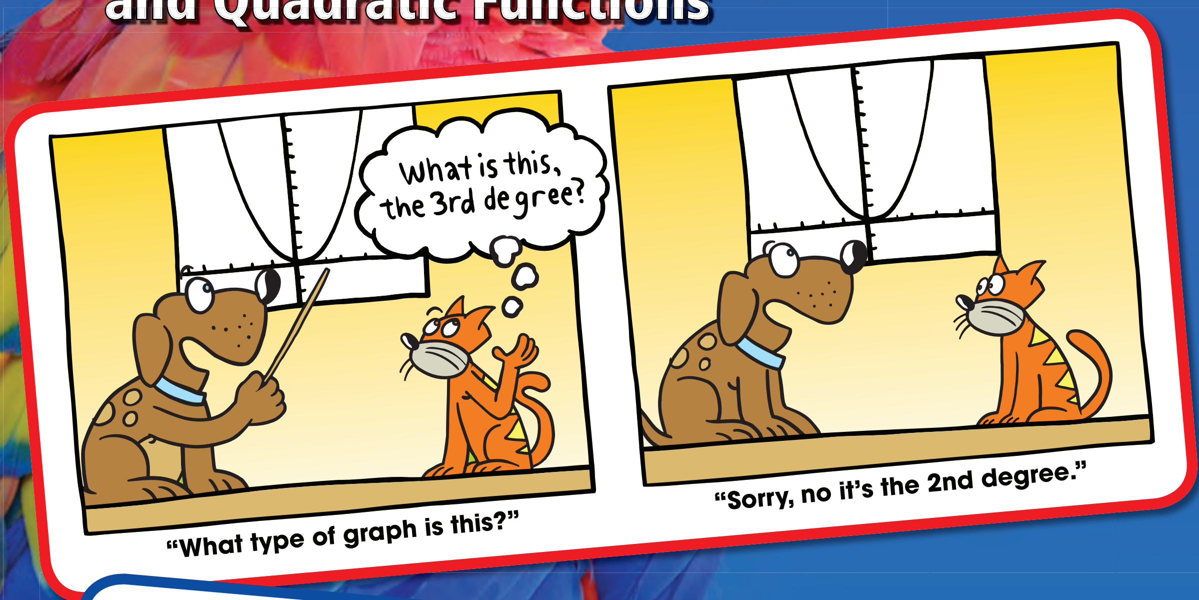
8 Graphing Quadratic Functions



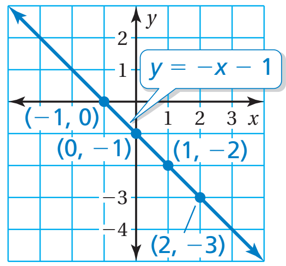
What You Learned Before

Graphing a Linear Equation (A.CED.2)

Example 1 **Graph $y = -x - 1$.**

Step 1: Make a table of values.

x	$y = -x - 1$	y	(x, y)
-1	$y = -(-1) - 1$	0	$(-1, 0)$
0	$y = -(0) - 1$	-1	$(0, -1)$
1	$y = -(1) - 1$	-2	$(1, -2)$
2	$y = -(2) - 1$	-3	$(2, -3)$



Step 2: Plot the ordered pairs.

Step 3: Draw a line through the points.

Try It Yourself

Graph the linear equation.

1. $y = 2x - 3$ **2.** $y = -3x + 4$ **3.** $y = x + 5$

Evaluating an Expression (6.EE.2c)

Example 2 **Evaluate $2x^2 + 3x - 5$ when $x = -1$.**

$2x^2 + 3x - 5 = 2(-1)^2 + 3(-1) - 5$ Substitute -1 for x.

$= 2(1) + 3(-1) - 5$ Evaluate the power.

$= 2 - 3 - 5$ Multiply.

$= -6$ Subtract.

Try It Yourself

Evaluate the expression when $x = -2$.

4. $-x^2 - 4x + 1$ **5.** $3x^2 + x - 2$ **6.** $-2x^2 - 4x + 3$

8.1 Graphing $y = ax^2$

Essential Question What are the characteristics of the graph of the quadratic function $y = ax^2$? How does the value of a affect the graph of $y = ax^2$?

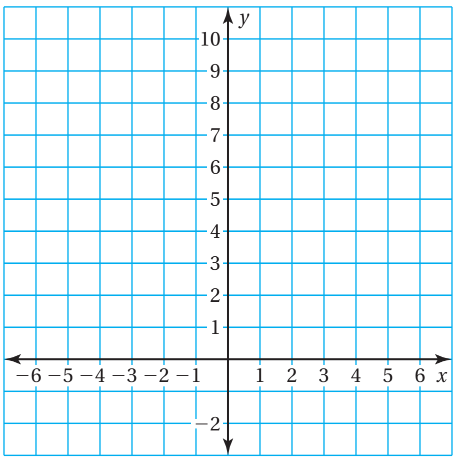
1 ACTIVITY: Graphing a Quadratic Function

Work with a partner.

- Complete the input-output table.
- Plot the points in the table.
- Sketch the graph by connecting the points with a smooth curve.
- What do you notice about the graphs?

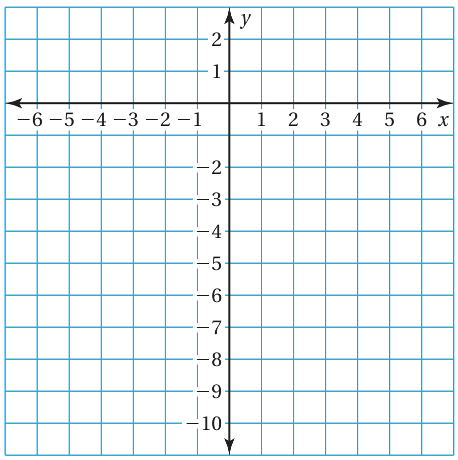
a.

x	$y = x^2$
−3	
−2	
−1	
0	
1	
2	
3	



b.

x	$y = -x^2$
−3	
−2	
−1	
0	
1	
2	
3	



Common Core

Graphing Quadratic Functions

In this lesson, you will
- identify characteristics of quadratic functions.
- graph quadratic functions.

Learning Standard
F.BF.3

2 ACTIVITY: Graphing a Quadratic Function

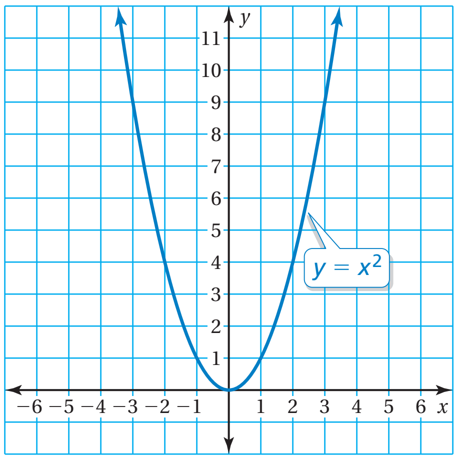
Math Practice 7

Look for Patterns

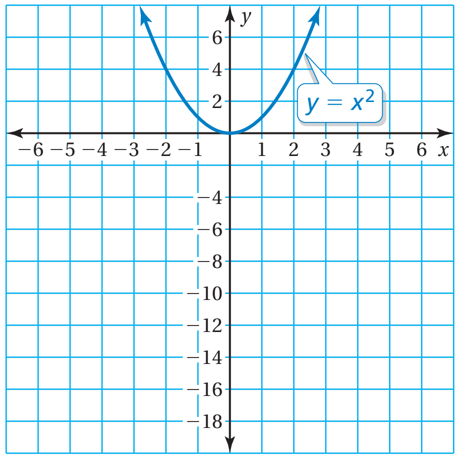
What pattern do you notice when comparing each equation with its graph?

Work with a partner. Graph each function. How does the value of *a* affect the graph of $y = ax^2$?

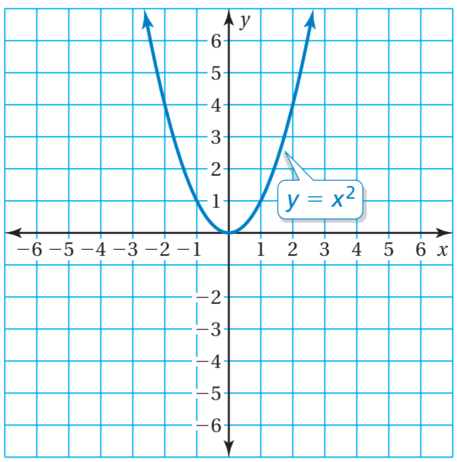
a. $y = 3x^2$



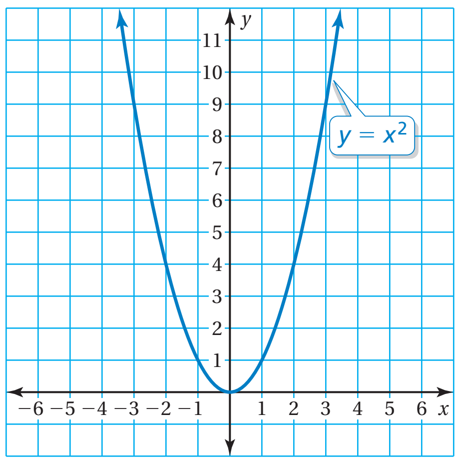
b. $y = -5x^2$



c. $y = -0.2x^2$

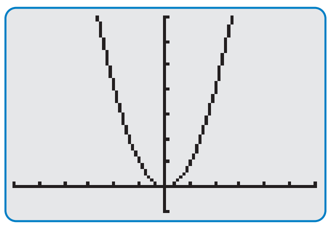


d. $y = \frac{1}{10}x^2$



What Is Your Answer?

3. **IN YOUR OWN WORDS** What are the characteristics of the graph of the quadratic function $y = ax^2$? How does the value of a affect the graph of $y = ax^2$? Consider $a < 0$, $|a| > 1$, and $0 < |a| < 1$ in your answer.



Practice Use what you learned about the graphs of quadratic functions to complete Exercises 5–7 on page 407.

8.1 Lesson

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Key Vocabulary
quadratic function, *p. 404*
parabola, *p. 404*
vertex, *p. 404*
axis of symmetry, *p. 404*

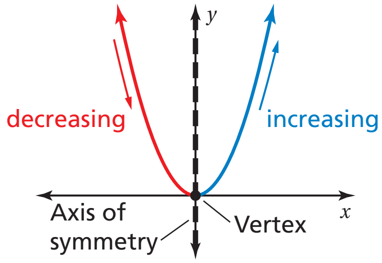
A **quadratic function** is a nonlinear function that can be written in the standard form $y = ax^2 + bx + c$, where $a \neq 0$. The U-shaped graph of a quadratic function is called a **parabola.**

Key Idea

Characteristics of Quadratic Functions

The most basic quadratic function is $y = x^2$.

The lowest or highest point on a parabola is the **vertex.**

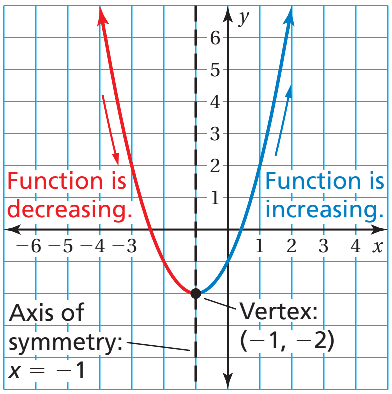


The vertical line that divides the parabola into two symmetric parts is the **axis of symmetry.** The axis of symmetry passes through the vertex.

EXAMPLE 1 Identifying Characteristics of a Quadratic Function

Consider the graph of the quadratic function.

Using the graph, you can identify the vertex, axis of symmetry, and the behavior of the graph as shown.



You can also determine the following:

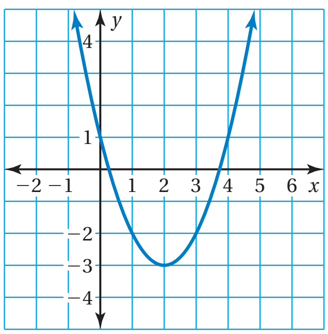
- The domain is all real numbers.
- The range is all real numbers greater than or equal to -2.
- When $x < -1$, y increases as x decreases.
- When $x > -1$, y increases as x increases.

On Your Own

Now You're Ready
Exercises 8–10

Identify characteristics of the graph of the quadratic function.

1.



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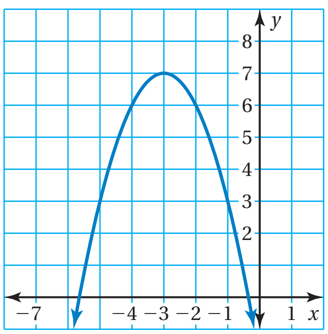

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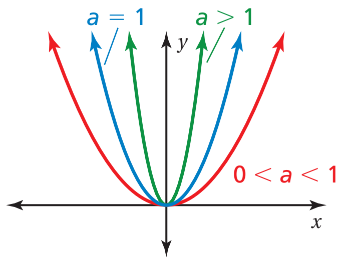
Key Ideas

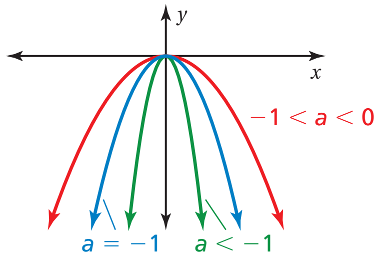
Graphing $y = ax^2$ When $a > 0$

- When $0 < a < 1$, the graph of $y = ax^2$ opens up and is wider than the graph of $y = x^2$.
- When $a > 1$, the graph of $y = ax^2$ opens up and is narrower than the graph of $y = x^2$.



Graphing $y = ax^2$ When $a < 0$

- When $-1 < a < 0$, the graph of $y = ax^2$ opens down and is wider than the graph of $y = x^2$.
- When $a < -1$, the graph of $y = ax^2$ opens down and is narrower than the graph of $y = x^2$.



EXAMPLE 2 Graphing $y = ax^2$ When $a > 0$

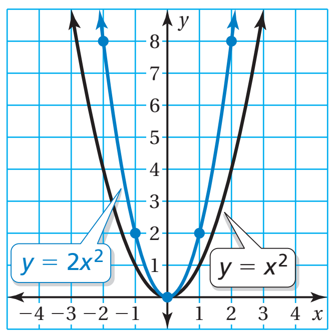
Graph $y = 2x^2$. Compare the graph to the graph of $y = x^2$.

Step 1: Make a table of values.

x	−2	−1	0	1	2
y	8	2	0	2	8

Step 2: Plot the ordered pairs.

Step 3: Draw a smooth curve through the points.



Both graphs open up and have the same vertex, (0, 0), and the same axis of symmetry, $x = 0$. The graph of $y = 2x^2$ is narrower than the graph of $y = x^2$.

On Your Own

Now You're Ready
Exercises 11–16

Graph the function. Compare the graph to the graph of $y = x^2$.

3. $y = 5x^2$

4. $y = \frac{1}{3}x^2$

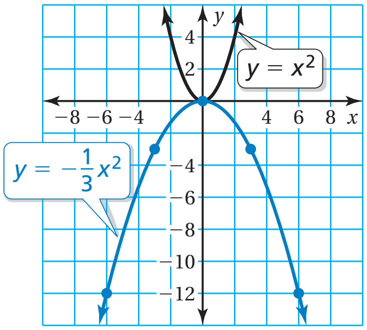
5. $y = \frac{3}{2}x^2$

EXAMPLE 3 Graphing $y = ax^2$ When $a < 0$

Graph $y = -\frac{1}{3}x^2$. Compare the graph to the graph of $y = x^2$.

Step 1: Make a table of values. Choose x-values that make the calculations simple.

x	−6	−3	0	3	6
y	−12	−3	0	−3	−12



Step 2: Plot the ordered pairs.

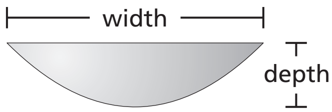
Step 3: Draw a smooth curve through the points.

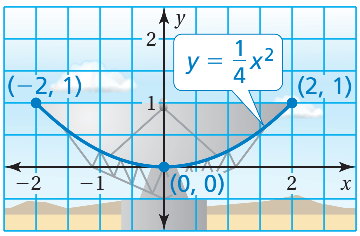
The graphs have the same vertex, (0, 0), and the same axis of symmetry, $x = 0$, but the graph of $y = -\frac{1}{3}x^2$ opens down. The graph of $y = -\frac{1}{3}x^2$ is wider than the graph of $y = x^2$.

EXAMPLE 4 Real-Life Application

The diagram shows the cross section of a satellite dish, where x and y are measured in meters. Find the width and depth of the dish.

Use the domain of the function to find the width of the dish. Use the range to find the depth.





The leftmost point on the graph is (−2, 1) and the rightmost point is (2, 1). So, the domain is $-2 \le x \le 2$, which represents 4 meters.

The lowest point on the graph is (0, 0) and the highest points on the graph are (−2, 1) and (2, 1). So, the range is $0 \le y \le 1$, which represents 1 meter.

So, the satellite dish is 4 meters wide and 1 meter deep.

