6.3 Radicals and Rational Exponents

Essential Question How can you write and evaluate an nth root of a number?

Recall that you cube a number as follows.

Symbol for cubing is 3rd power.

$$2^3 = 2 \cdot 2 \cdot 2$$
$$= 8 \qquad \text{2 cubed is 8.}$$

To "undo" this, take the cube root of the number.

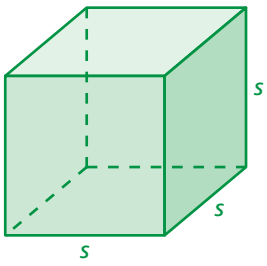
Symbol for cube root is $\sqrt[3]{\ }$.

$$\sqrt[3]{8} = \sqrt[3]{2^3} = 2 \qquad \text{The cube root of 8 is 2.}$$

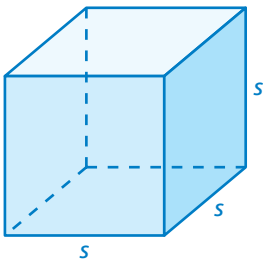
1 ACTIVITY: Finding Cube Roots

Work with a partner. Use a cube root symbol to write the side length of the cube. Then find the cube root. Check your answer by multiplying. Which cube is the largest? Which two are the same size? Explain your reasoning.

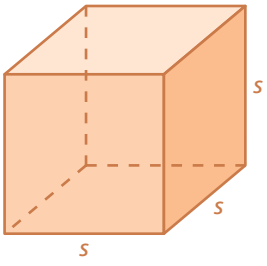
a. Volume = 27 ft^3



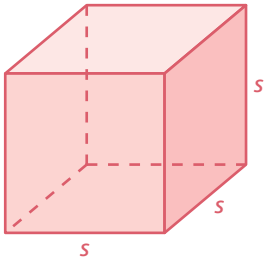
b. Volume = 125 cm^3



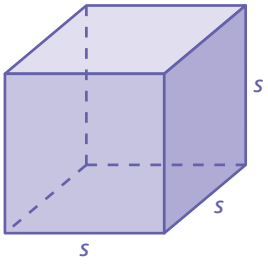
c. Volume = 3375 in.^{3}



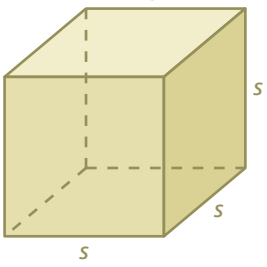
d. Volume = 3.375 m^3



e. Volume = 1 yd^3



f. Volume = $\frac{125}{8}$ mm^3



Cubes are not drawn to scale.

Common Core

Exponents

In this lesson, you will
- simplify expressions with rational exponents.

Learning Standards
N.RN.1
N.RN.2

2 ACTIVITY: Estimating *n*th Roots

Work with a partner. When you raise an *n*th root of a number to the *n*th power, you get the original number.

$$(\sqrt[n]{a})^n = a$$

Sample: The 4th root of 16 is 2 because $2^4 = 16$.

$$\sqrt[4]{16} = 2$$

Check: $2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$ ✓

Match the *n*th root with the point on the number line. Justify your answer.

a. $\sqrt[4]{25}$ **b.** $\sqrt{0.5}$ **c.** $\sqrt[5]{2.5}$

d. $\sqrt[3]{65}$ **e.** $\sqrt[3]{55}$ **f.** $\sqrt[6]{20{,}000}$

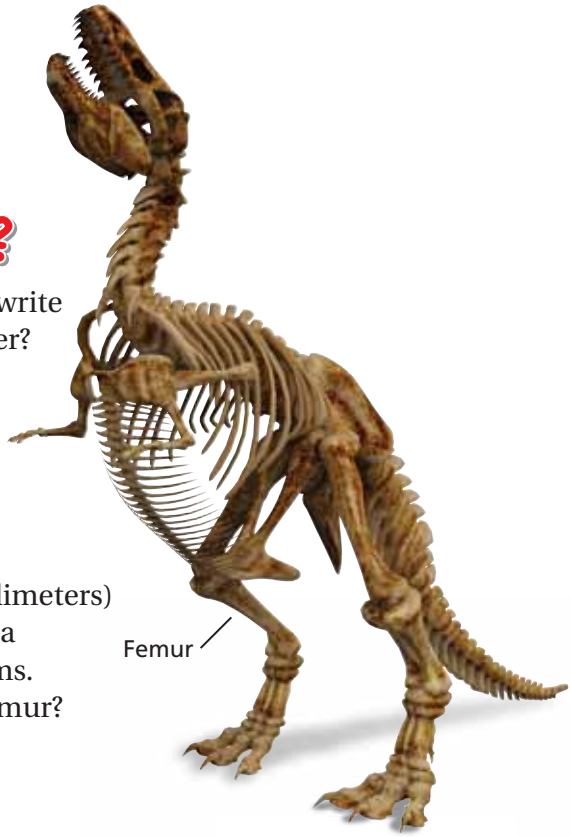
What Is Your Answer?

