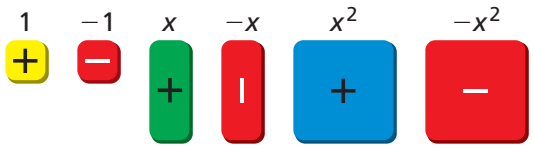
7.7 Factoring $x^2 + bx + c$

Essential Question How can you factor the trinomial $x^2 + bx + c$ into the product of two binomials?

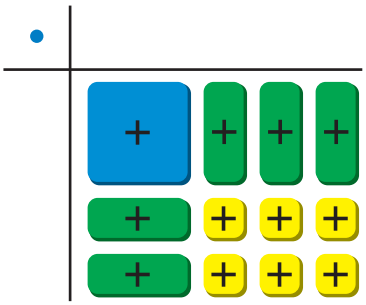
1 ACTIVITY: Finding Binomial Factors

Work with a partner. Six different algebra tiles are shown below.

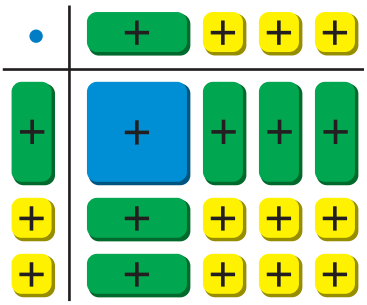


Sample:

Step 1: Arrange the algebra tiles into a rectangular array to model $x^2 + 5x + 6$.



Step 2: Use algebra tiles to label the dimensions of the rectangle.



Step 3: Write the polynomial in factored form by finding the dimensions of the rectangle.

width, length

$$\text{Area} = x^2 + 5x + 6 = (x + 2)(x + 3)$$

Common Core

Polynomial Equations

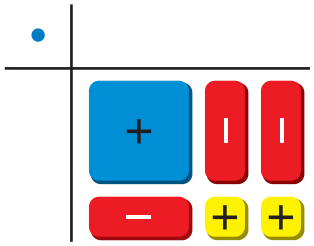
In this lesson, you will

- factor trinomials of the form $x^2 + bx + c$.

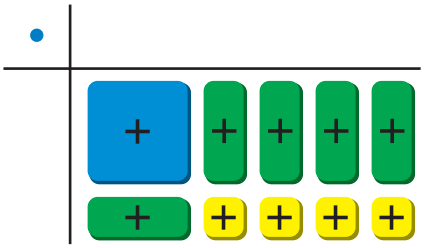
Learning Standards
A.REI.4b
A.SSE.3a

Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying.

a.



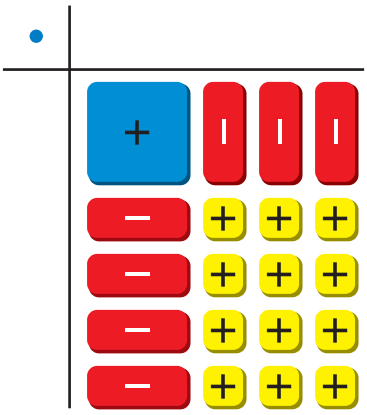
b.



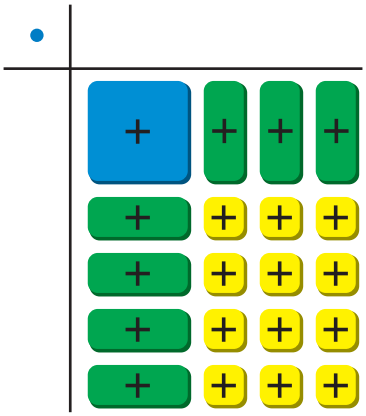
2 ACTIVITY: Finding Binomial Factors

Work with a partner. Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying.

a.



b.



3 ACTIVITY: Finding Binomial Factors

Work with a partner. Write each polynomial as the product of two binomials. Check your answer by multiplying.

a. $x^2 + 6x + 9$ **b.** $x^2 - 6x + 9$ **c.** $x^2 + 6x + 8$

d. $x^2 - 6x + 8$ **e.** $x^2 + 6x + 5$ **f.** $x^2 - 6x + 5$

Math Practice 2

Make Sense of Quantities

What is the relationship between a polynomial and its binomial factors?

What Is Your Answer?

4. IN YOUR OWN WORDS How can you factor the trinomial $x^2 + bx + c$ into the product of two binomials?

a. Describe a strategy that uses algebra tiles.

b. Describe a strategy that does not use algebra tiles.

