we change its shape, it is possible to get the cost of the truss under $2180.

Q7

Can you optimize the shape of a truss?

Starting with the standard Warren Deck Truss configuration, can you modify the shape, such that the total cost of the optimized design is less than $2200?

Choose the Optimum Design

In this learning activity, we have developed the following four major design alternatives:

Type	Cost
Standard Pratt Through Truss	$2297.62
Parker Through Truss	$2226.28
Standard Warren Deck Truss	$2457.67
Modified Warren Deck Truss	$2179.78

Of these four alternatives, the modified Warren Deck Truss is clearly the optimum. Nonetheless, we could certainly reduce the cost further, by considering a larger number of alternative configurations. (Indeed, a number of users of the West Point Bridge Designer have created successful designs costing under $1600 for this project.)

On an actual bridge project, however, this degree of optimization would probably be adequate. It wouldn't make much sense for the structural engineer to spend many additional hours refining the design, just to trim a few more dollars off the project cost. (Don't forget that the engineer's time costs money too!) The engineer's professional responsibility is to consider a sufficient number of alternatives to ensure that the final project cost is reasonable. Pursuit of the absolute lowest possible cost is seldom justified.

It is also important to note that selecting a "best" alternative is usually far more complicated than simply picking the design that costs the least. Cost is certainly important, but other factors like aesthetics, ease of construction, availability of materials and labor, type of traffic, soil conditions, environmental impact, and safety might be equally important in determining which design alternative actually gets built. For example, if Union Creek were a navigable waterway, a deck truss might not provide enough overhead clearance for vessels traveling under the bridge; thus a through truss might be required even if it is not the most economical configuration.

Record Your Design

At the conclusion of the design process, the Design Team prepares a detailed set of plans and specifications—drawings and documents that describe every aspect of the project. These documents are the principal means of communicating the design to the Constructor who will actually build the facility. Designers also maintain copies of all the engineering calculations on which the design is based. All of these documents serve as a permanent record of the design, as well as a valuable reference for future projects of a similar nature.

You should document the products of your design work in a similar manner. The West Point Bridge Designer provides three different ways to record your design:

To save your design in a specially formatted file, click the Save button on the main tool bar, enter a file name, and click OK. This bridge design file can be opened and modified later.

To print a drawing of your design, click the Print button on the main toolbar. A drawing showing the configuration and dimensions of the truss and a table listing the assigned member properties will be printed to your default printer.

To print a detailed report of your most recent load test results, click the Report Load Test Results button on the main toolbar, then click the Print button at the top of the report window.

Conclusion

In this activity, we learned about engineering design by doing it—by designing an actual truss bridge. Using a specially developed software package, we created a structural model, tested it to ensure that it was strong enough to carry its prescribed loading, then optimized it to minimize its cost. As we used the software, we observed that design is always iterative and that design always involves tradeoffs. We also saw how the computer can be used to enhance the effectiveness and efficiency of the design process. What we *didn't* do was to actually build and test the bridge we designed. We're getting pretty good at engineering and construction, but building a 24-meter steel truss bridge is probably still a bit beyond our capability. In Learning Activity #5, however, we'll work through a complete design of a *model* truss bridge, and we will build and test it.

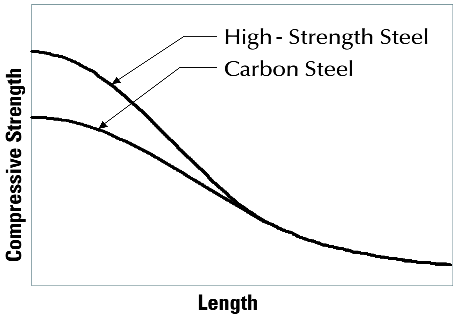
Answers to the Questions

1) **What portion of the engineering design process does WPBD address?** When you use the West Point Bridge Designer, you begin with Phase 4 of the engineering design process. You select one particular subsystem—a main truss—for a highway bridge. Then you proceed with multiple cycles of analysis and design—Phases 5 and 6—until an optimal design for that particular subsystem is achieved. It is important to recognize that, on an actual bridge project, many other subsystems would have to be designed before the final plans and specifications could be produced.

2) **Why are these member selections not symmetrical?** Even though the shape of this truss is perfectly symmetrical, the loading is not. The highway load used in the West Point Bridge Designer load test is a standard AASHTO H20-44 truck loading. The H20-44 has a heavy rear axle and a lighter front axle. Because the loading is not symmetrical, the maximum internal member forces are unequal in symmetrical members, like the two end posts. Since the internal member forces are not symmetrical, the member sizes are not either. Is this realistic? Well, no. The bridge will only carry load safely when the truck crosses from left to right. If the truck reversed direction (with the light axle on the left and the heavy axle on the right), the structure would fail the load test. This is an inaccuracy in the West Point Bridge Designer and one of many reasons why the software should be used for educational purposes only!

3) **Why does Member 11 pass the load test as a 180mm tube?** Early in the optimization process, a 190mm tube was required for Member 11. But at that time, all of the compression members had not yet been changed to tubes, and none of the tension members had been optimized. The optimization of these members caused a significant reduction in the weight of the truss itself. And even though the weight of the truss is quite small in comparison with the other loads applied to this structure, the lighter weight of the optimized truss was just enough to reduce the internal force in Member 11 slightly below the compressive strength of a 180mm tube.

4) **Why did high-strength steel produce no benefit for compression members?** This graph shows the compressive strength vs. length curves for two bars that have identical dimensions but are made of two different types of steel. Note that using a stronger material only improves the compressive strength of relatively short members. The strength of the steel has no effect at all on the compressive strength of longer members.



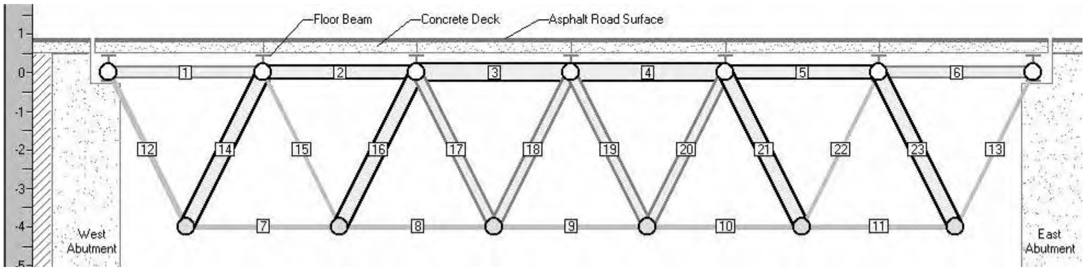
To verify this observation yourself, start up the West Point Bridge Designer, and click the **Report Member Properties button** on the toolbar. You will see a graph of strength vs. length for the member size, cross-section, and material currently displayed in the Member Properties lists. Now select a new material from the drop-down list, and watch how the graph changes.

In our design project, changing the compression members to higher strength steels produced no cost reduction because these members are all relatively long. If the change to stronger steel produced any increase in member strength, that increase was not enough to offset the increased cost of the stronger steel.

5) **How would the cost of the design change if we change the height of the truss?** This is a trick question. There's no way to answer it without actually changing the height of the truss, optimizing member selections for the new configuration, and comparing the new cost with the old one. At first glance, you might guess that reducing the height will cause the cost to decrease. Reducing the height causes the verticals and diagonals to get shorter, and shorter members cost less. But reducing the height also causes the internal member forces in the top and bottom chords to increase. (See Trusses 1 through 4 in the Gallery of Structural Analysis Results, Appendix B.) To pass the load test, you'll need to increase the size of these members, which will add to the total cost. So decreasing the height of the truss causes two competing effects—one that tends to decrease the cost and one that tends to increase it. You'll see these same competing effects in reverse, if you *increase* the height of the truss. As the truss gets higher, the internal member forces in the top and bottom chords get smaller, and you can reduce their cost by using smaller members. But increasing the height also makes the verticals and diagonals longer, which makes them more expensive. Clearly there is a trade-off between (1) the member force in the top and bottom chords and (2) the length of the verticals and diagonals. Every truss has an optimum height, which represents the best compromise between these two competing effects. The best way to find the optimum height for your design is through trial and error.

Design almost always involves these sorts of tradeoffs. It is a rare case indeed when a design change has only positive consequences. More often than not, design changes result in some positive and some negative consequences. The engineer's challenge is to figure our when the positives outweigh the negatives.

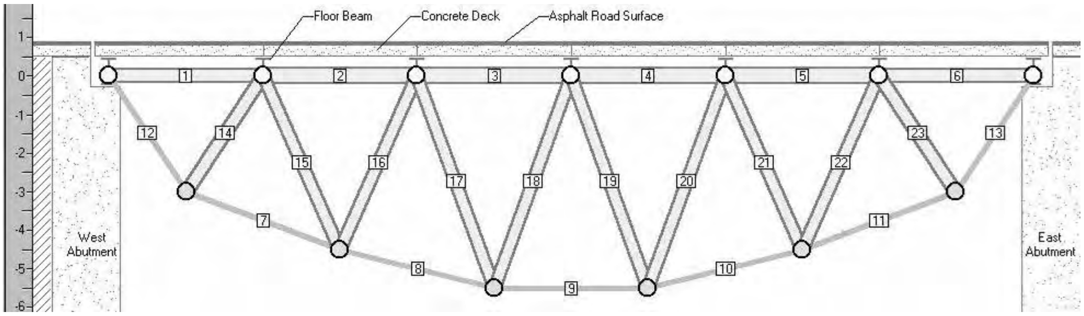
6) **Can you optimize member properties for a truss?** By following a methodical procedure to optimize the selection of members for the standard Warren deck truss, you should be able to achieve the following result.



#	Material	Sec.	Size (mm)	#	Material	Sec.	Size (mm)	#	Material	Sec.	Size (mm)
1	Carbon Steel	Tube	120	**9**	Q&T Steel	Bar	55	**17**	Carbon Steel	Tube	120
2	HS Steel	Tube	150	**10**	Q&T Steel	Bar	55	**18**	Carbon Steel	Tube	120
3	HS Steel	Tube	170	**11**	Q&T Steel	Bar	40	**19**	Carbon Steel	Tube	120
4	HS Steel	Tube	170	**12**	Q&T Steel	Bar	55	**20**	Carbon Steel	Tube	120
5	HS Steel	Tube	150	**13**	Q&T Steel	Bar	40	**21**	HS Steel	Tube	150
6	Carbon Steel	Tube	120	**14**	HS Steel	Tube	150	**22**	Q&T Steel	Bar	40
7	Q&T Steel	Bar	40	**15**	Q&T Steel	Bar	40	**23**	HS Steel	Tube	150
8	Q&T Steel	Bar	55	**16**	HS Steel	Tube	150				

The total cost of this truss is $2,457.67.

7) **Can you optimize the shape of a truss?** The picture below shows one of many possible ways to reduce the cost of the design by changing its shape. The six bottom-chord joints have been moved vertically to increase the height of the truss and to give the bottom chord a more rounded shape. By methodically optimizing the member properties for this new configuration, the total cost can be reduced to $2179.78.



#	Material	Sec.	Size (mm)	#	Material	Sec.	Size (mm)	#	Material	Sec.	Size (mm)
1	Carbon Steel	Tube	130	**9**	Q&T Steel	Bar	45	**17**	Carbon Steel	Tube	130
2	Carbon Steel	Tube	160	**10**	Q&T Steel	Bar	45	**18**	Carbon Steel	Tube	130
3	Carbon Steel	Tube	160	**11**	Q&T Steel	Bar	45	**19**	Carbon Steel	Tube	130
4	Carbon Steel	Tube	160	**12**	Q&T Steel	Bar	45	**20**	Carbon Steel	Tube	130
5	Carbon Steel	Tube	160	**13**	Q&T Steel	Bar	45	**21**	Carbon Steel	Tube	130
6	Carbon Steel	Tube	130	**14**	Carbon Steel	Tube	130	**22**	Carbon Steel	Tube	130
7	Q&T Steel	Bar	45	**15**	Carbon Steel	Tube	130	**23**	Carbon Steel	Tube	130
8	Q&T Steel	Bar	45	**16**	Carbon Steel	Tube	130				

Some Ideas for Enhancing This Learning Activity

The West Point Bridge Designer has proved to be very popular with students, no doubt because it resembles a video game in some ways. Graphical creation of the structural model, the load test animation, and the use of a single easily understood measure of performance all contribute to this resemblance. Like a video game, the Bridge Designer lends itself well to competition. Students generally enjoy competing against each other and against the "best scores" posted the West Point Bridge Designer web page (http://bridgecontest.usma.edu).

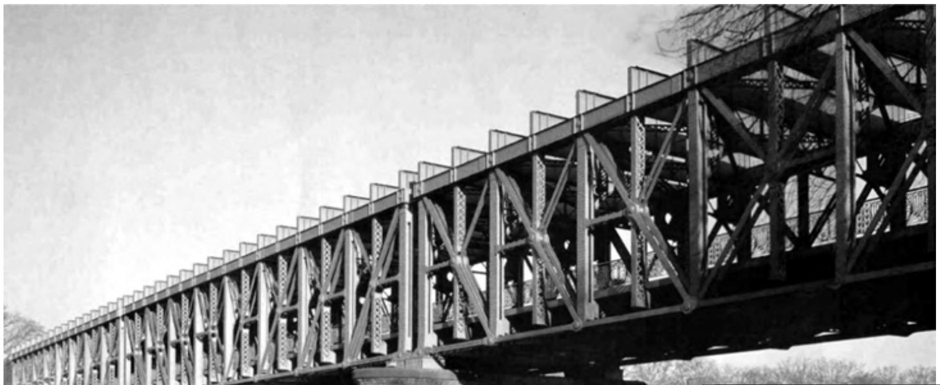
This resemblance to a video game has both positive and negative implications. Students who get engaged in "playing the game" often discover important principles of structural engineering in the process. It is virtually impossible to develop a highly optimal design purely by chance. The student must work through many design iterations, and each new iteration must be informed by careful observations of cause and effect. *When I made the member shorter, how did its compressive strength change? When I changed the height of the truss, how did the internal member forces change?* The answers to these sorts of questions will inevitably lead to important insights about structural engineering. *Shorter members are stronger in compression. Increasing the height of the truss causes the internal forces to decrease in the top and bottom chords*.

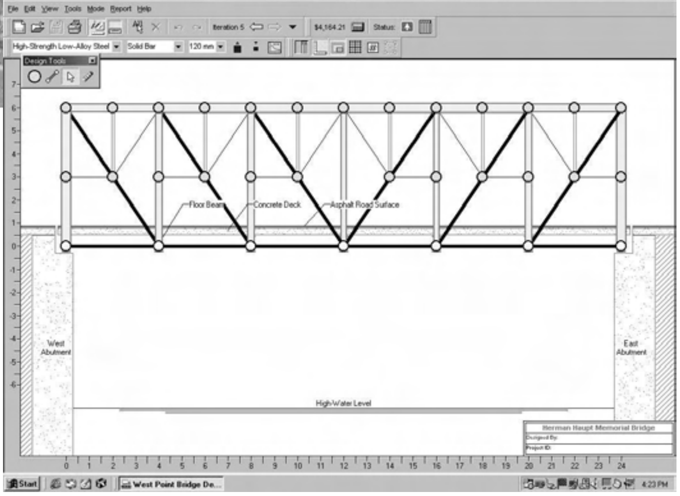
Nonetheless, using the West Point Bridge Designer as a video game has some obvious dangers. Students who get totally absorbed in "the game" may lose sight of the all-important connection between the simulation and the real-world structure it represents. Students might also develop mistaken impressions of how the engineering design process works on a real project. Design is usually not performed in a competitive mode, and a practicing structural engineer would almost certainly not do hundreds of design iterations just for the sake of shaving a few dollars off of the construction cost.

With these positives and negatives in mind, the teacher might enhance the use of the West Point Bridge Designer as a classroom activity in the following ways:

- Select one of the seven projects in the West Point Bridge Designer, and use it as the basis for a design competition. Offer a prize or bonus points for the design with the lowest cost. Organize the class into teams of two or three students, with *only one computer per team*. This organization will encourage students to interact with each other and to decide among themselves how each new design iteration should proceed. This interaction will make them more aware of their own decision-making process and will enhance their opportunities to discover important insights about structural engineering from each other.
- You can eliminate some of the unhealthy aspects of competition by setting absolute standards of performance, instead of a relative one. For example, instead of giving a reward to the best design in the class (a relative standard), you might offer a 5% bonus to all designs under $2500, a 10% bonus to all designs under $2250, and a 20% bonus to all designs under $2000. This format is still competitive, but students compete against a fixed set of standards, rather than against each other. This format tends to be particularly effective for less gifted students, who might give up more readily when in direct competition with their peers. In a sense, this format is also more realistic, because most actual projects have a fixed project budget—an absolute standard—that the designer must satisfy.
- Rather than holding a competition, have students explore the relative efficiency of various different truss configurations. Divide the class into teams, and assign a *different* truss configuration to each team. The easiest way to do this is to have each team use one of the standard truss templates provided by the West Point Bridge Designer (see page 4-16). Each team then optimizes the member selections for its assigned truss configuration, *without changing the shape of the truss*. At the end of the project, each team reports its optimum cost, and these costs are compared to determine which configuration is most economical.

- Students can be asked to model and design an actual bridge—preferably one familiar to them. If there is an appropriate truss bridge near the school, the teacher can take photographs of it and provide copies to the students. The students can use the Bridge Designer to model this structure as accurately as possible, then test and optimize it. (Since the West Point Bridge Designer does not allow the user to change the span length of a design, students will only be able to model the shape and configuration of the actual bridge, not its specific dimensions.) This sort of project helps to reinforce the connection between the computer simulation and the actual structure it represents. If no appropriate truss bridges are located near the school, the teacher can use the ones provided in the Gallery of Truss Bridges (Appendix A).





West Point Bridge Designer - C:\My Documents\Scholarship\Bicentennial\LAM\FF Bridge Project\Learning Activity 4\Falls Bridge.bd4

Iteration 5

$4,164.21

High-Strength Low-Alloy Steel

Solid Bar

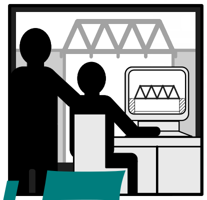
120 mm

Herman Haupt Memorial Bridge

Designed By:

Project ID:

- No matter how the West Point Bridge Designer is used, students are more likely to learn from the experience if they are asked to write a reflective essay at the end of the project. In their essays, they can be asked to comment on what they learned about bridges, about how structures carry load, and about the engineering design process.



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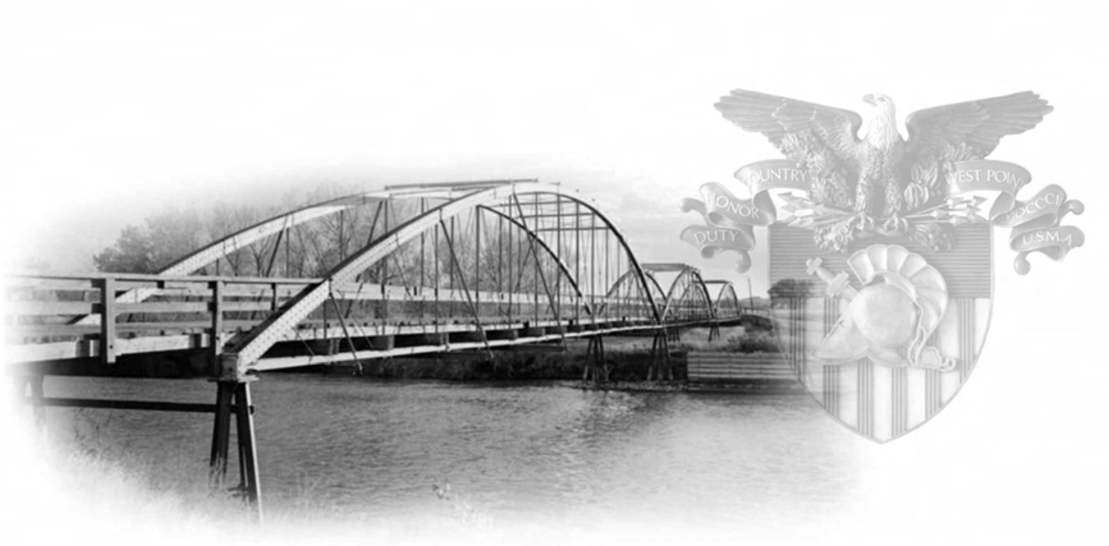


Learning Activity #5:

Design and Build a Model Truss Bridge

Overview of the Activity

In this learning activity, we will design, build, and test a model truss bridge. We will analyze the Owner's needs, then formulate specific design requirements. We will develop a truss configuration, analyze the structure, design each individual member and connection, then develop plans and specifications. Finally, we will build the bridge and test it to verify that it can carry load safely.

Why?

In Learning Activity #1, we played the role of the Constructor and built a model bridge that had been designed by someone else. In Learning Activity #5, we will assume the role of the Design Professional and design a new bridge with the same span length but with a different loading and a very different geometric configuration. In doing so, we will learn a process that can be used to design a bridge with practically *any* span length, loading, or configuration.

This project provides an opportunity to apply *everything* we have seen in the previous four learning activities. We will see how the various elements of the engineering design process *fit together*—how scientific principles, mathematic tools, engineering concepts, experimental data, and practical considerations contribute to the final product. We'll see how the truss configuration is tailored to the Owner's needs; how the structural model is derived from the truss configuration; how structural analysis results and experimental data contribute to the design of structural members; how the size and shape of connections are determined; how constructability considerations affect the final design; and how engineering computations are translated into the drawings and schedules required for construction. Finally, we will build the bridge we designed—a great way to check the validity of the design and the accuracy of the plans and specifications.

Learning Objectives

As a result of this learning activity, you will be able to do the following:

- Explain how *design-build* project delivery differs from *design-bid-build* project delivery.
- Explain how the *factor of safety* is used in design.
- Explain how scientific principles, mathematic tools, engineering concepts, experimental data, and practical considerations contribute to the engineering design process.
- Design a model truss bridge to meet a set of design requirements.
- Build a model truss bridge, consistent with a set of plans and specifications.

Key Terms

To successfully complete this learning activity, you must understand the following key terms and concepts from previous learning activities:

truss
member
top chord
bottom chord
diagonal
deck
abutment
deck truss
through truss
gusset plate
joint
reaction
load
equilibrium
internal force
tension
compression
strength
factor of safety
static determinacy
stability
right triangle
hypotenuse
Pythagorean theorem
Owner
Design Professional
Constructor
plans & specifications

If you need to refresh your memory on any of these terms, see the Glossary in Appendix D.

Information

Using the Factor of Safety in Design

When we *analyzed* a structure in Learning Activity #3, we used the following definition for the factor of safety:

$$Factor\ of\ Safety = \frac{Strength}{Internal\ Member\ Force}$$

To use this equation, we first determined the internal force in each member (by doing a structural analysis) and the strength of each member (by using our experimental data from Learning Activity #2). Then we used these numbers to calculate a unique factor of safety for every member in the structure. In short, we used *known* values of internal force and strength to calculate *unknown* factors of safety.

When we *design* a structure, we need to select members that are strong enough to carry load safely. Thus, in design, the *unknown* quantity in the equation above is the strength. The *known* quantities are the internal forces and the factor of safety. As before, the internal member forces are determined by a structural analysis; but in design, we will simply specify the factor of safety. We might use a design code as the basis for deciding what the factor of safety should be, or we might simply use our experience and judgment. In either case, we will choose a value that appropriately reflects the level of safety required for our structure.

