## Ghapter Reviewy

## Review Key Vocabulary

monomial, p. 330
degree of a monomial, p. 330
polynomial, p. 331
binomial, p. 331
trinomial, p. 331
degree of a polynomial, p. 331
FOIL Method, p. 343
factored form, p. 358
Zero-Product Property, p. 358
root, p. 358
factoring by grouping, p. 388
prime polynomial, p. 389
factored completely, p. 389

## Review Examples and Exercises

## 7. 1$]$ Polynomials (pp. 328-333)

a. Find the degree of $4 x^{2} y$.

The exponent of $x$ is 2 and the exponent of $y$ is 1 .
The sum of the exponents is $2+1=3$.
$\therefore$ So, the degree of the monomial is 3 .
b. Write $x+1+2 x^{3}$ in standard form. Identify the degree and classify the polynomial by the number of terms.

| Polynomial | Standard Form | Degree | Type of Polynomial |
| :---: | :---: | :---: | :---: |
| $x+1+2 x^{3}$ | $2 x^{3}+x+1$ | 3 | trinomial |

## Exercises

Write the polynomial in standard form. Identify the degree and classify the polynomial by the number of terms.

1. $2 w^{3}+3-4 w$
2. $-6 y^{2}$
3. $-6.2+3 t^{5}$

## 7,2 Adding and Subtracting Polynomials (pp. 334-339)

a. $\left(2 d^{2}-3\right)+\left(4 d^{2}+2\right)$

$$
\begin{aligned}
\left(2 d^{2}-3\right)+\left(4 d^{2}+2\right) & =\left(2 d^{2}+4 d^{2}\right)+(-3+2) \\
& =6 d^{2}-1
\end{aligned}
$$

b. $\left(c^{2}+5 c+1\right)-\left(c^{2}-2\right)$

$$
\begin{aligned}
\left(c^{2}+5 c+1\right)-\left(c^{2}-2\right) & =\left(c^{2}+5 c+1\right)+\left(-c^{2}+2\right) \\
& =\left[c^{2}+\left(-c^{2}\right)\right]+5 c+(1+2)=5 c+3
\end{aligned}
$$

## Exercises

Find the sum or difference.
4. $(3 a+7)+(a-1)$
5. $\left(x^{2}+4 x-2\right)+\left(6 x^{2}+6\right)$
6. $\left(-y^{2}+y+2\right)-\left(y^{2}-5 y-2\right)$
7. $(p-9)-\left(-8 p^{2}+7\right)$

## 7. 3 Multiplying Polynomials (pp. 340-347)

Find $(x+1)(x-4)$.

$$
\begin{aligned}
& \text { First Outer Inner Last } & & \\
(x+1)(x-4) & =x(x)+x(-4)+(1)(x)+(1)(-4) & & \text { Use the FOIL Method. } \\
& =x^{2}+(-4 x)+(x)+(-4) & & \text { Multiply. } \\
& =x^{2}-3 x-4 & & \text { Combine like terms. }
\end{aligned}
$$

## Exercises

## Find the product.

8. $(y+4)(y-2)$
9. $(q-3)(2 q+7)$
10. $(-3 v+1)\left(v^{2}-v-2\right)$

## 7. 43 Special Products of Polynomials (pp. 348-353)

## Find each product.

a. $(x+3)(x-3)$

$$
\begin{aligned}
(a+b)(a-b) & =a^{2}-b^{2} & & \text { Sum and Difference Pattern } \\
(x+3)(x-3) & =x^{2}-3^{2} & & \text { Use pattern. } \\
& =x^{2}-9 & & \text { Simplify. }
\end{aligned}
$$

b. $(y+2)^{2}$

$$
\begin{aligned}
(a+b)^{2} & =a^{2}+2 a b+b^{2} & & \text { Square of a Binomial Pattern } \\
(y+2)^{2} & =y^{2}+2(y)(2)+2^{2} & & \text { Use pattern. } \\
& =y^{2}+4 y+4 & & \text { Simplify. }
\end{aligned}
$$

