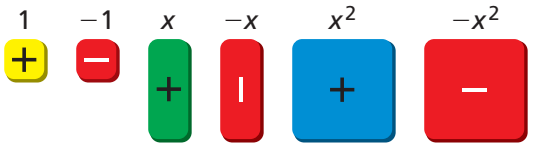
7.8 Factoring $ax^2 + bx + c$

Essential Question How can you factor the trinomial $ax^2 + bx + c$ into the product of two binomials?

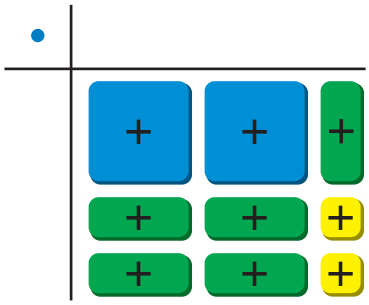
1 ACTIVITY: Finding Binomial Factors

Work with a partner. Six different algebra tiles are shown below.

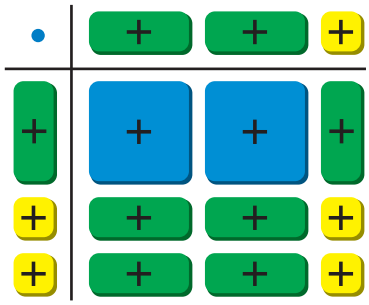


Sample:

Step 1: Arrange the algebra tiles into a rectangular array to model $2x^2 + 5x + 2$.



Step 2: Use algebra tiles to label the dimensions of the rectangle.

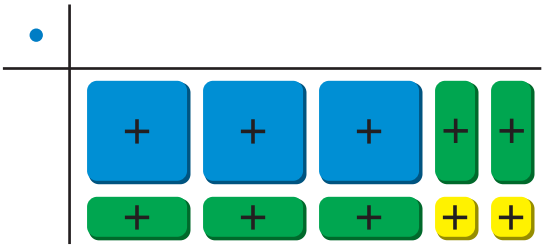


Step 3: Write the polynomial in factored form by finding the dimensions of the rectangle.

length, width

$$\text{Area} = 2x^2 + 5x + 2 = (2x + 1)(x + 2)$$

Use algebra tiles to write the polynomial as the product of two binomials. Check your answer by multiplying.



Common Core

Polynomial Equations

In this lesson, you will

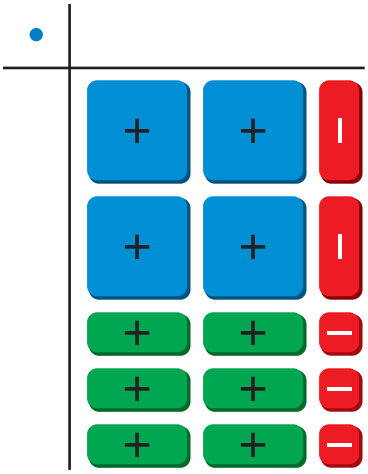
- factor trinomials of the form $ax^2 + bx + c$.

Learning Standards
A.REI.4b
A.SSE.3a

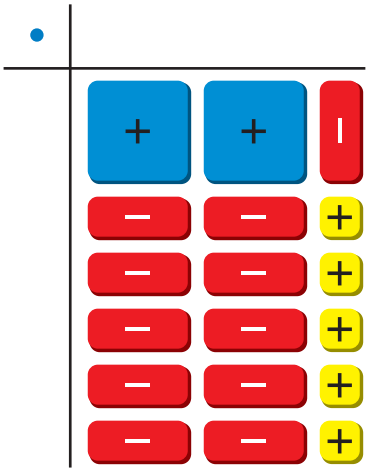
2 ACTIVITY: Finding Binomial Factors

Work with a partner. Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying.

a.



b.



3 ACTIVITY: Finding Binomial Factors

Work with a partner. Write each polynomial as the product of two binomials. Check your answer by multiplying.

a. $2x^2 + 5x - 3$ **b.** $3x^2 + 10x - 8$ **c.** $4x^2 + 4x - 3$

d. $2x^2 + 11x + 15$ **e.** $9x^2 - 6x + 1$ **f.** $4x^2 + 11x - 3$

Math Practice 1

Find Entry Points

What should you do first when factoring a polynomial using algebra tiles?

What Is Your Answer?

4. IN YOUR OWN WORDS How can you factor the trinomial $ax^2 + bx + c$ into the product of two binomials?

5. Use your strategy to factor each trinomial.

a. $4x^2 + 4x + 1$ **b.** $3x^2 + 5x - 2$ **c.** $2x^2 - 13x + 15$

Practice Use what you learned about factoring trinomials to complete Exercises 3–5 on page 380.