Since strength is the unknown quantity, it makes sense to algebraically rearrange the equation above by multiplying both sides by the internal member force. The result is

$$Strength = (Factor\ of\ Safety)(Internal\ Member\ Force)$$

To use this equation for design purposes, we will change the "equal sign" to a "greater than or equal sign," like this:

$$Strength \geq (Factor\ of\ Safety)(Internal\ Member\ Force)$$

The product on the right-hand side of this expression—the factor of safety times the internal member force—is called the **required strength**. This expression tells us that the *actual strength* of a member must be greater than or equal to its *required strength*. We use $\geq$ because it's always OK for a member to be "too strong." Indeed, as we saw in Learning Activity #3, sometimes it makes good economic sense for some members in a structure to be stronger than they really need to be.

We will use the expression above as the basis for determining the size of each structural member in our design.

Design-Build Project Delivery

As we discussed in Learning Activity #4, most public works projects in the United Sates use *design-bid-build project delivery*. In this system, (1) the Design Professional develops a complete design and provides it to the Owner, (2) the Owner advertises the project, (3) construction contractors submit bids, and (4) the Owner awards the construction contract to the lowest responsive, responsible bidder. Owners typically use design-bid-build project delivery because the competitive bidding process tends to keep the construction cost low. However, this system has some significant disadvantages as well:

- In design-bid-build project delivery, the Design Professional often has only minimal involvement in the construction phase of the project. Thus the designer is not able to ensure that that structure is built as intended.
- The Constructor is never involved in the design process. Thus constructability issues may not be fully considered in the design.
- The period of time required for advertising, collecting contractors' bids, and awarding the construction contract can be quite substantial. At this point in the process, the design is complete, and construction activity has not yet begun. Thus this entire period is essentially non-productive.

For these reasons (and others), an alternative system called **design-build project delivery** is becoming increasingly popular. In a design-build project, a single firm contracts with the Owner to do an entire project—both design and construction. Thus, in a design-build project, there is no break in continuity between design and construction. Coordination between the Design Professional and the Constructor is likely to be more effective, because one firm has overall responsibility for the project. Eliminating the bidding phase may also speed up the project. Indeed, with design-build project delivery it is possible for construction to begin even before the design is complete—a procedure called "fast-tracking."

Of course, design-build project delivery also has its disadvantages. Thus the best means of project delivery always depends on the nature of the project.

The Learning Activity

The Problem

The Need

Recently a tractor-trailer truck lost its brakes while driving on Grant Road. The driver lost control of the vehicle, and it collided with one of the end posts on the west end of the Grant Road Bridge. Fortunately, no one was hurt; but the bridge was damaged beyond repair. Grant Road is now closed, and the Town of Hauptville has initiated a project to replace the structure as quickly as possible.

Design Requirements

The Town of Hauptville is the Owner for this project. On behalf of the Owner, the Town Engineer has again hired Thayer Associates to provide design services. Thayer Associates has sent a team of engineers to begin working on the needs analysis. The engineers meet with the Mayor, the Town Council, the Town Engineer, and other Hauptville residents to work out the functional and aesthetic requirements for the new structure. At the meeting, the engineers receive the following input:

- The Mayor says, "I don't want another bridge failure in my town. I want you to ensure that this new bridge is not as vulnerable to a vehicular collision as the old one was."
- The President of the Town Council adds, "We didn't plan on having to replace a bridge when we developed this year's budget. The cost of this project must be kept as low as possible."
- Another member of the Town Council adds, "The residents of Hauptville are very upset about the closure of Grant Road. We need to get this project completed as soon as possible."
- A member of the Hauptville Historical Society says, "I know money is tight. But it would be a terrible mistake to build an ugly bridge, just to save some money. We at the Historical Society think it's important to the preserve the historic character of the town so, if possible, we'd like the new bridge to be a truss."
- Finally, the Town Engineer adds his own input: "I am still very concerned with the ever-increasing number of heavy trucks using Grant Road. To give us an added margin of safety, I'd like the new structure to be designed for a 20% higher vehicular loading than the AASHTO bridge design code requires."

Based on this input, as well as data gathered from a thorough investigation of the project site, the engineers from Thayer Associates develop the following design requirements:

- The replacement bridge will be constructed on the existing abutments, which are 24 meters apart.
  [Again our 1/40 scale model bridge will have a span of 60 centimeters.]
- Like the previous bridge, the new structure will carry two lanes of traffic. However, the width of the deck will be increased by 20% to provide more space for larger vehicles.
  [Our model bridge will have a roadway width of 11 centimeters—2 centimeters wider than the first Grant Road Bridge model.]

- The bridge will be designed for a vehicular loading 20% larger than that required by the AASHTO bridge design code.
  [Our model bridge will be designed for a "traffic load" consisting of a 6 kilogram mass placed on the structure at mid-span; the first Grant Road Bridge model was designed for only 5 kilograms.]
- The **factor of safety** will be 2.0.
- The bridge will be made of steel.
  [Again, our model will use cardboard from standard manila file folders.]
- The bridge configuration will be a *deck truss*. With no portion of the structure extending above the roadway, the bridge will be invulnerable to a vehicular collision.
- Because of the limited project budget, the cost of the new bridge must be kept to a minimum.
- To get the bridge into service as quickly as possible, **design-build project delivery** will be used for this project. Consistent with this requirement, Thayer Associates enters into a partnership with Mahan Construction Company, a local contractor, to do the project.

Your Job

You are the Chief Engineer for Thayer Associates. You are the Design Professional for this project. Your responsibility is to design a replacement for the Grant Road Bridge that meets all of the Owner's requirements. Once the design is complete, you will continue to work with Mahan Construction Company to ensure that the bridge is built correctly.

The Solution

The Plan

Our plan to design the new Grant Road Bridge consists of the following major activities:

- Decide on a truss configuration.
- Create the structural model.
- Check static determinacy and stability.
- Calculate reactions.
- Calculate internal member forces.
- Determine member sizes.
- Check member sizes for constructability.
- Draw plans.
- Create a schedule of truss members and a schedule of gusset plates.
- Build the bridge.

Decide on a Truss Configuration

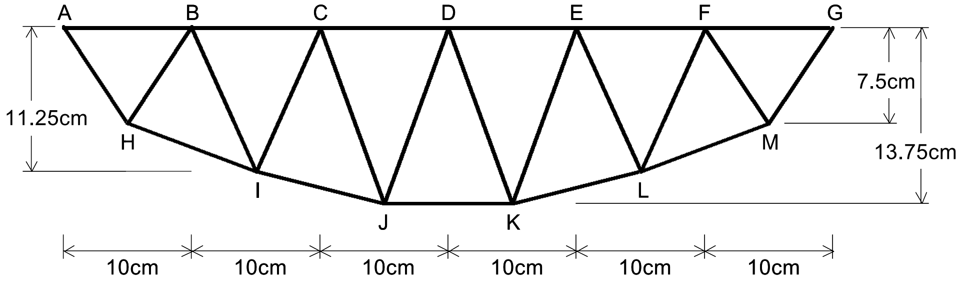
In general, when you design a truss bridge, you may use any stable truss configuration that satisfies the project requirements. Of course, for any given set of project requirements, some configurations are bound to be more efficient than others. An experienced engineer might be able to choose an efficient configuration based simply on what has worked well for previous projects. If you lack experience, you might try several different alternative configurations, develop a preliminary design for each one, and select the configuration that costs the least. You might also base your selection on aesthetics or constructability, rather than on structural efficiency.

For this specific project, the only constraint on the selection of a truss configuration is that it must be a deck truss.

Fortunately, we do have previous experience with designing this particular bridge type. In Learning Activity #4, we used the West Point Bridge Designer software to design a Warren Deck Truss that proved to be quite efficient. Let's use this same configuration for our Grant Road Bridge replacement. This configuration is also included as Truss 16 in the Gallery of Structural Analysis Results (Appendix B). By using a configuration that is included in the Gallery, we will be able to save considerable effort in our structural analysis.

Create the Structural Model

Having selected a truss configuration, we will now model the structure, by defining (1) the geometry of the truss, (2) the loads, and (3) the supports and reactions—just as we did in Learning Activity #3. We idealize the three-dimensional bridge as a pair of identical two-dimensional trusses. The geometry of one main truss is shown below. The dimensions indicate the locations of the member *centerlines*. Joints are identified with the letters A through M.



Geometry of the main truss.

Note that the dimensions of our structural model are all consistent with the dimensions shown for Truss 16 in the Gallery of Structural Analysis Results. The Gallery shows that each of the six top chord members has a length **L**. To achieve a total span length of 60cm, as the design requirements specify, we must use **L**=10cm. Now the remaining dimensions are calculated using this same value of **L**. For example, the Gallery shows the overall height of the truss as **1.375L**. Since we have defined L as 10cm, the height of our structural model is

$$Height = \mathbf{1.375L} = \mathbf{1.375}(\mathbf{10}cm) = \mathbf{13.75}cm$$

Once we have determined the geometry of the truss, we can calculate the loads. According to the design requirements, the bridge must be capable of safely carrying a 6-kilogram mass placed on the structure at mid-span. The weight of a 6-kilogram mass is

$$\mathbf{W} = \mathbf{mg} = (\mathbf{6}kg)\left(\mathbf{9.81}\frac{m}{\sec^{\mathbf{2}}}\right) = \mathbf{58.86}N$$

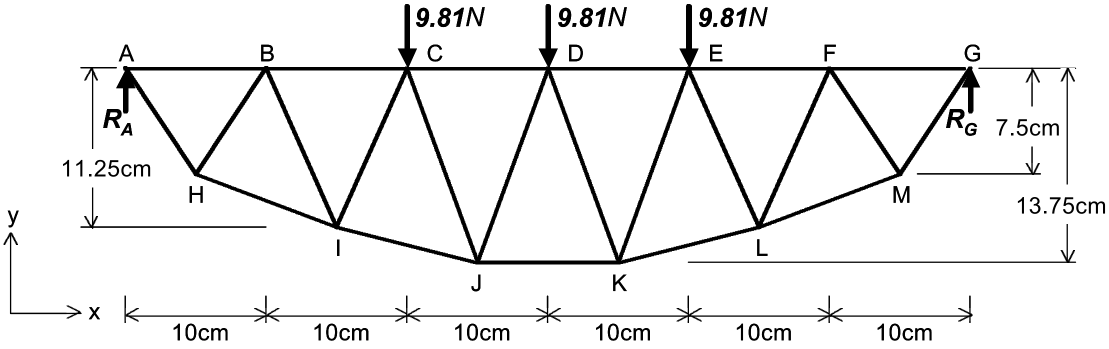
Again we will apply this load by placing a stack of books onto the top chord of the truss. The weight of the stack will be supported on six joints—C, D, and E on each of the two main trusses. Assuming that the weight of the books will be distributed equally to these six joints, the downward force applied to each joint is

$$Load\ per\ Joint = \frac{Total\ Load}{Number\ of\ Joints} = \frac{\mathbf{58.86}N}{\mathbf{6}} = \mathbf{9.81}N$$

Note that we could have gotten this same result directly from the Gallery of Structural Analysis Results. The diagram for Truss 16 shows that a downward load of **0.1667*W*** is applied to each of the three center top-chord joints. For a total load ***W****=58.86N*, the load at each joint is

$$Load\ per\ Joint = \mathbf{0.1667W} = (\mathbf{0.1667})(\mathbf{58.86}N) = \mathbf{9.81}N$$

A complete free body diagram of the truss looks like this:



Free body diagram of the main truss.

The bridge will be supported only at its ends; thus, the reactions R_A and R_G are shown at Joints A and G.

Check Static Determinacy and Stability

Before we can use the equations of equilibrium to analyze a truss, we must first verify that it is statically determinate and stable. As we saw in Learning Activity #3, the mathematical condition for static determinacy and stability is

$$2j = m + 3$$

where j is the number of joints and m is the number of members. Our structural model has 13 joints and 23 members. Substituting these numbers into the equation above, we find that $2j$ and $m+3$ are both equal to 26, so the mathematical condition for static determinacy and stability is satisfied.

Calculate Reactions

Now we can begin the structural analysis of our truss by calculating its unknown reactions. Since all loads and reactions act in the vertical direction, we can use the sum of forces in the y-direction (ΣF_y) to find the unknown reactions R_A and R_G.

$$\sum F_y = 0$$

$$R_A + R_G - 9.81 - 9.81 - 9.81 = 0$$

Since the structure, the loads, and the reactions are all symmetrical about the centerline of the truss, the two reactions R_A and R_G must be equal. Substituting $R_A = R_G$ into the equilibrium equation above, we get

$$R_A + R_A - 29.43 = 0$$

$$2R_A = 29.43$$

$$R_A = 14.7N \uparrow$$

And since $R_A = R_G$, then

$$R_G = 14.7N \uparrow$$

Note once again that we could have gotten this same result directly from the Gallery of Structural Analysis Results. The diagram for Truss 16 indicates that each reaction has a magnitude of **0.25*W***. For a total load ***W***=*58.86N*, each reaction is

$$0.25W = 0.25(58.86N) = 14.7N \uparrow$$

Calculate Internal Member Forces

At this point in the design process, we must determine the internal force in each member of the truss. As long as the truss is statically determinate, we can always calculate internal member forces by applying the Method of Joints, just as we did in Learning Activity #3. However, when we use a truss configuration from the Gallery of Structural Analysis Results, we can determine these forces with considerably less effort.

Each truss in the Gallery is presented with a complete set of internal member forces, calculated for the loading shown. The internal forces are shown in terms on the total applied load ***W***. To determine the internal member forces for our specific loading, we just substitute W=58.86N for each member. For example, the Gallery indicates that Member AB in our structural model has an internal force of ***-0.167W***. Therefore, the force in Member AB (F_{AB}) is

$$F_{AB} = -0.167W = -0.167(58.86N) = -9.83N = 9.83N \text{ (compression)}$$

Similarly, the Gallery shows that the internal force in Members CD and AH are ***–0.394W*** and ***+0.301W***. Therefore,

$$F_{CD} = -0.394W = -0.394(58.86N) = -23.2N = 23.2N \text{ (compression)}$$

$$F_{AH} = +0.301W = +0.301(58.86N) = +17.7N = 17.7N \text{ (tension)}$$

Recall that a minus sign indicates compression, while a plus sign indicates tension.

Q1

Can you calculate the remaining internal member forces?

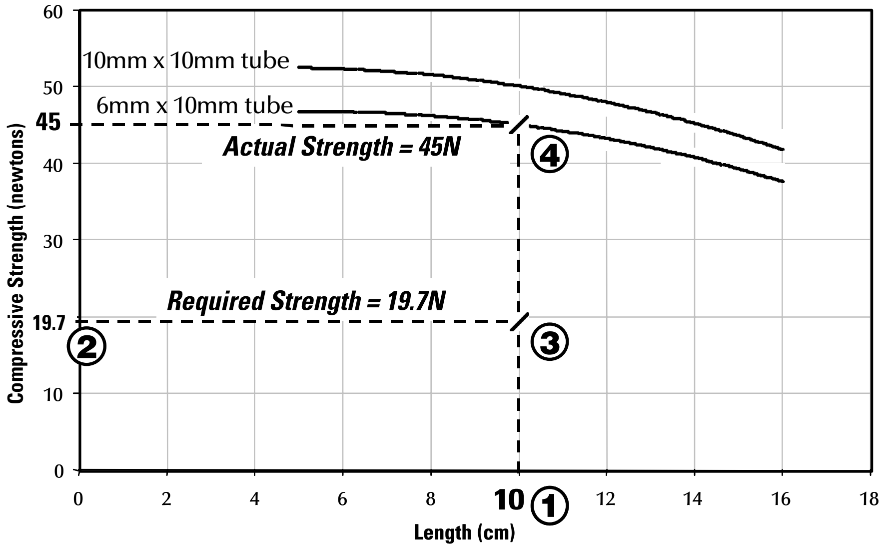
Use the Gallery of Structural Analysis Results to calculate the internal member forces for all remaining members in our truss. Use a total load W=58.86N.

Determine Member Sizes

Now we will determine the size of each member in our structure. Our objective is to ensure that each member is strong enough to safely carry its internal force. If the internal force is compression, we'll use a tube for the member. If the internal force is tension, we'll use a doubled bar, just as we did on the original Grant Road Bridge in Learning Activity #1.

Tubes

Member AB carries load in compression, so we will use a tube for this member. To determine the required size of the tube, we will use the compressive strength vs. length graph we created in Learning Activity #2. That graph is shown below.



Selecting the required tube size for Member AB.

Selecting the required tube size is a four-step process:

1) Determine the member length. Member AB is 10cm long.

2) Calculate the required strength, using the equation

$$\textit{Required Strength} = (\textit{Factor of Safety})(\textit{Internal Member Force})$$

The design requirements specify that the factor of safety will be 2.0, and above we determined that the internal member force F_{AB} is 9.83N (compression). So the required strength is

$$Required\ Strength = \mathbf{2.0}(\mathbf{F}_{AB}) = \mathbf{2.0}(\mathbf{9.83}) = \mathbf{19.7}N\ (compression)$$

This calculation tells us that Member AB must be a tube with a compressive strength of *at least* 19.7 newtons.

3) Now plot the point corresponding to **Length=10cm** and the **Strength=19.7N**, as shown above.

4) Finally determine the smallest available tube size that has a *strength larger than 19.7* for the *same length*. To do this, start at the point you plotted in Step 3, and draw a line straight upward to the closest strength curve. In this case, a 6mm x 10mm tube with a length of 10cm has a compressive strength of about 45 newtons—considerably greater than the required strength of 19.7N. Therefore, we can safely use a 6mm x 10mm tube for Member AB.

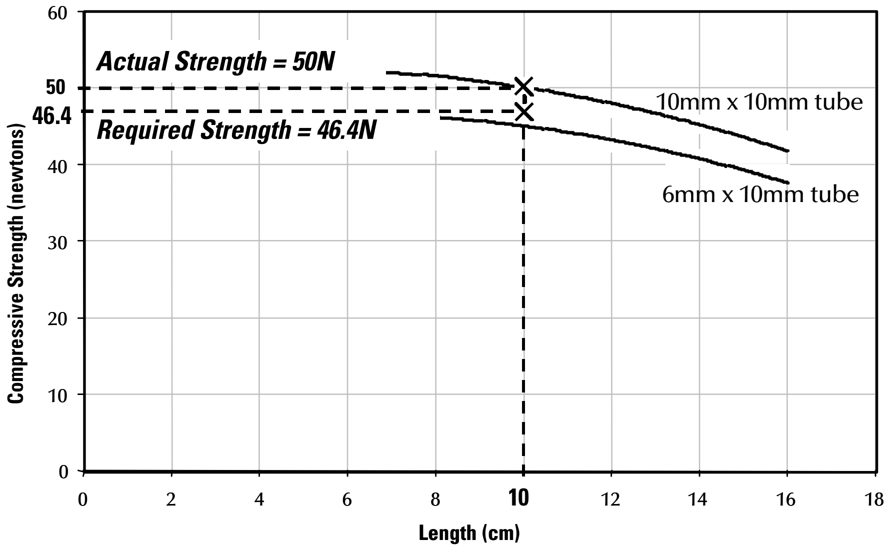
Note that we also *could* use a 10mm x 10mm tube for Member AB. With a compressive strength of about 50 newtons, this member is even stronger than the 6mm x 10mm tube—but quite a bit stronger (and more expensive) than it really needs to be. Note also that we could probably use a tube that is considerably smaller than 6mm x 10mm, except that we have no test data available for any smaller member sizes. Any tube with a compressive strength greater than 19.7N for a 10cm length would be perfectly acceptable for Member AB.

Whatever member size we decide to use for Member AB, we should use the same one for its twin, Member FG on the opposite side of the truss. Members AB and FG have the same internal force and the same length, so they should use the same member size. By taking advantage of symmetry in this manner, we can save a lot of work, because we only really need to determine member sizes for half of the members in the truss.

Now let's use the same procedure for Member CD. Like Member AB, the length of Member CD is 10cm. Its required strength is

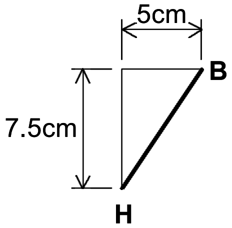
$$Required\ Strength = \mathbf{2.0}(\mathbf{F}_{CD}) = \mathbf{2.0}(\mathbf{23.2}) = \mathbf{46.4}N\ (compression)$$

When we plot **Length=10cm** and **Strength=46.4N** on the graph, the point falls *between* the two strength curves, as shown below. Therefore, the 6mm x 10mm tube, with its compressive strength of 45N, is *not* strong enough for Member CD. Only the 10mm x 10mm tube will work, because its actual strength of 50N exceeds to required strength. We will use a 10mm x 10mm tube for Member CD and for its twin, Member DE.



Selecting the required tube size for Member CD.

The sizes of the remaining compression members can be determined in exactly the same manner. The only aspect of this process that presents a new challenge is finding the lengths of the diagonal members. Consider Member BH as an example. If we visualize this member as the hypotenuse of a right triangle, as shown here, then we can use the Pythagorean theorem to find its length, BH:



$$BH = \sqrt{(7.5)^2 + (5)^2} = 9.01cm$$

This is Step 1 of the four-step process. Once you have determined the length BH, you can complete the remaining three steps exactly as we did above.

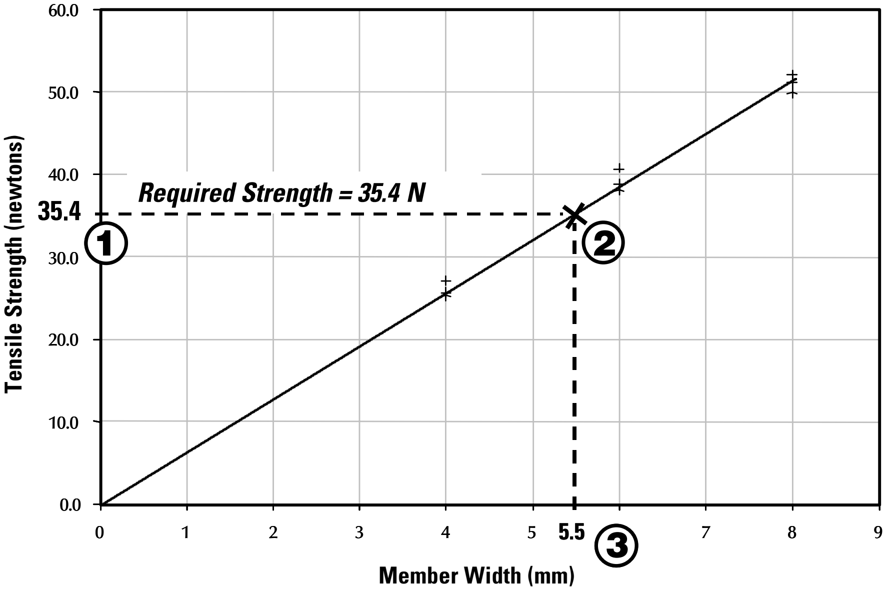
2

Can you determine the sizes of the remaining compression members?

Using our graph of compression strength vs. length, determine appropriate member sizes for Members BC, BH, CI, and CJ. Then use symmetry to determine the member sizes for all remaining compression members.

Tension Members

We will use bars for all members that carry load in tension. To determine the sizes of the bars, we will use the tensile strength vs. member width graph we created in Learning Activity #2.



Selecting the required width of Member AH.

Let's use Member AH as an example. The procedure for determining the size of this member is as follows:

1) Calculate the required strength just as we did for compression members:

$$\text{Required Strength} = \mathbf{2.0}(\mathbf{F}_{AH}) = \mathbf{2.0}(\mathbf{17.7}) = \mathbf{35.4}\text{N (tension)}$$

This calculation tells us that Member AH must have a tensile strength of *at least* 35.4 newtons.

2) Now start at **Strength=35.4N** on the vertical axis, and draw a horizontal line to the strength vs. width line, as shown above.

3) Then draw a vertical line *down* to the horizontal axis. This value, **5.5mm**, is the smallest width that will safely carry a tension force of 35.4N.