## Exercises

## Find the product.

11. $(y+9)(y-9)$
12. $(2 x+4)(2 x-4)$
13. $(h+4)^{2}$
14. $(-1+2 d)^{2}$

### 7.5 Solving Polynomial Equations in Factored Form (pp. 356-361)

Solve $(x+4)(x-3)=0$.

$$
\begin{array}{rlrl}
(x+4)(x-3)=0 & \text { Write equation. } \\
x+4=0 & \text { or } & x-3=0 & \text { Use Zero-Product Property. } \\
x=-4 & \text { or } & x=3 & \text { Solve for } x .
\end{array}
$$

$\because \quad$ The roots are $x=-4$ and $x=3$.

## Exercises

## Solve the equation.

15. $x(x+2)=0$
16. $(t-3)(t-8)=0$
17. $(a+10)^{2}=0$
18. $2 s(s+1)(s-4)=0$

### 7.6 Factoring Polynomials Using the GCF (pp. 362-367)

Factor $4 z^{2}+32$.
Step 1: Find the GCF of the terms.

$$
\begin{aligned}
4 z^{2} & =2 \cdot\left(\begin{array}{l}
2 \cdot z \cdot z \\
32
\end{array}=2 \cdot 2 \cdot 2 \cdot 2\right.
\end{aligned}
$$

The GCF is $2 \cdot 2=4$.
Step 2: Write the polynomial as a product of the GCF and its remaining factors.

$$
\begin{aligned}
4 z^{2}+32 & =4\left(z^{2}\right)+4(8) & & \text { Factor out GCF. } \\
& =4\left(z^{2}+8\right) & & \text { Distributive Property }
\end{aligned}
$$

## Exercises

Factor the polynomial.
19. $6 t^{2}+36$
20. $2 x^{2}-20 x$
21. $15 y^{3}+3 y^{2}$

### 7.7 Factoring $\mathbf{x}^{\mathbf{2}}+\mathbf{b x}+\mathbf{c}$ (pp. 368-375)

Factor $x^{2}+12 x+27$.
Notice that $b=12$ and $c=27$.

- Because $c$ is positive, the factors $p$ and $q$ must have the same sign so that $p q$ is positive.
- Because $b$ is also positive, $p$ and $q$ must each be positive so that $p+q$ is positive.

Find two positive integer factors of 27 whose sum is 12 .

| Factors of 27 | Sum of Factors |
| :---: | :---: |
| 1,27 | 28 |
| 3,9 | 12 |

The values of $p$ and $q$ are 3 and 9 .
$\therefore$ So, $x^{2}+12 x+27=(x+3)(x+9)$.

## Exercises

## Factor the polynomial.

22. $p^{2}+2 p-35$
23. $b^{2}+9 b+20$
24. $z^{2}-4 z-21$

### 7.8 Factoring $\mathbf{a} \mathbf{x}^{\mathbf{2}}+\mathbf{b} \mathbf{x}+\mathbf{c}$ (pp. 376-381)

a. Factor $2 x^{2}+13 x+15$.

Consider the possible factors of $a=2$ and $c=15$.

```
Factors are 1 and 2. }\longrightarrow2\mp@subsup{x}{}{2}+13x+15\longleftarrow~Factors are 1, 3, 5, and 15
```

These factors lead to the following possible products.

$$
\begin{array}{ll}
(1 x+1)(2 x+15) & (1 x+3)(2 x+5) \\
(1 x+15)(2 x+1) & (1 x+5)(2 x+3)
\end{array}
$$

Multiply to find the product that is equal to the original polynomial.

$$
\begin{gathered}
(x+1)(2 x+15)=2 x^{2}+17 x+15 \\
(x+15)(2 x+1)=2 x^{2}+31 x+15 \\
(x+3)(2 x+5)=2 x^{2}+11 x+15 \\
(x+5)(2 x+3)=2 x^{2}+13 x+15 \\
\therefore \quad \text { So, } 2 x^{2}+13 x+15=(x+5)(2 x+3) .
\end{gathered}
$$

b. Factor $5 x^{2}+4 x-9$.