On Your Own

Now You're Ready
Exercises 18–23 and 34

Graph the function. Compare the graph to the graph of $y = x^2$.

6. $y = -3x^2$ **7.** $y = -0.1x^2$ **8.** $y = -\frac{1}{4}x^2$

9. The cross section of a spotlight can be modeled by the graph of $y = 0.5x^2$, where x and y are measured in inches and $-2 \le x \le 2$. Find the width and depth of the spotlight.

8.1 Exercises

Check It Out
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Vocabulary and Concept Check

1. **VOCABULARY** Describe the vertex and axis of symmetry of the graph of $y = ax^2$.
2. **VOCABULARY** What is the U-shaped graph of a quadratic function called?
3. **WRITING** Without graphing, which graph is wider, $y = 6x^2$ or $y = \frac{1}{6}x^2$? Explain your reasoning.
4. **WRITING** When does the graph of a quadratic function open up? open down?

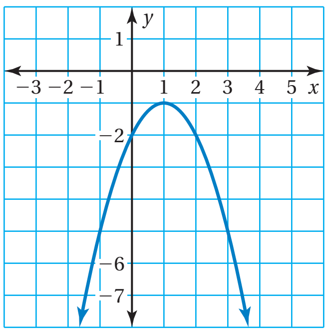
Practice and Problem Solving

Use a graphing calculator to graph the function. Compare the graph to the graph of $y = -4x^2$.

5. $y = -0.4x^2$
6. $y = -0.04x^2$
7. $y = -0.004x^2$

Identify characteristics of the graph of the quadratic function.

1 **8.**



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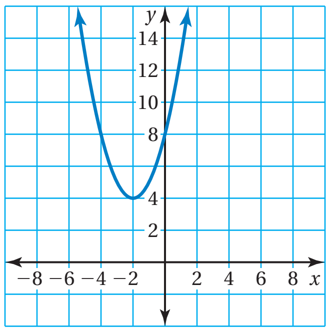

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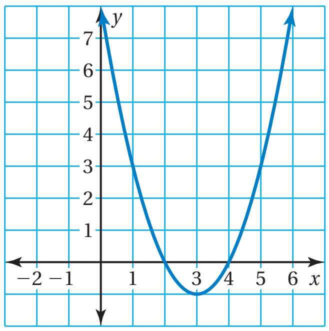


Graph the function. Compare the graph to the graph of $y = x^2$.

2 **11.** $y = 6x^2$

12. $y = 8x^2$

13. $y = \frac{1}{4}x^2$

14. $y = \frac{3}{4}x^2$

15. $y = \frac{5}{2}x^2$

16. $y = \frac{7}{5}x^2$

17. WATERFALL A fish swims over the waterfall. The distance y (in feet) that the fish falls is given by the function $y = 16t^2$, where t is the time (in seconds).

a. Describe the domain and range of the function.

b. Graph the function using the domain in part (a).

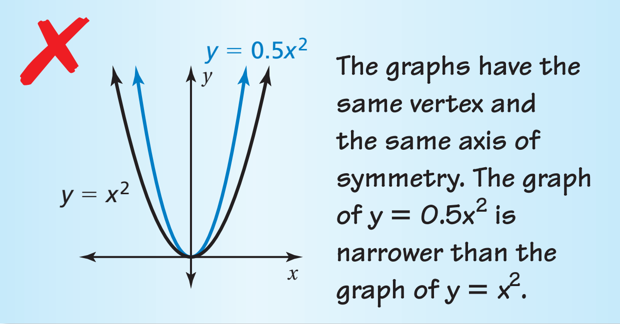
c. Use the graph to determine when the fish lands in the water below.

Graph the function. Compare the graph to the graph of $y = x^2$.

18. $y = -2x^2$ **19.** $y = -7x^2$ **20.** $y = -\frac{1}{5}x^2$

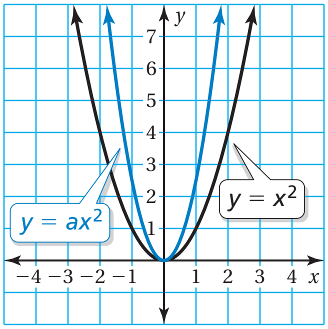
21. $y = -\frac{5}{8}x^2$ **22.** $y = -\frac{5}{3}x^2$ **23.** $y = -\frac{9}{4}x^2$

24. ERROR ANALYSIS Describe and correct the error in graphing and comparing $y = x^2$ and $y = 0.5x^2$.



Describe the possible values of *a*.

25.



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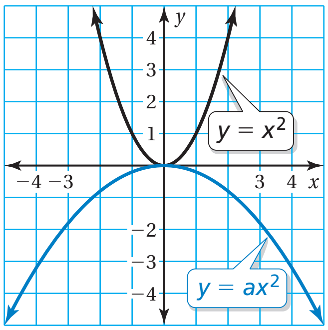

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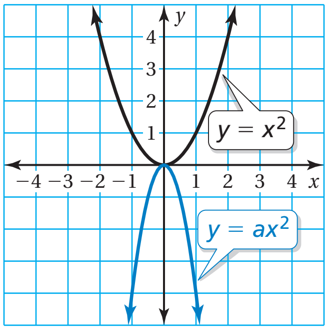


28. REASONING A parabola opens up and passes through $(-4, 2)$ and $(6, -3)$. How do you know that $(-4, 2)$ is not the vertex?

29. REASONING Describe the domain and range of the function $y = ax^2$ when (a) $a > 0$ and (b) $a < 0$.

Determine whether the statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

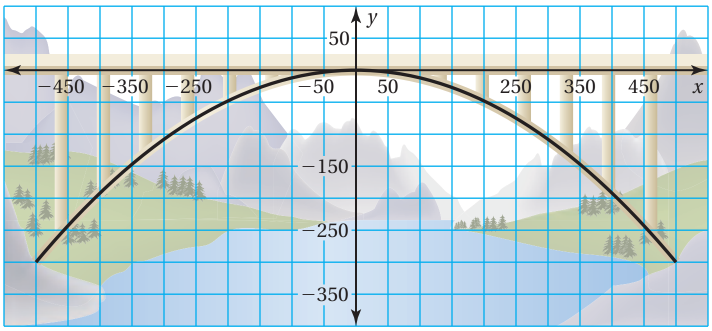
30. The graph of $y = ax^2$ is narrower than the graph of $y = x^2$ when $a > 0$.

31. The graph of $y = ax^2$ is narrower than the graph of $y = x^2$ when $|a| > 1$.

32. The graph of $y = ax^2$ is wider than the graph of $y = x^2$ when $0 < |a| < 1$.

33. The graph of $y = ax^2$ is wider than the graph of $y = dx^2$ when $|a| > |d|$.

34. BRIDGE The arch support of a bridge can be modeled by $y = -0.0012x^2$, where x and y are measured in feet. Find the height and width of the arch.

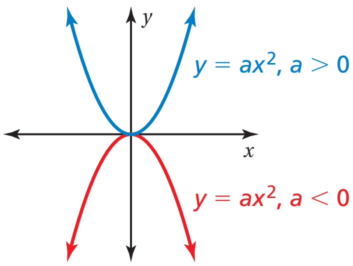


In Exercises 35–37, use the graph of the function $y = ax^2$.

35. When is the function increasing?

36 When is the function decreasing?

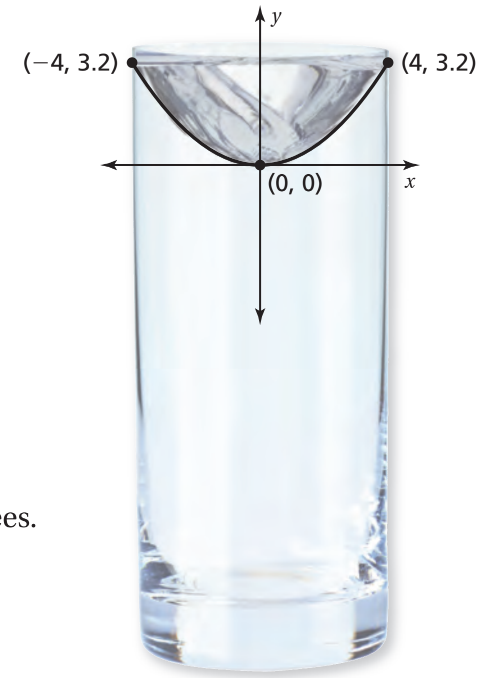
37. Find the value of a when the graph passes through $(-2, 3)$.



38. **MODELING** The diagram shows the cross section of a swirling glass of water, where x and y are measured in centimeters. The surface of the cross section of the rotating liquid is a parabola.

a. About how wide is the mouth of the glass?

b. Suppose the rotational speed of the liquid changes. The cross section can now be modeled by $y = 0.1x^2$. Did the rotational speed *increase* or *decrease*? Explain your reasoning.



39. **ASSEMBLY LINE** The number y of units an assembly line can produce in 1 hour can be modeled by the function $y = 0.5x^2$, where x is the number of employees. The assembly line has a capacity of 10 employees.

a. Describe the domain of the function.

b. Graph the function using the domain in part (a).

c. Is it better for the company to run one assembly line at full capacity or two assembly lines at half capacity? Explain your reasoning.

40. **Logic** Is the x-intercept of the graph of $y = ax^2$ always 0? Justify your answer.

Fair Game Review What you learned in previous grades & lessons

Solve the proportion using the Cross Products Property. *(Skills Review Handbook)*

41. $\frac{x}{6} = \frac{13}{2}$ **42.** $\frac{5}{3} = \frac{n}{9}$ **43.** $\frac{4}{b} = \frac{6}{21}$ **44.** $\frac{14}{9} = \frac{7}{y}$

45. **MULTIPLE CHOICE** What is the completely factored form of $2x^5 - 8x^3$? *(Section 7.9)*

Ⓐ $2x^3(x - 2)^2$ Ⓑ $2x^3(x - 2)(x + 2)$

Ⓒ $2x^3(x + 2)^2$ Ⓓ $2x^3(x^2 - 4)$

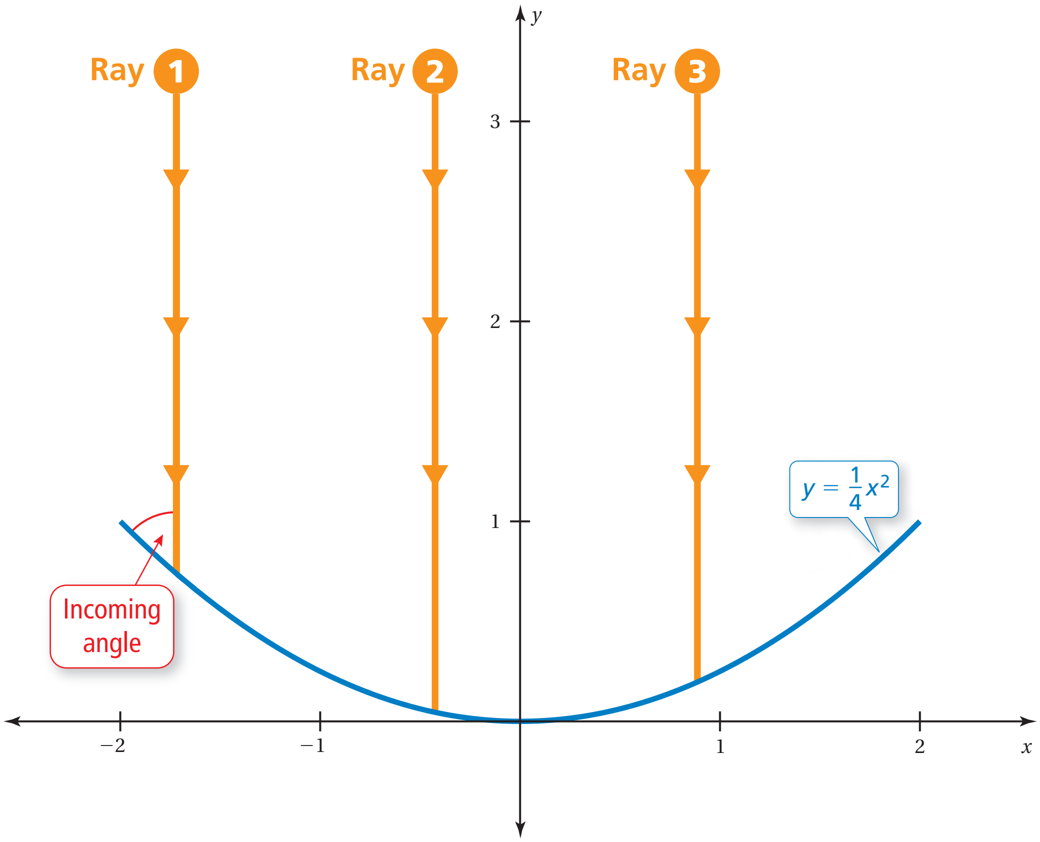
8.2 Focus of a Parabola

Essential Question Why do satellite dishes and spotlight reflectors have parabolic shapes?

1 ACTIVITY: A Property of Satellite Dishes

Work with a partner. Rays are coming straight down. When they hit the parabola, they reflect off at the same angle at which they entered.

- Draw the outgoing part of each ray so that it intersects the y-axis.
- What do you notice about where the reflected rays intersect the y-axis?
- Where is the receiver for the satellite dish? Explain.



COMMON CORE

Graphing Quadratic Functions

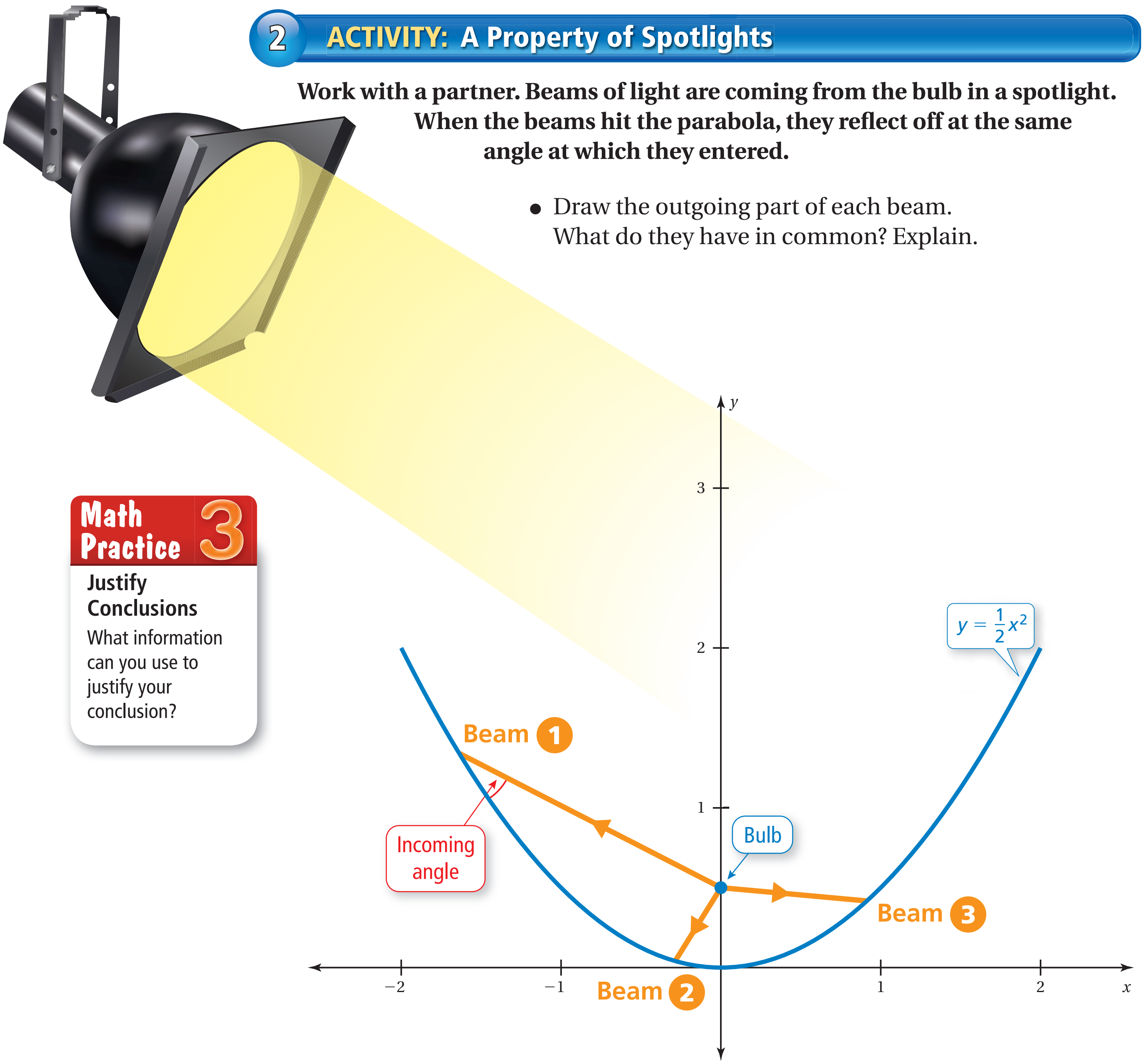
In this lesson, you will
- find the foci of parabolas.
- write equations of parabolas with vertices at the origin given the foci.

