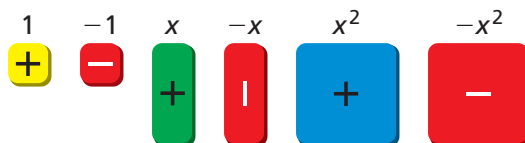**Essential Question** How can you recognize and factor special products?

1 ACTIVITY: Factoring Special Products

Work with a partner. Six different algebra tiles are shown below.
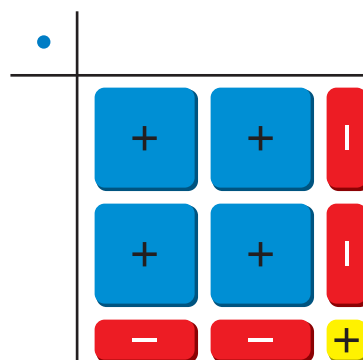
1 -1 x $-x$ x^2 $-x^2$

Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying. State whether the product is a "special product" that you studied in Lesson 7.4.
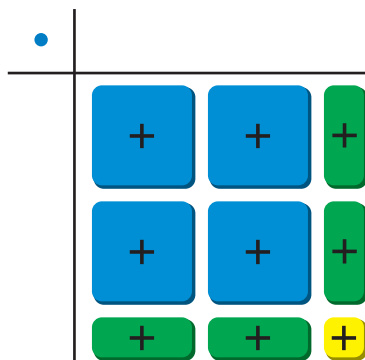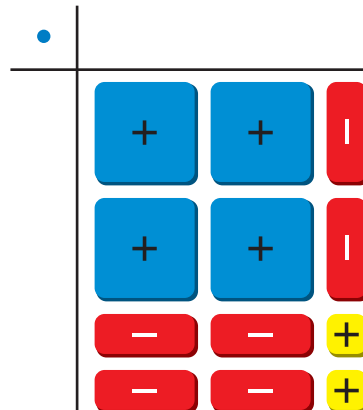
a.
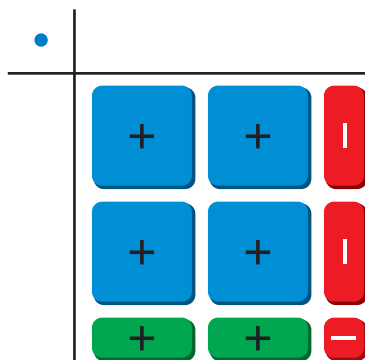
b.

c.

d.

2 ACTIVITY: Factoring Special Products

Work with a partner. Use algebra tiles to complete the rectangular array in three different ways, so that each way represents a different special product. Write each special product in polynomial form and also in factored form.
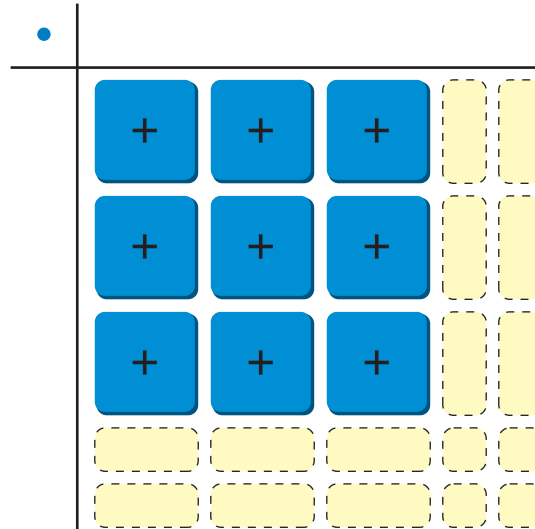
3 ACTIVITY: Finding Binomial Factors

Math Practice 8

Maintain Oversight

How is factoring a special product similar to factoring a polynomial?

Work with a partner. Write each polynomial as the product of two binomials. Check your answer by multiplying.

a. $4x^2 - 12x + 9$ **b.** $4x^2 - 9$ **c.** $4x^2 + 12x + 9$

What Is Your Answer?

4. IN YOUR OWN WORDS How can you recognize and factor special products? Describe a strategy for recognizing which polynomials can be factored as special products.

5. Use your strategy to factor each polynomial.

a. $25x^2 + 10x + 1$ **b.** $25x^2 - 10x + 1$ **c.** $25x^2 - 1$

Practice Use what you learned about factoring polynomials as special products to complete Exercises 4–6 on page 386.

You can use special product patterns to factor polynomials.