Consider the possible factors of $a=5$ and $c=-9$. Because $b$ is positive and $c$ is negative, the factors of $c$ must have different signs.

$$
\text { Factors are } 1 \text { and } 5 . \longrightarrow 5 x^{2}+4 x-9 \longleftarrow \quad \text { Factors are } \pm 1, \pm 3 \text {, and } \pm 9 \text {. }
$$

These factors lead to the following possible products.

$$
\begin{array}{lll}
(1 x+1)(5 x-9) & (1 x-1)(5 x+9) & (1 x-3)(5 x+3) \\
(1 x+9)(5 x-1) & (1 x-9)(5 x+1) & (1 x+3)(5 x-3)
\end{array}
$$

Multiply to find the product that is equal to the original polynomial.

$$
\begin{aligned}
(x+1)(5 x-9) & =5 x^{2}-4 x-9 \mathbf{X} \\
(x+9)(5 x-1) & =5 x^{2}+44 x-9 \quad \mathbf{X} \\
(x-1)(5 x+9) & =5 x^{2}+4 x-9 \text { Х } \\
(x-9)(5 x+1) & =5 x^{2}-44 x-9 X \\
(x-3)(5 x+3) & =5 x^{2}-12 x-9 X \\
(x+3)(5 x-3) & =5 x^{2}+12 x-9 X \\
\therefore \quad \text { So, } 5 x^{2}+4 x-9 & =(x-1)(5 x+9) .
\end{aligned}
$$

## Exercises

## Factor the polynomial.

25. $10 a^{2}+11 a+3$
26. $4 z^{2}+11 z+6$
27. $2 x^{2}-27 x-14$
28. $-2 p^{2}+2 p+4$
29. OUTSIDE PATIO You are installing new tile on an outside patio. The area (in square feet) of the rectangular patio can be represented by $8 x^{2}+33 x+4$. Write the expressions that represent the dimensions of the patio.


### 7.9 Factoring Special Products (pp. 382-389)

## Factor each polynomial.

a. $x^{2}-16$

$$
\begin{aligned}
x^{2}-16 & =x^{2}-4^{2} & & \text { Write as } a^{2}-b^{2} . \\
& =(x+4)(x-4) & & \text { Difference of Two Squares Pattern }
\end{aligned}
$$

b. $x^{2}-2 x+1$

$$
\begin{aligned}
x^{2}-2 x+1 & =x^{2}-2(x)(1)+1^{2} & & \text { Write as } a^{2}-2 a b+b^{2} . \\
& =(x-1)^{2} & & \text { Perfect Square Trinomial Pattern }
\end{aligned}
$$

c. $x^{3}+4 x^{2}+3 x+12$

$$
x^{3}+4 x^{2}+3 x+12=\left(x^{3}+4 x^{2}\right)+(3 x+12) \quad \text { Group terms with common factors. }
$$

Common binomial factor is $x+4 . \longrightarrow=x^{2}(x+4)+3(x+4) \quad$ Factor out GCF of each pair of terms. $=(x+4)\left(x^{2}+3\right) \quad$ Factor out $(x+4)$.
d. $2 x^{4}-8 x^{2}$

$$
\begin{aligned}
2 x^{4}-8 x^{2} & =2 x^{2}\left(x^{2}-4\right) & & \text { Factor out } 2 x^{2} . \\
& =2 x^{2}\left(x^{2}-2^{2}\right) & & \text { Write as } a^{2}-b^{2} . \\
& =2 x^{2}(x+2)(x-2) & & \text { Difference of Two Squares Pattern }
\end{aligned}
$$

## Exercises

Factor the polynomial.
30. $x^{2}-9$
31. $y^{2}-100$
32. $z^{2}+6 z+9$
33. $m^{2}+16 m+64$
34. $x^{2}-3 x+4 a x-12 a$
35. $n^{3}-9 n$