Learning Standard
F.IF.4

2 ACTIVITY: A Property of Spotlights

Work with a partner. Beams of light are coming from the bulb in a spotlight. When the beams hit the parabola, they reflect off at the same angle at which they entered.

- Draw the outgoing part of each beam. What do they have in common? Explain.



Math Practice 3

Justify Conclusions

What information can you use to justify your conclusion?

What Is Your Answer?

3. **IN YOUR OWN WORDS** Why do satellite dishes and spotlight reflectors have parabolic shapes?

4. Design and draw a parabolic satellite dish. Label the dimensions of the dish. Label the receiver.

Practice Use what you learned about parabolas to complete Exercises 4–6 on page 414.

8.2 Lesson

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Lesson Tutorials
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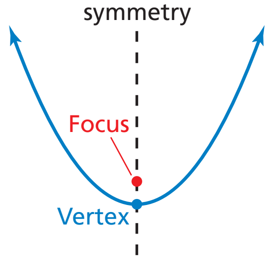
Key Vocabulary
focus, *p. 412*

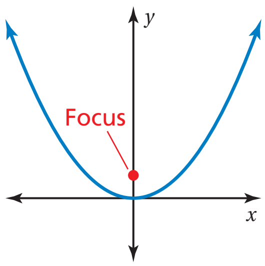
Key Idea

The Focus of a Parabola

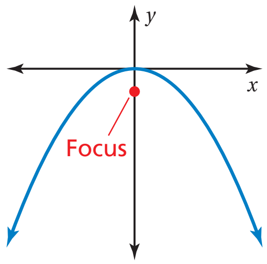
The **focus** of a parabola is a fixed point on the interior of a parabola that lies on the axis of symmetry. A parabola "wraps" around the focus.



For functions of the form $y = ax^2$, the focus is $\left(0, \frac{1}{4a}\right)$.



$y = ax^2,\ a > 0$



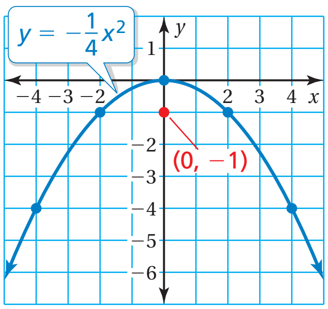
$y = ax^2,\ a < 0$

EXAMPLE 1 Finding the Focus of a Parabola

Graph $y = -\frac{1}{4}x^2$. Identify the focus.

Step 1: Make a table of values. Then graph.

x	-4	-2	0	2	4
y	-4	-1	0	-1	-4



Step 2: Identify the focus. The function is of the form $y = ax^2$, so $a = -\frac{1}{4}$.

$$\frac{1}{4a} = \frac{1}{4\left(-\frac{1}{4}\right)} \qquad \text{Substitute } -\frac{1}{4} \text{ for } a.$$

$$= \frac{1}{-1}, \text{ or } -1 \qquad \text{Multiply.}$$

So, the focus of the function $y = -\frac{1}{4}x^2$ is $(0, -1)$.

On Your Own

Now You're Ready
Exercises 7–12

Graph the function. Identify the focus.

1. $y = 2x^2$
2. $y = \frac{1}{6}x^2$
3. $y = -3x^2$

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EXAMPLE 2 Writing an Equation of a Parabola

Write an equation of the parabola with focus (0, 4) and vertex at the origin.

For $y = ax^2$, the focus is $\left(0, \frac{1}{4a}\right)$. Use the given focus, (0, 4), to write an equation to find a.

$\frac{1}{4a} = 4$ — Equate the y-coordinates.

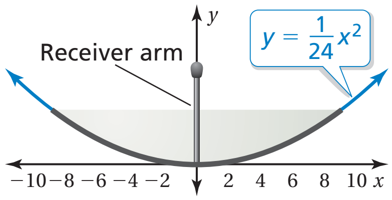
$1 = 16a$ — Multiply each side by $4a$.

$\frac{1}{16} = a$ — Divide each side by 16.

An equation of the parabola is $y = \frac{1}{16}x^2$.

EXAMPLE 3 Real-Life Application

A birdwatcher uses a parabolic microphone to collect and record bird sounds. The cross section of the microphone can be modeled by $y = \frac{1}{24}x^2$, where x and y are measured in inches. The focus is located at the end of the receiver arm. What is the length of the receiver arm?



The arm length is the distance from the focus to the vertex.

Identify the focus. For the function $y = \frac{1}{24}x^2$, $a = \frac{1}{24}$.

$\frac{1}{4a} = \frac{1}{4\left(\frac{1}{24}\right)}$ — Substitute $\frac{1}{24}$ for a.

$= \frac{1}{\frac{1}{6}}$ — Multiply.

$= 6$ — Divide.

The focus is (0, 6). The vertex is (0, 0). The distance from (0, 0) to (0, 6) is 6 units.

So, the length of the receiver arm is 6 inches.

On Your Own

Now You're Ready
Exercises 14–19

4. Write an equation of the parabola with focus (0, −3) and vertex at the origin.

5. **WHAT IF?** In Example 3, the cross section of the microphone can be modeled by $y = \frac{1}{40}x^2$. What is the length of the receiver arm?

8.2 Exercises

Vocabulary and Concept Check

1. **VOCABULARY** What is the relationship between the focus and the axis of symmetry of a parabola?
2. **WRITING** When the focus of a parabola lies below the vertex, does the parabola open up or down? Is $a > 0$ or $a < 0$? Explain.
3. **OPEN-ENDED** Write an equation of a parabola whose focus is below the x-axis.

Practice and Problem Solving

Determine whether the shape is parabolic.

4. 

5. 

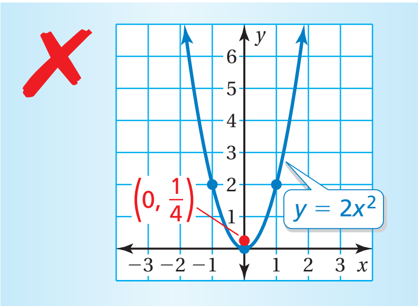
6. 

Graph the function. Identify the focus.

1 7. $y = x^2$ 8. $y = 4x^2$ 9. $y = -12x^2$

10. $y = \frac{1}{4}x^2$ 11. $y = \frac{1}{2}x^2$ 12. $y = -0.75x^2$

13. **ERROR ANALYSIS** Describe and correct the error in identifying the focus.



Write an equation of the parabola with a vertex at the origin and the given focus.

2 14. (0, 1) 15. (0, −2) 16. (0, 5)

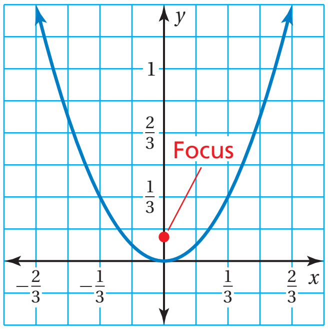
17. $\left(0, -\frac{1}{4}\right)$ 18. (0, −1) 19. (0, 0.5)

20. **COMET** A comet travels along a parabolic path around the Sun. The Sun is the focus of the path. When the comet is at the vertex of the path, it is 60,000,000 kilometers from the Sun. Write an equation that represents the path of the comet. Assume the focus is on the positive y-axis and the vertex is (0, 0).

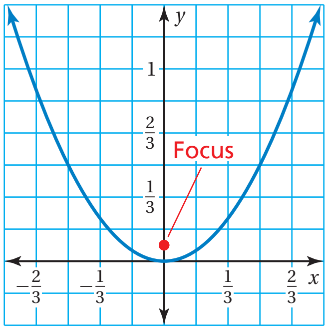
Match the equation with its graph.

21. $y = 2x^2$ **22.** $y = 3x^2$ **23.** $y = 2.5x^2$

A.



B.



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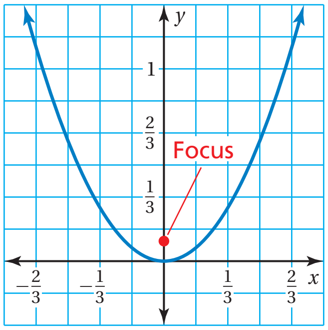


Determine whether the statement is *sometimes*, *always*, or *never* true for the function $y = ax^2$. Explain your reasoning.

24. The vertex and focus of a parabola can occur at the same point.

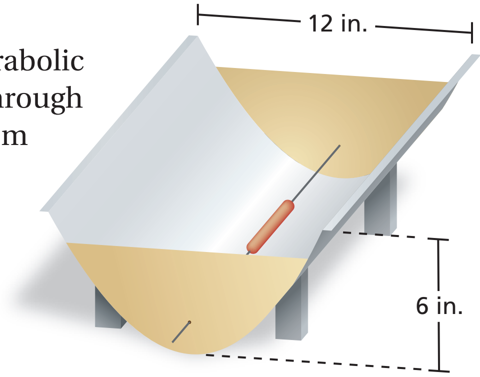
25. If a parabola opens down, then the focus lies below the x-axis.

26. The y-coordinate of the focus of a parabola is greater than the y-coordinate of the vertex.

27. **REASONING** Describe how the graph of $y = ax^2$ changes as the distance between the vertex and focus increases.

28. **WHISPER DISH** Whisper dishes are parabolic sound reflectors that transmit and receive sound waves from opposite ends of a room. For the best sound reception, you place your ear at the focus of the dish. Write an equation for the cross section of a dish when your ear is 3 feet from the vertex.

29. **SOLAR COOKING** You make a solar cooker using a parabolic reflective surface. You suspend a sausage with wire through the focus of each end piece of the cooker. How far from the bottom should you place the wire?



30. **Structure** For what values of a will the distance between the focus and the vertex of the graph of $y = ax^2$ be less than the distance between the focus and the vertex of the graph of $y = (ax)^2$?

Fair Game Review What you learned in previous grades & lessons

Evaluate the expression when $x = -3$ and $n = 2$. *(Skills Review Handbook)*

31. $x^2 + 7$ **32.** $3n^2 - 2$ **33.** $x + n^2$

34. **MULTIPLE CHOICE** Which number is equivalent to $32^{3/5}$? *(Section 6.3)*

Ⓐ 2 Ⓑ 4 Ⓒ 8 Ⓓ 10

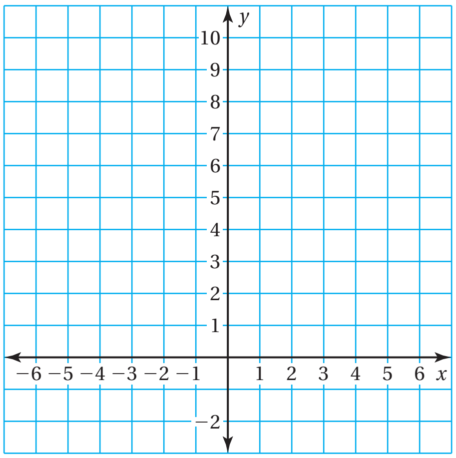
8.3 Graphing $y = ax^2 + c$

Essential Question How does the value of c affect the graph of $y = ax^2 + c$?

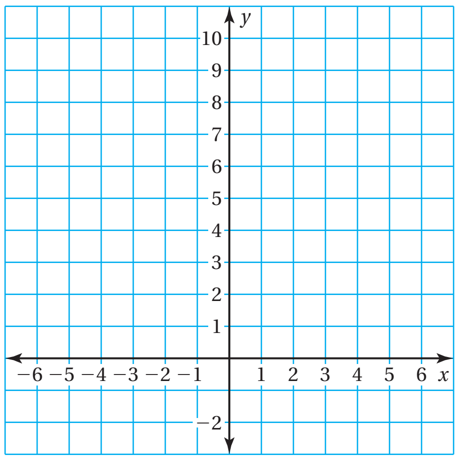
1 ACTIVITY: Graphing $y = ax^2 + c$

Work with a partner. Sketch the graphs of both functions in the same coordinate plane. How does the value of c affect the graph of $y = ax^2 + c$?

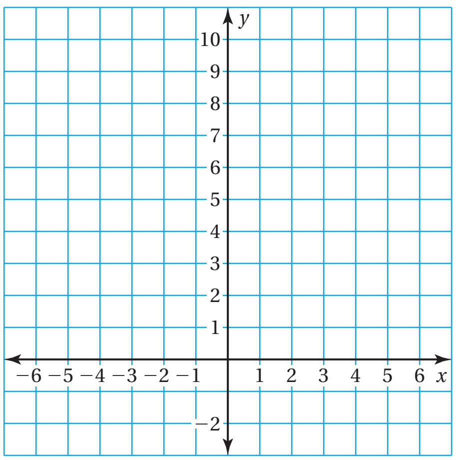
a. $y = x^2$ and $y = x^2 + 2$



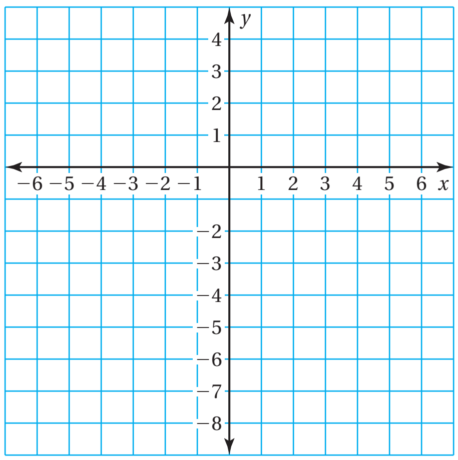
b. $y = 2x^2$ and $y = 2x^2 - 2$



c. $y = -x^2 + 4$ and $y = -x^2 + 9$



d. $y = \frac{1}{2}x^2$ and $y = \frac{1}{2}x^2 - 8$



COMMON CORE

Graphing Quadratic Functions

In this lesson, you will

- graph quadratic functions of the form $y = ax^2 + c$ and compare to the graph of $y = x^2$.

Learning Standard
F.BF.3

2 ACTIVITY: Finding x-Intercepts of Graphs

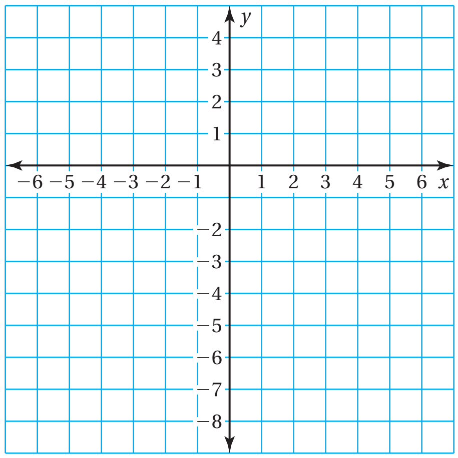
Math Practice 6

Communicate Precisely

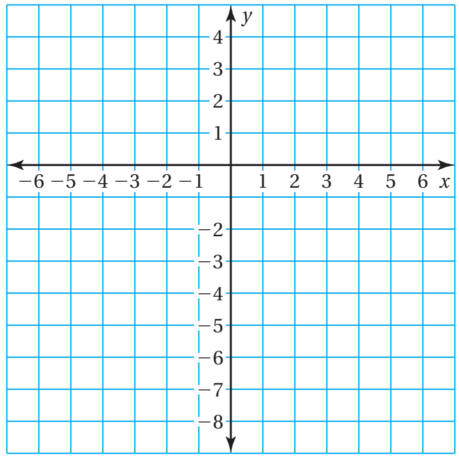
What have you included in your answer to make sure your explanation is precise?

Work with a partner. Graph each function. Find the x-intercepts of the graph. Explain how you found the x-intercepts.

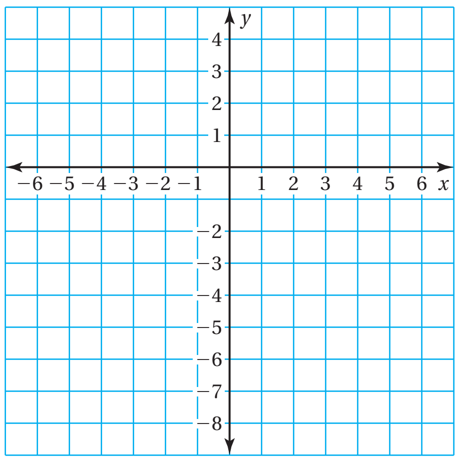
a. $y = x^2 - 4$



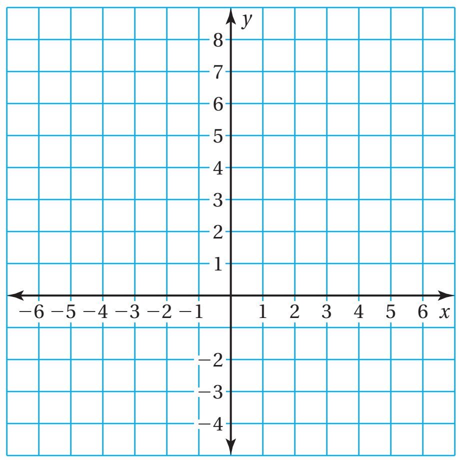
b. $y = 2x^2 - 8$



c. $y = -x^2 + 1$

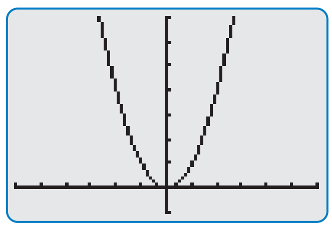


d. $y = \frac{1}{3}x^2 - 3$



What Is Your Answer?