5. Use one of your strategies to factor each trinomial.

a. $x^2 + 6x - 16$ **b.** $x^2 - 6x - 16$ **c.** $x^2 + 6x - 27$

Practice Use what you learned about factoring trinomials to complete Exercises 3–5 on page 373.

7.7 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Consider the polynomial $x^2 + bx + c$, where b and c are integers. To factor this polynomial as $(x + p)(x + q)$, you need to find integers p and q such that $p + q = b$ and $pq = c$.

$$(x + p)(x + q) = x^2 + px + qx + pq$$
$$= x^2 + (p + q)x + pq$$

Key Idea

Factoring $x^2 + bx + c$ When c Is Positive

Algebra $x^2 + bx + c = (x + p)(x + q)$ when $p + q = b$ and $pq = c$.

When c is positive, p and q have the same sign as b.

Examples $x^2 + 6x + 5 = (x + 1)(x + 5)$

$x^2 - 6x + 5 = (x - 1)(x - 5)$

EXAMPLE 1 Factoring $x^2 + bx + c$ When b and c Are Positive

Factor $x^2 + 10x + 16$.

Notice that $b = 10$ and $c = 16$.

- Because c is positive, the factors p and q must have the same sign so that pq is positive.
- Because b is also positive, p and q must each be positive so that $p + q$ is positive.

Find two positive integer factors of 16 whose sum is 10.

Factors of 16	Sum of Factors
1, 16	17
2, 8	10
4, 4	8

The values of p and q are 2 and 8.

∴ So, $x^2 + 10x + 16 = (x + 2)(x + 8)$.

Check

Use the FOIL Method.

$(x + 2)(x + 8)$

$= x^2 + 8x + 2x + 16$

$= x^2 + 10x + 16$ ✓

On Your Own

Now You're Ready
Exercises 3–8

Factor the polynomial.

1. $x^2 + 2x + 1$
2. $x^2 + 9x + 8$
3. $y^2 + 6y + 8$
4. $z^2 + 11z + 24$

EXAMPLE 2 Factoring $x^2 + bx + c$ When b Is Negative and c Is Positive

Factor $x^2 - 8x + 12$.

Notice that $b = -8$ and $c = 12$.

- Because c is positive, the factors p and q must have the same sign so that pq is positive.
- Because b is negative, p and q must each be negative so that $p + q$ is negative.

Find two negative integer factors of 12 whose sum is -8.

Factors of 12	$-1, -12$	$-2, -6$	$-3, -4$
Sum of Factors	-13	-8	-7

The values of p and q are -2 and -6.

∴ So, $x^2 - 8x + 12 = (x - 2)(x - 6)$.

Check

Use the FOIL Method.

$(x - 2)(x - 6)$

$= x^2 - 6x - 2x + 12$

$= x^2 - 8x + 12$ ✓

On Your Own

Now You're Ready
Exercises 10–15

Factor the polynomial.

5. $w^2 - 4w + 3$ **6.** $n^2 - 12n + 35$ **7.** $x^2 - 14x + 24$

Key Idea

Factoring $x^2 + bx + c$ When c Is Negative

Algebra $x^2 + bx + c = (x + p)(x + q)$ when $p + q = b$ and $pq = c$.

When c is negative, p and q have different signs.

Example $x^2 - 4x - 5 = (x + 1)(x - 5)$

EXAMPLE 3 Factoring $x^2 + bx + c$ When c Is Negative

Factor $x^2 + 4x - 21$.

Notice that $b = 4$ and $c = -21$. Because c is negative, the factors p and q must have different signs so that pq is negative.

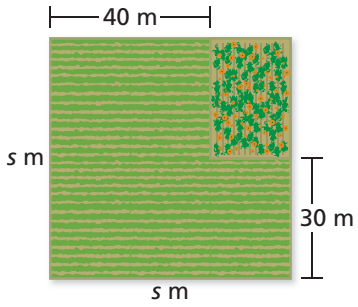
Find two integer factors of -21 whose sum is 4.

Factors of -21	$-21, 1$	$-1, 21$	$-7, 3$	$-3, 7$
Sum of Factors	-20	20	-4	4

The values of p and q are -3 and 7.

∴ So, $x^2 + 4x - 21 = (x - 3)(x + 7)$.

EXAMPLE 4 Real-Life Application



A farmer plants a rectangular pumpkin patch in the northeast corner of the square plot of land. The area of the pumpkin patch is 600 square meters. What is the area of the square plot of land?

The length of the pumpkin patch is $(s - 30)$ meters and the width is $(s - 40)$ meters. Write and solve an equation for its area.

$600 = (s - 30)(s - 40)$ Write an equation.