4) Finally, decide on the actual bar size you will use. Member AH must have a *total width* of at least 5.5mm. But since we are using *doubled bars* for all tension members, the width of each individual bar must be half of 5.5, or 2.75mm. In practice, it would be very difficult to measure and cut cardboard bars precisely 2.75mm wide, so let's round up to the next whole millimeter. We will use doubled 3mm-wide bars for Member AH.

Q3

Can you determine the sizes of the remaining tension members?

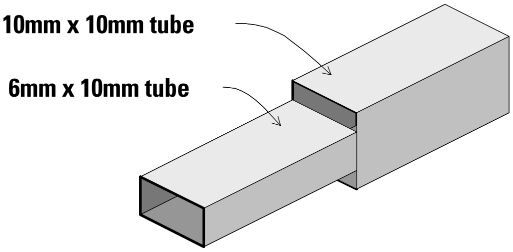
Using our graph of tensile strength vs. member width, determine appropriate member sizes for Members BI, HI, IJ, and JK. Then use symmetry to determine the member sizes for all remaining tension members.

Check Member Sizes for Constructability

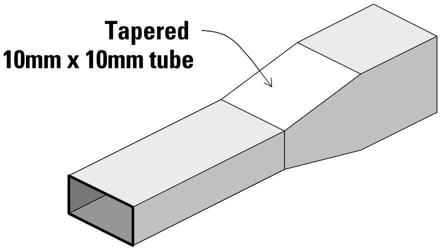
At this point in the design process, we have determined member sizes for the entire truss. For each member, we selected the smallest size tube or bar that can safely carry the corresponding internal force. In doing so, we have effectively minimized the *material cost* of the truss. However, as we saw in Learning Activity #4, minimizing the material cost does not necessarily minimize the *total cost* of the structure. Using many different member sizes might increase the costs of fabrication and construction, because it can sometimes be difficult to connect different sized members together.

We can see this situation in our own truss design. For the top chord, we have selected a 6mm x 10mm tube for members AB, BC, EF, and FG and a 10mm x 10mm tube for members CD and DE. Thus at Connections C and E we will have to join two different member sizes together, as shown at right. This will create some serious challenges for the Constructor. (That's you!)

When two compression members are spliced together, it is best for both tubes to be in contact with each other *on all four sides*, so the internal compression force can be effectively transmitted from one member to the other. Here only two of four sides are in contact. The only way we could ensure that all four sides are in contact would be to taper the



At Connections C and E, two different tube sizes need ot be joined.



The larger tube should be tapered, so that both tubes are connected along all four sides.

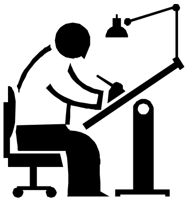
end of 10mm x 10mm tube, as shown. Building this joint will take a lot of time, and building it *well* will be quite difficult. Furthermore, recall that in Learning Activity #1, we attached the two main trusses together by placing them upside down on the lateral bracing subassembly, which was pinned to the building board. That procedure won't work if the top surface of the top chord is not entirely at the same elevation.

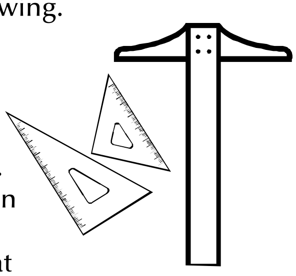
For these reasons, we can greatly simplify the construction of our bridge by using 10mm x 10mm tubes for *all* top chord members. As a result, Members AB, BC, EF, and FG will be considerably stronger—and somewhat more expensive—than they really need to be. But the benefits gained from using a single tube size for the entire top chord will greatly outweigh the small additional cost of using four slightly oversized members.

Draw Plans

Having decided what the size of each truss member should be, we're now ready to draw the plans. Specifically, we will create the full-scale layout drawing on which we will actually build the main trusses and lateral bracing subassembly for the Grant Road Bridge. In Learning Activity #1 this drawing was provided to you. Now you're smart enough to do it yourself!

Before you can create the drawing, get the necessary tools and supplies. First, you'll need a big sheet of paper—at least 30 centimeters wide and 65 centimeters long. Craft paper, shelf paper, or even wrapping paper will work fine. You'll also need a metric ruler, a sharp pencil, and an eraser to do the drawing.





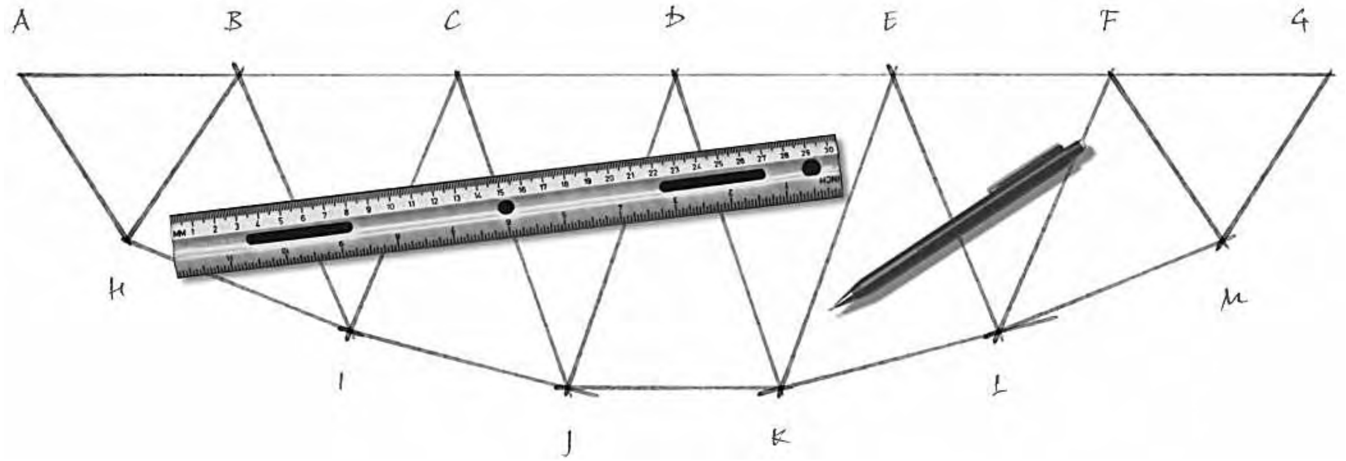
If available, a drawing board, a T-square, and some draftsman's triangles will also be very helpful. These tools will help you to draw parallel and perpendicular lines accurately. If these tools are not available, it would be a good idea to do your drawings on graph paper. Try to find graph paper that is large enough to do the entire drawing on one sheet. Your local office supply store might have large-size graph paper. If not, tape several standard-size sheets together, being very careful to ensure that all of the grid lines on adjacent sheets are aligned.

As an alternative, if you have access to appropriate computer-aided drawing software and know how to use it, you can use a computer to do the layout drawing. (The layout drawing we used in Learning Activity #1 was done entirely by computer.) But if you decide to use a computer, you must ensure that you have the right software and the right hardware to do the job. First, the software must be a true *technical drawing* or *computer-aided drafting* package like AutoCAD, IntelliCAD, or TurboCAD. Presentation graphics programs like PowerPoint are not appropriate for this job, because they don't provide the necessary degree of precision. Second, you must have access to a printer or plotter capable of producing a *full-scale* hard copy of your drawing.

Once all of the necessary supplies and tools are on hand, tape the paper to your drawing board or to a smooth flat tabletop. Now sharpen your pencil, and let's get to work.

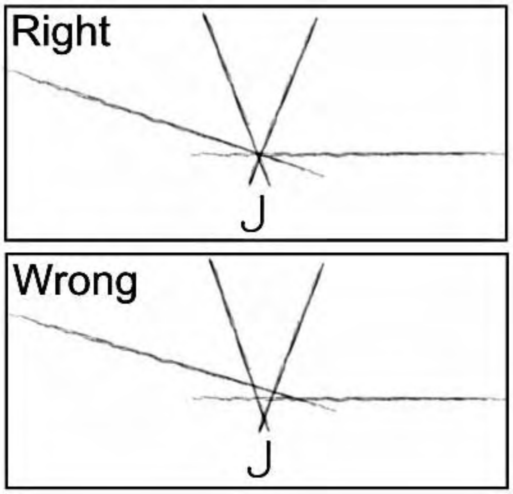
Lay Out Centerlines

We'll begin by drawing one main truss, as shown below. The drawing must be *exactly* full size—60cm long and 13.75cm high. Use the dimensions provided in the sketch on page 5-7 to ensure that all joints and members are at their correct positions. Label the joints with letters, as shown. Use only a single line for each member. These lines are the *centerlines* of the members.



Start the drawing by carefully laying out the centerlines of all members in one main truss.

It is critically important that all member centerlines intersect precisely at the joints, as shown at right. Otherwise, one of our basic assumptions about trusses will be violated. When we analyze a truss, we assume that its members carry load primarily in tension or compression; but if the centerlines do not all intersect at a common point, the members will *bend* when the truss is loaded. Bending may cause some members to fail at a lower load than we designed them for. So the performance of our structure will depend heavily of the accuracy of the member centerlines. Draw them with care!

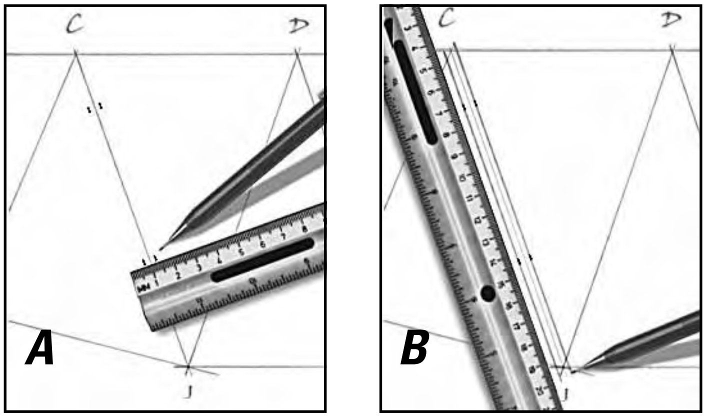


Member centerlines should intersect at the joints.

Draw Members

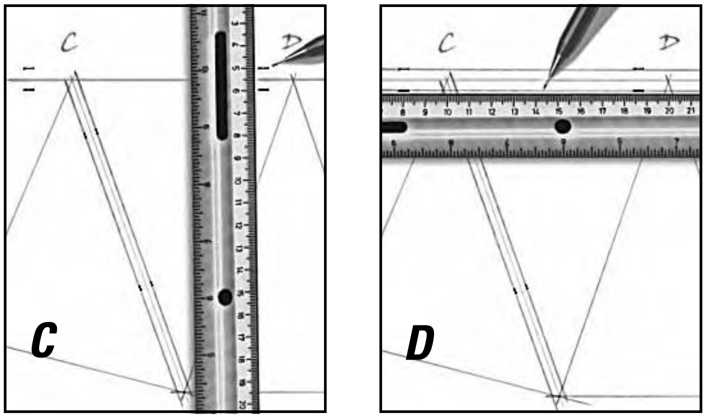
Now we will draw the truss members, using the member sizes we determined earlier in the design process.

Let's use Member CJ as an example. Begin by placing your ruler perpendicular to the member centerline, as shown in **A**. Make two pencil marks to indicate the actual width of the member. Since Member CJ is a 6mm x 10mm tube, the two marks should be 6mm apart. Each mark should be 3mm from the centerline. Then make an identical pair of marks at the opposite end of the same member. Finally, use your ruler to draw two parallel lines connecting the marks, as shown in **B**. These two lines represent the outer edges of Member CJ. If you measure accurately and draw your lines carefully, the centerline will be exactly midway between the two edges.



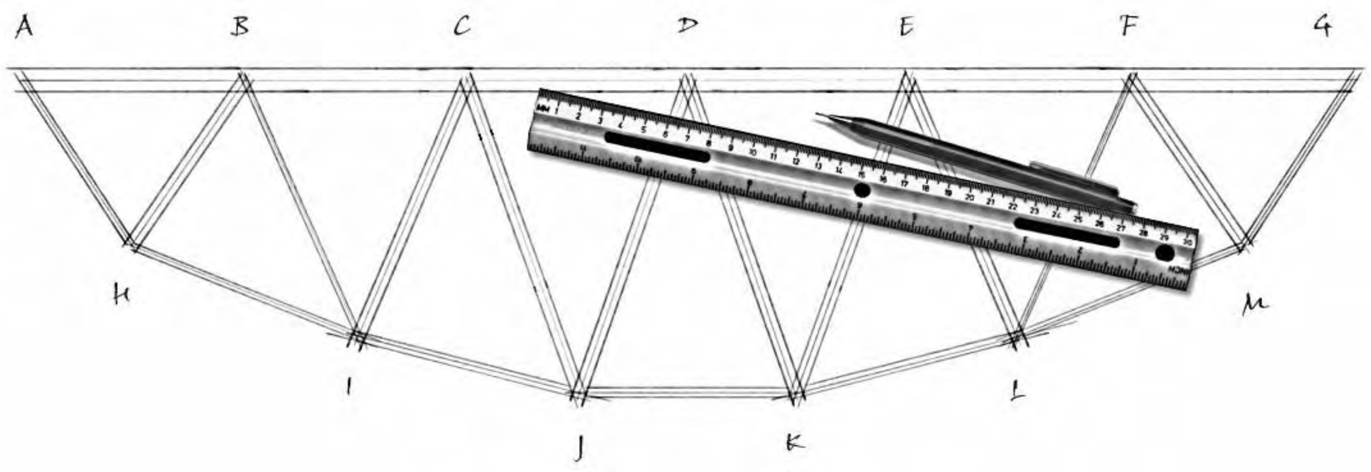
To draw a member, first mark its width (A), then draw two parallel lines (B).

Now let's follow this same procedure to draw Member CD, a top chord. Since this member is a 10mm x 10mm tube, the pairs of pencil marks shown in **C** are 10mm apart, with each mark located 5mm from the centerline. Again, we draw the edges of the member by "connect the dots" with two parallel lines, as shown in **D**. Since the entire top chord is made of identical 10mm x 10mm tubes, we can actually draw the edges of the chord as two continuous lines running all the way from Joint A to joint G.



Use the same procedure to draw the top chord.

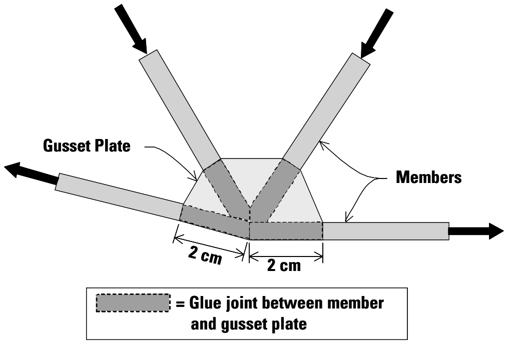
With all members drawn, the truss layout should look like this.



Layout drawing of the main truss with all members completed.

Draw Gusset Plates

A gusset plate is a structural element that connects two or more members together at a joint. For a truss to carry load safely, the connections between members and gusset plates should be stronger than the members themselves. In our cardboard model, we can achieve sufficient strength by connecting each member and gusset plate with a glue joint at least 2 centimeters long, as shown at right. These glue joints are very similar to welds in an actual steel connection. By using 2 centimeters as our standard "weld length," we will ensure not only that the connections are sufficiently strong, but also that the gusset plates look reasonably realistic.



The glue joint connecting a member to a gusset plate should be at least 2 centimeters long.

On an Actual Bridge Project

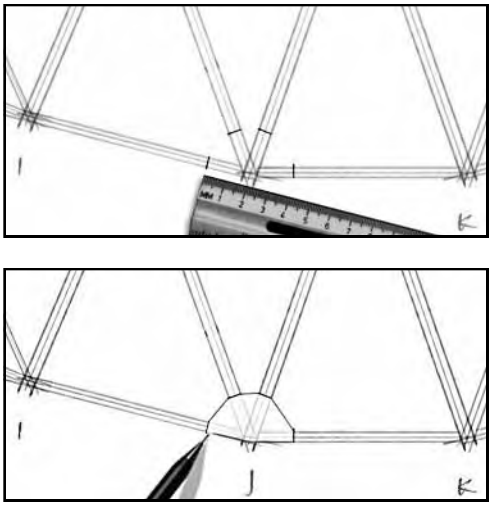
How are gusset plate connections designed?

On modern structures, members are connected to gusset plates with bolts or welds. The strength of a bolted connection depends on the number of bolts used. The strength of a welded connection depends on the length of the weld. To design a connection, the engineer first determines how much force the connection must be able to carry safely. Based on this force, the engineer determines the required number of bolts or the required length of the weld. Finally the gusset plate is sized, so that it is large enough to accommodate the required number of bolts or the required length of weld.

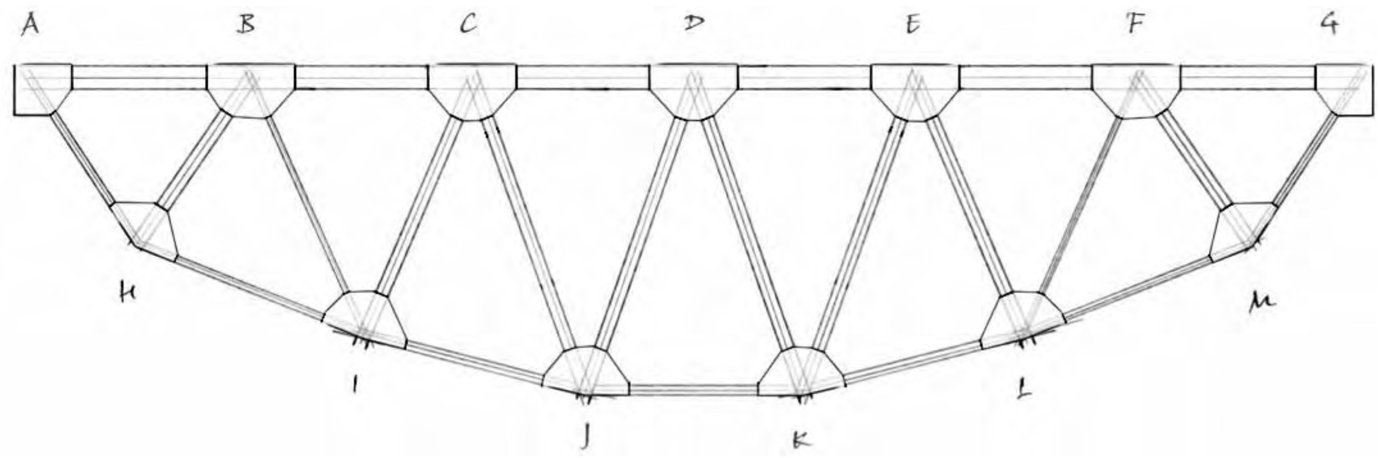
As an example, let's lay out the gusset plate for Joint J. This connection joins Members IJ, JK, CJ, and DJ together. To start the layout, measure 2 centimeters from the center of the joint (the point where the centerlines intersect) outward along the centerline of Member HI, and make a pencil mark perpendicular to the centerline. Do the same for the other three members, as shown.

Now connect the four marks with a series of straight lines. The gusset plate for Joint J is now complete.

Repeat this process for the remaining joints. The completed main truss layout drawing should look like the illustration on the following page.



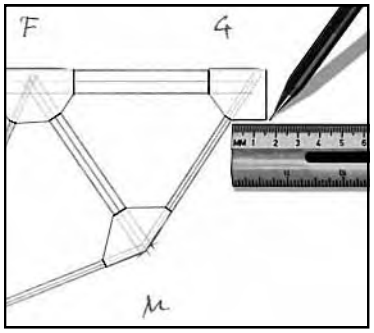
Laying out a gusset plate.



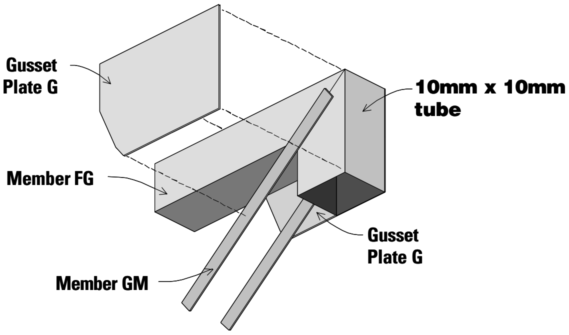
Layout drawing of the main truss with all members completed.

Note that the gusset plates at Joints A and G have been squared off on the bottom and outside edges. The bottom edges of these two gusset plates will serve as the supports for the bridge. These edges should be perfectly horizontal and about 1.5 centimeters long, as shown below (left).

These gusset plates alone won't be strong enough to support the bridge. Recall that we calculated the reactions at Joints A and G as 14.7 newtons each. Since these reactions are directed upward, they will cause compression in the gusset plates at A and G. These gussets are just flat sheets of cardboard; thus they will buckle with only a slight application of compressive force. To keep them from buckling, we'll need to reinforce them with a short section of 10mm x 10mm tube oriented vertically, as shown below (right).



The Gusset plates at Joints A and G also serve as supports.

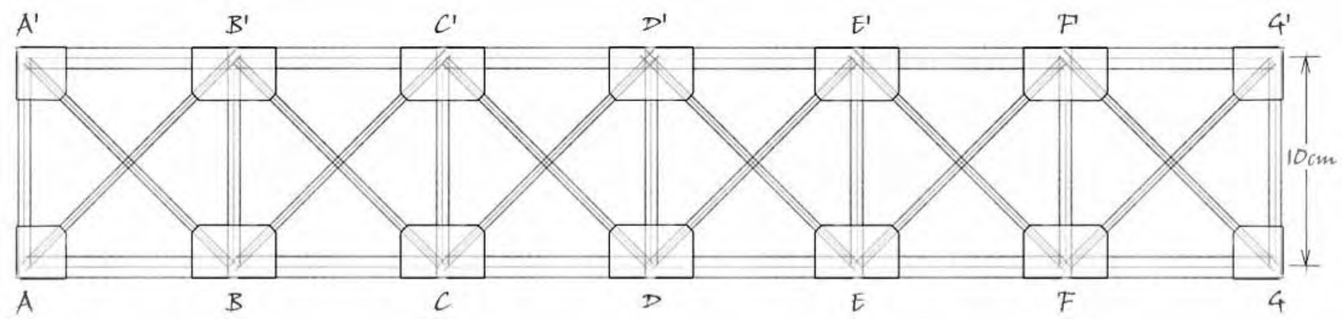


Reinforce the support with a short length of 10mm x 10mm tube oriented vertically.

You may recall that the layout drawing we used in Learning Activity #1 included two identical copies of the main truss layout. With two copies available, we were able to build both halves of each truss simultaneously, resulting in a considerable time saving during construction. Of course, the drawing you used in Learning Activity #1 was done with a computer, so the second copy of the truss was easily created and guaranteed to be identical to the first. If you are doing your layout drawing by hand, it is best to use just one truss layout. If you try to draw a second copy by hand, it probably won't be identical to the first, and the two halves of the truss won't line up correctly. Just remember that, if you draw only one truss layout, you'll need to use it four times—once for each half of the two main trusses.

Lay Out Top Lateral Bracing

The layout drawing for our design must also include the top lateral bracing subassembly, shown below. Begin by drawing the member centerlines, as we did for the main trusses. The centerlines of the two top chords should be 10 centimeters apart. This will ensure that the total width of the bridge, from one outside edge to the other, will be 11 centimeters—the roadway width specified in the design requirements. Because this bridge is a deck truss, the struts (Members AA', BB', CC', and so forth) must also serve as floor beams. Thus we will use 6mm x 10mm tubes for these members, rather than the 6mm x 6mm tubes used for the struts in the original Grant Road Bridge. We'll use single 3mm bars for the diagonal bracing.