3. IN YOUR OWN WORDS How does the value of c affect the graph of $y = ax^2 + c$? Use a graphing calculator to verify your conclusions.



Practice Use what you learned about the graphs of quadratic functions to complete Exercises 7–9 on page 420.

8.3 Lesson

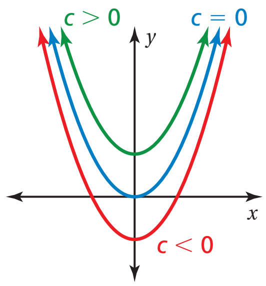
Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
zero, *p. 419*

Key Idea

Graphing $y = x^2 + c$

- When $c > 0$, the graph of $y = x^2 + c$ is a vertical translation c units up of the graph of $y = x^2$.
- When $c < 0$, the graph of $y = x^2 + c$ is a vertical translation $|c|$ units down of the graph of $y = x^2$.



EXAMPLE 1 Graphing $y = x^2 + c$

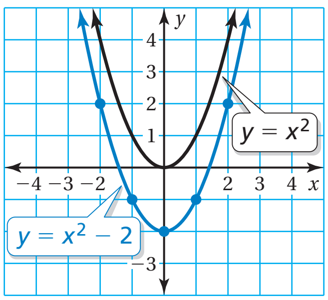
Graph $y = x^2 - 2$. Compare the graph to the graph of $y = x^2$.

Step 1: Make a table of values.

x	−2	−1	0	1	2
y	2	−1	−2	−1	2

Step 2: Plot the ordered pairs.

Step 3: Draw a smooth curve through the points.



Both graphs open up and have the same axis of symmetry, $x = 0$. The graph of $y = x^2 - 2$ is a translation 2 units down of the graph of $y = x^2$.

EXAMPLE 2 Graphing $y = ax^2 + c$

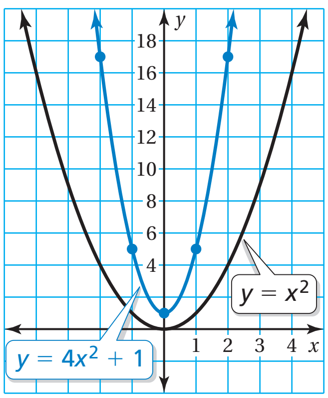
Graph $y = 4x^2 + 1$. Compare the graph to the graph of $y = x^2$.

Step 1: Make a table of values.

x	−2	−1	0	1	2
y	17	5	1	5	17

Step 2: Plot the ordered pairs.

Step 3: Draw a smooth curve through the points.



Both graphs open up and have the same axis of symmetry, $x = 0$. The graph of $y = 4x^2 + 1$ is narrower than the graph of $y = x^2$. The vertex of the graph of $y = 4x^2 + 1$ is a translation 1 unit up of the vertex of the graph of $y = x^2$.

On Your Own

Now You're Ready
Exercises 7–15

Graph the function. Compare the graph to the graph of $y = x^2$.

1. $y = x^2 + 3$ **2.** $y = 2x^2 - 5$ **3.** $y = -\frac{1}{2}x^2 + 4$

Multi-Language Glossary at BigIdeasMath.com

EXAMPLE 3 Translating the Graph of $y = x^2 + c$

Which of the following is true when you translate the graph of $y = x^2 - 5$ to the graph of $y = x^2 + 2$?

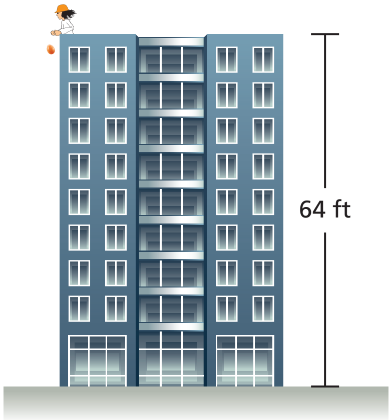
Ⓐ The graph shifts 3 units up.
Ⓑ The graph shifts 7 units up.
Ⓒ The graph shifts 7 units down.
Ⓓ The graph shifts 3 units down.

Both graphs open up and have the same axis of symmetry, $x = 0$. The vertex of $y = x^2 - 5$ is $(0, -5)$. The vertex of $y = x^2 + 2$ is $(0, 2)$. To move the vertex from $(0, -5)$ to $(0, 2)$, you must translate the graph 7 units up.

∴ The correct answer is Ⓑ.

A **zero** of a function $f(x)$ is an x-value for which $f(x) = 0$. A zero is located at the x-intercept of the graph of the function.

EXAMPLE 4 Real-Life Application

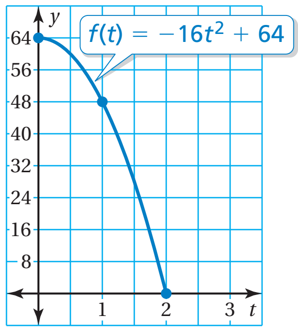


The function $f(t) = -16t^2 + s_0$ gives the approximate height (in feet) of a falling object t seconds after it is dropped from an initial height s_0 (in feet). An egg is dropped from a height of 64 feet. When does the egg hit the ground?

The initial height is 64 feet. So, the function $f(t) = -16t^2 + 64$ gives the height of the egg after t seconds. It hits the ground when $f(t) = 0$.

Step 1: Make a table of values and sketch the graph.

t	0	1	2
$f(t)$	64	48	0



Step 2: Find the zero of the function. When $t = 2$, $f(t) = 0$. So, the zero is 2.

∴ The egg hits the ground 2 seconds after it is dropped.

Common Error

The graph in Example 4 shows the height of the object over time, not the path of the object.

On Your Own

Now You're Ready
Exercises 16–18, 21

4. The graph of $y = 2x^2 + 1$ is shifted to $y = 2x^2 - 1$. Describe the translation.
5. **REASONING** Explain why only nonnegative values of t are used in Example 4.
6. **WHAT IF?** In Example 4, the egg is dropped from a height of 100 feet. When does the egg hit the ground?

8.3 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

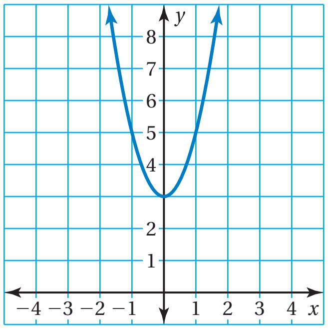
1. **VOCABULARY** Describe the vertex and axis of symmetry of the graph of $y = ax^2 + c$. How is the value of c related to the vertex of the graph?
2. **NUMBER SENSE** Without graphing, which graph has the greater y-intercept, $y = x^2 + 4$ or $y = x^2 - 4$? Explain your reasoning.
3. **WRITING** How does the graph of $y = ax^2 + c$ compare to the graph of $y = ax^2$?

Practice and Problem Solving

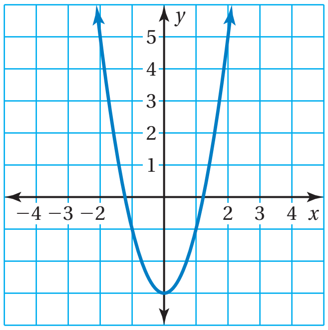
Match each function with its graph.

4. $y = 2x^2 + 3$
5. $y = -2x^2 + 3$
6. $y = 2x^2 - 3$

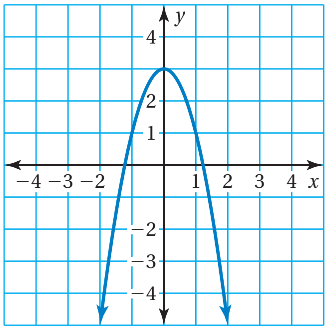
A.



B.



C.



Graph the function. Compare the graph to the graph of $y = x^2$.

1 2

7. $y = x^2 + 4$
8. $y = x^2 - 3$
9. $y = -x^2 + 5$
10. $y = -x^2 - 9$
11. $y = 2x^2 - 4$
12. $y = \frac{1}{2}x^2 + 2$

Use a graphing calculator to graph the function. Compare the graph to the graph of $y = x^2$.

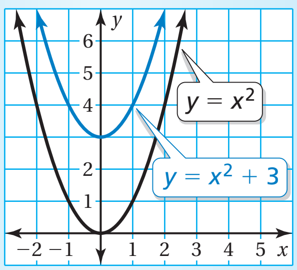
13. $y = 3x^2 + 4$
14. $y = -2x^2 - 1$
15. $y = -\frac{1}{4}x^2 - \frac{1}{2}$

Describe how to translate the graph of $y = x^2 + 2$ to the graph of the given function.

3

16. $y = x^2 + 4$
17. $y = x^2 - 1$
18. $y = x^2 - 4.5$

19. **ERROR ANALYSIS** Describe and correct the error in comparing the graphs.



The graph of $y = x^2 + 3$ is a translation 3 units down of the graph of $y = x^2$.

20. **REASONING** The domain of $y = ax^2 + c$ is all real numbers. Describe the range when (a) $a > 0$ and (b) $a < 0$.

4

21. **WATER BALLOON** A water balloon is dropped from a height of 16 feet. The function $h = -16x^2 + 16$ gives the height h of the balloon after x seconds. When does it hit the ground?

22. **APPLE** The function $y = -16x^2 + 36$ gives the height y (in feet) of an apple after falling x seconds. Find and interpret the x- and y-intercepts.

Find the zeros of the function.

23. $y = x^2 - 1$

24. $y = x^2 - 4$

25. $y = -x^2 + 9$

Sketch a quadratic function with the given characteristics.

26. The parabola opens up and the vertex is (0, 3).

27. The parabola opens down, the vertex is (0, 4), and one of the x-intercepts is 2.

28. The function is increasing when $x < 0$ and the x-intercepts of the parabola are -1 and 1.

29. The graph is below the x-axis and the highest point on the parabola is (0, -5).

30. **REASONING** Describe two algebraic methods you can use to find the zeros of the function $f(t) = -16t^2 + 64$.

31. **REASONING** Can the focus and the vertex of a parabola lie on opposite sides of the x-axis? Explain your reasoning.

32. **PROBLEM SOLVING** The paths of water from three different garden waterfalls are given below. Each function gives the height h (in feet) and the horizontal distance d (in feet) of the water.

Waterfall 1: $h = -3.1d^2 + 4.8$

Waterfall 2: $h = -3.5d^2 + 1.9$

Waterfall 3: $h = -1.1d^2 + 1.6$

a. Which waterfall drops water from the highest point?

b. Which waterfall follows the narrowest path?

c. Which waterfall sends water the farthest?

33. **Logic** Let $f(x)$ be a quadratic function of the form $f(x) = ax^2 + c$.

a. How does the graph of $f(x) + k$ compare to the graph of $f(x)$ when $k < 0$? when $k > 0$?

b. Let k be a real number not equal to 0 or 1. How does the graph of $k \cdot f(x)$ compare to the graph of $f(x)$?

Fair Game Review *What you learned in previous grades & lessons*

Factor the polynomial. *(Section 7.7 and Section 7.8)*

34. $x^2 - 2x - 8$

35. $2x^2 - 7x + 3$

36. $x^2 + 2x - 35$

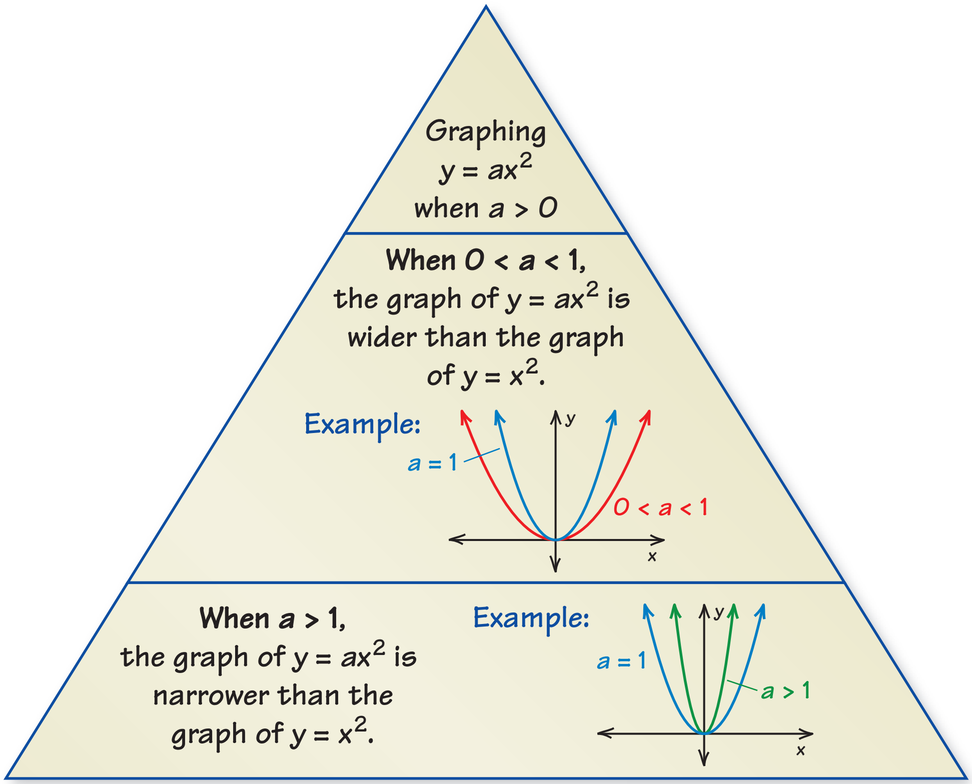
37. **MULTIPLE CHOICE** What is the product of $(x - 2)$ and $(x - 4)$? *(Section 7.3)*

Ⓐ $x^2 - 2$ Ⓑ $x^2 - 6x + 8$ Ⓒ $x^2 - 2x - 6$ Ⓓ $x^2 - 4x - 8$

Check It Out
Graphic Organizer
BigIdeasMath.com

8 Study Help

You can use a **summary triangle** to explain a topic. Here is an example of a summary triangle for graphing a quadratic function of the form $y = ax^2$ when $a > 0$.



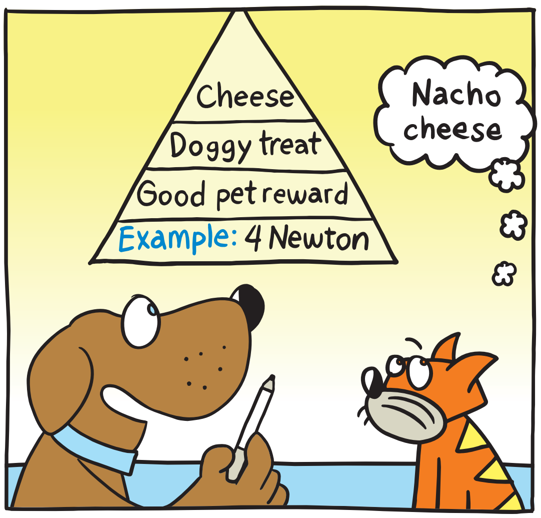
On Your Own

Make summary triangles to help you study these topics.

1. graphing $y = ax^2$ when $a < 0$
2. identifying the focus of a parabola
3. graphing $y = ax^2 + c$

After you complete this chapter, make summary triangles for the following topics.

4. graphing $y = ax^2 + bx + c$
5. graphing $y = a(x - h)^2 + k$



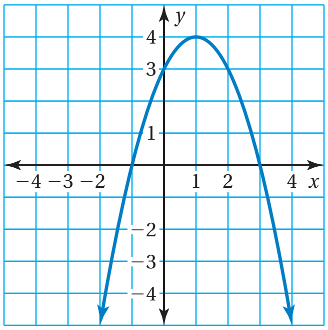
"What do you call a cheese summary triangle that isn't yours?"

8.1–8.3 Quiz

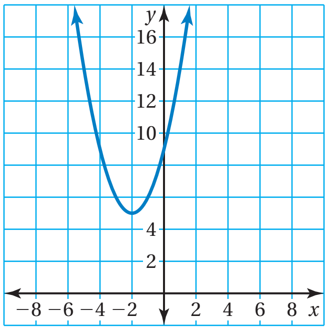
Check It Out
Progress Check
BigIdeasMath.com

Identify characteristics of the graph of the quadratic function. *(Section 8.1)*

1.



2.