🔑 Key Idea

Difference of Two Squares Pattern

Algebra

$a^2 - b^2 = (a + b)(a - b)$

Example

$$x^2 - 9 = x^2 - 3^2$$
$$= (x + 3)(x - 3)$$

EXAMPLE 1   Factoring the Difference of Two Squares

Factor each polynomial.

a. $x^2 - 25$

$$x^2 - 25 = x^2 - 5^2$$ Write as $a^2 - b^2$.
$$= (x + 5)(x - 5)$$ Difference of Two Squares Pattern

b. $64 - y^2$

$$64 - y^2 = 8^2 - y^2$$ Write as $a^2 - b^2$.
$$= (8 + y)(8 - y)$$ Difference of Two Squares Pattern

Remember

You can check your answers using the FOIL Method.

c. $4z^2 - 1$

$$4z^2 - 1 = (2z)^2 - 1^2$$ Write as $a^2 - b^2$.
$$= (2z + 1)(2z - 1)$$ Difference of Two Squares Pattern

🔴 On Your Own

Now You're Ready
Exercises 4–8

Factor the polynomial.

1. $x^2 - 36$ **2.** $100 - m^2$ **3.** $9n^2 - 16$ **4.** $16h^2 - 49$

🔑 Key Idea

Perfect Square Trinomial Pattern

Algebra

$a^2 + 2ab + b^2 = (a + b)^2$

$a^2 - 2ab + b^2 = (a - b)^2$

Example

$$x^2 + 6x + 9 = x^2 + 2(x)(3) + 3^2$$
$$= (x + 3)^2$$

$$x^2 - 6x + 9 = x^2 - 2(x)(3) + 3^2$$
$$= (x - 3)^2$$

EXAMPLE **2**   **Factoring Perfect Square Trinomials**

Factor each polynomial.

a. $n^2 + 8n + 16$

$$n^2 + 8n + 16 = n^2 + 2(n)(4) + 4^2 \qquad \text{Write as } a^2 + 2ab + b^2.$$
$$= (n + 4)^2 \qquad\qquad\qquad \text{Perfect Square Trinomial Pattern}$$

b. $x^2 - 18x + 81$

$$x^2 - 18x + 81 = x^2 - 2(x)(9) + 9^2 \qquad \text{Write as } a^2 - 2ab + b^2.$$
$$= (x - 9)^2 \qquad\qquad\qquad \text{Perfect Square Trinomial Pattern}$$

On Your Own

Now You're Ready
Exercises 9–12

Factor the polynomial.

5. $m^2 - 2m + 1$ **6.** $d^2 - 10d + 25$ **7.** $z^2 + 20z + 100$

EXAMPLE **3**   **Real-Life Application**

A bird picks up a golf ball and drops it while flying. The function represents the height y (in feet) of the golf ball t seconds after it is dropped. The ball hits the top of a 32-foot tall pine tree. After how many seconds does the ball hit the tree?

Substitute 32 for y and solve for t.

$$y = 81 - 16t^2 \qquad\qquad\qquad \text{Write equation.}$$
$$32 = 81 - 16t^2 \qquad\qquad\qquad \text{Substitute 32 for } y.$$
$$0 = 49 - 16t^2 \qquad\qquad\qquad \text{Subtract 32 from each side.}$$
$$0 = 7^2 - (4t)^2 \qquad\qquad\qquad \text{Write as } a^2 - b^2.$$
$$0 = (7 + 4t)(7 - 4t) \qquad\qquad \text{Difference of Two Squares Pattern}$$
$$7 + 4t = 0 \quad \text{or} \quad 7 - 4t = 0 \qquad \text{Use Zero-Product Property.}$$
$$t = -\frac{7}{4} \quad \text{or} \qquad t = \frac{7}{4} \qquad \text{Solve for } t.$$

$y = 81 - 16t^2$

A negative time does not make sense in this situation.

So, the golf ball hits the tree after $\frac{7}{4}$, or 1.75 seconds.

On Your Own

8. **WHAT IF?** The golf ball does not hit the pine tree. After how many seconds does the ball hit the ground?

Check It Out
Help with Homework
BigIdeasMath ✓com
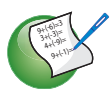
✓ Vocabulary and Concept Check

1. **WRITING** Describe two ways to show that $x^2 - 16$ is equal to $(x + 4)(x - 4)$.

2. **REASONING** Can you use the perfect square trinomial pattern to factor $y^2 + 16y + 64$? Explain.