Top Lateral Bracing Layout

Transfer Gusset Plates

Now that the layout drawing is complete, we'll need to transfer the gusset plate outlines to file-folder cardboard without cutting up the drawing. The easiest way to do this is with tracing paper. Place a sheet of tracing paper on top of the layout drawing, and trace the outline of each gusset plate onto the tracing paper. Then photocopy the tracing paper to transfer the gusset plate outlines onto cardboard, as described in Learning Activity #1 (Page 1-24).

As an alternative, you can also use carbon paper to do the transfer. Place a file folder *beneath* the layout drawing, with a sheet of carbon paper face down between the drawing and the file folder. Then carefully trace over the outline of each gusset plate to transfer it directly to the cardboard.

Actually, you don't really need to transfer *every* gusset plate. On the main truss layout, note that Joint A is identical to Joint G; B is identical to F; H is identical to M; and so forth. You can transfer *only* the gusset plates at Joints A, B, C, D, H, J, and I, then make a set of identical copies for the corresponding joints on the opposite side of the truss. Similarly, on the lateral bracing subassembly, you only need to transfer the gusset plates at Joints A and B. All remaining gusset plates in the subassembly are identical to one or the other of these two.

Create a Schedule of Truss Members and a Schedule of Gusset Plates

The Schedule of Truss Members is an important part of the plans and specifications for our bridge design. It summarizes the type, size, and length of every structural member in the bridge and thus serves as an important reference for the Constructor. In formulating the schedule, we note that the 10mm x 10mm top chord is perfectly straight from one end of the bridge to the other. Thus we could actually make each top chord from a single tube—if we could find a file folder 62 centimeters long. We can't, of course, so we'll have to build each top chord in three segments, each about 21 centimeters long. (If you use a legal size file folder, you can make the top chord in only two segments.)

Component	Members	Type	Approx. Length	# Req'd
Bottom Chords	HI, LM, H′I′, L′M′	3mm bar (double)	11cm	8
Bottom Chords	IJ, JK, KL, I′J′, J′K′, K′L′	4mm bar (double)	11cm	12
Diagonals	AH, GM, A′H′, G′M′	3mm bar (double)	10cm	8
Diagonals	BI, FL, B′I′, F′L′	2mm bar (double)	13cm	8
Top Lateral Bracing	AB′, A′B, BC′, B′C, CD′, C′D, DE′, D′E, EF′, E′F, FG′, F′G	3mm bar (single)	12cm	12
Top Chords	AC, CE, EG, A′C′, C′E′, E′G′	10mm x 10mm tube	21cm	6
Diagonals	BH, FM, B′H′, F′M′	6mm x 10mm tube	9cm	4
Diagonals	CI, EL, C′I′, E′L′	6mm x 10mm tube	13cm	4
Diagonals	CJ, EK, C′J′, E′K′	6mm x 10mm tube	15cm	4
Diagonals	DJ, DK, D′J′, D′K′	6mm x 10mm tube	15cm	4
Floor Beams	AA′, BB′, CC′, DD′, E E′, FF′, GG′	6mm x 10mm tube	10cm	7
Bottom Struts	HH′, II′, JJ′, KK′, LL′, MM′	6mm x 6mm tube	10cm	6

The lengths provided in the Schedule of Truss Members are only approximate. The Constructor will cut them to their exact lengths as the trusses are built.

The Schedule of Gusset Plates shows the number of each type of gusset plate that will be used for the truss connections.

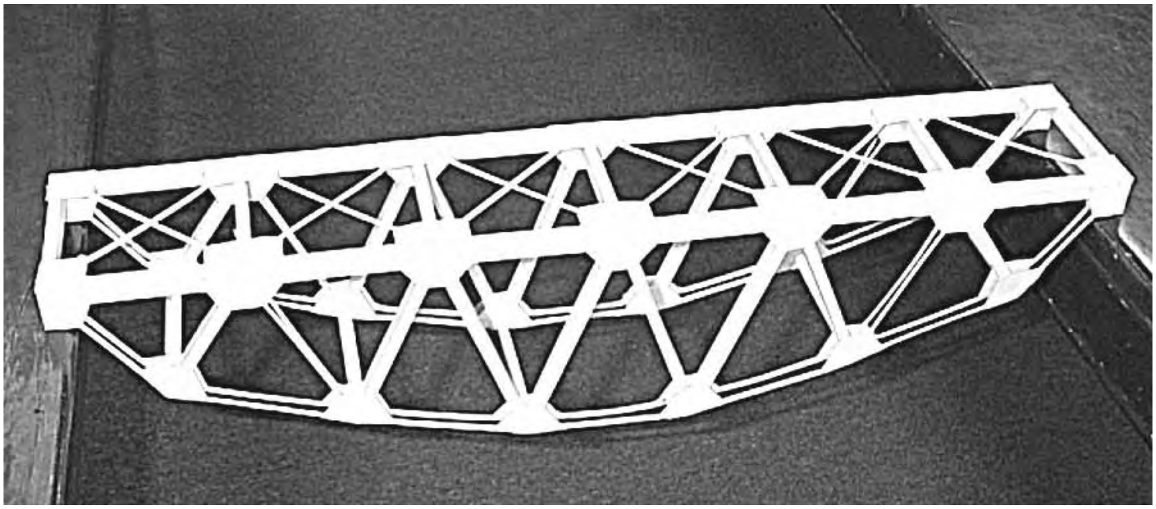
Connection	# Req'd
A, A′, G, G′	8
B, B′, F, F′	8
C, C′, E, E′	8
D, D′	4
H, H′, M, M′	8
I, I′, L, L′	8
J, J′, K, K′	8
A, A′, G, G′ (top)	4
B, B′, C, C′, D, D′, E, E′, F, F′ (top)	10

Build the Bridge

The plans and specifications for the new Grant Road Bridge are complete. If this were a traditional *design-bid-build* project, your work as the Design Professional would be mostly done. You would present your completed design to the Owner, who would then procure a Constructor through competitive bidding. But because we are using *design-build* project delivery, you have full responsibility for *both* design and construction.

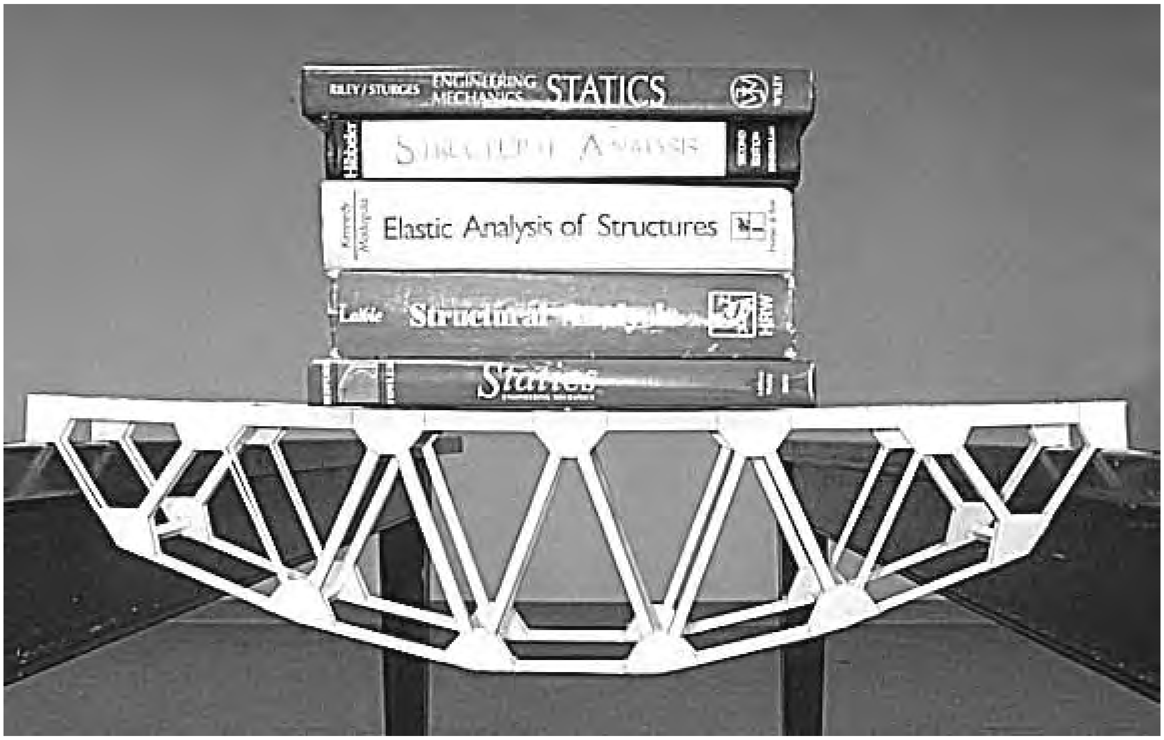
There's no time to waste. The residents of Hauptville are anxious to have Grant Road reopened to traffic. It's up to you to make it happen.

To build the new Grant Road Bridge, follow the same procedure we used in Learning Activity #1. The result should look like the photo below.



The completed bridge model.

Once you have completed your quality control inspection, place the bridge on two desks positioned 58 centimeters apart. Place six coins on the top chord at Connections C, C′, D, D′, E, and E′. Then place books one at a time on top, until the total mass of the stack is 6 kilograms. The new Grant Road Bridge is now open for traffic. Congratulations of the successful completion of the project!



The completed Grant Road Bridge, with 6-kilogram loading in place.

Q4

How are math, science, and computer technology used in the engineering design process?

How did mathematical tools, scientific principles, and computer technology contribute to the creation of our design for the Grant Road Bridge? Give at least two examples in each area.

Conclusion

In this learning activity, we "put it all together." We applied scientific principles, mathematic tools, engineering concepts, experimental data, and practical considerations to design a truss bridge. Our design was carefully tailored to meet the needs of the Town of Hauptville, to be safe, to be constructable, and to be aesthetically pleasing. We validated the design by building and testing it—and it worked!

We conclude this project—and this book—with a strong sense of accomplishment, not only for the bridge we designed and built, but also for the skills we acquired along the way. Learning engineering isn't easy, but learning engineering is worth every ounce of effort you put into it. To learn engineering is to open a new door—one that leads to a world of limitless possibilities for creative accomplishment and service to society. The door is now open. But how do you enter?

Every bridge begins in the mind of an engineer. There's probably one in your mind right now. Build it!

Answers to the Questions

1) **Can you calculate the remaining internal member forces?** The table below provides all internal member forces for the truss.

Members	Internal Force (newtons)
AB, FG	9.83 (compression)
BC, EF	19.6 (compression)
CD, DE	23.2 (compression)
BH, FM	10.6 (compression)
CI, EL	9.65 (compression)
CJ, EK	1.06 (compression)
DJ, DK	5.24 (compression)
AH, GM	17.7 (tension)
HI, LM	16.8 (tension)
IJ, KL	24.3 (tension)
JK	25.0 (tension)
BI, FL	9.65 (tension)

2) **Can you determine the sizes of the remaining compression members?** The table below shows the tube sizes you should get if you follow the four-step process outlined on page 5-10 for compression members. The two intermediate computations—required strength and member length—are included as well.

Members	Req'd Strength (newtons)	Length (cm)	Member Size
AB, FG	19.7 (compression)	10	6mm x 10mm tube
BC, EF	39.2 (compression)	10	6mm x 10mm tube
CD, DE	46.4 (compression)	10	10mm x 10mm tube
BH, FM	21.2 (compression)	9.01	6mm x 10mm tube
CI, EL	19.3 (compression)	12.3	6mm x 10mm tube
CJ, EK	2.11 (compression)	14.6	6mm x 10mm tube
DJ, DK	10.5 (compression)	14.6	6mm x 10mm tube

3) **Can you determine the sizes of the remaining tension members?** The table below shows the member widths you should get if you follow the four-step process outlined on page 5-13 for tension members. The two intermediate computations—required strength and required width—are included as well.

Members	Req'd Strength (newtons)	Req'd Width (cm)	Member Size
AH, GM	35.4 (tension)	5.5	3mm bar (doubled)
HI, LM	33.6 (tension)	5.2	3mm bar (doubled)
IJ, KL	48.5 (tension)	7.5	4mm bar (doubled)
JK	49.9 (tension)	7.7	4mm bar (doubled)
BI, FL	19.3 (tension)	3.0	2mm bar (doubled)

4) How are math, science, and computer technology used in the engineering design process?

We used the following mathematical tools to develop our design:

- The Pythagorean Theorem – to find the lengths of diagonal truss members.
- Trigonometry – to write equations of equilibrium in our structural analysis.
- Algebra – to solve equilibrium equations for unknown internal member forces.
- Vectors – to represent forces in our structural model.

We applied the following scientific principles and concepts to develop our design:

- Force – to represent loads, reactions, and internal member forces.
- Compression and tension – to represent the direction of internal member forces.
- Equilibrium – to calculate reactions and internal member forces.
- Relationship between force and mass – to convert the load from kilograms to newtons.

We used computer technology to develop our design, as follows:

- Spreadsheet – to analyze and graph the experimental data from strength testing.
- The West Point Bridge Designer – to find an efficient truss configuration.
- Computer-aided drafting software – to draw the plans (an alternative to drawing by hand).

Some Ideas for Enhancing This Learning Activity

The best way to enhance this learning activity is to provide students with many and varied opportunities to design and build bridges. If the students do not have the algebra or trigonometry background necessary to do a truss analysis using the Method of Joints, then their choices for truss configurations must necessarily be limited to those provided in the Gallery of Structural Analysis Results (Appendix B). If students do have the mathematical skills to apply the Method of Joints, then they can design bridges of practically any span or configuration.

It can be particularly effective to combine Learning Activities #4 and #5 into a single project. Students begin by designing a 24-meter bridge with the West Point Bridge Designer. Once they have found an efficient or interesting truss configuration, they design and build a 1/40th scale model of the same structure, using the procedures described in Learning Activity #5. This two-phase project uses the computer in the role for which it is best suited—exploring many different design alternatives relatively efficiently. But it also ensures that students do not merely use the computer as a "black box." By working though the design of the 1/40th scale model manually, they gain an appreciation for the challenging mathematical calculations the computer performs so effortlessly.

A

Appendix A

A Gallery of Truss Bridges

Description

The Gallery of Truss Bridges is a collection of photographs showing thirty truss bridges from all over the United States. These bridges represent a wide variety of configurations and sizes. Each photo is identified with the name and location of the bridge. In a few cases, two photographs of the same bridge are provided. In these cases, the second photo contains an interesting detail or alternate view of the structure.

All of these photographs were obtained from the Historic American Engineering Record (HAER), a division of the National Park Service, U. S. Department of the Interior. HAER is a long-range program to document significant engineering and industrial works in the United States with measured and interpretive drawings, photographs, and written histories.

How to Use this Resource

Here are a few suggestions for using the Gallery of Truss Bridges to enhance your understanding of structures and engineering design:

- Select any bridge from the Gallery, and identify its major component parts, using the illustration on page 1-3 as a guide. (Learning Activity #1)
- Identify the configuration of each bridge, using the illustration on page 1-6 as a guide. Also note whether each bridge is a *through truss*, *deck truss*, or *pony truss*. (Learning Activity #1)
- Select a bridge from the Gallery, then model it with the West Point Bridge Designer software, and optimize your design. (Learning Activity #4)
- Select a bridge from the Gallery, then design, build, and test a cardboard model of it. (Learning Activity #5)

1. State Highway Bridge No. 16 over the Kickapoo River, Vernon County, WI

2. Rock Island Bridge, Rock Island Arsenal, IL

3. Albion Bridge over Blackstone River, Cumberland, RI

4. Railroad Bridge over Lin Branch Creek, Clinton County, MO

5. **Hamden Bridge over Raritan River, Hunterdon County, NJ**

6. **State Rt. 256 Bridge over Middle River, Augusta County, VA**

7. **Denver South Park & Pacific Railroad Bridge, Romley, CO**

8. **Harrison Road Bridge over Great Miami River, Miamitown, OH**

9. **Wabash River Bridge, State Rt. 316, Wells County, IN**

10. Cheyenne River Bridge, County Rd. 46, Niobrara County, WY

11. New Fork River Bridge, County Rd. 136, Sublette County, WY

12. Gospel Street Bridge, over Lick Creek, Paoli, IN

13. Boston & Maine Railroad Bridge over Connecticut River, Northampton, MA

14. Boston & Maine Railroad Bridge, with State Route 9 highway bridge in background, Northampton, MA

15. Arvada Bridge over Power River, County Rd. No. 38, Sheridan County, WY

16. Baltimore & Ohio Railroad Bridges, Cattaraugus County, NY

17. Big Island Bridge over Green River, County Road No. 4, Sweetwater County, WY

18. Bridge over Blackledge River, Colchester, New London County, CT

19. **Enterprise Bridge over Smoky Hill River, Highway K-43, Enterprise, Dickinson County, KS**

20. **Four Mile Bridge over Big Horn River, County Road No. 173, Hot Springs County, WY**

21. **Main Street Bridge over Yellow Creek, Poland, OH**

22. **North Platte River Bridge, Fort Laramie, Goshen County, WY**

23. **Shell Creek Bridge, County Road 57, Big Horn County, WY**

24. **Powder River Bridge, County Rd. 269, Sheridan County, WY**

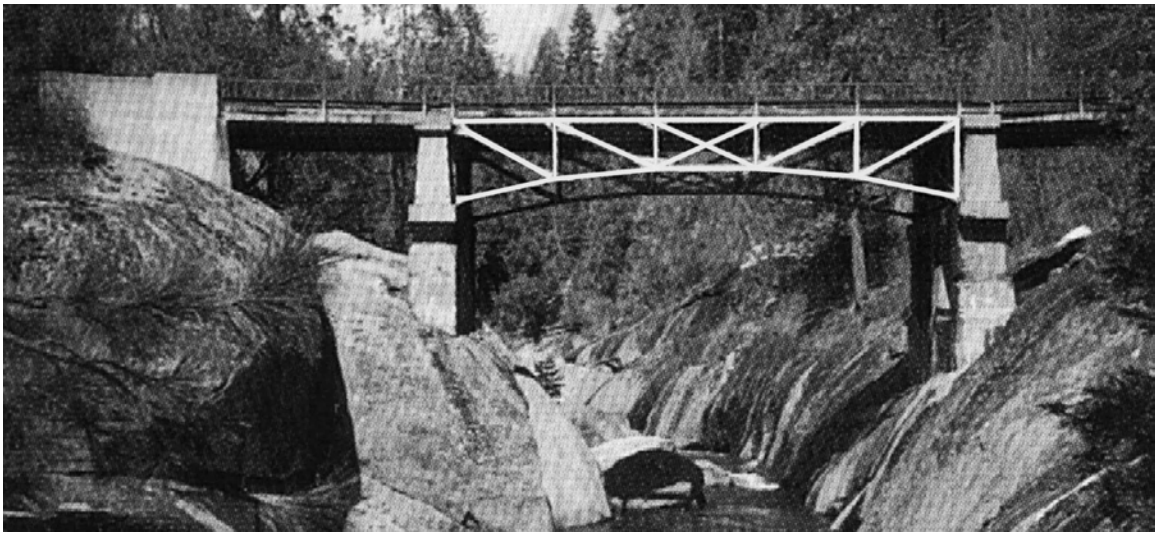
25. Roaring Run Bridge, State Route 657, Bedford County, VA

26. Powder River Bridge, Arvada, Sheridan County, WY

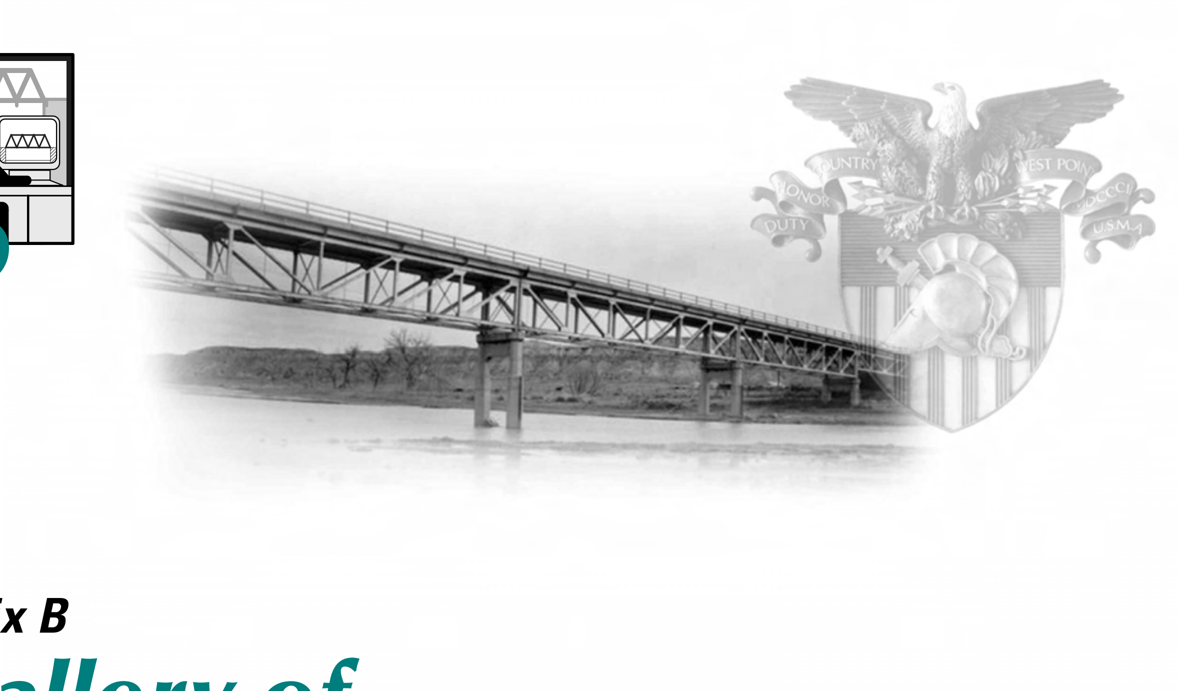
27. Shoshone River Bridge, County Road 111, Big Horn County, WY

28. Wind River Bridge, Wyoming Highway 132, Fremont County, WY

29. Navaho Bridge, Lees Ferry, AZ



30. Marble Fork Bridge, Sequoia National Park, CA



Appendix B

A Gallery of Structural Analysis Results

Description

The Gallery of Structural Analysis Results provides internal member forces for 18 different truss configurations. All of these trusses are statically determinate, and all have a span length of ***6L***—six panels of equal length ***L***. In all cases, the loading is assumed to be a weight, ***W***, placed at the center of the span. The weight is placed on the top chord of the bridge, except in two cases where the rounded shape of the top chord provides no flat surface on which to place the load. In these cases (Trusses 5 and 11), the loading is applied to the bottom chord. Each truss is assumed to be one of two main trusses in a bridge; thus each truss carries exactly half of the load ***W***. All internal member forces are given in terms of ***W***, so the structural analysis results can be easily determined for any magnitude of load.

How to Use this Resource

Here are a few suggestions for using the Gallery of Structural Analysis Results to enhance your understanding of structures and engineering design:

- Select any bridge configuration from the Gallery, and calculate its internal member forces using the Method of Joints. Then use the analysis results provided in the Gallery to check your work. (Learning Activity #3)
- Compare the analysis results from various truss configurations in the Gallery to determine how the height and shape of a truss affect its internal member forces. (Learning Activity #4)
- Select a bridge from the Gallery, then design, build, and test a cardboard model of it. To determine the internal member forces—an essential part of the design process—use the Gallery in lieu of the Method of Joints. (Learning Activity #5)

How to Determine Internal Member Forces

To determine the internal member forces for a given truss:

1) Select a truss from the Gallery of Structural Analysis Results.
2) Decide on the total amount of weight that the bridge will carry. This is ***W***. Note that the total load applied to each truss is only half of the total weight *(**0.5W**)*. The bridge is assumed to consist of two main trusses; therefore, each truss carries half of the total load.
3) Next to each member on the diagram, you will see a decimal number. For each member, multiply the decimal number by the value of ***W*** you determined in Step 2. This product is the internal member force, expressed in the same units you used for ***W***. If the decimal value is positive, the member is in tension. If it is negative, the member is in compression.