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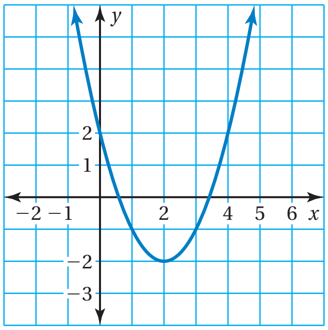

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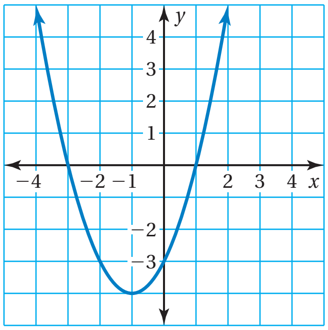


Graph the function. Compare the graph to the graph of $y = x^2$. *(Section 8.1)*

5. $y = -x^2$

6. $y = 4x^2$

7. $y = \frac{2}{5}x^2$

Graph the function. Identify the focus. *(Section 8.2)*

8. $y = 5x^2$

9. $y = -6x^2$

10. $y = \frac{1}{3}x^2$

Write an equation of the parabola with a vertex at the origin and the given focus. *(Section 8.2)*

11. $(0, -4)$

12. $(0, 2)$

13. $\left(0, \frac{1}{5}\right)$

Graph the function. Compare the graph to the graph of $y = x^2$. *(Section 8.3)*

14. $y = x^2 + 5$

15. $y = 2x^2 - 2$

16. $y = -x^2 + 3$

17. **PINEAPPLE** The distance y (in feet) that a pineapple falls is given by the function $y = 16t^2$, where t is the time (in seconds). Use a graph to determine how many seconds it takes for the pineapple to fall 32 feet. *(Section 8.1)*

18. **SMARTPHONE** A new smartphone application is available for download. The number y of downloads can be modeled by the function $y = 6.3x^2 + 3000$, where x is the number of hours since the new application was released. How many hours does it take for the number of downloads to reach 3630? *(Section 8.3)*

8.4 Graphing $y = ax^2 + bx + c$

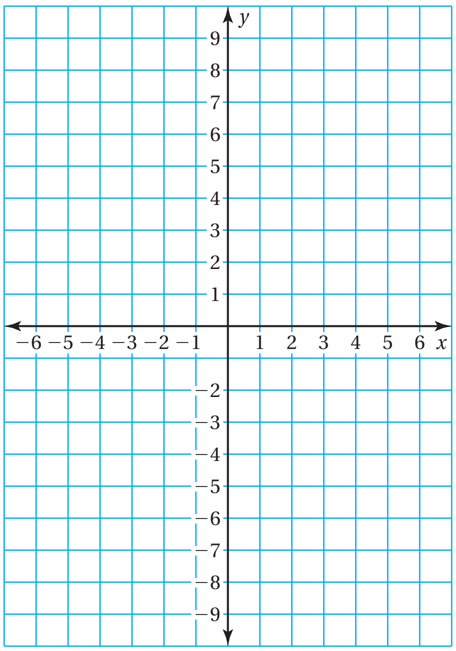
Essential Question How can you find the vertex of the graph of $y = ax^2 + bx + c$?

1 ACTIVITY: Comparing Two Graphs

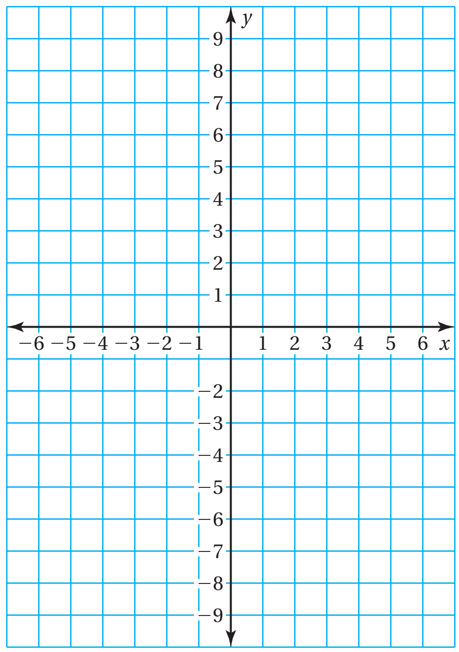
Work with a partner.

- Sketch the graphs of $y = 2x^2 - 8x$ and $y = 2x^2 - 8x + 6$.
- What do you notice about the x-value of the vertex of each graph?

$y = 2x^2 - 8x$



$y = 2x^2 - 8x + 6$



Common Core

Graphing Quadratic Functions

In this lesson, you will

- find the axes of symmetry and the vertices of graphs.
- graph quadratic functions of the form $y = ax^2 + bx + c$.
- find maximum and minimum values.

Learning Standards
F.BF.3
F.IF.4
F.IF.7a

2 ACTIVITY: Comparing x-Intercepts with the Vertex

Work with a partner.

- Use the graph in Activity 1 to find the x-intercepts of the graph of $y = 2x^2 - 8x$. Verify your answer by solving $0 = 2x^2 - 8x$.
- Compare the location of the vertex to the location of the x-intercepts.

3 ACTIVITY: Finding Intercepts

Math Practice 3

Use Prior Results

How can you use results from the previous activities to complete the table?

Work with a partner.

- Solve $0 = ax^2 + bx$ by factoring.
- What are the x-intercepts of the graph of $y = ax^2 + bx$?
- Copy and complete the table to verify your answer.

x	$y = ax^2 + bx$
0	
$-\frac{b}{a}$	

4 ACTIVITY: Deductive Reasoning

Work with a partner. Complete the following logical argument.

The x-intercepts of the graph of $y = ax^2 + bx$ are 0 and $-\frac{b}{a}$.

The vertex of the graph of $y = ax^2 + bx$ occurs when $x =$ ______.

The vertices of the graphs of $y = ax^2 + bx$ and $y = ax^2 + bx + c$ have the same x-value.

The vertex of $y = ax^2 + bx + c$ occurs when $x =$ ______.

What Is Your Answer?

5. **IN YOUR OWN WORDS** How can you find the vertex of the graph of $y = ax^2 + bx + c$?
6. Without graphing, find the vertex of the graph of $y = x^2 - 4x + 3$. Check your result by graphing.

Practice Use what you learned about the vertices of the graphs of quadratic functions to complete Exercises 6–8 on page 429.

8.4 Lesson

Check It Out
Lesson Tutorials
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Key Vocabulary

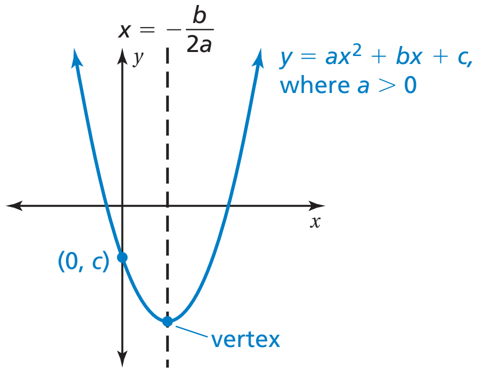
maximum value, *p. 427*

minimum value, *p. 427*

Key Idea

Properties of the Graph of $y = ax^2 + bx + c$

- The graph opens up when $a > 0$ and the graph opens down when $a < 0$.
- The y-intercept is c.
- The x-coordinate of the vertex is $-\frac{b}{2a}$.
- The axis of symmetry is $x = -\frac{b}{2a}$.



EXAMPLE 1 Finding the Axis of Symmetry and the Vertex of a Graph

Find (a) the axis of symmetry and (b) the vertex of the graph of $y = 2x^2 + 8x - 1$.

a. Find the axis of symmetry when $a = 2$ and $b = 8$.

$x = -\frac{b}{2a}$ — Write the equation for the axis of symmetry.

$x = -\frac{8}{2(2)}$ — Substitute 2 for a and 8 for b.

$x = -2$ — Simplify.

∴ The axis of symmetry is $x = -2$.

b. The axis of symmetry is $x = -2$, so the x-coordinate of the vertex is -2. Use the function to find the y-coordinate of the vertex.

$y = 2x^2 + 8x - 1$ — Write the function.

$= 2(-2)^2 + 8(-2) - 1$ — Substitute -2 for x.

$= -9$ — Simplify.

∴ The vertex is $(-2, -9)$.

On Your Own

Now You're Ready
Exercises 6–11

Find (a) the axis of symmetry and (b) the vertex of the graph of the function.

1. $y = 3x^2 - 2x$ **2.** $y = x^2 + 6x + 5$ **3.** $y = -\frac{1}{2}x^2 + 7x - 4$

Multi-Language Glossary at BigIdeasMath.com

EXAMPLE 2 Graphing $y = ax^2 + bx + c$

Graph $y = 3x^2 - 6x + 5$. Describe the domain and range.

Step 1: Find and graph the axis of symmetry.

$$x = -\frac{b}{2a} = -\frac{(-6)}{2(3)} = 1$$

Step 2: Find and plot the vertex.

The axis of symmetry is $x = 1$, so the x-coordinate of the vertex is 1. Use the function to find the y-coordinate of the vertex.

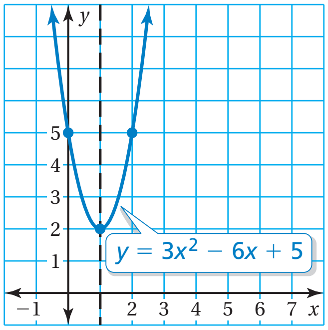
$y = 3x^2 - 6x + 5$	Write the function.
$= 3(1)^2 - 6(1) + 5$	Substitute 1 for x.
$= 2$	Simplify.

So, the vertex is (1, 2).

Step 3: Use the y-intercept to find two more points on the graph.

The y-intercept is 5. So, (0, 5) lies on the graph. Because the axis of symmetry is $x = 1$, the point (2, 5) also lies on the graph.

Step 4: Draw a smooth curve through the points.



The domain is all real numbers. The range is $y \geq 2$.

On Your Own

Now You're Ready
Exercises 13–18

Graph the function. Describe the domain and range.

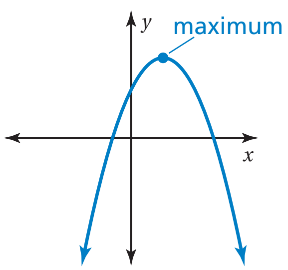
4. $y = 2x^2 + 4x + 1$ **5.** $y = x^2 - 8x + 7$ **6.** $y = -5x^2 - 10x - 2$

Key Ideas

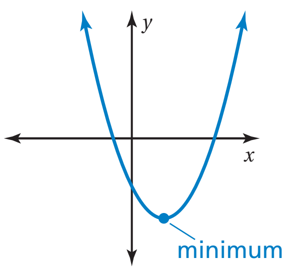
Maximum and Minimum Values

The y-coordinate of the vertex of the graph of $y = ax^2 + bx + c$ is the **maximum value** of the function when $a < 0$ or the **minimum value** of the function when $a > 0$.

$y = ax^2 + bx + c,\ a < 0$



$y = ax^2 + bx + c,\ a > 0$



EXAMPLE 3 Finding Maximum and Minimum Values

Tell whether the function $f(x) = -4x^2 - 24x - 19$ has a minimum value or a maximum value. Then find the value.

For $f(x) = -4x^2 - 24x - 19$, $a = -4$ and $-4 < 0$. So, the parabola opens down and the function has a maximum value. To find the maximum value, find the y-coordinate of the vertex.

$$x = -\frac{b}{2a} = -\frac{(-24)}{2(-4)} = -3 \qquad \text{The } x\text{-coordinate of the vertex is } -\frac{b}{2a}.$$

$$f(-3) = -4(-3)^2 - 24(-3) - 19 \qquad \text{Substitute } -3 \text{ for } x.$$

$$= 17 \qquad \text{Simplify.}$$

The maximum value is 17.

On Your Own

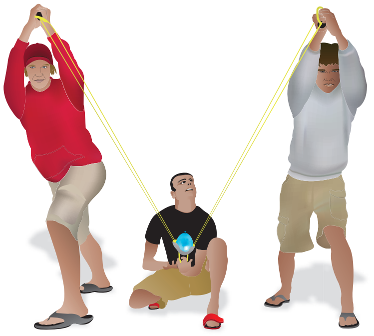
Now You're Ready
Exercises 21–26

Tell whether the function has a minimum value or a maximum value. Then find the value.

7. $g(x) = 8x^2 - 8x + 6$

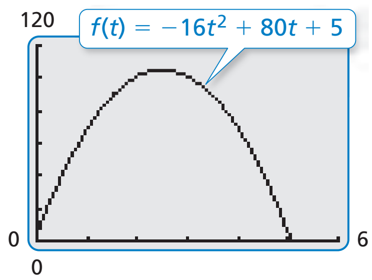
8. $h(x) = -\frac{1}{4}x^2 + 3x + 1$

EXAMPLE 4 Real-Life Application

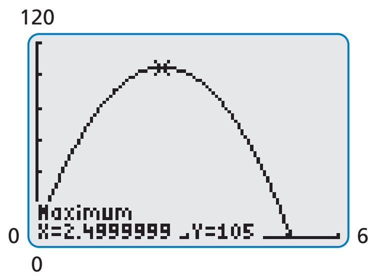


The function $f(t) = -16t^2 + 80t + 5$ gives the height (in feet) of a water balloon t seconds after it is launched. Use a graphing calculator to find the maximum height of the water balloon.

Step 1: Enter the function $f(t) = -16t^2 + 80t + 5$ into your calculator and graph it. Because time cannot be negative, use only nonnegative values of t.



Step 2: Use the *maximum* feature to find the maximum value of the function.



The maximum height of the water balloon is 105 feet.

Study Tip

The *minimum* feature of a graphing calculator can be used for parabolas that open up.

On Your Own

9. When does the water balloon reach its maximum height?

8.4 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **VOCABULARY** Explain how you can tell whether a quadratic function has a maximum value or a minimum value without graphing the function.

2. **DIFFERENT WORDS, SAME QUESTION** Consider the quadratic function $y = -2x^2 + 8x + 24$. Which is different? Find "both" answers.

   What is the maximum value of the function?

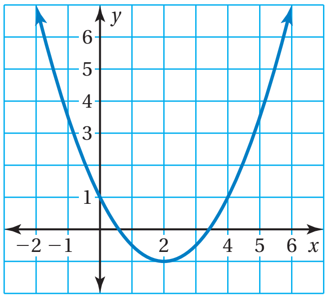
   What is the y-coordinate of the vertex of the graph?

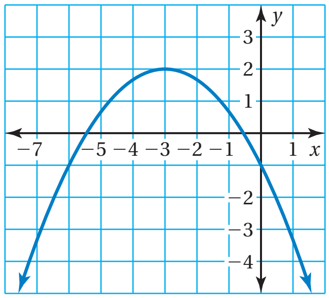
   What is the greatest number in the range of the function?

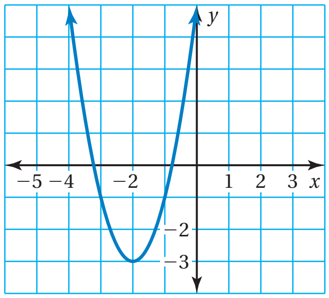
   What is the axis of symmetry of the graph of the function?

Practice and Problem Solving

Find the vertex, the axis of symmetry, and the y-intercept of the graph.

3. 

4. 

5. 

Find (a) the axis of symmetry and (b) the vertex of the graph of the function.

① 6. $y = 2x^2 - 4x$ 7. $y = 3x^2 + 12x$ 8. $y = -8x^2 - 16x - 1$

9. $y = -6x^2 + 24x - 20$ 10. $y = \frac{2}{5}x^2 - 4x + 14$ 11. $y = -\frac{3}{4}x^2 + 6x - 18$

12. **ERROR ANALYSIS** Describe and correct the error in finding the axis of symmetry of the graph of $y = 3x^2 - 12x + 11$.

✗ $x = -\frac{b}{2a} = \frac{-12}{2(3)} = -2$

The axis of symmetry is $x = -2$.

Graph the function. Describe the domain and range.

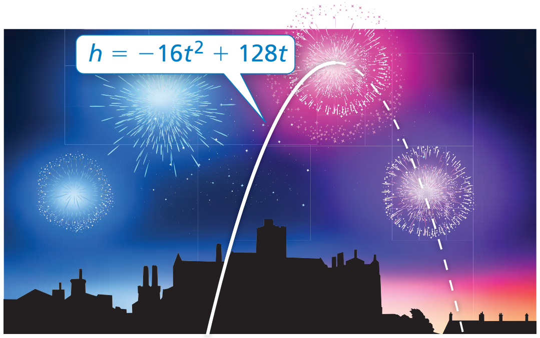
② 13. $y = 2x^2 + 12x + 14$ 14. $y = 4x^2 + 24x + 31$ 15. $y = -8x^2 - 16x - 9$

16. $y = -5x^2 + 30x - 47$ 17. $y = \frac{2}{3}x^2 - 8x + 19$ 18. $y = -\frac{1}{2}x^2 - 8x - 25$

19. **FIREWORK** The function shown represents the height h (in feet) of a firework t seconds after it is launched. The firework explodes at its highest point.

a. When does the firework explode?

b. Describe the domain and range of h.



20. **REASONING** Given the quadratic equation $y = ax^2 + bx + c$, find the axis of symmetry when $b = 0$.

Tell whether the function has a minimum value or a maximum value. Then find the value.