7.8 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

In Section 7.7, you factored polynomials of the form $ax^2 + bx + c$, where $a = 1$. To factor polynomials of the form $ax^2 + bx + c$, where $a \neq 1$, first look for the GCF of the terms of the polynomial.

EXAMPLE 1 Factoring Out the GCF

Factor $5x^2 + 15x + 10$.

Notice that the GCF of the terms $5x^2$, $15x$, and 10 is 5.

$$5x^2 + 15x + 10 = 5(x^2 + 3x + 2) \qquad \text{Factor out GCF.}$$
$$= 5(x + 1)(x + 2) \qquad \text{Factor } x^2 + 3x + 2.$$

∴ So, $5x^2 + 15x + 10 = 5(x + 1)(x + 2)$.

When there is no GCF, consider the possible factors of a and c.

EXAMPLE 2 Factoring $ax^2 + bx + c$ When ac Is Positive

a. Factor $4x^2 + 13x + 3$.

Consider the possible factors of $a = 4$ and $c = 3$.

Factors are 1, 2, and 4. → $4x^2 + 13x + 3$ ← Factors are 1 and 3.

These factors lead to the following possible products.

$$(1x + 1)(4x + 3) \qquad (1x + 3)(4x + 1) \qquad (2x + 1)(2x + 3)$$

Multiply to find the product that is equal to the original polynomial.

$(x + 1)(4x + 3) = 4x^2 + 7x + 3$ ✗ $(2x + 1)(2x + 3) = 4x^2 + 8x + 3$ ✗

$(x + 3)(4x + 1) = 4x^2 + 13x + 3$ ✓

∴ So, $4x^2 + 13x + 3 = (x + 3)(4x + 1)$.

b. Factor $3x^2 - 7x + 2$.

Consider the possible factors of $a = 3$ and $c = 2$. Because b is negative and c is positive, both factors of c must be negative.

Factors are 1 and 3. → $3x^2 - 7x + 2$ ← Factors are -2 and -1.

These factors lead to the following possible products.

$$(1x - 1)(3x - 2) \qquad (1x - 2)(3x - 1)$$

Multiply to find the product that is equal to the original polynomial.

$(x - 1)(3x - 2) = 3x^2 - 5x + 2$ ✗ $(x - 2)(3x - 1) = 3x^2 - 7x + 2$ ✓

∴ So, $3x^2 - 7x + 2 = (x - 2)(3x - 1)$.

Study Tip

When ac is positive, the sign of b determines whether the factors of c are positive or negative.

Now You're Ready
Exercises 6–11
and 13–15

On Your Own

Factor the polynomial.

1. $8x^2 - 56x + 48$
2. $2x^2 + 11x + 5$
3. $2x^2 - 7x + 5$
4. $3x^2 - 14x + 8$

EXAMPLE 3 Factoring $ax^2 + bx + c$ When ac Is Negative

Study Tip

For polynomials of the form $ax^2 + bx + c$, where a is negative, factor out -1 first to make factoring easier. Just be sure to put -1 back in your final answer.

Factor $2x^2 - 5x - 7$.

Consider the possible factors of $a = 2$ and $c = -7$. Because b and c are both negative, the factors of c must have different signs.

Factors are 1 and 2. → $2x^2 - 5x - 7$ ← Factors are ± 1 and ± 7.

These factors lead to the following possible products.

$(x + 1)(2x - 7)$ $(x + 7)(2x - 1)$ $(x - 1)(2x + 7)$ $(x - 7)(2x + 1)$

Multiply to find the product that is equal to the original polynomial.

$(x + 1)(2x - 7) = 2x^2 - 5x - 7$ ✓ $(x - 1)(2x + 7) = 2x^2 + 5x - 7$ ✗

$(x + 7)(2x - 1) = 2x^2 + 13x - 7$ ✗ $(x - 7)(2x + 1) = 2x^2 - 13x - 7$ ✗

∴ So, $2x^2 - 5x - 7 = (x + 1)(2x - 7)$.

EXAMPLE 4 Real-Life Application

The length of a rectangular game reserve is 1 mile longer than twice the width. The area of the reserve is 55 square miles. How wide is the reserve?

(A) 2 mi (B) 2.5 mi (C) 5 mi (D) 5.5 mi

Write an equation that represents the area of the reserve. Then solve by factoring. Let w represent the width. Then $2w + 1$ represents the length.

$w(2w + 1) = 55$	Area of the reserve
$2w^2 + w - 55 = 0$	Multiply. Then subtract 55 from each side.
$(w - 5)(2w + 11) = 0$	Factor left side of the equation.
$w - 5 = 0$ *or* $2w + 11 = 0$	Use Zero-Product Property.
$w = 5$ *or* $w = -\frac{11}{2}$	Solve for w. Use the positive solution.

∴ The correct answer is (C).