For example, let's assume that Truss 1 (on the following page) has a total load ***W***=10 newtons. Then the bottom chord member on the left-hand side of the truss has an internal force of

$$F = +0.333W = +0.333(10\ N) = +3.33\ N = \underline{\underline{3.33\ N\ (tension)}}$$

For ***W***=10 newtons, the two top-chord members at the center of the span have an internal force of

$$F = -0.778W = -0.778(10\ N) = -7.78\ N = \underline{\underline{7.78\ N\ (compression)}}$$

Note that the internal member forces do not depend on the length ***L***. They are valid for a truss of *any* span—as long as the span length and height of the truss are in the proportions shown on the diagram.

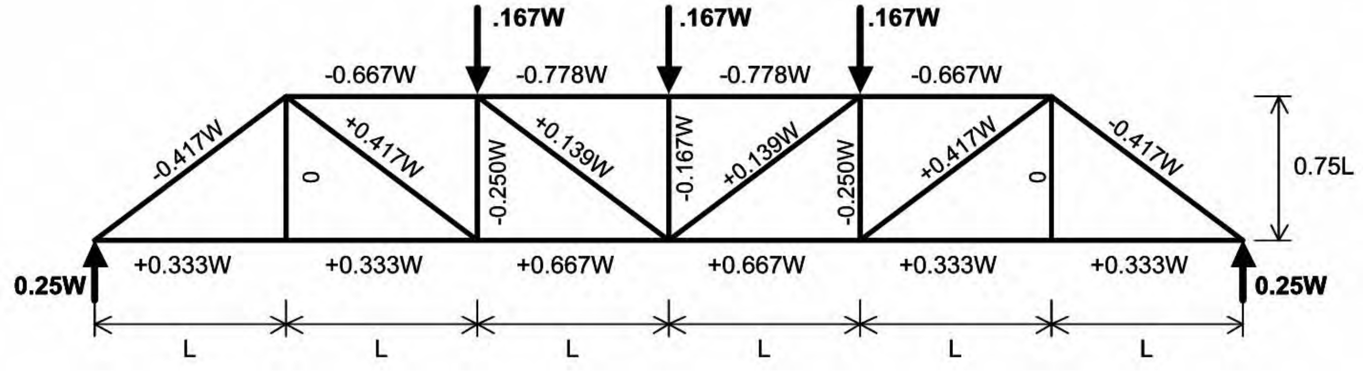
To determine ***L***, divide the total span length of the bridge by six. Once you know ***L***, you can use this number to determine the actual height of the truss.

Again, as an example, let's use Truss 1 and assume that its total span length is 60 centimeters. Then

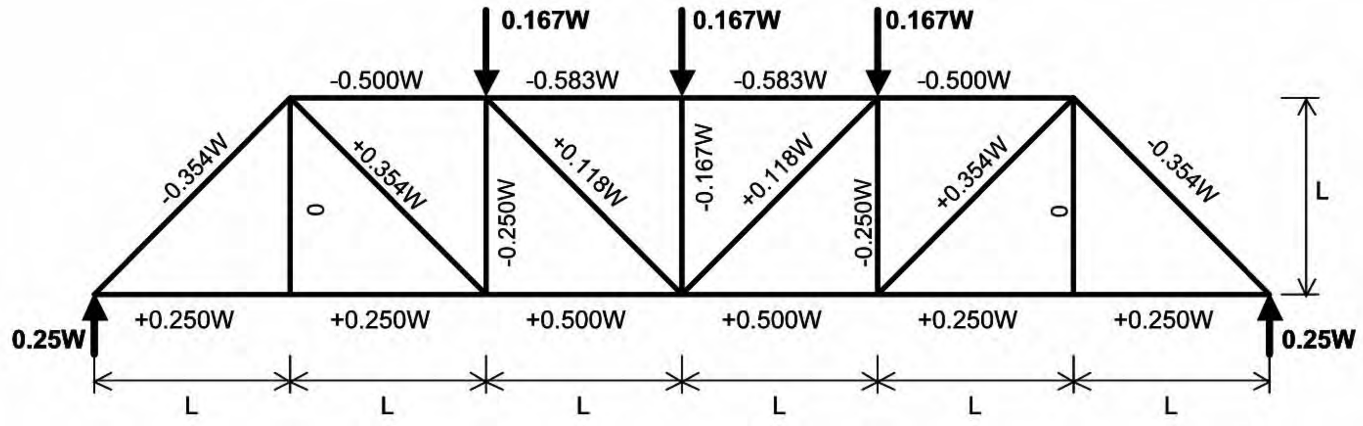
$$L = \frac{Total\ Span}{6} = \frac{60\ cm}{6} = \underline{\underline{10\ cm}}$$

On the picture, the height of Truss 1 is shown as ***0.75L***. Therefore the actual height is

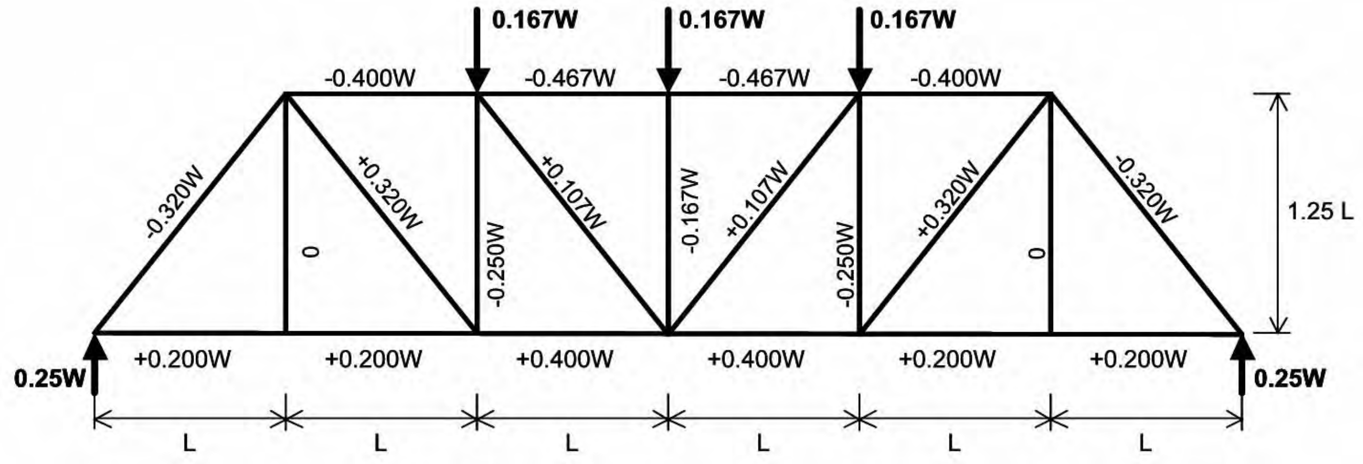
$$Height = 0.75L = 0.75(10\ cm) = \underline{\underline{7.5\ cm}}$$



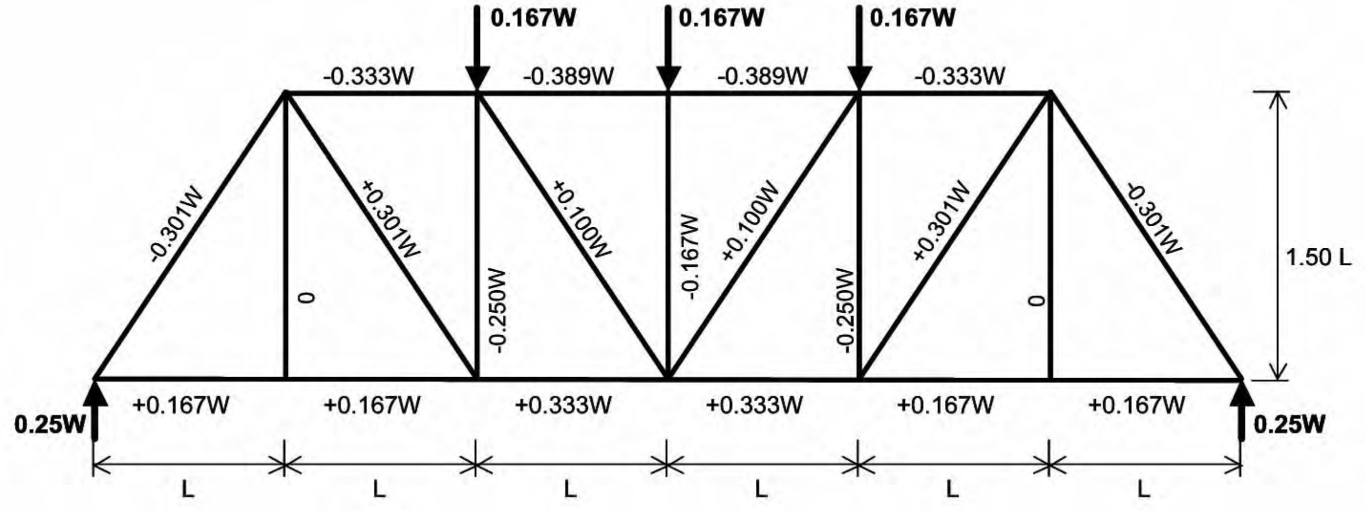
1. Pratt Through Truss (Span=6L, Height=0.75L).



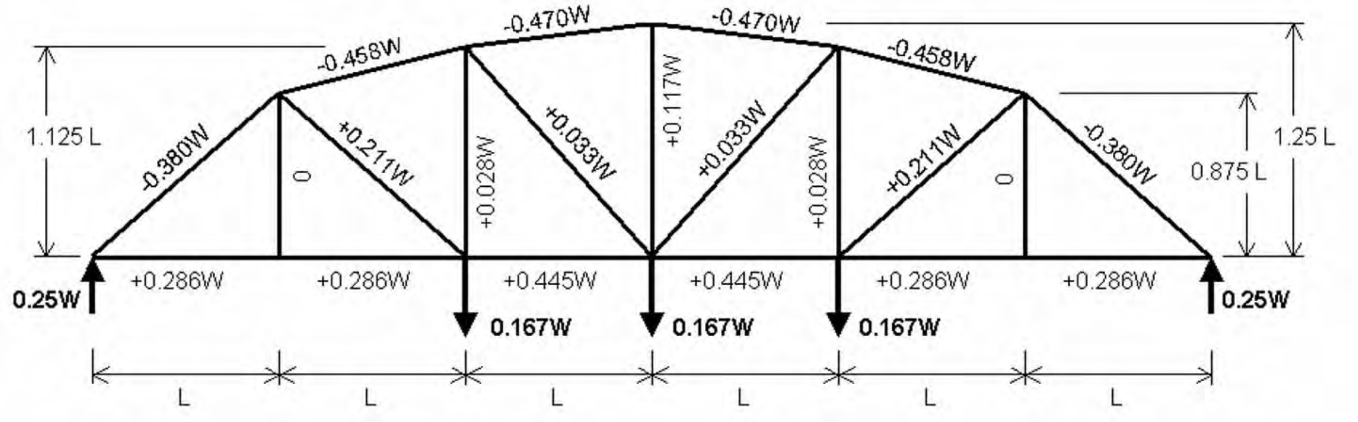
2. Pratt Through Truss (Span=6L, Height=L).



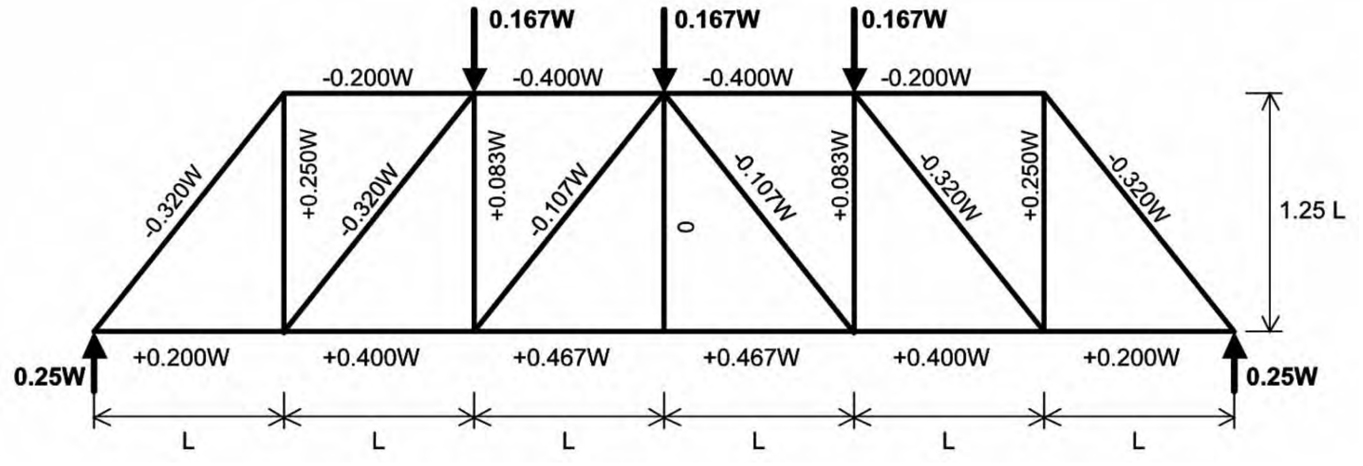
3. Pratt Through Truss (Span=6L, Height=1.25L).



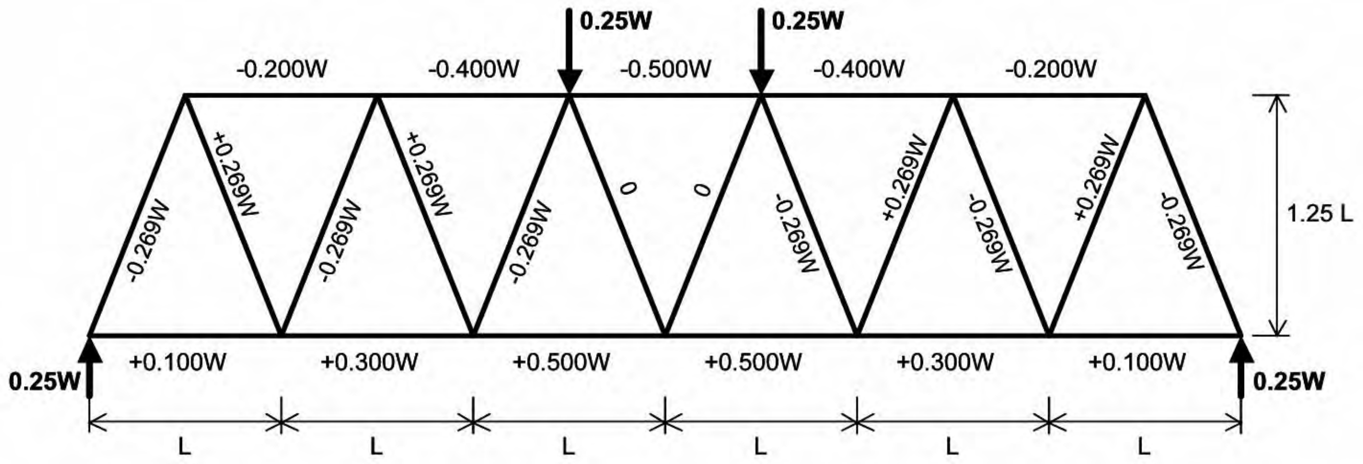
4. Pratt Through Truss (Span=6L, Height=1.50L).



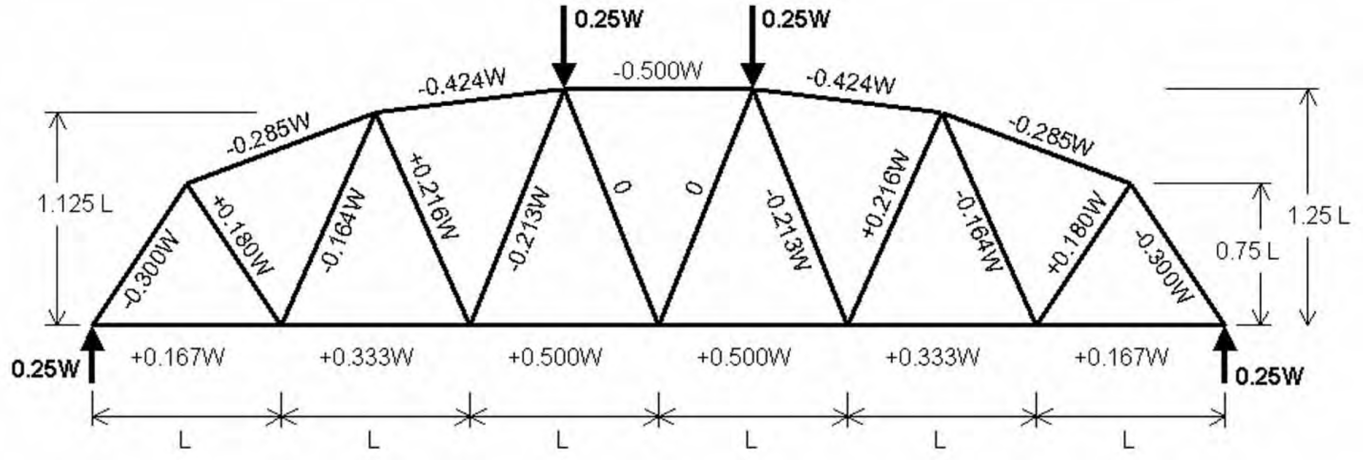
5. Parker Through Truss (Span=6L, Height=1.25L).



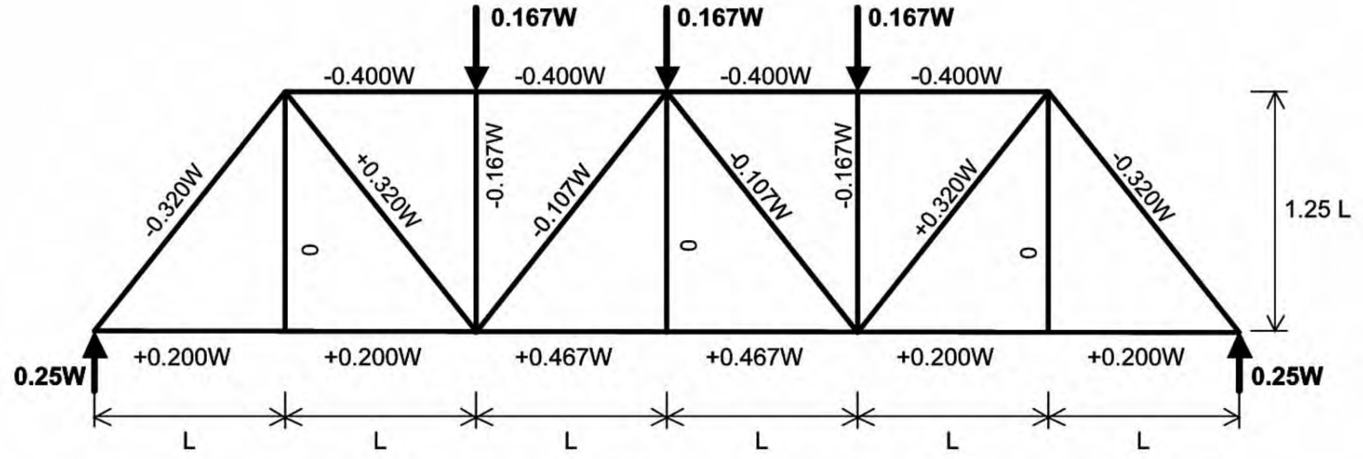
6. Howe Through Truss (Span=6L, Height=1.25L).



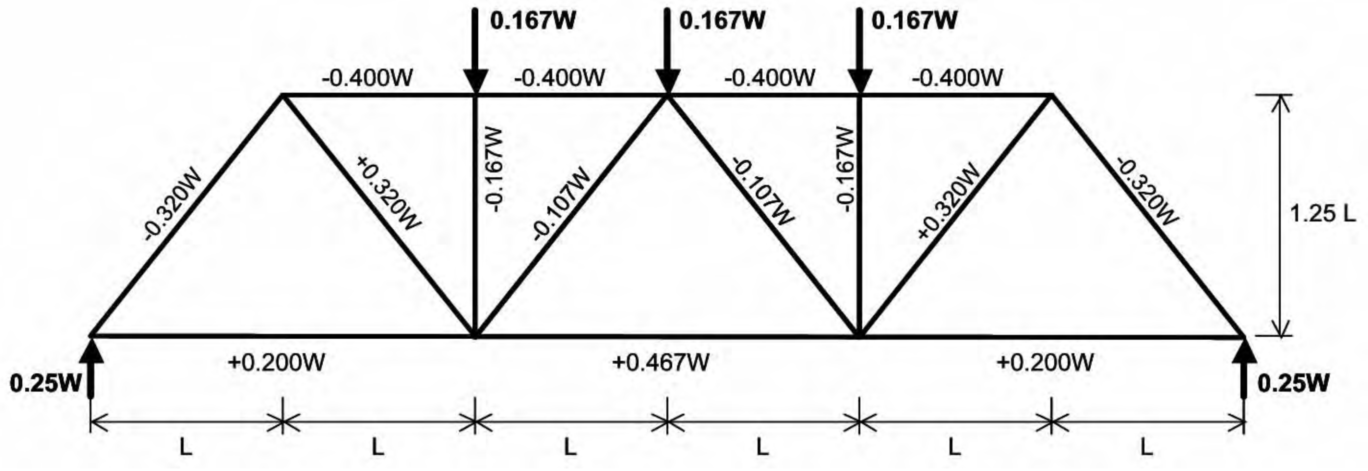
7. Warren Through Truss (Span=6L, Height=1.25L).



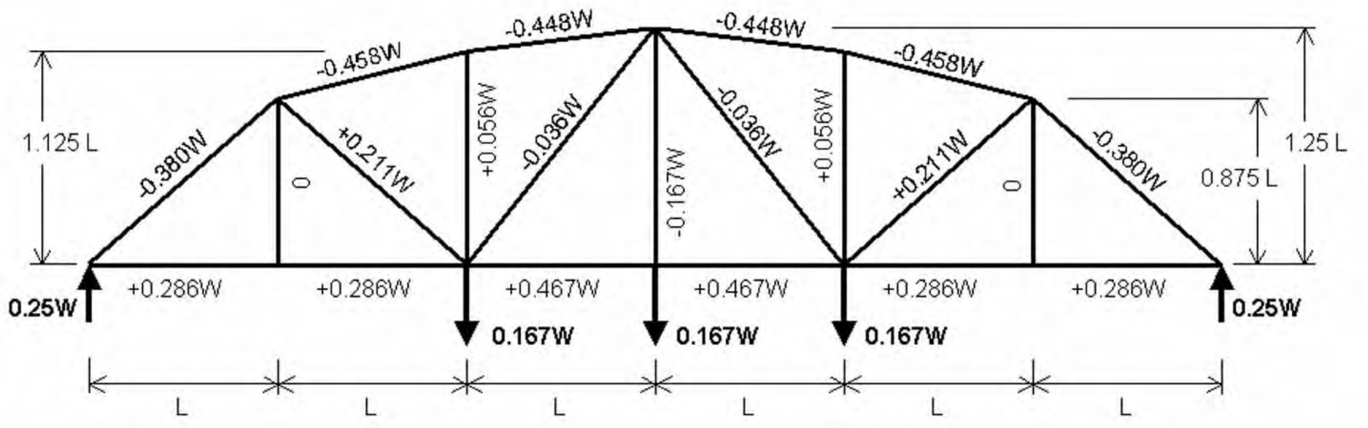
8. Warren Through Truss (Span=6L, Height=1.25L).