3 **21.** $y = 3x^2 - 18x + 15$

22. $y = -5x^2 + 10x + 7$

23. $y = -4x^2 + 4x - 2$

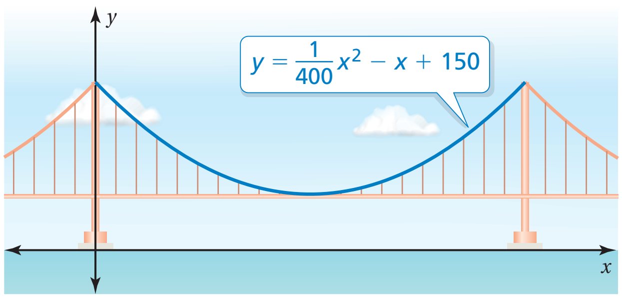
24. $y = 2x^2 - 10x + 13$

25. $y = -\frac{1}{2}x^2 + 8x + 20$

26. $y = \frac{1}{5}x^2 - 12x + 27$

27. **PRECISION** The vertex of a graph of a quadratic function is $(3, -1)$. One point on the graph is $(6, 8)$. Find another point on the graph. Justify your answer.

28. **SUSPENSION BRIDGE** The cables between the two towers of a suspension bridge can be modeled by $y = \frac{1}{400}x^2 - x + 150$, where x and y are measured in feet. The cables are at road level midway between the towers. How high is the road above the water?



29. **STEEPLECHASE** The function $h(t) = -16t^2 + 16t$ gives the height h (in feet) of a horse t seconds after it jumps during a steeplechase.

a. When does the horse reach its maximum height?

b. Can the horse clear a fence that is 3.5 feet tall? If so, by how much?

c. How long is the horse in the air?

Use the *minimum* or *maximum* feature of a graphing calculator to approximate the vertex of the graph of the function.

30. $y = -6.2x^2 + 4.8x - 1$ **31.** $y = 0.5x^2 + \sqrt{2}x - 3$ **32.** $y = \pi x^2 + 3x$

33. **CHOOSE TOOLS** The graph of a quadratic function passes through (4, 0), (5, 3), and (6, 4). Does the graph open up or down? Explain your reasoning.

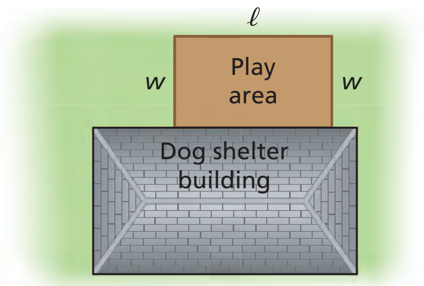
34. **REASONING** For a quadratic function f, what does $f\left(-\frac{b}{2a}\right)$ represent? Explain your reasoning.

35. **CALCULATORS** An office supply store sells about 60 graphing calculators per month for \$100 each. For each \$5 decrease in price, the store expects to sell 5 more calculators.

a. Write a quadratic function that represents the revenue from calculator sales. (*Note:* revenue = units sold × unit price)

b. How much should the store charge per calculator to maximize monthly revenue?

36. **AIR CANNON** At a basketball game, an air cannon is used to launch T-shirts into the crowd. The function $y = -\frac{1}{8}x^2 + 4x$ gives the path of a T-shirt. The function $3y = 2x - 14$ gives the height of the bleachers. In both functions, y represents height (in feet) and x represents horizontal distance (in feet). At what height does the T-shirt land in the bleachers?



37. **DOG SHELTER** The owners of a dog shelter want to enclose a rectangular play area on the side of their building. They have k feet of fencing. What is the maximum area of the outside enclosure in terms of k? (*Hint:* Find the y-coordinate of the vertex of the graph of the area function.)

Fair Game Review What you learned in previous grades & lessons

Graph the function. *(Section 2.4 and Section 6.5)*

38. $-4x + y = 3$ **39.** $y = 20(1.2)^t$ **40.** $r(t) = 400(1.05)^t$

41. **MULTIPLE CHOICE** What is the value of $3(4)^x$ when $x = 2$? *(Section 6.4)*

Ⓐ 6 Ⓑ 24 Ⓒ 48 Ⓓ 144

Graphing $y = a(x - h)^2 + k$

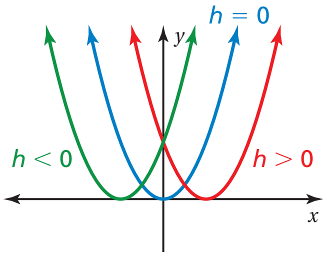
Key Vocabulary
vertex form, *p. 432*

The **vertex form** of a quadratic function is $y = a(x - h)^2 + k$, where $a \neq 0$. The vertex of the parabola is (h, k).

Key Idea

Graphing $y = (x - h)^2$

- When $h > 0$, the graph of $y = (x - h)^2$ is a horizontal translation h units to the right of the graph of $y = x^2$.
- When $h < 0$, the graph of $y = (x - h)^2$ is a horizontal translation h units to the left of the graph of $y = x^2$.



EXAMPLE 1 Graphing $y = (x - h)^2$

Common Core
Graphing Quadratic Functions
In this extension, you will
- graph quadratic functions of the form $y = a(x - h)^2 + k$ and compare to the graph of $y = x^2$.

Learning Standards
F.BF.3
F.IF.4
F.IF.7a

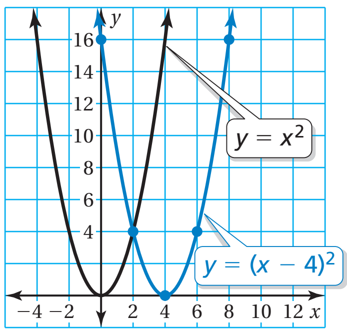
Graph $y = (x - 4)^2$. Compare the graph to the graph of $y = x^2$.

Step 1: Make a table of values.

x	0	2	4	6	8
y	16	4	0	4	16

Step 2: Plot the ordered pairs.

Step 3: Draw a smooth curve through the points.



The graph of $y = (x - 4)^2$ is a translation 4 units to the right of the graph of $y = x^2$.

Practice

Graph the function. Compare the graph to the graph of $y = x^2$. Use a graphing calculator to check your answer.

1. $y = (x + 3)^2$ **2.** $y = (x - 1)^2$ **3.** $y = (x - 6)^2$

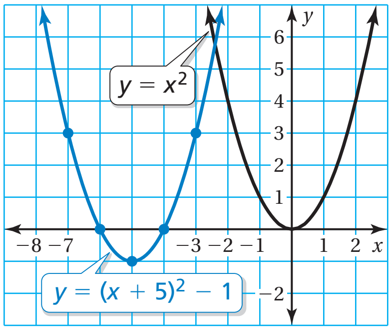
4. $y = (x + 10)^2$ **5.** $y = (x - 1.5)^2$ **6.** $y = \left(x + \frac{5}{2}\right)^2$

7. REASONING Compare the graphs of $y = x^2 + 6x + 9$ and $y = x^2$ without graphing the functions. How can factoring help you compare the parabolas? Explain.

8. STRUCTURE Write the function in Example 1 in the form $y = ax^2 + bx + c$. Describe advantages and disadvantages of writing the function in each form.

EXAMPLE 2 Graphing $y = (x - h)^2 + k$

Graph $y = (x + 5)^2 - 1$. Compare the graph to the graph of $y = x^2$.



Step 1: Make a table of values.

x	−7	−6	−5	−4	−3
y	3	0	−1	0	3

Step 2: Plot the ordered pairs.

Step 3: Draw a smooth curve through the points.

The graph of $y = (x + 5)^2 - 1$ is a translation 5 units to the left and 1 unit down of the graph of $y = x^2$.

EXAMPLE 3 Graphing $y = a(x - h)^2 + k$

Graph $y = -2(x + 2)^2 + 3$. Compare the graph to the graph of $y = x^2$.

Study Tip

Notice what the values in vertex form represent:

a: opens up or down, and is wider or narrower

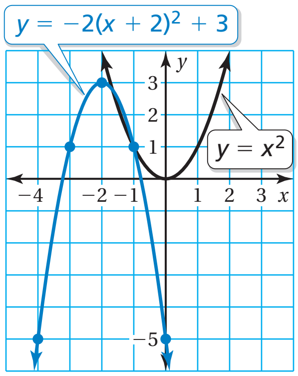
h: horizontal translation

k: vertical translation

(*h*, *k*): vertex

Step 1: Make a table of values.

x	−4	−3	−2	−1	0
y	−5	1	3	1	−5



Step 2: Plot the ordered pairs.

Step 3: Draw a smooth curve through the points.

The graph of $y = -2(x + 2)^2 + 3$ opens down and is narrower than the graph of $y = x^2$. The vertex of the graph of $y = -2(x + 2)^2 + 3$ is a translation 2 units to the left and 3 units up of the vertex of the graph of $y = x^2$.

Practice

Graph the function. Compare the graph to the graph of $y = x^2$. Use a graphing calculator to check your answer.

9. $y = (x - 2)^2 + 4$

10. $y = (x + 1)^2 - 7$

11. $y = (x - 8)^2 - 8$

12. $y = 3(x - 1)^2 + 6$

13. $y = -(x - 3)^2 - 5$

14. $y = \frac{1}{2}(x + 4)^2 - 2$

Describe how the graph of $g(x)$ compares to the graph of $f(x)$.

15. $g(x) = f(x) - 7$

16. $g(x) = f(x + 10)$

17. $g(x) = 5f(x)$

18. $g(x) = f(2x)$

19. **REASONING** The graph of $y = x^2$ is translated 2 units right and 5 units down. Write an equation for the function in vertex form and in standard form.

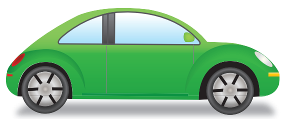
20. **REASONING** Does k represent the y-intercept of the graph of $y = a(x - h)^2 + k$? Explain.

8.5 Comparing Linear, Exponential, and Quadratic Functions

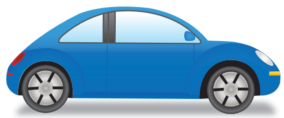
Essential Question How can you compare the growth rates of linear, exponential, and quadratic functions?

1 ACTIVITY: Comparing Speeds

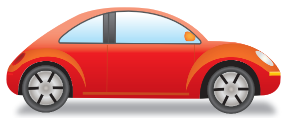
Work with a partner. Three cars start traveling at the same time. The distance traveled in *t* minutes is *y* miles. Complete each table and sketch all three graphs in the same coordinate plane. Compare the speeds of the three cars. Which car has a constant speed? Which car is accelerating the most? Explain your reasoning.



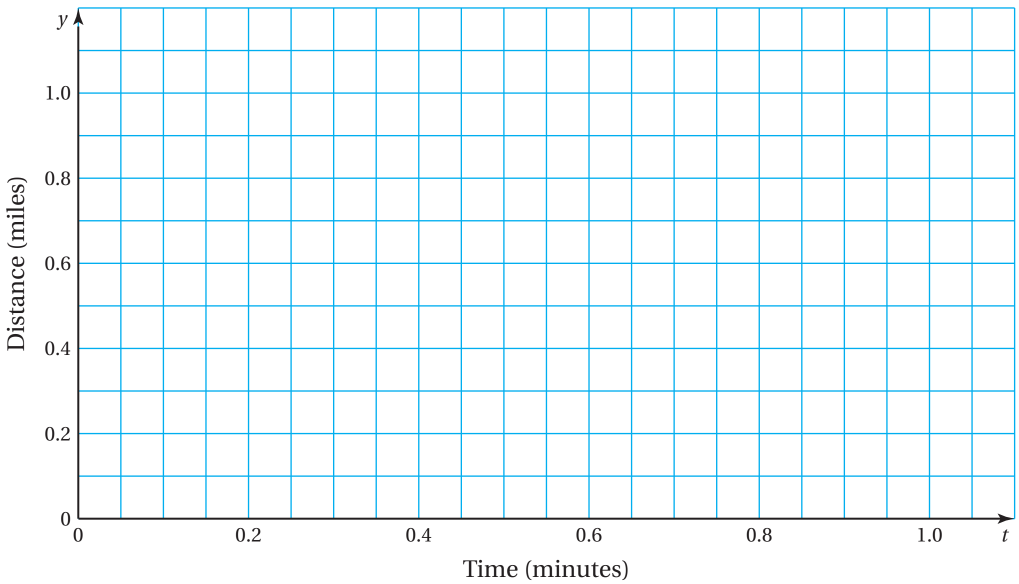
t	$y = t$
0	
0.2	
0.4	
0.6	
0.8	
1.0	



t	$y = 2^t - 1$
0	
0.2	
0.4	
0.6	
0.8	
1.0	



t	$y = t^2$
0	
0.2	
0.4	
0.6	
0.8	
1.0	



COMMON CORE

Linear, Quadratic, and Exponential Functions

In this lesson, you will

- identify linear, quadratic, and exponential functions using graphs or tables.

Learning Standards
F.IF.4
F.IF.6
F.IF.7a
F.LE.3

2 ACTIVITY: Comparing Speeds

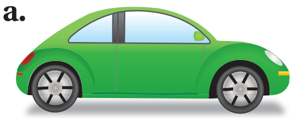
Math Practice 4

Analyze Relationships

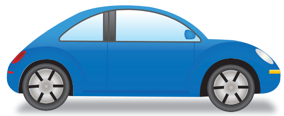
What is the relationship between the speeds of the cars?

Work with a partner. Analyze the speeds of the three cars over the given time periods. The distance traveled in t minutes is y miles. Which car eventually overtakes the others?

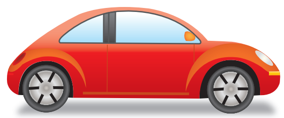
a.



t	$y = t$
1	
2	
3	
4	

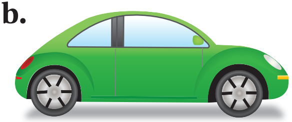


t	$y = 2^t - 1$
1	
2	
3	
4	

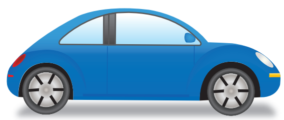


t	$y = t^2$
1	
2	
3	
4	

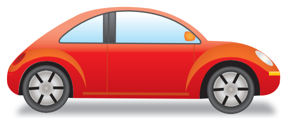
b.



t	$y = t$
4	
5	
6	
7	
8	
9	



t	$y = 2^t - 1$
4	
5	
6	
7	
8	
9	



t	$y = t^2$
4	
5	
6	
7	
8	
9	

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you compare the growth rates of linear, exponential, and quadratic functions? Which type of growth eventually leaves the other two in the dust? Explain your reasoning.

Practice Use what you learned about comparing functions to complete Exercises 3–5 on page 439.

8.5 Lesson

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Lesson Tutorials
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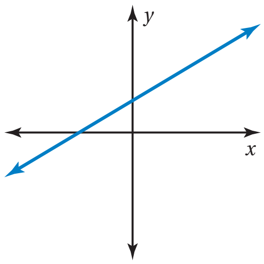
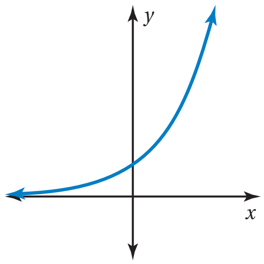
Key Idea

Linear Function



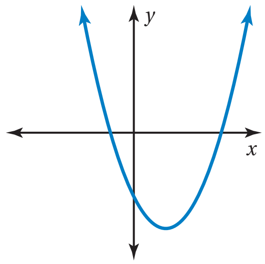
Line
$y = mx + b$

Exponential Function



Curve
$y = ab^x$

Quadratic Function



Parabola
$y = ax^2 + bx + c$

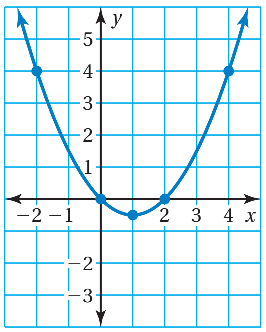
EXAMPLE 1 Identifying Functions Using Graphs

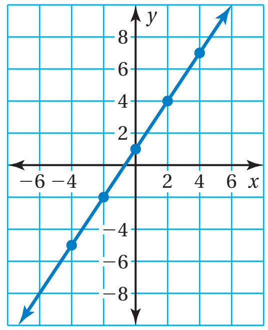
Plot the points. Tell whether the points represent a *linear*, an *exponential*, or a *quadratic* function.

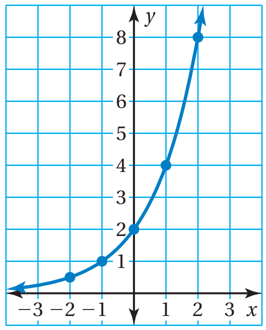
a. $(4, 4), (2, 0), (0, 0), \left(1, -\frac{1}{2}\right), (-2, 4)$

b. $(0, 1), (2, 4), (4, 7), (-2, -2), (-4, -5)$

c. $(0, 2), (2, 8), (1, 4), (-1, 1), \left(-2, \frac{1}{2}\right)$







∴ Quadratic function ∴ Linear function ∴ Exponential function

On Your Own

Now You're Ready
Exercises 9–12

Plot the points. Tell whether the points represent a *linear*, an *exponential*, or a *quadratic* function.

