Extension 7.9 Factoring Polynomials Completely

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Lesson Tutorials
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Key Vocabulary
factoring by grouping, *p. 388*
prime polynomial, *p. 389*
factored completely, *p. 389*

To factor polynomials with four terms, group the terms into pairs, factor the GCF out of each pair of terms, and look for a common binomial factor. This process is called **factoring by grouping**.

EXAMPLE 1 Factoring by Grouping

Factor each polynomial.

a. $x^3 + 3x^2 + 2x + 6$

$x^3 + 3x^2 + 2x + 6 = (x^3 + 3x^2) + (2x + 6)$ — Group terms with common factors.

Common binomial factor is $x + 3$. $\rightarrow = x^2(x + 3) + 2(x + 3)$ — Factor out GCF of each pair of terms.

$= (x + 3)(x^2 + 2)$ — Factor out $(x + 3)$.

b. $x^3 - 7 - x^2 + 7x$

The terms x^3 and -7 do not have a common factor. Rearrange the terms of the polynomial so you can group terms with common factors.

$x^3 - 7 - x^2 + 7x = x^3 - x^2 + 7x - 7$ — Rewrite polynomial.

$= (x^3 - x^2) + (7x - 7)$ — Group terms with common factors.

Common binomial factor is $x - 1$. $\rightarrow = x^2(x - 1) + 7(x - 1)$ — Factor out GCF of each pair of terms.

$= (x - 1)(x^2 + 7)$ — Factor out $(x - 1)$.

c. $x^2 + y + x + xy$

$x^2 + y + x + xy = x^2 + x + xy + y$ — Rewrite polynomial.

$= (x^2 + x) + (xy + y)$ — Group terms with common factors.

$= x(x + 1) + y(x + 1)$ — Factor out GCF of each pair of terms.

$= (x + 1)(x + y)$ — Factor out $(x + 1)$.

Practice

Factor the polynomial by grouping.

1. $n^3 + 2n^2 + 5n + 10$

2. $p^3 - 7p^2 + 3p - 21$

3. $2y^3 + 8y^2 + 3y + 12$

4. $6s^3 - 16s^2 + 21s - 56$

5. $8v^3 + 48v - 5v^2 - 30$

6. $2w^3 - w^2 - 18w + 9$

7. $x^2 + xy + 3x + 3y$

8. $a - ab + a^2 - b$

9. $4xy + 20y + 3x + 15$

A **prime polynomial** is a polynomial that cannot be factored as a product of polynomials with integer coefficients. A factorable polynomial with integer coefficients is said to be **factored completely** when no more factors can be found and it is written as the product of prime factors.

EXAMPLE 2 Factoring Completely

Common Core

Polynomial Equations

In this extension, you will

- factor polynomials by grouping.
- factor polynomials completely.

Learning Standards
A.REI.4b
A.SSE.2
A.SSE.3a

Factor each polynomial completely.

a. $3x^3 - 18x^2 + 24x$

$3x^3 - 18x^2 + 24x = 3x(x^2 - 6x + 8)$	Factor out $3x$.
$= 3x(x-2)(x-4)$	Factor $x^2 - 6x + 8$.

b. $7x^4 - 28x^2$

$7x^4 - 28x^2 = 7x^2(x^2 - 4)$	Factor out $7x^2$.
$= 7x^2(x^2 - 2^2)$	Write as $a^2 - b^2$.
$= 7x^2(x+2)(x-2)$	Difference of Two Squares Pattern

c. $p^2 + 4p - 2$

The terms of $p^2 + 4p - 2$ have no common factors. There are no integer factors of -2 whose sum is 4. So, this polynomial is already factored completely.

EXAMPLE 3 Solving an Equation by Factoring Completely

$2x^3 + 8x^2 = 10x$	Original equation
$2x^3 + 8x^2 - 10x = 0$	Subtract $10x$ from each side.
$2x(x^2 + 4x - 5) = 0$	Factor out $2x$.
$2x(x+5)(x-1) = 0$	Factor $x^2 + 4x - 5$.
$2x = 0$ *or* $x + 5 = 0$ *or* $x - 1 = 0$	Use Zero-Product Property.
$x = 0$ *or* $x = -5$ *or* $x = 1$	Solve for x.

The solutions are $x = -5$, $x = 0$, and $x = 1$.

Practice

Factor the polynomial completely, if possible.

10. $2x^3 + 10x^2 - 48x$

11. $5z^4 - 5z^2$

12. $20c + 4c^3 - 24c^2$

13. $y^2 + 6y - 5$

14. $q^2 - q + 7$

15. $3n^4 - 48n^2$

Solve the equation.

16. $k^3 - 6k^2 + 9k = 0$

17. $3x^3 + 6x^2 = 72x$

18. $4y^3 - 12y^2 - 40y = 0$