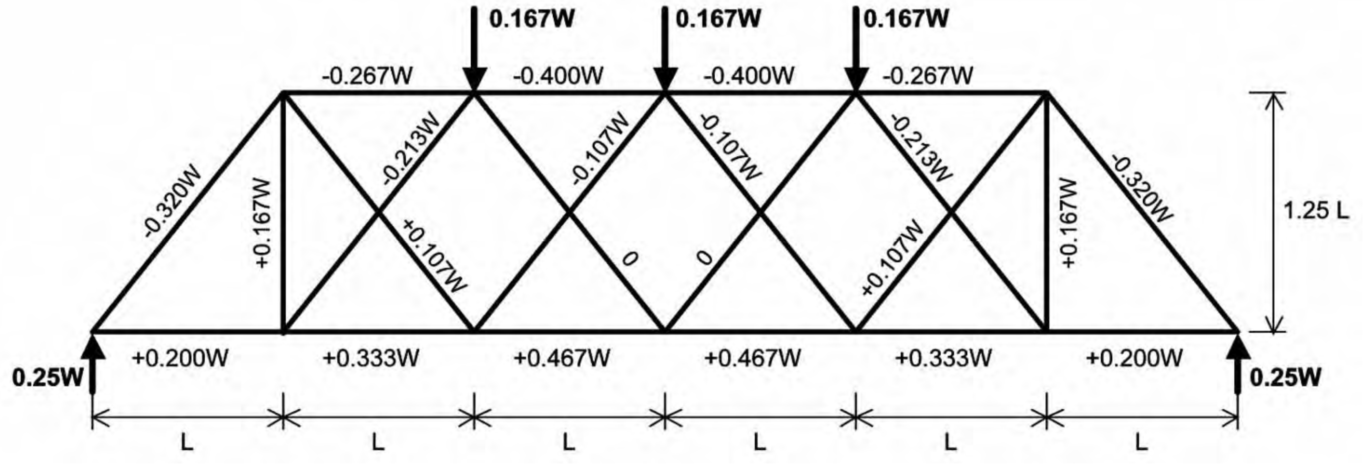
9. Warren Through Truss with Verticals (Span=6L, Height=1.25L).



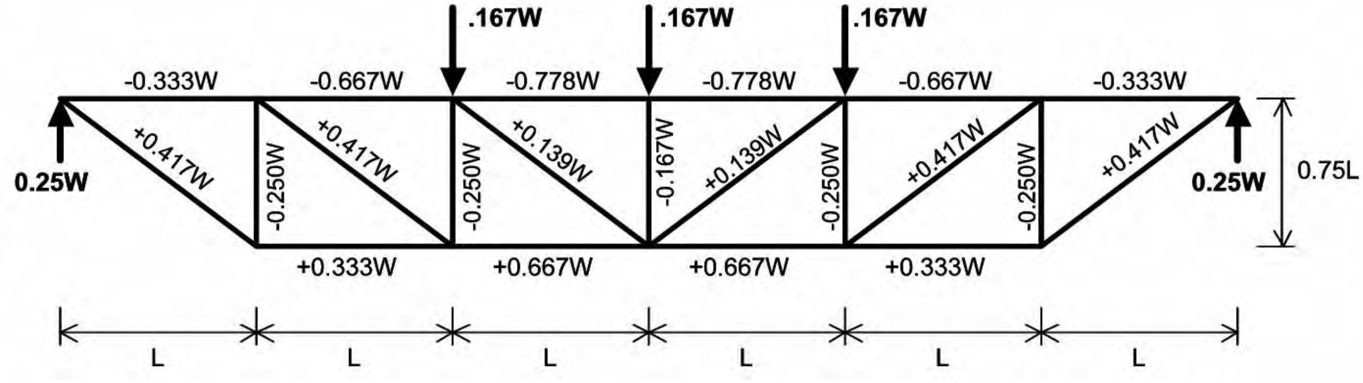
10. Warren Through Truss with Verticals (Span=6L, Height=1.25L).



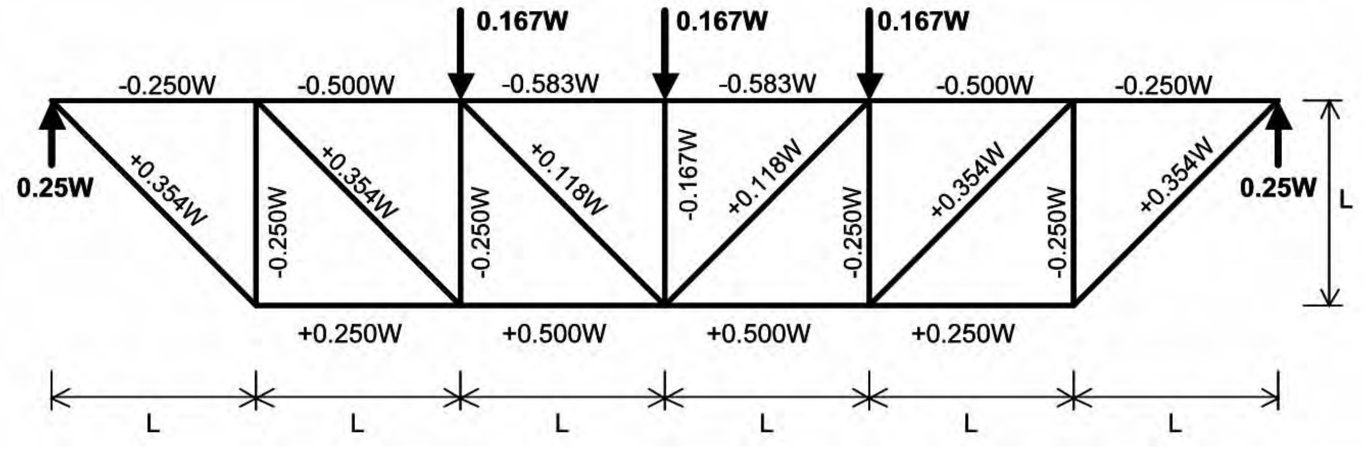
11. Warren Through Truss with Verticals (Span=6L, Height=1.25L).



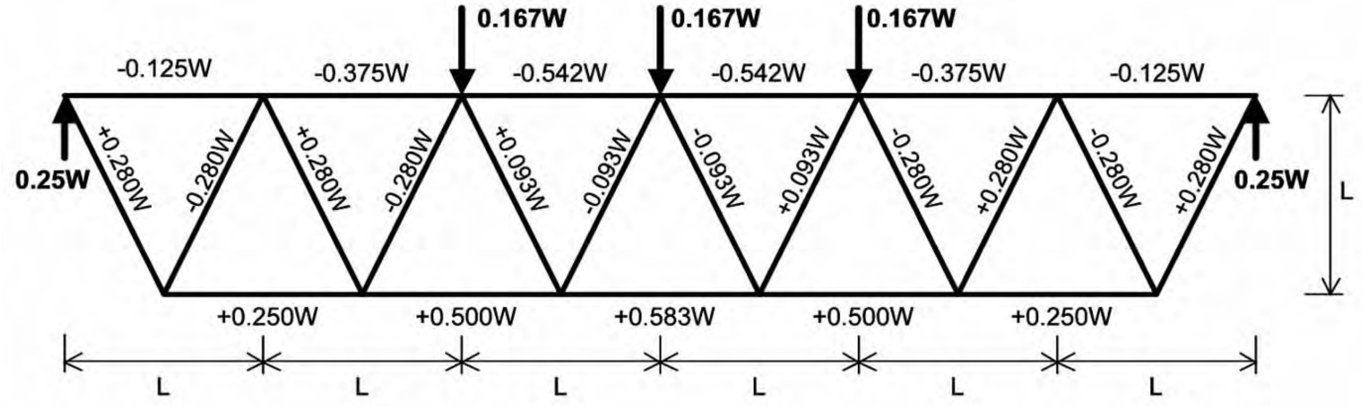
12. Double-Intersection Warren Through Truss (Span=6L, Height=1.25L). On this truss, the points where pairs of diagonals cross over each other are *not* joints. The members are not physically connected to each other at these points.



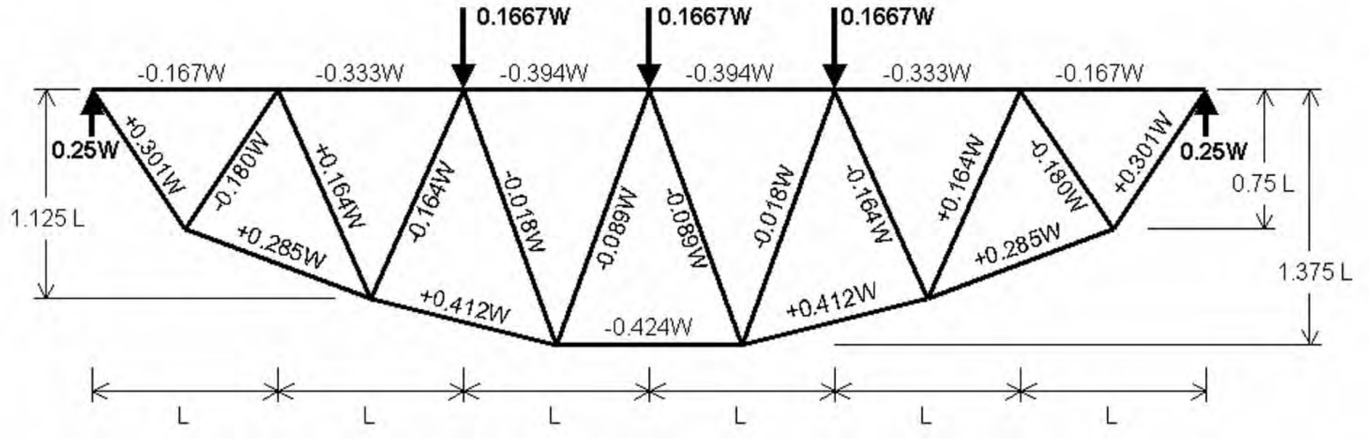
13. Pratt Deck Truss (Span=6L, Height=0.75L).



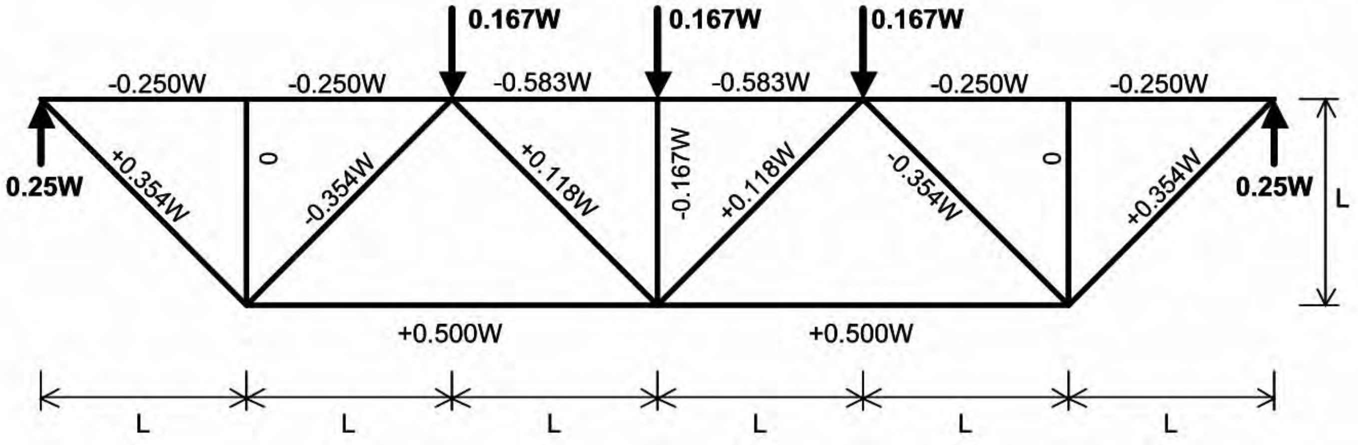
14. Pratt Deck Truss (Span=6L, Height=L).



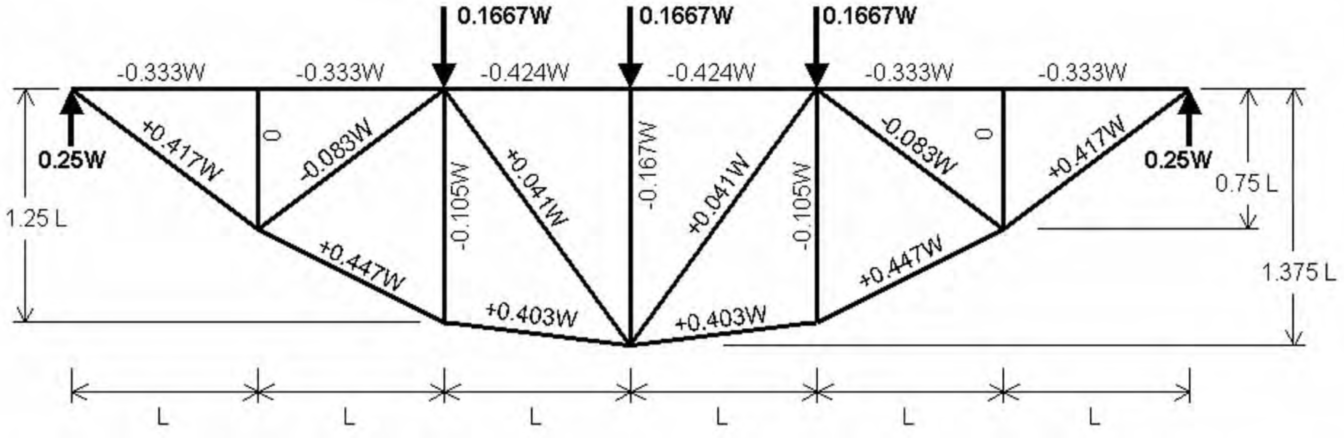
15. Warren Deck Truss (Span=6L, Height=L).



16. Warren Deck Truss (Span=6L, Height=1.375L).



17. Warren Deck Truss with Verticals (Span=6L, Height=L).



18. Warren Deck Truss with Verticals (Span=6L, Height=1.375L).

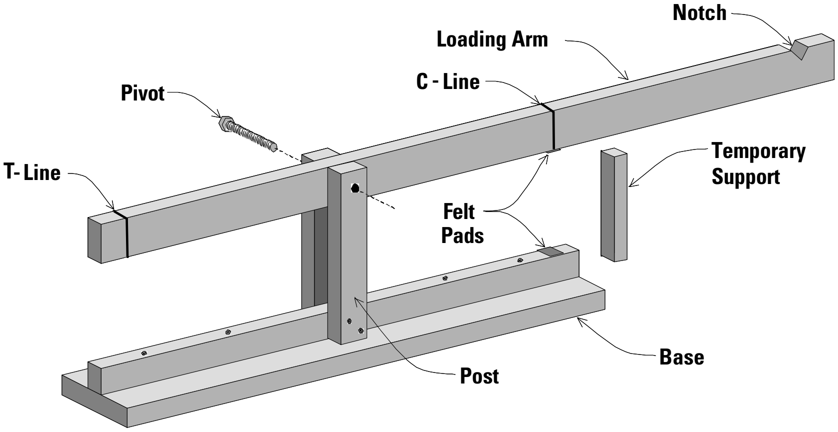
Appendix C

Building the Testing Machine

Description

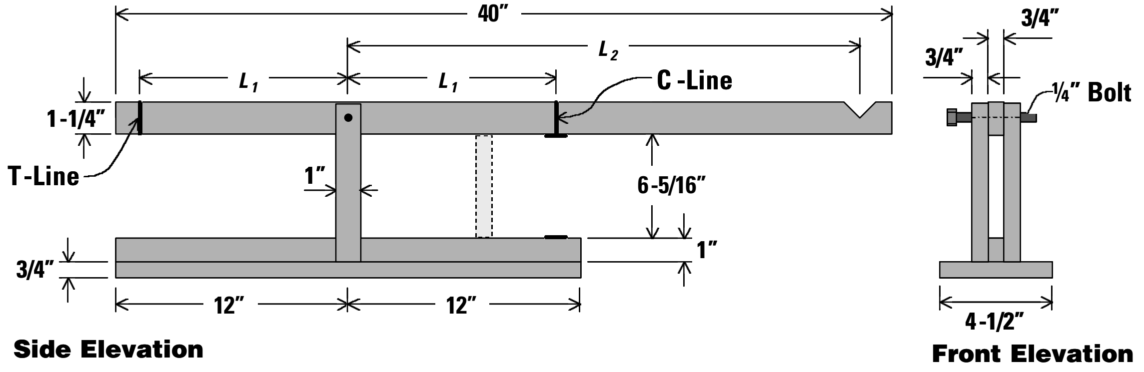
This simple, lever-based testing machine is designed to apply a controlled tension or compression force to a cardboard test specimen and measure that force with reasonable accuracy. Its use is described in Learning Activity #2. Only a moderate level of woodworking skill is required to build it.

The configuration and component parts of the testing machine are illustrated in the drawing below. The *loading arm*, *posts*, and *base* are made of wood. Pine was used on the original device, but any wood will do. The posts and base are all connected together with glue and woodscrews, while the loading arm is fastened to the posts with only a single steel bolt, which serves as a pivot. The arm should be free to rotate about the pivot. The *T-Line* and *C-Line* are vertical marks on the loading arm, indicating the points where the tension and compression specimens will be fastened for testing. *Felt pads* are glued to the underside of the loading arm and the top side of the base at the C-Line. These pads will ensure that compression test specimens are uniformly loaded. The *temporary support* is a wooden post that is used to support the loading arm while a tension specimen is being clamped into position.



Isometric View of the Testing Machine

The drawing below shows the dimensions the testing machine, in inches. Of all the dimensions provided, only the 6-5/16″ distance between the loading arm and the base must be exact. (This is just slightly larger than 16 centimeters, the length of our longest compression specimens). All other dimensions can be adapted to the sizes of available lumber, storage space requirements, etc. The dimension L_1 is the distance from the pivot to the T-Line *and* from the pivot to the C-Line. The dimension L_2 is the distance from the pivot to the center of the notch. These dimensions will be determined during the process of balancing the loading arm, as describ below.



Elevation View of the Testing Machine

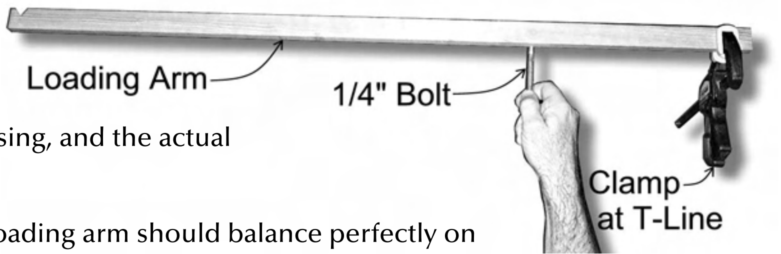
Test specimens will be fastened into the testing machine with two woodworker's clamps. 6″ Quick-Grip® clamps are highly recommended. These clamps are available at most hardware stores. They work well for this project, because they can be put in place with one hand, and their rubber pads are very effective in preventing the test specimen from slipping.

Balancing the Loading Arm

To get accurate experimental results from the testing machine, the loading arm must be properly balanced. The objective of the balancing process is to ensure that the weight of the arm does not place any load on the test specimen. It is best to accomplish this task during construction, before the hole for the pivot is drilled in the loading arm. Here's how to do it:

- Cut the loading arm to size, and cut out the notch at one end.
- Mark the T-Line about 2 centimeters from the end opposite the notch.
- Place one Quick-Grip® clamp on the T-Line. Ensure that the clamp is centered on the line.
- Using the ¼″ bolt that will eventually be used for the pivot, find the point where the arm (with one clamp attached) balances perfectly. See the photo below.
- Mark the balance point, and drill a ¼″ hole for the pivot through the center of the arm at this location.
- Measure L_1, the distance from the T-Line to the pivot hole. Then measure the same distance on the opposite side of the pivot, and mark the location of the C-Line.
- Finally measure L_2, the distance from the pivot to the center of the notch. Record both L_1 and L_2 for future reference.

On the original testing device, L_1 was 25 cm and L_2 was 69.5 cm. However, these dimensions could vary substantially, depending on the weight of your clamp, the type of wood you are using, and the actual dimensions of your loading arm.



Once the testing machine is assembled, the loading arm should balance perfectly on the pivot, with one clamp attached at the T-line. If not, add weight to one end until it does balance.

Appendix D

Glossary

Abutment—A concrete wall that supports the end of a bridge and holds back the soil that is filled in behind it.

Aesthetic Requirement—A design requirement that describes the desired appearance of a completed structure.

Analysis—An examination of a complex system, usually conducted by breaking the system down into its component parts.

Approach—A roadway leading up to the end of a bridge.

Bid—An estimate of the construction cost, submitted by a construction contractor as part of the design-bid-build project delivery method.

Bid Opening—A public meeting, at which all bids submitted for a construction project are opened and read aloud.

Bottom Chord—A type of structural member used in a truss. (See diagram on page 1-3.)

Brittle—A characteristic of a material that fails by rupturing suddenly and without warning, when loaded in tension. The opposite of brittle is ductile.

Buckling—A failure that occurs when compression causes a member to suddenly bend sideways, perpendicular to the direction of the applied load.

Carbon Steel—A common type of steel, composed of iron and a very small amount of carbon.

Compression—An internal force that tends to make a structural member shorter.

Compressive Strength—The maximum compression force a structural member can carry before it fails.

Conceptual Design—A preliminary design that describes the appearance and general configuration of a structure.

Concrete—A structural material made by mixing Portland cement, sand, gravel, and water. When concrete cures, it becomes a solid, rock-like substance that is very strong in compression, but relatively weak in tension.

Connection—An assembly of structural components (plates, angles, bolts, welds, etc.) that join two or more structural members together.

Cosine—A trigonometric function of an angle. The cosine of an angle in a right triangle is calculated by dividing the length of the *adjacent side* by the length of the *hypotenuse*. The cosine function is abbreviated "***cos***." (See page 3-3.)

Construction Contractor—A company that performs the role of the Constructor, under the terms of a formal contract with the Owner.

Constraint—A design requirement that limits some aspect of the design. An example of a constraint is a requirement for a bridge to be built above some minimum specified height, so that boats on the waterway below can pass beneath the structure unhindered.

Constructor—One of the four key players in the Project Team. The Constructor is responsible for planning, managing, and performing the construction of a facility.

Cross Section—The two-dimensional shape you see when you look at the end of a structural member.

Deck—The flat surface that forms the floor of a bridge and supports the roadway. (See diagram on page 1-3.)

Deck Truss—A truss configuration for which the deck is located at the level of the top chord.

Deflection—The distance a structure moves when it is loaded.

Deformation—A change in the size or shape of a structural member that occurs when the member is loaded.

Design—To devise or create something that meets a need.

Design Professional—One of the four key players in the Project Team. The Design Professional is responsible for conceiving, planning, and providing a high-quality design solution to the Owner.

Design-Bid-Build Project Delivery—A method of project delivery, in which the Constructor is selected by a competitive bidding process.

Design-Build Project Delivery—A method of project delivery, in which a single firm is responsible for both design and construction.

Design Review—A technique used by the Owner to monitor the conduct of the design process. The Design Professional submits a complete draft of the design for review, at specified levels of completion.

Diagonal—A type of structural member used in a truss. (See diagram on page 1-3.)

Ductility—The capacity of a material to undergo very large plastic deformation before rupturing. A ductile material provides ample warning of failure. The opposite of ductile is brittle.

Elastic—Behavior characterized by the capability of a structural member to return to its original size and shape after its load is removed.

Elevation View—A drawing showing an object viewed from the side.

End Post—The diagonal member at each end of a through truss. (See diagram on page 1-3.)