1. $(-1, 5), (2, -1), (0, -1), (3, 5), (1, -3)$

2. $(-1, 2), (-2, 8), (-3, 32), \left(0, \frac{1}{2}\right), \left(1, \frac{1}{8}\right)$

3. $(-3, 5), (0, -1), (2, -5), (-4, 7), (1, -3)$

Key Idea

Differences and Ratios of Functions

Linear Function: $y = 2x + 5$

+1 +1 +1 +1

x	−2	−1	0	1	2
y	1	3	5	7	9

+2 +2 +2 +2

The y-values have a common *difference* of 2.

Exponential Function: $y = 4(2)^x$

+1 +1 +1 +1

x	−2	−1	0	1	2
y	1	2	4	8	16

×2 ×2 ×2 ×2

The y-values have a common *ratio* of 2.

Quadratic Function: $y = x^2 + 2x - 1$

+1 +1 +1 +1

x	−2	−1	0	1	2
y	−1	−2	−1	2	7

−1 +1 +3 +5 ← First differences

+2 +2 +2 ← Second differences

For quadratic functions, the second differences are constant.

Study Tip

For a linear function, the first differences are constant.

EXAMPLE 2 Identifying Functions Using Differences or Ratios

Tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function.

a.

+1 +1 +1 +1

x	−3	−2	−1	0	1
y	11	8	5	2	−1

−3 −3 −3 −3

The y-values have a common difference of −3. So, the table represents a linear function.

b.

+1 +1 +1 +1

x	−1	0	1	2	3
y	0	−1	2	9	20

−1 +3 +7 +11

+4 +4 +4

The second differences are constant. So, the table represents a quadratic function.

Study Tip

For a quadratic function, the y-values will increase, then decrease, or the y-values will decrease, then increase.

On Your Own

Now You're Ready
Exercises 14–17

4. Tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function.

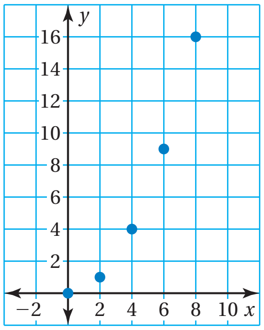
x	−1	0	1	2	3
y	1	3	9	27	81

EXAMPLE 3 Identifying and Writing a Function

x	y
0	0
2	1
4	4
6	9
8	16

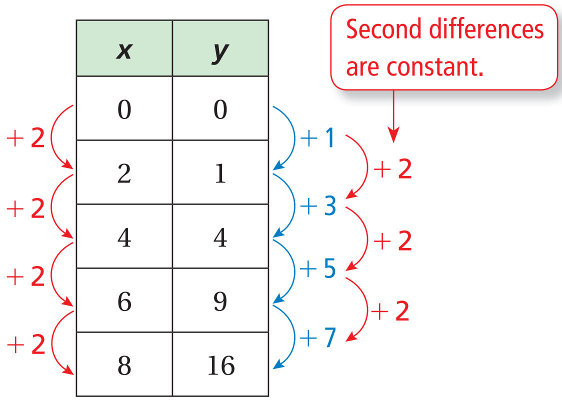
Tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function. Then write an equation for the function using the form $y = mx + b$, $y = ab^x$, or $y = ax^2$.

Step 1:
Graph the data. The function appears to be exponential or quadratic.



Step 2:
Check the y-values. If there is no common difference or ratio, check the second differences.

x	y
0	0
2	1
4	4
6	9
8	16



The function is quadratic.

Step 3: Use the form $y = ax^2$.

$1 = a(2)^2$ Use the point (2, 1). Substitute 2 for x and 1 for y.

$\frac{1}{4} = a$ Solve for a.

So, an equation for the quadratic function is $y = \frac{1}{4}x^2$.

Study Tip

To check your function in Example 3, substitute the other points from the table to see if they satisfy the function.

On Your Own

Now You're Ready
Exercises 19–24

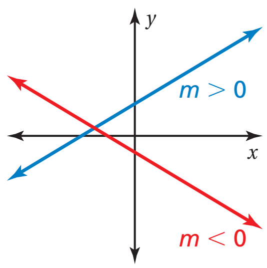
5. Tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function. Then write an equation for the function using the form $y = mx + b$, $y = ab^x$, or $y = ax^2$.

x	−1	0	1	2	3
y	16	8	4	2	1

Summary

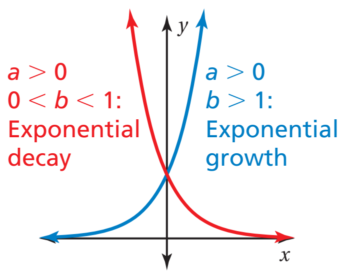
Linear Function

$y = mx + b$



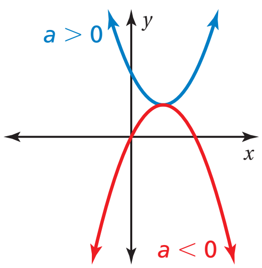
Exponential Function

$y = ab^x$, $a \neq 0$, $b \neq 1$, and $b > 0$



Quadratic Function

$y = ax^2 + bx + c$, $a \neq 0$



8.5 Exercises

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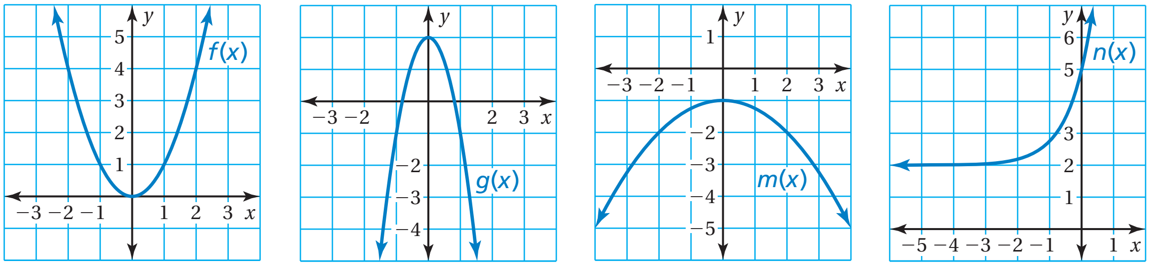
Vocabulary and Concept Check

1. **VOCABULARY** How can you decide whether to use a linear, a quadratic, or an exponential function to model a data set?

2. **WHICH ONE DOESN'T BELONG?** Which graph does *not* belong with the other three? Explain your reasoning.



Practice and Problem Solving

Find the values of x when $f(x)$ is greater than $g(x)$.

3. $f(x) = x^2$
   $g(x) = x$

4. $f(x) = 3^x$
   $g(x) = 2x$

5. $f(x) = 4^x$
   $g(x) = 2x^2$

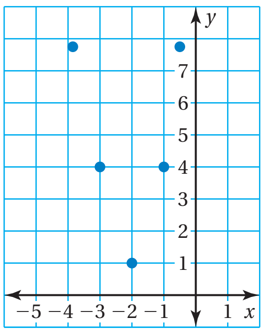
Match the function type with its graph.

6. Linear function

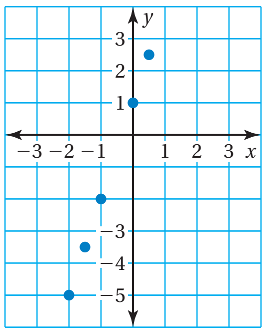
7. Exponential function

8. Quadratic function

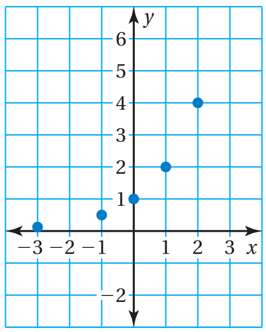
A.



B.



C.



Plot the points. Tell whether the points represent a *linear*, an *exponential*, or a *quadratic* function.

1 9. $(-2, -1), (-1, 0), (1, 2), (2, 3), (0, 1)$

10. $(1, 8), \left(-4, \frac{1}{4}\right), \left(-3, \frac{1}{2}\right), (-2, 1), (-1, 2)$

11. $(0, -3), (1, 0), (2, 9), (-2, 9), (-1, 0)$

12. $(-1, -3), (-3, 5), (0, -1), (1, 5), (2, 15)$

13. **SUBWAY** A student takes a subway to a public library. The table shows the distance d (in miles) the student travels in t minutes. Tell whether the data can be modeled by a *linear*, an *exponential*, or a *quadratic* function.

Time, t	0.5	1	3	5
Distance, d	0.335	0.67	2.01	3.35

Tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function.

2 **14.**

x	−2	−1	0	1	2
y	0	0.5	1	1.5	2

15.

x	−1	0	1	2	3
y	0.2	1	5	25	125

16.

x	−2	−1	0	1	2
y	0.75	1.5	3	6	12

17.

x	2	3	4	5	6
y	2	4.5	8	12.5	18

18. REASONING Can the y-values of a data set have both a common difference and a common ratio? Explain your reasoning.

Tell whether the data values represent a *linear*, an *exponential*, or a *quadratic* function. Then write an equation for the function using the form $y = mx + b$, $y = ab^x$, or $y = ax^2$.

3 **19.** (−2, 8), (−1, 2), (0, 0), (1, 2), (2, 8)

20. (−3, 8), (−2, 4), (−1, 2), (0, 1), (1, 0.5)

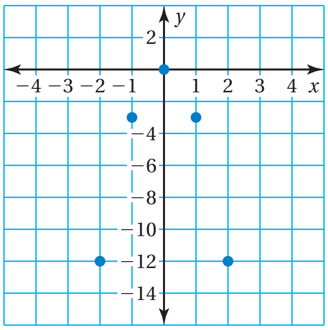
21.

x	−2	−1	0	1	2
y	4	1	−2	−5	−8

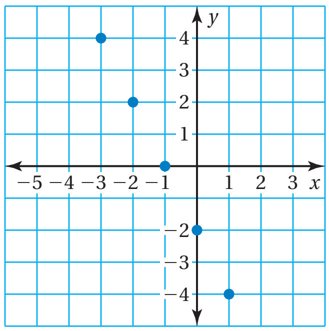
22.

x	−1	0	1	2	3
y	2.5	5	10	20	40

23.



24.



25. ERROR ANALYSIS Describe and correct the error in writing an equation for the function represented by the ordered pairs.

(−1, 4), (0, 0), (1, 4), (2, 16), (3, 36)

✗

+1 +1 +1 +1

x	−1	0	1	2	3
y	4	0	4	16	36

−4 +4 +12 +20 ← First differences

+8 +8 +8 ← Second differences

The ordered pairs represent a quadratic function.

$y = ax^2$

$1 = a(4)^2$

$\frac{1}{16} = a$

So, an equation is $y = \frac{1}{16}x^2$.

26. **HIGH SCHOOL FOOTBALL** The table shows the number of people attending the first five football games at a high school.

a. Plot the points.

b. Does a *linear*, an *exponential*, or a *quadratic* function represent this situation? Explain.

Game, g	1	2	3	4	5
Number of People, p	252	325	270	249	310

27. **CRITICAL THINKING** Is the graph of a set of points enough to determine whether the points represent a *linear*, an *exponential*, or a *quadratic* function? Justify your answer.

28. **RECORDING STUDIO** The table shows the amount of money (in dollars) that a musician pays for using a recording studio.

Number of Hours, h	1	2	3	4
Amount, m (dollars)	110	145	180	215

a. Plot the points. Then determine the type of function that best represents this situation.

b. Write a function that models the data.

c. How much does it cost to use the studio for 10 hours?

29. **TOURNAMENT** At the beginning of a basketball tournament, there are 64 teams. After each round, one-half of the remaining teams are eliminated.

a. Make a table showing the number of teams remaining after each round.

b. Determine the type of function that best represents this situation.

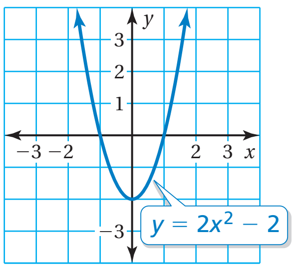
c. Write a function that models the data.

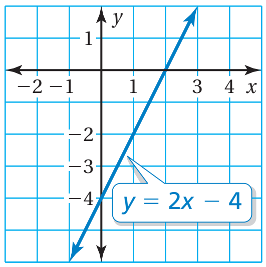
d. After which round do you know the team that won the tournament?

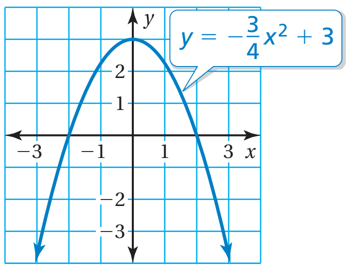
30. Repeated Reasoning Write a function that has constant second differences of 3.

Fair Game Review What you learned in previous grades & lessons

Find the x-intercept(s) of the graph. *(Section 2.3 and Section 8.3)*

31.


32.


33.


34. **MULTIPLE CHOICE** What is the factored form of $8x^3 - 18x$? *(Section 7.9)*

Ⓐ $2x(2x + 3)^2$ Ⓑ $(2x + 3)(2x - 3)$ Ⓒ $(2x - 3)^2$ Ⓓ $2x(2x + 3)(2x - 3)$

Extension 8.5 Comparing Graphs of Functions

Check It Out
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You have already learned that the average rate of change (or slope) between any two points on a line is the change in *y* divided by the change in *x*. You can find the average rate of change between two points of a nonlinear function using the same method.

ACTIVITY 1 Rates of Change of a Quadratic Function

In Example 4 on page 428, the function $f(t) = -16t^2 + 80t + 5$ gives the height (in feet) of a water balloon *t* seconds after it is launched.

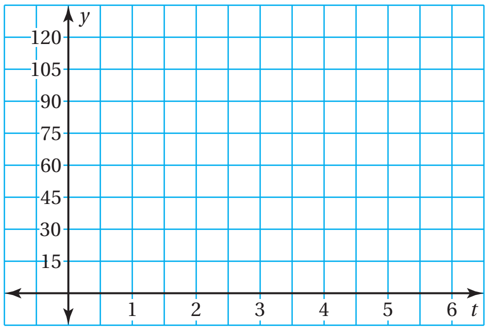
a. Copy and complete the table for $f(t)$.

t	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
$f(t)$											

b. Graph the ordered pairs from part (a). Then draw a smooth curve through the points.

c. For what values is the function increasing? decreasing?

d. Copy and complete the tables to find the average rate of change for each interval.



Common Core

Linear, Quadratic, and Exponential Functions

In this extension, you will

- compare graphs of linear, quadratic, and exponential functions.
- solve real-life problems.

Learning Standards
F.IF.4
F.IF.6
F.IF.7a
F.LE.3

Time Interval	0 to 0.5 sec	0.5 to 1 sec	1 to 1.5 sec	1.5 to 2 sec	2 to 2.5 sec
Average Rate of Change (ft/sec)					

Time Interval	2.5 to 3 sec	3 to 3.5 sec	3.5 to 4 sec	4 to 4.5 sec	4.5 to 5 sec
Average Rate of Change (ft/sec)					

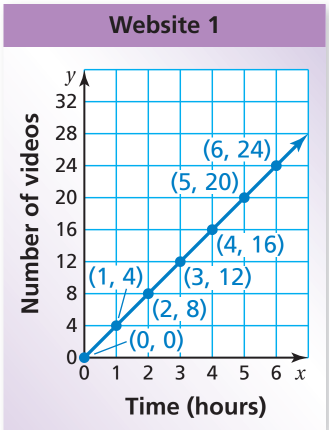
Practice

1. Compared to the average rate of change of a linear function, what do you notice about the average rate of change in part (d) of Activity 1?
2. Is the average rate of change increasing or decreasing from 0 to 2.5 seconds? How can you use the graph to justify your answer?
3. What do you notice about the average rate of change when the function is increasing and when the function is decreasing?
4. In Example 4 on page 419, the function $f(t) = -16t^2 + 64$ gives the height of an egg *t* seconds after it is dropped. (a) Make a table of values. Use the domain $0 \le t \le 2$ with intervals of 0.5 second. (b) Graph the ordered pairs and draw a smooth curve through the points. (c) Describe where the function is increasing and decreasing. (d) Find the average rate of change for each interval in the table. What do you notice?

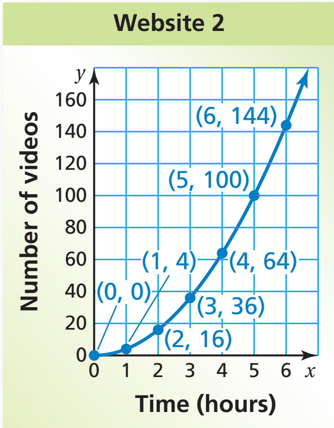
ACTIVITY 2 Rates of Change of Different Functions

The graphs show the numbers y of videos on three video-sharing websites x hours after the websites are launched.

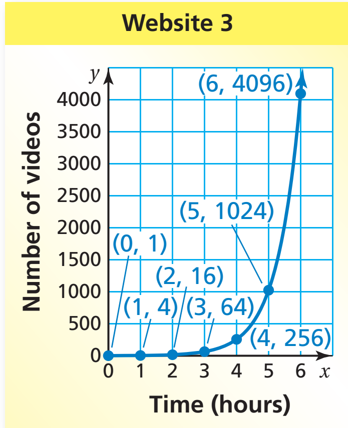
Linear



Quadratic



Exponential



a. Do the three websites ever have the same number of videos?

b. Copy and complete the table for each function.