3. **WHICH ONE DOESN'T BELONG?** Which polynomial does *not* belong with the other three? Explain your reasoning.

$$n^2 - 4 \qquad g^2 - 6g + 9 \qquad r^2 + 12r + 36 \qquad k^2 + 25$$

Practice and Problem Solving

Factor the polynomial.

① ② 4. $m^2 - 49$ 5. $9 - r^2$ 6. $4x^2 - 25$

7. $81d^2 - 64$ 8. $121 - 16t^2$ 9. $h^2 + 12h + 36$

10. $x^2 - 4x + 4$ 11. $w^2 - 14w + 49$ 12. $g^2 + 24g + 144$

13. **ERROR ANALYSIS** Describe and correct the error in factoring the polynomial.

✗ $n^2 - 16n + 64 = n^2 - 2(n)(8) + 8^2$
    $= (n + 8)^2$

Solve the equation.

14. $z^2 - 4 = 0$ 15. $s^2 + 20s + 100 = 0$ 16. $k^2 - 16k + 64 = 0$

17. $4x^2 = 49$ 18. $n^2 + 9 = -6n$ 19. $y^2 = 12y - 36$

20. **REASONING** Tell whether the polynomial can be factored. If not, change the constant term so that the polynomial can be factored using the perfect square trinomial pattern.

   a. $w^2 + 18w + 84$
   b. $y^2 - 10y + 23$
   c. $x^2 - 14x + 50$

21. **COASTER** The area (in square centimeters) of a square coaster can be represented by $d^2 + 8d + 16$. Write an expression that represents the side length of the coaster.
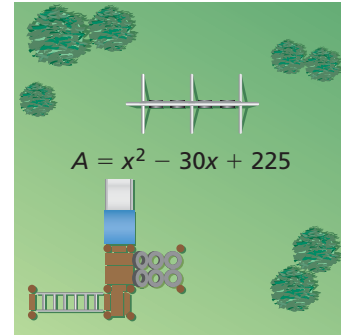
Factor the polynomial.

22. $3z^2 - 27$

23. $2m^3 - 50m$

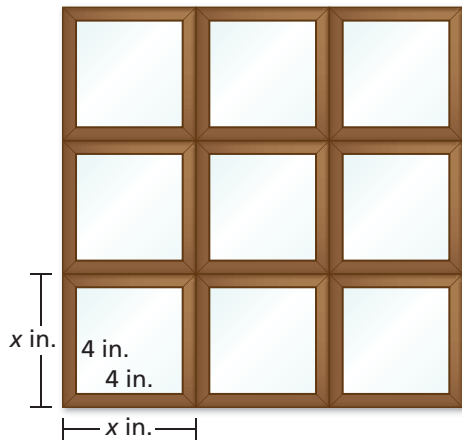**24.** $x^4 + 8x^3 + 16x^2$

25. $5f^3 - 20f^2 + 20f$

26. PROBLEM SOLVING The polynomial represents the area (in square feet) of the square playground.

    a. Write a polynomial that represents the side length of the playground.

    b. Write an expression for the perimeter of the playground.

$A = x^2 - 30x + 225$

27. NUMBER SENSE Solve $28 = 64 - 9x^2$ in two ways.

28. INTERIOR DESIGN You hang 9 identical square picture frames on a wall.

    a. Write a polynomial that represents the area of the picture frames, not including the pictures.

    b. The area in part (a) is 81 square inches. What is the side length of one of the picture frames? Explain your reasoning.
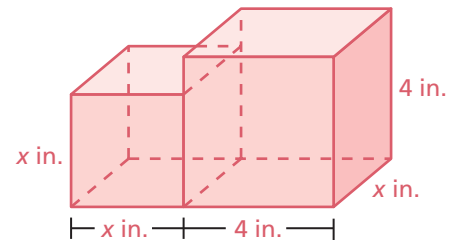
Factor the polynomial.

29. $4y^2 + 4y + 1$

30. $16v^2 - 24v + 9$

31. $9m^2 + 36m + 36$

32. Geometry A composite solid is made up of a cube and a rectangular prism.

    a. Write a polynomial that represents the volume of the composite solid.

    b. The volume of the composite solid is equal to $25x$. What is the value of x? Explain your reasoning.

Fair Game Review What you learned in previous grades & lessons

Factor the polynomial. *(Section 7.7)*

33. $w^2 + w - 12$

34. $x^2 - 5x - 36$

35. $d^2 - 4d - 60$

36. MULTIPLE CHOICE You deposit $3000 in a savings account. The account earns 4% simple interest per year. What is the balance after 2 years? *(Skills Review Handbook)*

    A $240   **B** $3000   **C** $3240   **D** $5400