Engineering Design—The application of math, science, and technology to create something that meets a human need.

Equations of Equilibrium—Equations describing the condition that the total force acting on an object in equilibrium is equal to zero.

Equilibrium—The condition that occurs when the total force acting on an object is zero. If an object is not moving, then it is in equilibrium.

Factor of Safety—A number representing the margin of safety in a structural design. The factor of safety is used to allow for uncertainty in loads, member strengths, and structural analysis results.

Failure—The condition that occurs when the internal force in a structural member becomes larger than the strength of that member.

Floor Beam—A structural member that supports the deck of a bridge. On a truss bridge, floor beams also help to connect the two main trusses together. (See diagram on page 1-3.)

Free Body Diagram—A sketch of a "body" (a structure or a portion of a structure) showing all of the forces acting on it.

Force—A push or a pull applied to an object. A force always has both magnitude and direction.

Foundation—A component of a structure that distributes the weight of the structure to the soil or rock below it.

Fulcrum—The pivot about which a lever rotates.

Functional Requirement—A design requirement that describes how the completed structure will do its job. An example of a functional requirement for a bridge is the required number of traffic lanes.

Geotechnical Engineer—A civil engineer with special expertise in soils and foundations.

Gusset Plate—A metal plate used to connect structural members together in a truss.

Gusset Plate Connection—A type of connection that uses gusset plates to join two or more members of a truss together.

Hip Vertical—The outermost vertical member at each end of a Pratt through truss. The hip vertical carries tension, while all of the remaining verticals in a Pratt through truss carry compression. (See diagram on page 1-3.)

Hypotenuse—The longest of the three sides of a right triangle. The hypotenuse is always the side opposite the 90° angle.

Internal Force—The tension or compression force developed in a structural member when loads are applied to the structure.

Isometric View—A drawing showing a three-dimensional view of an object.

Iteration—A cycle of analysis and design, performed as part of the engineering design process.

Joint—The point at which two or more members are joined together in a structure.

Lateral Bracing—A series of diagonal structural members that help a bridge resist lateral loads, like wind. The lateral bracing members and struts also help to prevent the top chords of a truss bridge from buckling sideways. (See diagram on page 1-3.)

Load—A force applied to a structure.

Load-Deformation Curve—A graph that shows how a member deforms when a load is applied to it.

Lever—A simple machine consisting of a bar or rod that rotates on a pivot.

Mechanics of Materials—The scientific study of structural members and materials.

Member—A load-carrying component of a structure.

Notice to Proceed—An official authorization for the Constructor to start work on a project.

Optimize—To maximize the efficiency of a design. Generally, a structural design is optimized by minimizing its cost without compromising its safety.

Owner—One of the four key players in the Project Team. The Owner identifies the need for the project, provides funding, puts together the Project Team, and establishes the project requirements.

Pier—A foundation element that supports a bridge in the middle of the gap.

Pile—A component of a structural foundation, consisting of a long steel or concrete shaft that is driven downward through weak soil into stronger soil or rock.

Pinned Connection—A type of connection that uses a single large metal pin to join two or more structural members together.

Plans and Specifications—The products of the design process, created by the Design Professional. Plans are drawings, and specifications are highly detailed written descriptions of every aspect of the project.

Plastic Deformation—The permanent elongation of a material under load. Plastic deformation occurs after a material has yielded.

Pony Truss— A truss configuration for which the deck is located at (or slightly above) the level of the bottom chord. A pony truss looks very similar to a through truss, except it is not as high and has no struts or lateral bracing between the top chords.

Portal Bracing—An assembly of struts and diagonal bracing members that connects the end posts of a through truss bridge together. The portal bracing prevents the two main trusses from falling over sideways. (See diagram on page 1-3.)

Procurement—A phase of the design process in which a Constructor is selected and construction materials are purchased for the project.

Project Manager—One of the four key players in the Project Team. The Project Manager has overall responsibility for managing both the design and construction of the facility. The Project Manager represents the Owner and looks after the Owner's interests on all aspects of the project, to include scheduling, financial management, and construction quality.

Project Team—A team of specialists who are brought together to design and build a facility. The principal members of the Project Team are the Owner, the Design Professional, the Constructor, and the Project Manager.

Pythagorean Theorem—A mathematical relationship between the lengths of the sides of a right triangle and the length of the hypotenuse. (See page 3-3.)

Quality Control—The process of routinely inspecting and testing materials and workmanship on a project and taking corrective action when problems are found.

Reaction—A force developed at a support, to keep the structure in equilibrium.

Required Strength—The actual strength a member must be, in order to carry load safely. In structural design, the required strength can be calculated by multiplying the factor of safety by the internal member force.

Reinforced Concrete—Concrete that has been strengthened for structural applications by embedding steel reinforcing bars inside it, before the concrete cures. The steel reinforcement compensates for the low tensile strength and low ductility of plain concrete.

Right Triangle—A triangle with one of its three angles measuring exactly 90 degrees.

Rupture—A failure mode that occurs when a member subjected to a tension force physically breaks into two pieces.

Schedule—A tabular listing of members, connections, or other components of a structural design, normally provided as part of the plans and specifications for a project.

Shop Drawings—Detailed drawings of every component that will be part of a completed structure. Shop Drawings are normally prepared by the Constructor and approved by the Design Professional.

Sine—A trigonometric function of an angle. The sine of an angle in a right triangle is calculated by dividing the length of the *opposite side* by the length of the *hypotenuse*. The sine is abbreviated "***sin***." (See page 3-3.)

Spread Footing—A type of foundation consisting of a flat slab of concrete placed directly on firm soil or rock.

Stable—A rigid structural configuration in which no member or members can move or rotate freely. A structure must be stable to carry load. A truss is generally stable when it is composed entirely of interconnected triangles.

Statically Determinate—A structural configuration that can be analyzed using the equations of equilibrium alone. Only statically determinate trusses can be analyzed with the Method of Joints.

Statically Indeterminate—A structural configuration that cannot be analyzed using the equations of equilibrium alone. Statically indeterminate trusses cannot be analyzed with the Method of Joints.

Steel Fabricator—A company that specializes in prefabricating steel structural members and connections before they are delivered to the construction site. The steel fabricator cuts all structural components to size, drills or punches holes for bolts, and bolts or welds some of the components together to form subassemblies. The steel fabricator is a member of the Construction Team.

Strength—The largest internal force that a structural component can experience before it fails.

Structural Analysis—A mathematical examination of a structure to determine its reactions, internal member forces, and deflections.

Structural Engineer—A civil engineer with special expertise in structural analysis and design.

Structural Model—A mathematical idealization of a structure, including a series of simplifying assumptions about the structure's configuration and loading that allow us to predict its behavior mathematically.

Strut—A structural member that connects the two main trusses together on a truss bridge. The struts work together with the lateral bracing to resist lateral loads, like wind, and to prevent the top chords of a truss bridge from buckling sideways. (See diagram on page 1-3.)

Subcontractor—A company hired by a construction contractor to perform a specialized part of a construction project.

Subsystem—A part of a larger system. If a truss bridge is considered to be a system, then its subsystems include the deck, the main trusses, the lateral bracing, the foundations, the approaches, and others.

Support—A point at which a structure is physically in contact with its surroundings.

Tensile Strength—The maximum tension force a member can carry before it fails.

Tension—An internal force that tends to make a structural member longer.

Through Truss—A truss configuration for which the deck is located at the level of the bottom chord. A through truss looks very similar to a pony truss, except it is higher and has struts and lateral bracing connecting the top chords together.

Trigonometry—The mathematical study of the properties of triangles.

Truss—A structure composed of members connected together to form a rigid framework.

Top Chord—A type of structural member used in a truss. (See diagram on page 1-3.)

Type Study—A report that describes the alternative configurations considered in a bridge design and explains the advantages and disadvantages of each one. The Design Professional prepares the type study for the Owner.

Unstable—A structural configuration that cannot carry load because one or more members can move or rotate without restraint. A truss is generally unstable when it is not made up entirely of interconnected triangles.

Ultimate Strength—The absolute maximum internal force a member can carry in tension before it fails.

Vector—A quantity that has both magnitude and direction.

Vertical—A type of structural member used in a truss. (See diagram on page 1-3.)

West Point Bridge Designer—A computer aided design program that will introduce you to the engineering design process and demonstrate how engineers use the computer as a problem-solving tool.

Yielding—The phenomenon that occurs when a ductile material undergoes very large plastic deformations with little change in load.

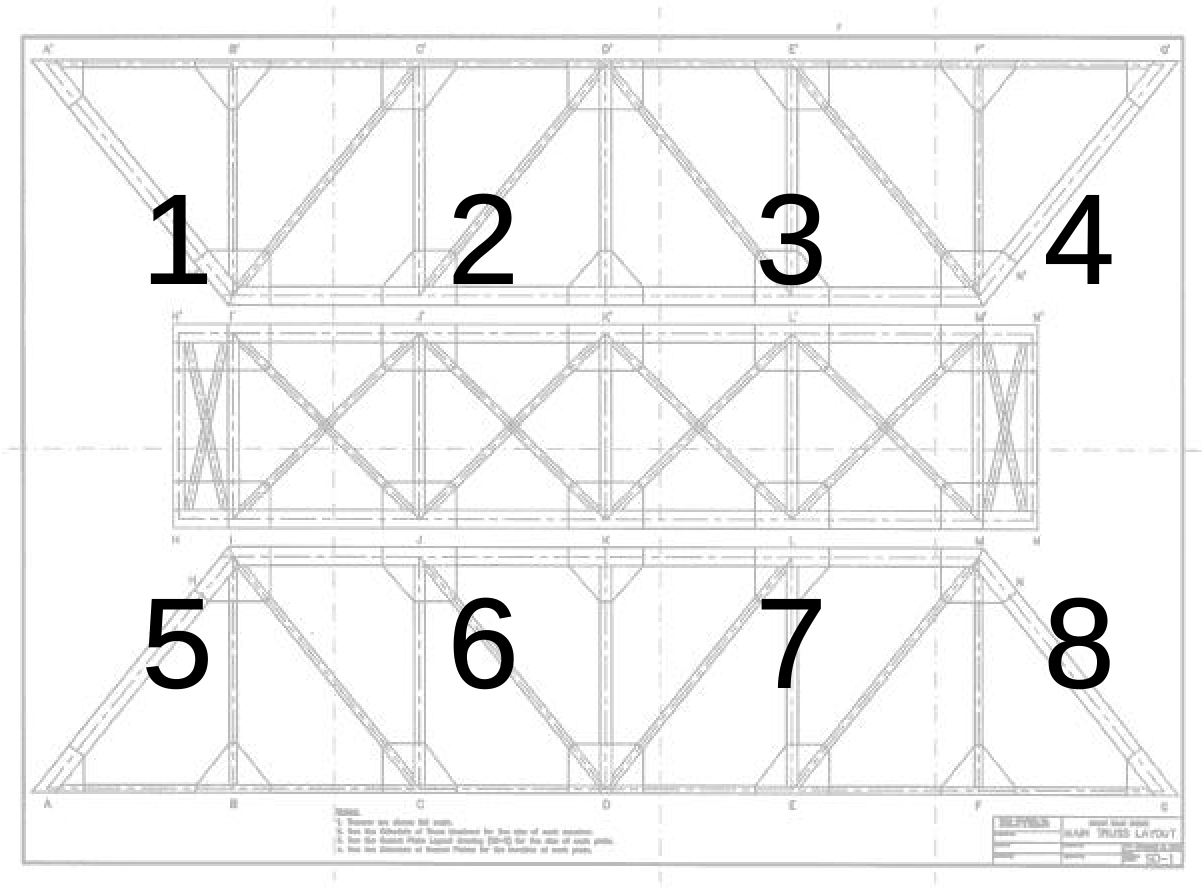
Yield Point—The point on a load-deformation curve at which yielding begins.

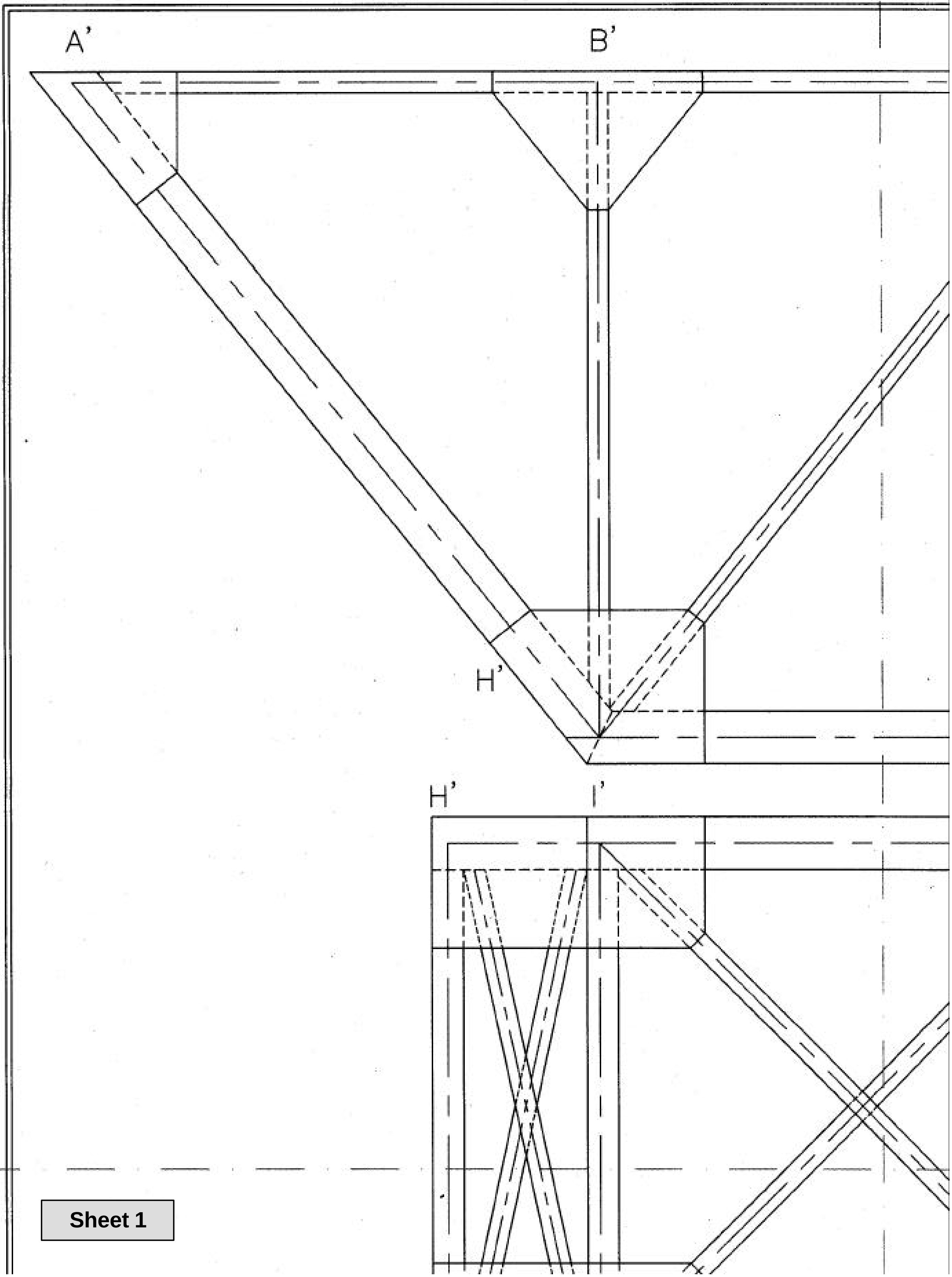
Yield Strength—The internal member force at which yielding occurs.

Full-Scale Layout Drawing
for
Learning Activity #1

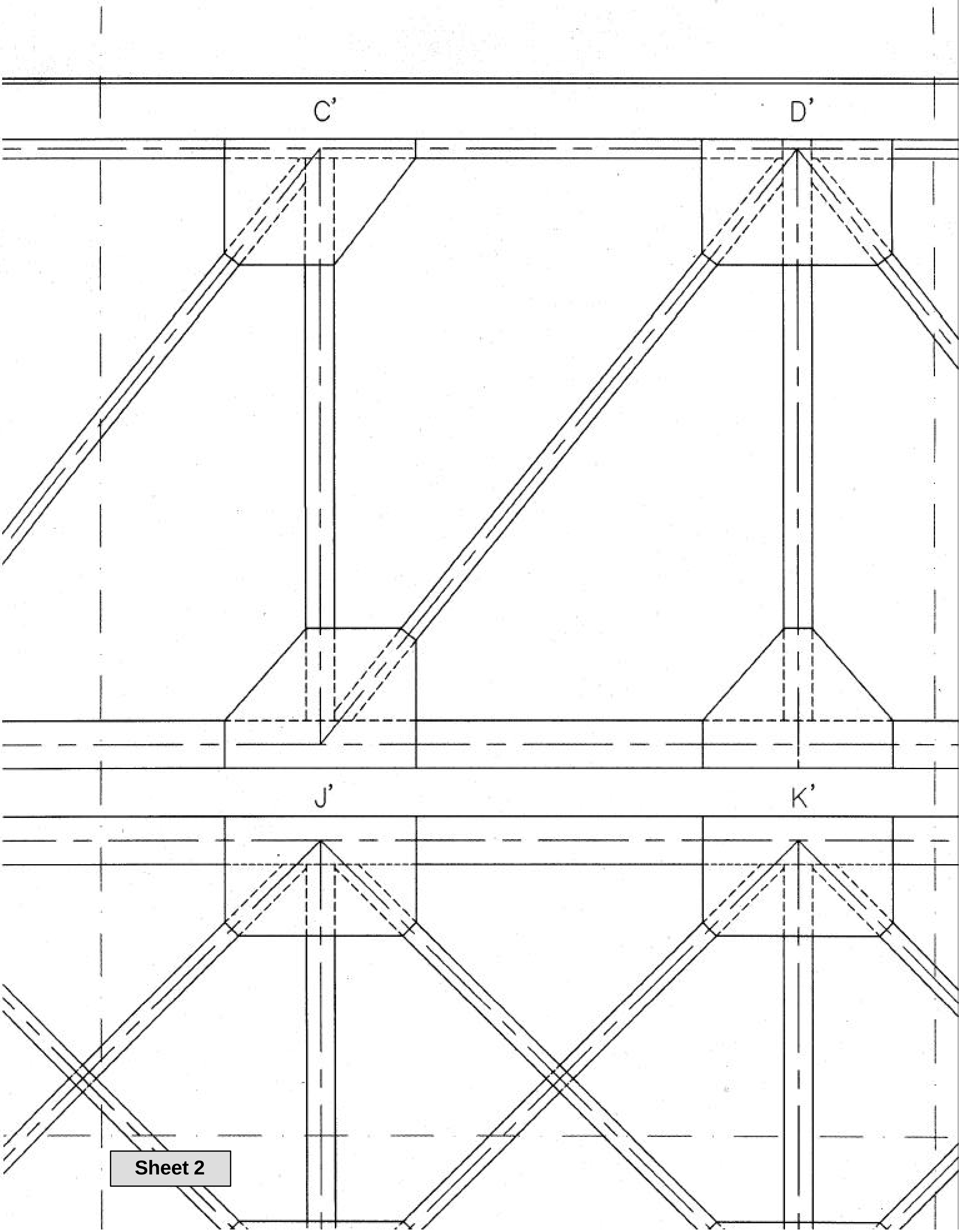
On the following eight pages, you will find the full-scale layout drawing for construction of the Grant Road Bridge in Learning Activity #1. To use the layout drawing, you will need to assemble it, by pasting or taping together these eight pages—labeled Sheet 1 through Sheet 8. Use the diagram below as a guide for the position of each sheet. The large dashed lines on each page are “match lines.” They show precisely where each sheet should line up with the adjacent sheets.

In Learning Activity #1, you will build the Grant Road Bridge directly on top of this layout drawing. Assemble the sheets with care! If they are not precisely aligned, the size and shape of your main trusses will be incorrect, and the bridge may not carry its prescribed load as a result.





Sheet 1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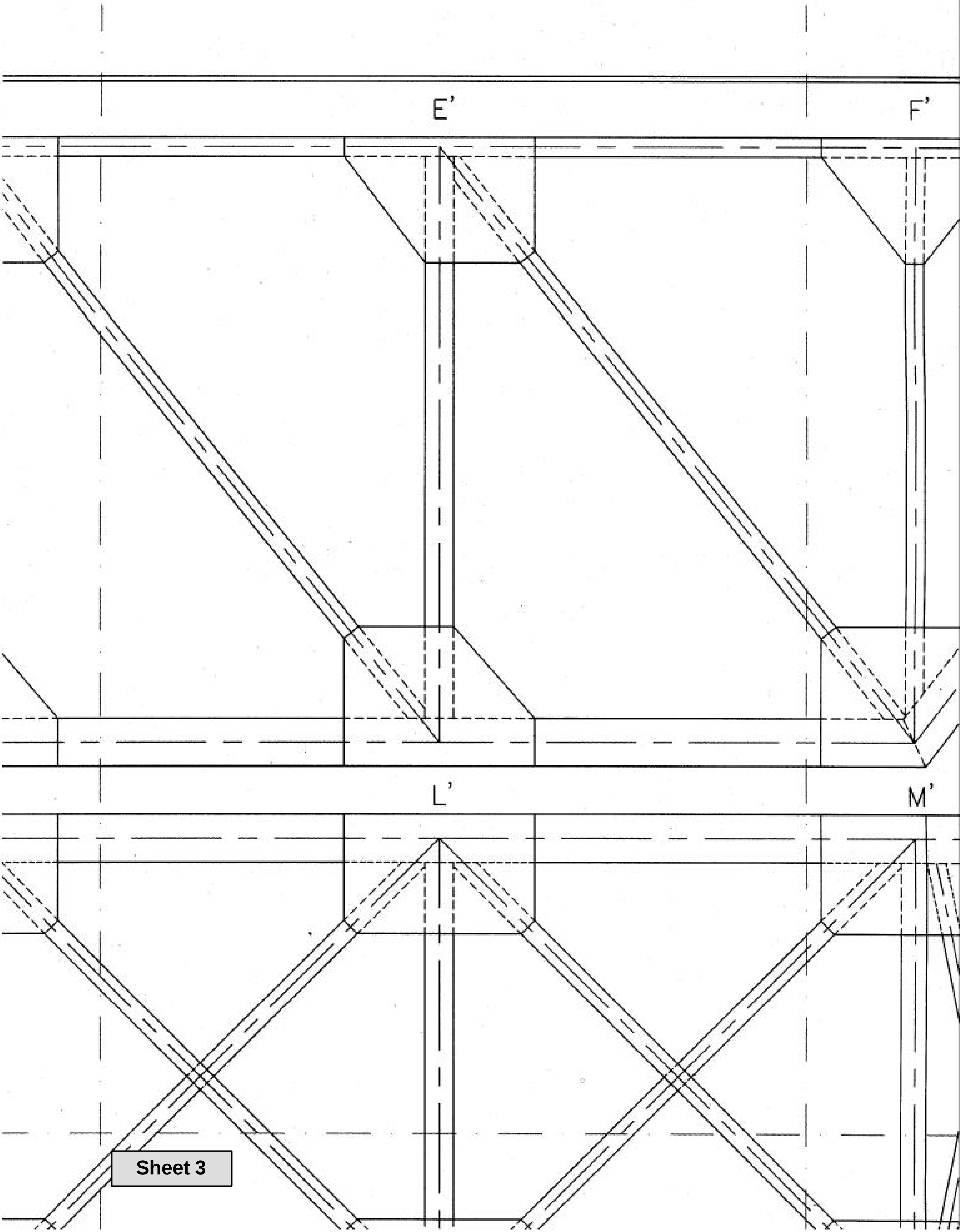