On Your Own

Now You're Ready
Exercises 16–21

Factor the polynomial.

5. $6x^2 + x - 12$
6. $4x^2 - 19x - 5$
7. **WHAT IF?** In Example 4, the area of the reserve is 136 square miles. How wide is the reserve?

7.8 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **WRITING** Describe how to factor polynomials of the form $ax^2 + bx + c$.
2. **WHICH ONE DOESN'T BELONG?** Which factored polynomial does *not* belong with the other three? Explain your reasoning.

$(2x - 3)(x + 2)$ | $x(2x - 3) + 2(2x - 3)$ | $(2x + 3)(x - 2)$ | $2x(x + 2) - 3(x + 2)$

Practice and Problem Solving

Use algebra tiles to write the polynomial as the product of two binomials.

3. $2x^2 - 3x + 1$
4. $3x^2 + x - 2$
5. $4x^2 + 11x + 6$

Factor the polynomial.

1 6. $3x^2 + 3x - 6$
7. $8v^2 + 8v - 48$
8. $4k^2 + 28k + 48$
9. $6y^2 - 24y + 18$
10. $9r^2 - 36r - 45$
11. $7d^2 - 63d + 140$

12. **ERROR ANALYSIS** Describe and correct the error in factoring the polynomial.

$$2x^2 + 2x - 4 = 2x(x + 1 - 2)$$
$$= 2x(x - 1)$$

Factor the polynomial.

2 3 13. $3h^2 + 11h + 6$
14. $6x^2 - 5x + 1$
15. $8m^2 + 30m + 7$
16. $18v^2 - 15v - 18$
17. $2n^2 - 5n - 3$
18. $4z^2 - 4z - 3$
19. $8g^2 - 10g - 12$
20. $10w^2 + 19w - 15$
21. $14d^2 + 3d - 2$

22. **ERROR ANALYSIS** Describe and correct the error in factoring the polynomial.

$$6x^2 - 7x - 3 = (3x - 3)(2x + 1)$$

23. **DANCE FLOOR** The area (in square feet) of a rectangular lighted dance floor can be represented by $8x^2 + 22x + 5$. Write the expressions that represent the dimensions of the dance floor.

Solve the equation.

24. $5x^2 - 5x - 30 = 0$ **25.** $2k^2 - 5k - 18 = 0$ **26.** $12m^2 + 11m = 15$

Factor the polynomial.

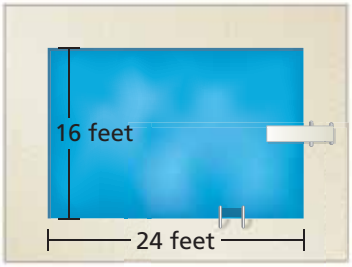
27. $-3w^2 - 2w + 8$ **28.** $-12x^2 + 48x + 27$ **29.** $-40n^2 + 70n - 15$

30. CLIFF DIVING The height h (in feet) above the water of a cliff diver is modeled by $h = -16t^2 + 8t + 80$, where t is the time (in seconds). How long is the diver in the air?

31. REASONING For what values of t can $2x^2 + tx + 10$ be written as the product of two binomials?

32. INVITATION The length of a rectangular birthday party invitation is 1 inch less than twice its width. The area of the invitation is 15 square inches. Will the invitation fit in a $3\frac{5}{8}$-inch by $5\frac{1}{8}$-inch envelope without being folded? Explain your reasoning.

33. SWIMMING POOL A rectangular swimming pool is bordered by a concrete patio. The width of the patio is the same on every side. The surface area of the pool is equal to the area of the patio border. What is the width of the patio border?



34. REASONING When is it *not* possible to factor $ax^2 + bx + c$, where $a \neq 1$? Give an example.

35. CHOOSE TOOLS A vendor can sell 50 bobbleheads per day when the price is \$40 each. For every \$2 decrease in price, 5 more bobbleheads are sold each day.

a. The revenue from yesterday was \$2160. What was the price per bobblehead? (*Note:* revenue = units sold $\times$ unit price)

b. How much should the vendor charge per bobblehead to maximize the daily revenue? Explain how you found your answer.

Structure **Factor the polynomial.**

36. $40k^3 + 6k^2 - 4k$ **37.** $6x^2 + 5xy - 4y^2$ **38.** $18m^3 + 39m^2n - 15mn^2$

Fair Game Review What you learned in previous grades & lessons

Find the product. *(Section 7.4)*

39. $(2x - 7)(2x + 7)$ **40.** $(k + 5)^2$ **41.** $(3b - 4)^2$

42. MULTIPLE CHOICE Two angles are supplementary. The measure of one of the angles is 58°. What is the measure of the other angle? *(Skills Review Handbook)*

Ⓐ 22° Ⓑ 32° Ⓒ 58° Ⓓ 122°