Time Interval	0 to 1 h	1 to 2 h	2 to 3 h	3 to 4 h	4 to 5 h	5 to 6 h
Average Rate of Change (videos/hour)						

c. What do you notice about the average rate of change of the linear function?

d. What do you notice about the average rate of change of the quadratic function?

e. What do you notice about the average rate of change of the exponential function?

f. Which average rate of change increases more quickly, the quadratic function or the exponential function?

Practice

5. **REASONING** How does a quantity that is increasing exponentially compare to a quantity that is increasing linearly or quadratically?

6. **REASONING** Explain why the average rate of change of a linear function is constant and the average rate of change of a quadratic or exponential function is not constant.

8.4–8.5 Quiz

Check It Out
Progress Check
BigIdeasMath.com

Find (a) the axis of symmetry and (b) the vertex of the graph of the function. *(Section 8.4)*

1. $y = x^2 - 2x - 3$

2. $y = -2x^2 + 12x + 5$

Graph the function. Describe the domain and range. *(Section 8.4)*

3. $y = -4x^2 - 4x + 7$

4. $y = 2x^2 + 8x - 5$

5. $y = -4x^2 - 8x + 12$

Tell whether the function has a minimum value or a maximum value. Then find the value. *(Section 8.4)*

6. $y = 5x^2 + 10x - 3$

7. $y = -\frac{1}{2}x^2 + 2x + 16$

8. $y = -2x^2 + 8x + 3$

Graph the function. Compare the graph to the graph of $y = x^2$. *(Section 8.4)*

9. $y = (x - 5)^2$

10. $y = (x + 6)^2 - 2$

Plot the points. Tell whether the points represent a *linear*, an *exponential*, or a *quadratic* function. *(Section 8.5)*

11. (3, 6), (4, 16), (5, 30), (0, 0), (−1, 6)

12. (1, 7.5), (3, 6.5), (4, 6), (2, 7), (5, 5.5)

Tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function. Then write an equation for the function using the form $y = mx + b$, $y = ab^x$, or $y = ax^2$. *(Section 8.5)*

13.

x	y
−1	1
0	3
1	9
2	27
3	81

14.

x	y
−3	−3
−2	−1
−1	1
0	3
1	5

15.

x	y
1	−5
2	−20
3	−45
4	−80
5	−125

16. **FOOTBALL** The function $h(t) = -16t^2 + 20t + 6$ gives the height (in feet) of a football t seconds after it is thrown. Describe the domain and range. Find the maximum height of the football. *(Section 8.4)*

17. **DOWNLOADING MUSIC** The table shows the amounts of money (in dollars) that you pay to download songs from a website. *(Section 8.5)*

Number of Songs, s	2	3	4	5
Amount, a (dollars)	2.58	3.87	5.16	6.45

a. Plot the points. Tell whether the points represent a *linear*, an *exponential*, or a *quadratic* function.

b. Write a function that models the data.

c. How much does it cost to download 15 songs?

8 Chapter Review

Check It Out
Vocabulary Help
BigIdeasMath.com

Review Key Vocabulary

quadratic function, *p. 404*	axis of symmetry, *p. 404*	maximum value, *p. 427*
parabola, *p. 404*	focus, *p. 412*	minimum value, *p. 427*
vertex, *p. 404*	zero, *p. 419*	vertex form, *p. 432*

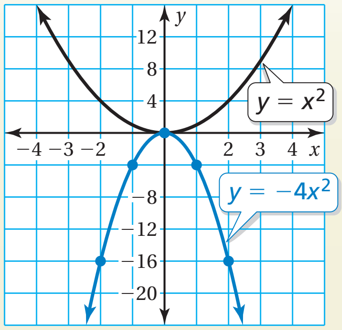
Review Examples and Exercises

8.1 Graphing $y = ax^2$ *(pp. 402–409)*

Graph $y = -4x^2$. Compare the graph to the graph of $y = x^2$.

Step 1: Make a table of values.

x	-2	-1	0	1	2
y	-16	-4	0	-4	-16



Step 2: Plot the ordered pairs.

Step 3: Draw a smooth curve through the points.

∴ The graphs have the same vertex, (0, 0), and the same axis of symmetry, $x = 0$, but the graph of $y = -4x^2$ opens down. The graph of $y = -4x^2$ is narrower than the graph of $y = x^2$.

Exercises

Graph the function. Compare the graph to the graph of $y = x^2$.

1. $y = 7x^2$

2. $y = \frac{1}{2}x^2$

3. $y = -\frac{3}{4}x^2$

8.2 Focus of a Parabola *(pp. 410–415)*

Write an equation of the parabola with focus (0, 2) and vertex at the origin.

For $y = ax^2$, the focus is $\left(0, \frac{1}{4a}\right)$. Use the given focus, (0, 2), to write an equation to find a.

$$\frac{1}{4a} = 2 \qquad \text{Equate the } y\text{-coordinates.}$$

$$\frac{1}{8} = a \qquad \text{Solve for } a.$$

∴ An equation of the parabola is $y = \frac{1}{8}x^2$.

Exercises

4. Graph $y = -\frac{1}{2}x^2$. Identify the focus.

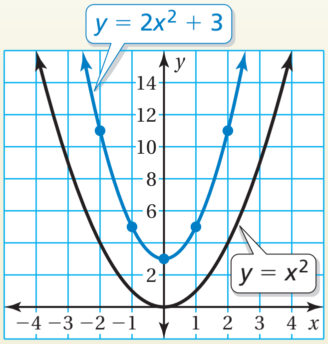
5. Write an equation of the parabola with focus (0, 10) and vertex at the origin.

8.3 Graphing $y = ax^2 + c$ (pp. 416–421)

Graph $y = 2x^2 + 3$. Compare the graph to the graph of $y = x^2$.

Step 1: Make a table of values.

x	−2	−1	0	1	2
y	11	5	3	5	11



Step 2: Plot the ordered pairs.

Step 3: Draw a smooth curve through the points.

Both graphs open up and have the same axis of symmetry, $x = 0$. The graph of $y = 2x^2 + 3$ is narrower than the graph of $y = x^2$. The vertex of the graph of $y = 2x^2 + 3$ is a translation 3 units up of the vertex of the graph of $y = x^2$.

Exercises

Graph the function. Compare the graph to the graph of $y = x^2$.

6. $y = x^2 + 6$ **7.** $y = -x^2 - 4$ **8.** $y = 3x^2 - 5$

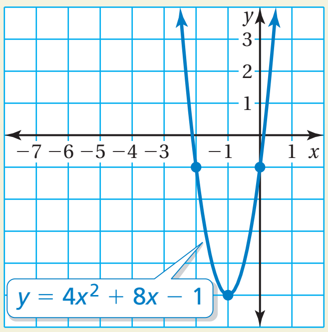
8.4 Graphing $y = ax^2 + bx + c$ (pp. 424–433)

Graph $y = 4x^2 + 8x - 1$. Describe the domain and range.

Step 1: Find and graph the axis of symmetry: $x = -\frac{b}{2a} = -\frac{8}{2(4)} = -1$.

Step 2: Find and plot the vertex. The x-coordinate of the vertex is −1. The y-coordinate is: $y = 4(-1)^2 + 8(-1) - 1 = -5$. So, the vertex is (−1, −5).

Step 3: Use the y-intercept to find two more points on the graph. The y-intercept is −1. So, (0, −1) lies on the graph. Because the axis of symmetry is $x = -1$, the point (−2, −1) also lies on the graph.



Step 4: Draw a smooth curve through the points.

The domain is all real numbers. The range is $y \geq -5$.

Exercises

Graph the function. Describe the domain and range.

9. $y = x^2 - 2x + 7$

10. $y = -3x^2 + 3x - 4$

11. $y = \frac{1}{2}x^2 - 6x + 10$

12. The function $f(t) = -16t^2 + 75t + 12$ gives the height (in feet) of a pumpkin t seconds after it is launched from a catapult. Use a graphing calculator to find the maximum height of the pumpkin. When does the pumpkin reach its maximum height?

Graph the function. Compare the graph to the graph of $y = x^2$. Use a graphing calculator to check your answer.

13. $y = (x + 5)^2$

14. $y = (x + 3)^2 - 2$

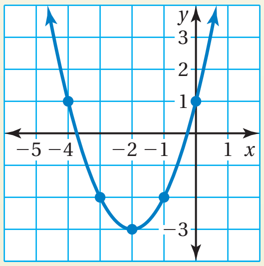
15. $y = -(x - 1)^2 + 1$

8.5 Comparing Linear, Exponential, and Quadratic Functions

(pp. 434–443)

Tell whether the data values represent a *linear*, an *exponential*, or a *quadratic* function.

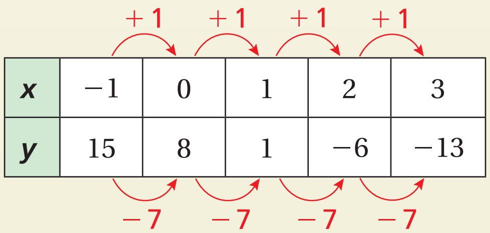
a. $(-4, 1), (-3, -2), (-2, -3), (-1, -2), (0, 1)$



The points represent a quadratic function.

b.

x	−1	0	1	2	3
y	15	8	1	−6	−13



The y-values have a common difference of −7. So, the table represents a linear function.

Exercises

16. Tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function. Then write an equation for the function using the form $y = mx + b$, $y = ab^x$, or $y = ax^2$.

x	−1	0	1	2	3
y	16	8	4	2	1

17. The function $f(t) = -16t^2 + 75t + 4$ gives the height (in feet) of a baseball t seconds after it is thrown. Sketch a graph of the function. Find the average rate of change from 0 to 1 second and from 1 to 2 seconds. Is the average rate of change increasing or decreasing? How does the graph justify your answer?

8 Chapter Test

Check It Out
Test Practice
BigIdeasMath.com

Graph the function. Compare the graph to the graph of $y = x^2$.

1. $y = 3x^2$

2. $y = 2x^2 + 2$

3. $y = -\frac{1}{2}x^2 - 1$

4. $y = (x - 3)^2$

5. $y = (x + 1)^2 - 1$

6. $y = -2(x - 5)^2$

Graph the function. Identify the focus.

7. $y = 6x^2$

8. $y = \frac{1}{5}x^2$

9. $y = -1.5x^2$

Graph the function. Describe the domain and range.

10. $y = x^2 + 2x - 1$

11. $y = -x^2 - 3x + 3$

12. $y = 2x^2 + 4x - 4$

13. Describe how the graph of $g(x) = f(x + 6)$ compares to the graph of $f(x)$.

Tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function. Then write an equation for the function using the form $y = mx + b$, $y = ab^x$, or $y = ax^2$.

14.

x	−1	0	1	2	3
y	4	8	16	32	64

15.

x	−2	−1	0	1	2
y	−8	−2	0	−2	−8

16. EARTH'S ORBIT The table shows the distance d (in miles) that the Earth moves in its orbit around the Sun after t seconds. Tell whether the data can be modeled by a *linear*, an *exponential*, or a *quadratic* function.

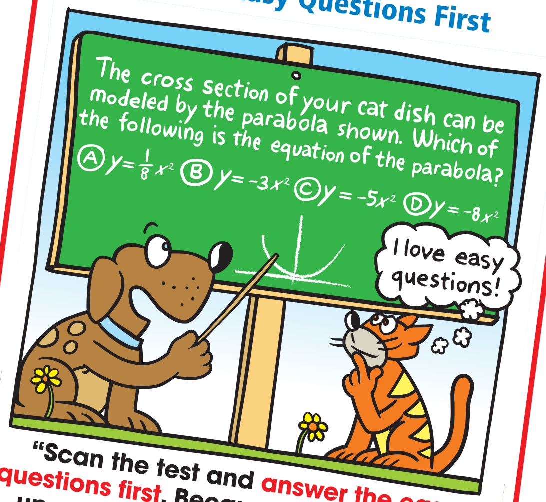
Time, *t*	1	2	3	4	5
Distance, *d*	19	38	57	76	95

17. RADIO TELESCOPE An observatory uses a radio telescope to collect data from another galaxy. The cross section of the telescope's dish can be modeled by $y = \frac{1}{120}x^2$, where x and y are measured in meters. The telescope's receiver is located at the focus of the parabola. What is the distance from the vertex of the parabola to the receiver?

18. REASONING Consider the function $f(x) = x^2 + 3$. Is the average rate of change increasing or decreasing from $x = 0$ to $x = 4$? Explain.

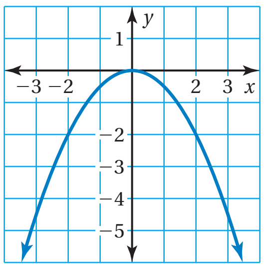
Test-Taking Strategy

Answer Easy Questions First



"Scan the test and **answer the easy questions first**. Because the graph opens up, you know the answer must be A."

1. What is the equation of the parabola shown in the graph? *(F.IF.4)*



A. $y = \frac{1}{2}x^2$

B. $y = 2x^2$

C. $y = -\frac{1}{2}x^2$

D. $y = -2x^2$

2. Which expression is equivalent to $(b^{-5})^{-4}$? *(N.RN.2)*

F. b^{-20}

G. b^{-9}

H. b^{9}

I. b^{20}

3. What is the axis of symmetry of the graph of $y = 3x^2 - 6x - 14$? *(F.IF.4)*

A. $x = -3$

B. $x = -1$

C. $x = 1$

D. $x = 3$

4. What is the minimum value of the function $y = 3x^2 + 12x + 6$? *(F.IF.7a)*

5. What are the solutions of $16a^2 - 49 = 0$? *(A.REI.4b)*

F. $a = -\frac{7}{4}$ and $a = \frac{7}{4}$

G. $a = -7$ and $a = 7$

H. $a = -4$ and $a = 4$

I. $a = -\frac{4}{7}$ and $a = \frac{4}{7}$

6. What is an equation of the parabola with focus $\left(0, \frac{1}{8}\right)$ and vertex at the origin? *(F.IF.4)*

A. $y = 2x^2$

B. $y = \frac{1}{2}x^2$

C. $y = \frac{1}{8}x^2$

D. $y = \frac{1}{32}x^2$

7. Which expression is equivalent to $(t - 4)^2$? *(A.APR.1)*

F. $t^2 + 16$

G. $t^2 - 16$

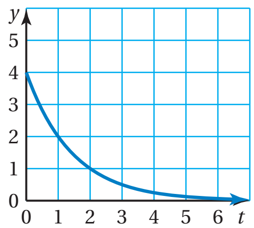
H. $t^2 - 8t + 16$

I. $t^2 - 8t - 16$

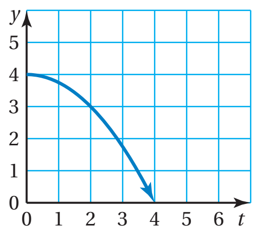
8. What is the value of $8^{2/3}$? *(N.RN.2)*

9. Which of the following is the graph of $y = 4(0.5)^t$? *(F.IF.7e)*

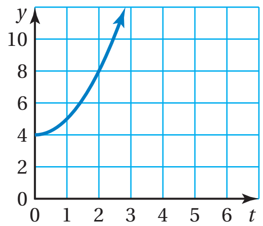
A.



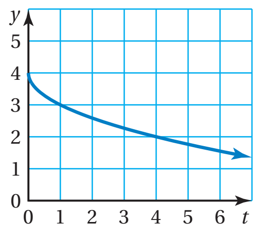
B.



C.



D.



10. The function $f(t) = -16t^2 + 32t + 4$ gives the height (in feet) of a softball t seconds after it is thrown. *(F.IF.4)*

Part A Does the function have a minimum value or a maximum value? Explain your reasoning.

Part B Find the value from Part A.

11. Which of the following is not a solution of the system of linear equations when $a \neq 0$, $b \neq 0$, and $c \neq 0$? *(8.EE.8a)*

$$ax + by = c$$

$$2ax + 2by = 2c$$

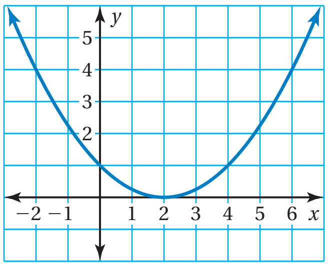
F. $\left(\frac{c}{2a}, \frac{c}{2b}\right)$

G. $\left(\frac{c}{a}, 0\right)$

H. $\left(c, \frac{c - ac}{b}\right)$

I. $\left(\frac{c}{b}, 0\right)$

12. Amanda was graphing $y = \frac{1}{4}x^2 + 2$. Her work is shown below. *(F.BF.3)*



What should Amanda do to correct the error that she made?

A. Shift the graph 2 units left.

B. Shift the graph 2 units up.

C. Shift the graph 2 units left and 2 units up.

D. Shift the graph 2 units left and 2 units down.

13. Which of the following is an equation of the line that passes through the points (4, 4) and (5, 7)? *(A.CED.2)*

F. $y = 3x - 8$

G. $y = \frac{1}{3}x + \frac{16}{3}$

H. $y = \frac{1}{3}x + \frac{8}{3}$

I. $y = -3x + 16$