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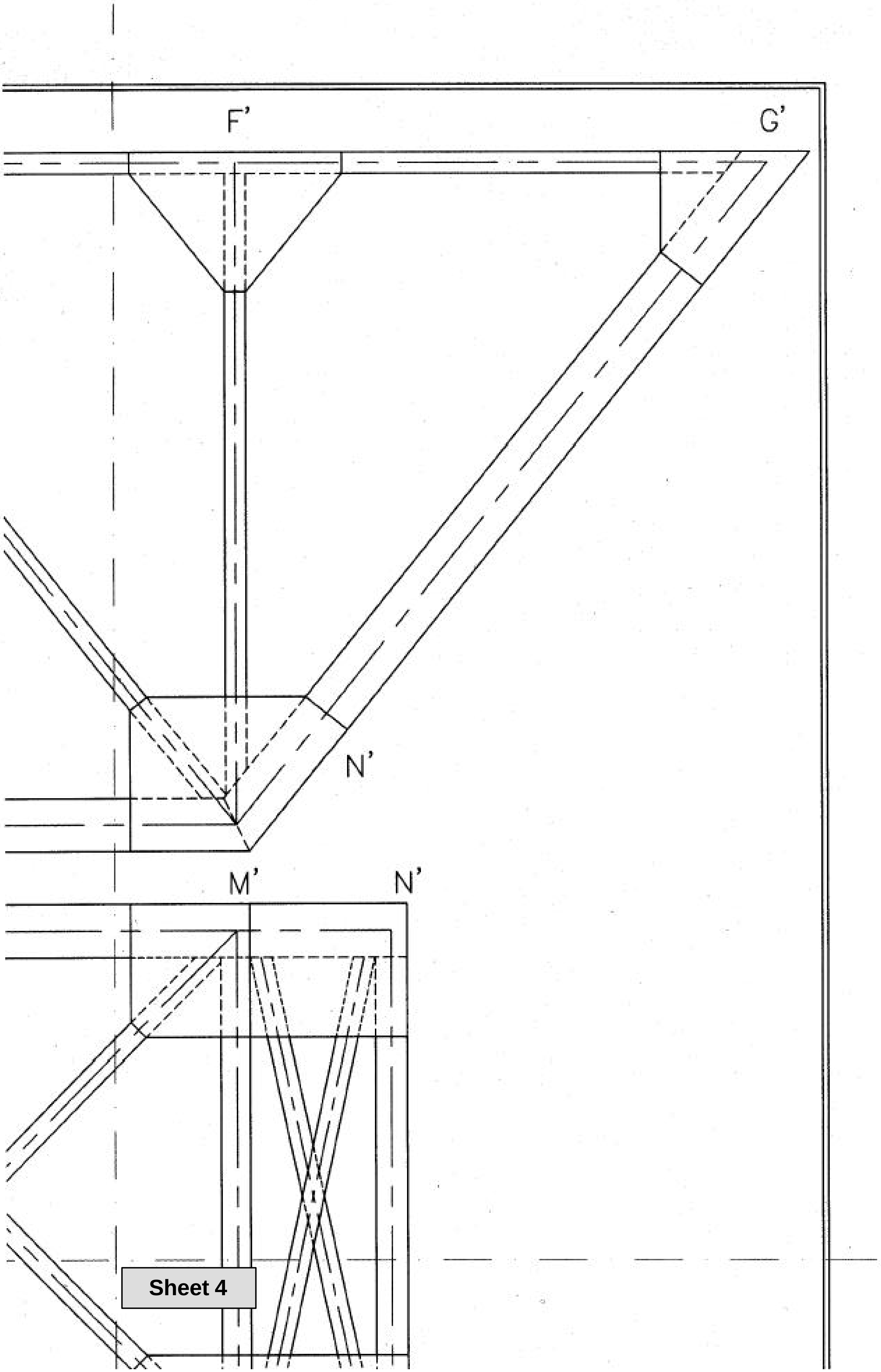

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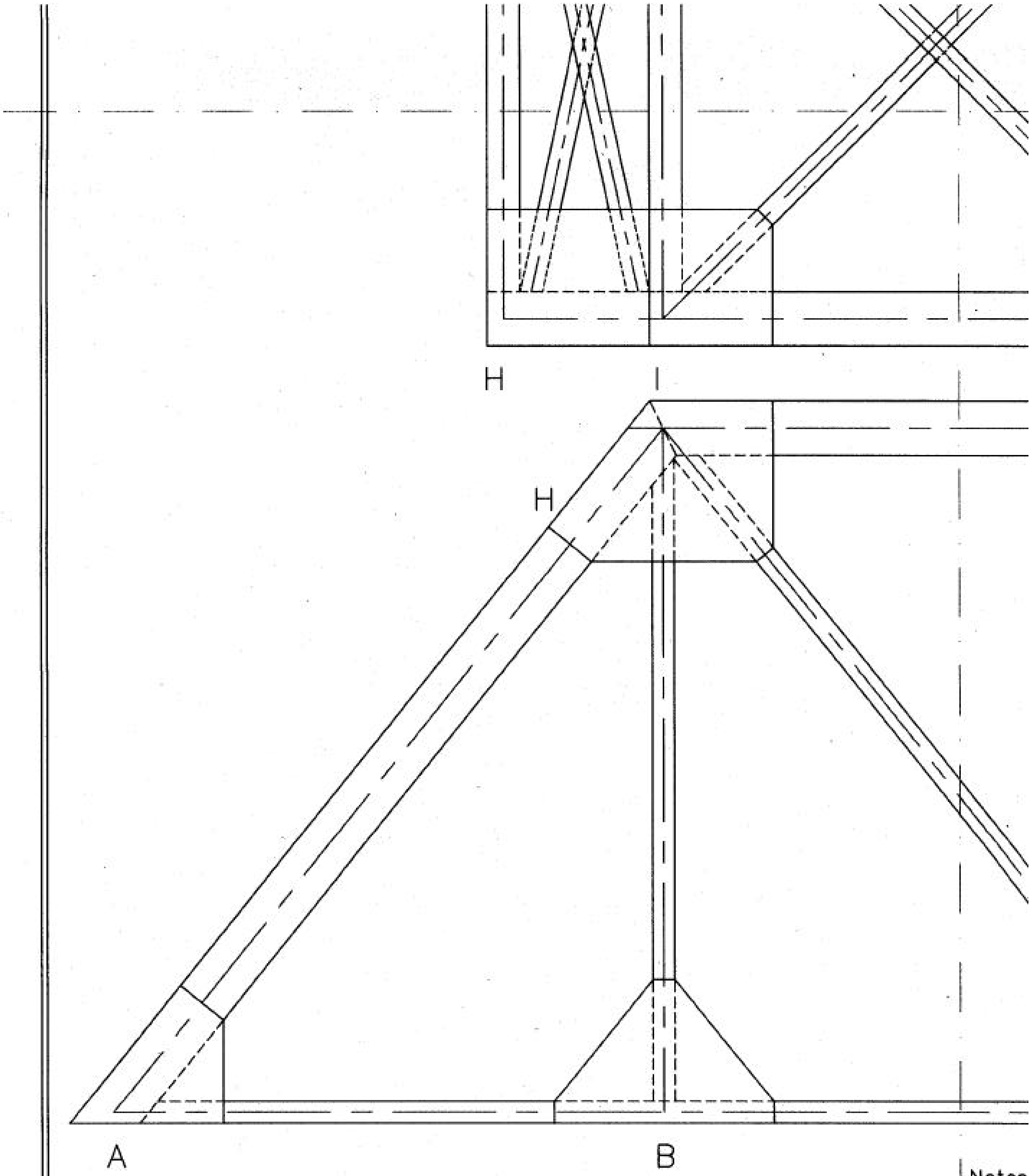

Notes

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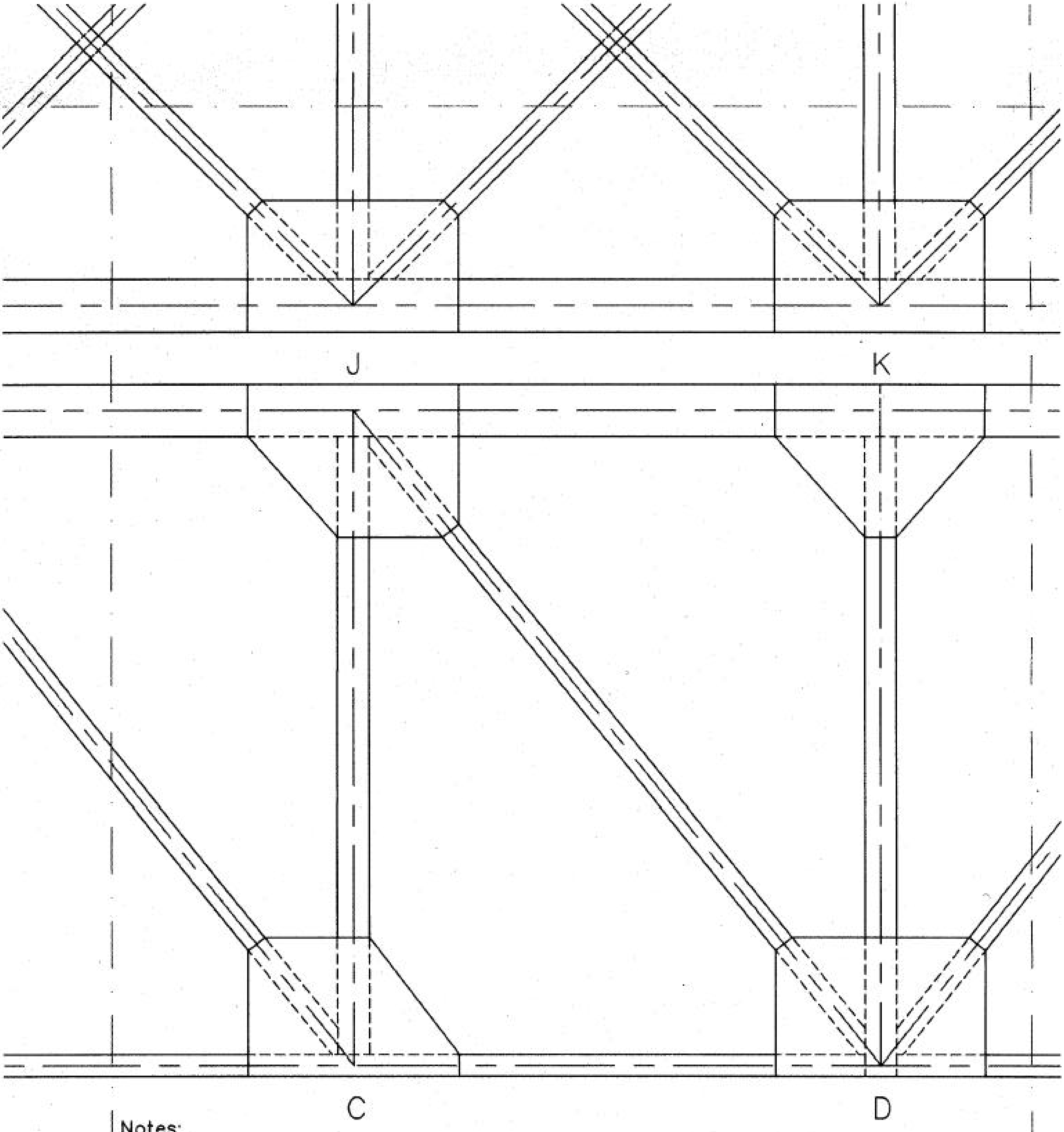
4. Se

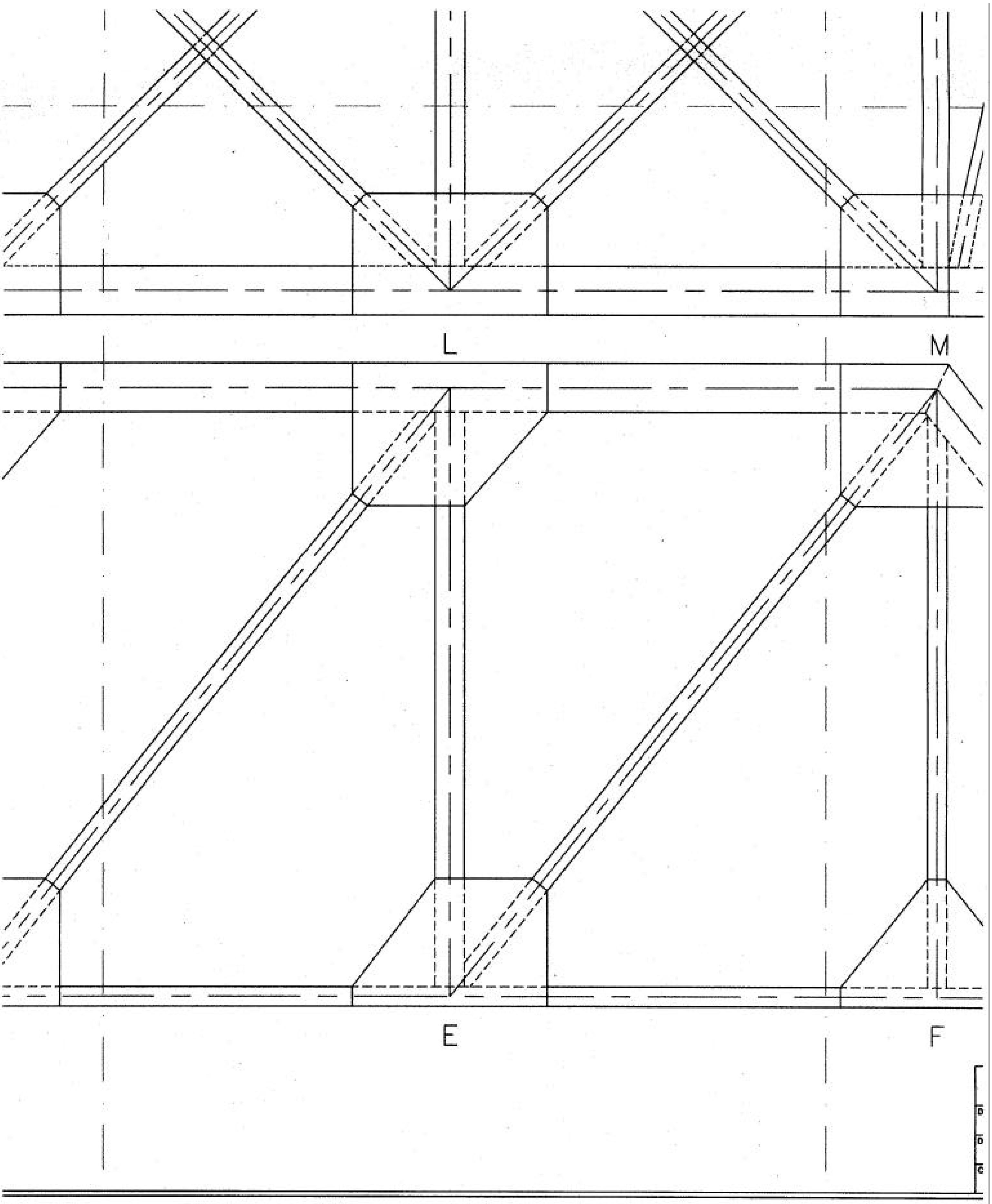
Sheet 5



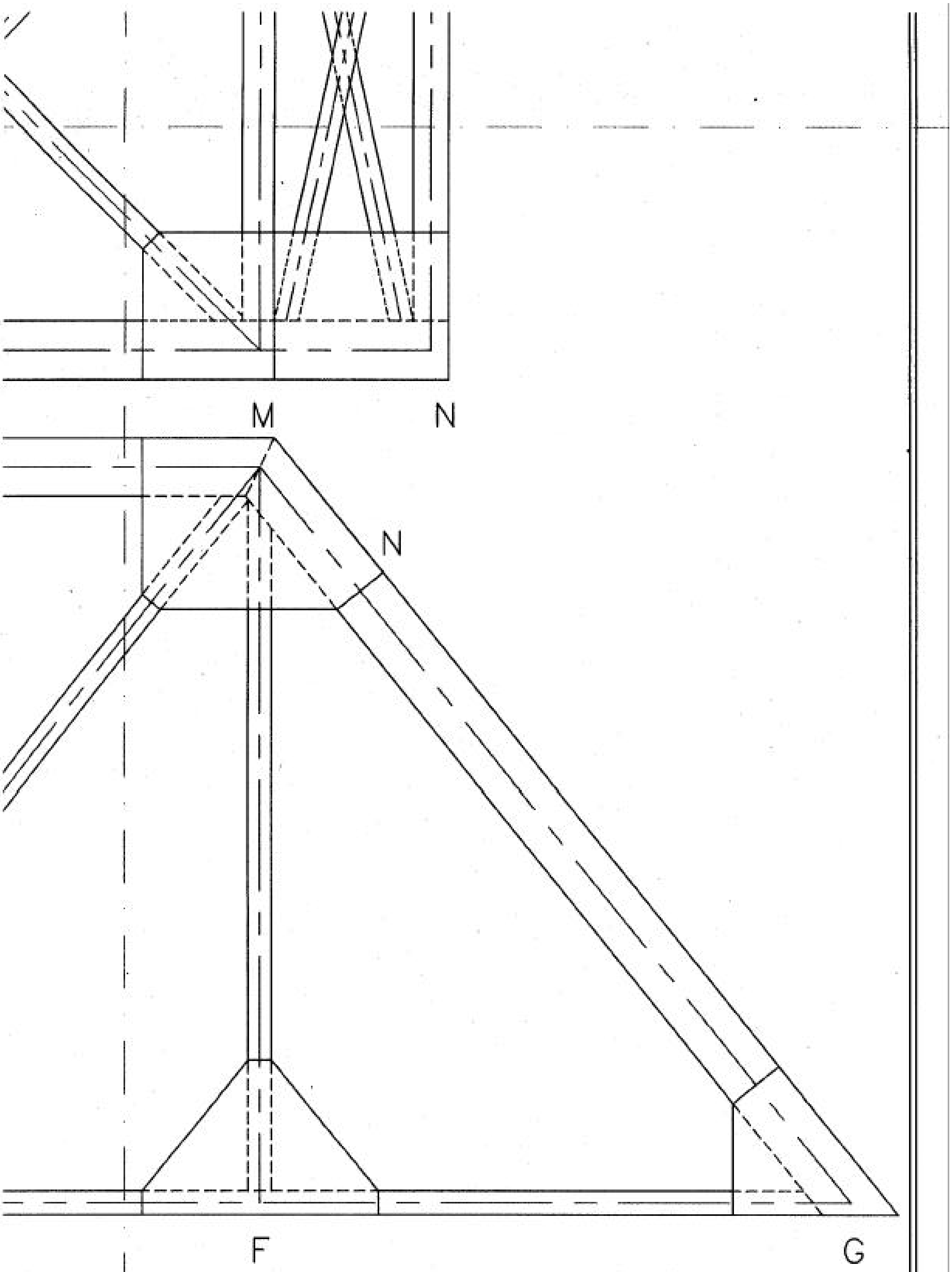
Notes:

1. Trusses are shown full scale.
2. See the Schedule of Truss Members for the size of each member.
3. See the Gusset Plate Layout drawing (SD-2) for the size of each plate.
4. See the Schedule of Gusset Plates for the location of each plate.

Sheet 6



Sheet 7



Thayer Associates, Inc.
Architects & Engineers

Designed by:

Drawn by:

Checked by:

GRANT ROAD BRIDGE

MAIN TRUSS LAYOUT

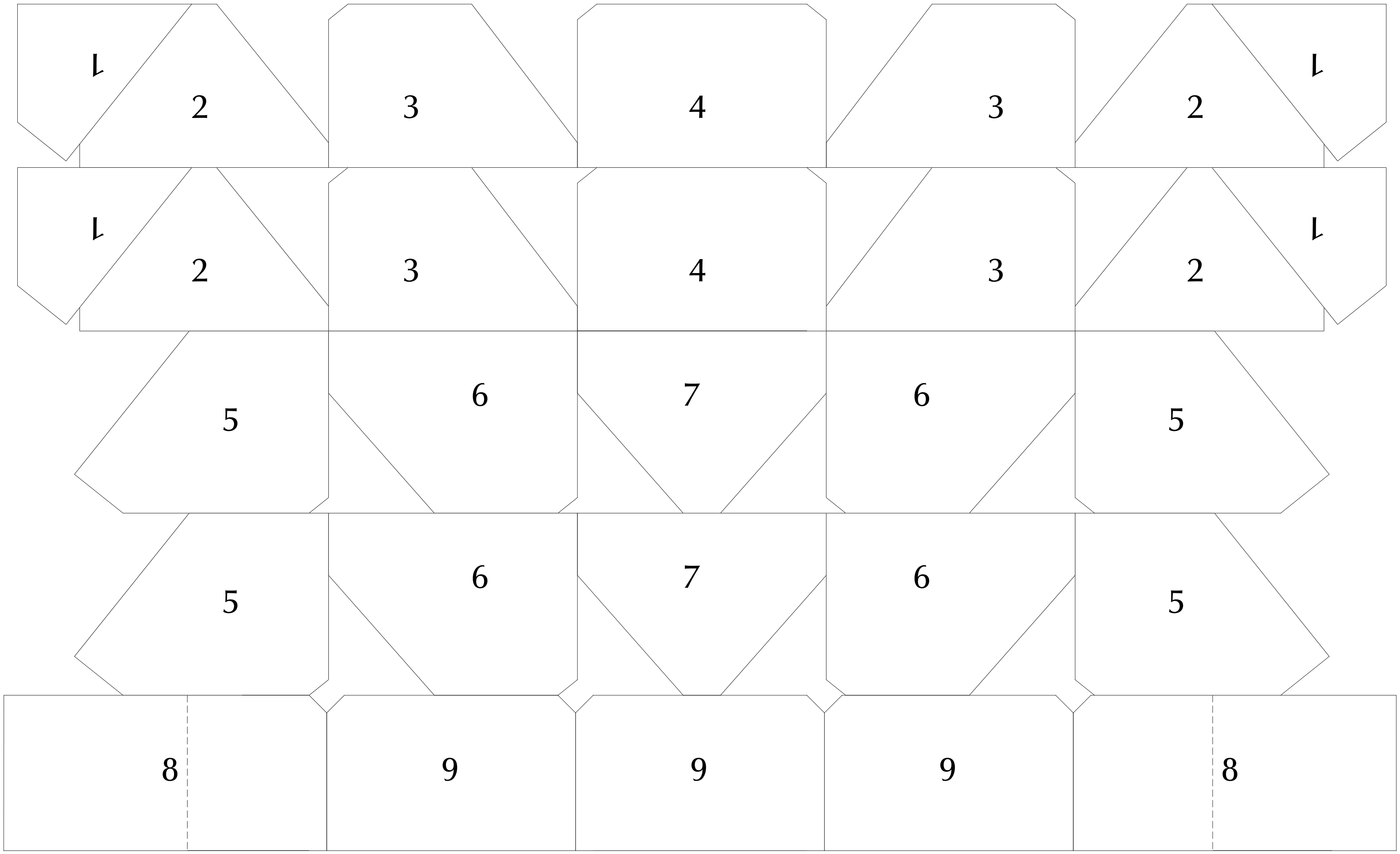
Reviewed by:

Approved by:

Date: NOVEMBER 10, 2000

Sheet Reference Number SD-1

Sheet 8



Notes:

1. All gusset plates are shown full scale.
2. This layout shows exactly half the number of gusset plates required for the bridge.
3. See the Schedule of Gusset Plates for the location of each plate.

Thayer Associates, Inc.
Architects & Engineers

GRANT ROAD BRIDGE

GUSSET PLATE LAYOUT

Designed by:

Drawn by:

Checked by:

Reviewed by:

Approved by:

Date: NOVEMBER 10, 2000

Sheet Reference Number SD-2