3. IN YOUR OWN WORDS How can you write and evaluate the *n*th root of a number?

4. The body mass m (in kilograms) of a dinosaur that walked on two feet can be modeled by

$$m = (0.00016)C^{2.73}$$

where C is the circumference (in millimeters) of the dinosaur's femur. The mass of a Tyrannosaurus rex was 4000 kilograms. What was the circumference of its femur?



Math Practice 3

Justify Conclusions

What information can you use to justify your answer?

Practice Use what you learned about cube roots to complete Exercises 3–5 on page 280.

6.3 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
*n*th root, *p. 278*

When $b^n = a$ for an integer n greater than 1, b is an **nth root** of a.

$\sqrt[n]{a}$ *n*th root of *a*

The nth roots of a number may be real numbers or *imaginary numbers*. You will study imaginary numbers in a future course.

EXAMPLE 1 Finding *n*th Roots

Simplify each expression.

a. $\sqrt[3]{64}$

$\sqrt[3]{64} = \sqrt[3]{4 \cdot 4 \cdot 4}$

$= 4$

b. $\sqrt[4]{81}$

$\sqrt[4]{81} = \sqrt[4]{3 \cdot 3 \cdot 3 \cdot 3}$

$= 3$

Study Tip

In Example 1b, although $3^4 = 81$ and $(-3)^4 = 81$, $\sqrt[4]{81} = 3$ because the radical symbol indicates the positive root.

Key Idea

Rational Exponents

Words The nth root of a positive number a can be written as a power with base a and an exponent of $1/n$.

Numbers $\sqrt[4]{81} = 81^{1/4}$ **Algebra** $\sqrt[n]{a} = a^{1/n}$

EXAMPLE 2 Simplifying Expressions with Rational Exponents

Simplify each expression.

a. $400^{1/2}$

$400^{1/2} = \sqrt{400}$ Write the expression in radical form.

$= \sqrt{20 \cdot 20}$ Rewrite.

$= 20$ Simplify.

b. $243^{1/5}$

$243^{1/5} = \sqrt[5]{243}$ Write the expression in radical form.

$= \sqrt[5]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$ Rewrite.

$= 3$ Simplify.

Reading

When $n = 2$, the 2 is typically not written with the radical sign.

On Your Own

Now You're Ready
Exercises 13–18

Simplify the expression.

1. $\sqrt[3]{216}$
2. $\sqrt[5]{32}$
3. $\sqrt[4]{625}$
4. $49^{1/2}$
5. $343^{1/3}$
6. $64^{1/6}$

You can use properties of exponents to simplify expressions involving rational exponents.

EXAMPLE 3 Using Properties of Exponents

a. $16^{3/4} = 16^{(1/4) \cdot 3}$ Rewrite the exponent.

$= (16^{1/4})^3$ Power of a Power Property

$= 2^3$ Evaluate the fourth root of 16.

$= 8$ Evaluate power.

b. $27^{4/3} = 27^{1/3 \cdot 4}$ Rewrite the exponent.

$= (27^{1/3})^4$ Power of a Power Property

$= 3^4$ Evaluate the third root of 27.

$= 81$ Evaluate power.

On Your Own

Now You're Ready
Exercises 20–25

Simplify the expression.

7. $64^{2/3}$ **8.** $9^{5/2}$ **9.** $256^{3/4}$

EXAMPLE 4 Real-Life Application

The radius r of a sphere is given by the equation $r = \left(\frac{3V}{4\pi}\right)^{1/3}$, where V is the volume of the sphere. Find the radius of the beach ball to the nearest foot. Use 3.14 for π.

$r = \left(\frac{3V}{4\pi}\right)^{1/3}$ Write the equation.

$= \left[\frac{3(113)}{4(3.14)}\right]^{1/3}$ Substitute 113 for V and 3.14 for π.

$= \left(\frac{339}{12.56}\right)^{1/3}$ Multiply.

≈ 3 Use a calculator.

∴ The radius of the beach ball is about 3 feet.

On Your Own

10. WHAT IF? In Example 4, the volume of the beach ball is 17,000 cubic inches. Find the radius to the nearest inch. Use 3.14 for π.

6.3 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **WRITING** Explain how to simplify $81^{1/4}$.

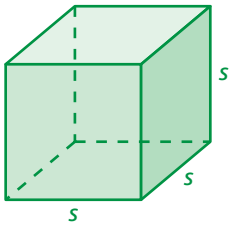
2. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$(\sqrt[3]{27})^2$ $\quad 27^{2/3}$ $\quad 3^2$ $\quad 27^{3/2}$

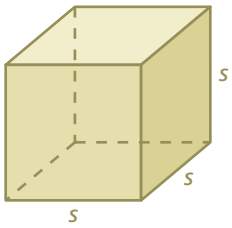
Practice and Problem Solving

Find the dimensions of the cube. Check your answer.

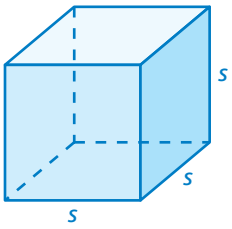
3. Volume = 64 in.^{3}



4. Volume = 216 cm^3



5. Volume = $\frac{343}{512}$ ft^3



Write the expression in rational exponent form.

6. $\sqrt[7]{5}$

7. $(\sqrt[3]{4})^2$

8. $(\sqrt[5]{8})^4$

Write the expression in radical form.

9. $15^{1/3}$

10. $140^{1/7}$

11. $78^{2/5}$

12. **ERROR ANALYSIS** Describe and correct the error in writing the expression in rational exponent form.

✗ $(\sqrt[3]{2})^4 = 2^{3/4}$

Simplify the expression.

13. $\sqrt[4]{256}$

14. $\sqrt[3]{125}$

15. $\sqrt[5]{1024}$

16. $128^{1/7}$

17. $1000^{1/3}$

18. $81^{1/2}$

19. **BAKE SALE** A math club is having a bake sale. Find the length and width of the bake sale sign.

$4^{1/2}$ ft

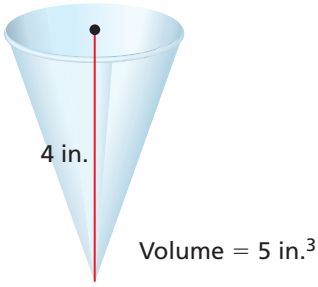
$\sqrt[6]{729}$ ft

Simplify the expression.

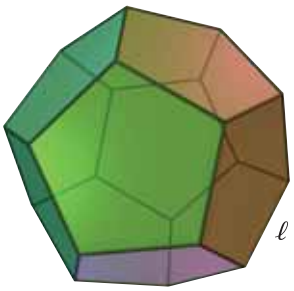
20. $32^{3/5}$ **21.** $125^{2/3}$ **22.** $36^{3/2}$

23. $243^{2/5}$ **24.** $128^{5/7}$ **25.** $343^{4/3}$

26. PAPER CUPS The radius r of the base of a cone is given by the equation $r = \left(\frac{3V}{\pi h}\right)^{1/2}$, where V is the volume of the cone and h is the height of the cone. Find the radius of the paper cup to the nearest inch. Use 3.14 for π.



27. WRITING Explain how to write $(\sqrt[n]{a})^m$ in rational exponent form.



28. PROBLEM SOLVING The formula for the volume of a regular dodecahedron is $V \approx 7.66\,\ell^3$, where ℓ is the length of an edge. The volume of the dodecahedron is 20 cubic feet. Estimate the edge length.

Logic **Determine whether the statement is *always*, *sometimes*, or *never* true. Let x be a nonnegative real number. Justify your answer.**

29. $(x^{1/3})^3 = x$ **30.** $x^{1/3} = x^{-3}$ **31.** $x^{1/3} = \sqrt[3]{x}$

32. $x^{1/3} = x^3$ **33.** $\frac{x^{2/3}}{x^{1/3}} = \sqrt[3]{x}$ **34.** $x = x^{1/3} \cdot x^3$

Fair Game Review What you learned in previous grades & lessons

Graph the linear equation. *(Section 2.3 and Section 2.4)*

35. $y = -2x + 1$ **36.** $4x - 2y = 6$ **37.** $y = -\frac{1}{3}x - 5$

38. MULTIPLE CHOICE Which equation is shown in the graph? *(Section 2.1)*

Ⓐ $y = -\frac{1}{2}x + 1$ Ⓑ $y = -\frac{1}{2}x - 1$

Ⓒ $y = \frac{1}{2}x - 1$ Ⓓ $y = \frac{1}{2}x + 1$