$600 = s^2 - 70s + 1200$ Multiply.

$0 = s^2 - 70s + 600$ Subtract 600 from each side.

$0 = (s - 10)(s - 60)$ Factor the polynomial.

$s - 10 = 0$ *or* $s - 60 = 0$ Use Zero-Product Property.

$s = 10$ *or* $s = 60$ Solve for s.

The diagram shows that the side length is at least 30 meters, so 10 meters does not make sense in this situation. The width is 60 meters.

∴ So, the area of the square plot of land is $60(60) = 3600$ square meters.

On Your Own

Now You're Ready
Exercises 21–29

Factor the polynomial.

8. $x^2 + 2x - 15$ **9.** $y^2 + 13y - 30$ **10.** $v^2 + v - 20$

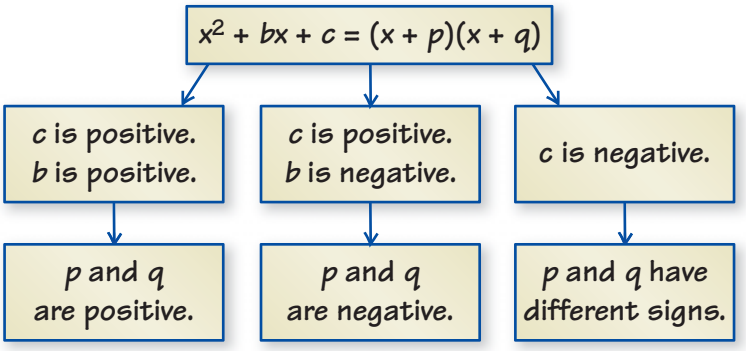
11. $z^2 - z - 12$ **12.** $m^2 - 11m - 26$ **13.** $x^2 - 3x - 40$

14. WHAT IF? In Example 4, the area of the pumpkin patch is 200 square meters. What is the area of the square plot of land?

Summary

Factoring $x^2 + bx + c$ as $(x + p)(x + q)$

The diagram shows the relationships between the signs of b and c and the signs of p and q.



7.7 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **WRITING** You are factoring $x^2 + 11x - 26$. What do the signs of the terms tell you about the factors? Explain.
2. **OPEN-ENDED** Write a trinomial that can be factored as $(x + p)(x + q)$ where p and q are positive.

Practice and Problem Solving

Factor the polynomial.

1 **3.** $x^2 + 8x + 7$ **4.** $z^2 + 7z + 12$ **5.** $n^2 + 8n + 12$

6. $s^2 + 11s + 30$ **7.** $h^2 + 11h + 18$ **8.** $y^2 + 13y + 40$

9. ERROR ANALYSIS Describe and correct the error in factoring the polynomial.

$t^2 + 14t + 48 = (t + 4)(t + 12)$

Factor the polynomial.

2 **10.** $v^2 - 5v + 4$ **11.** $x^2 - 9x + 20$ **12.** $d^2 - 5d + 6$

13. $k^2 - 10k + 24$ **14.** $w^2 - 17w + 72$ **15.** $j^2 - 13j + 42$

Solve the equation.

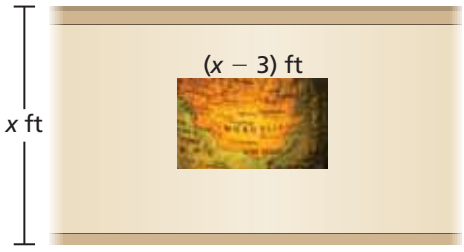
16. $m^2 + 3m + 2 = 0$ **17.** $x^2 + 11x + 28 = 0$ **18.** $n^2 - 9n + 18 = 0$

19. PROFIT A company's profit (in millions of dollars) can be represented by $x^2 - 6x + 8$, where x is the number of years since the company started. When did the company have a profit of \$3 million?

20. PROJECTION A projector displays an image on a wall. The area (in square feet) of the rectangular projection can be represented by $x^2 - 8x + 15$.

a. Write a binomial that represents the height of the projection.

b. Find the perimeter of the projection when the height of the wall is 8 feet.



Factor the polynomial.

21. $x^2 + 3x - 4$ **22.** $z^2 + 7z - 18$ **23.** $n^2 + 4n - 12$

24. $s^2 + 3s - 40$ **25.** $h^2 + 6h - 27$ **26.** $y^2 + 2y - 48$

27. $m^2 - 6m - 7$ **28.** $x^2 - x - 20$ **29.** $t^2 - 6t - 16$

Solve the equation.

30. $v^2 + 3v - 4 = 0$ **31.** $x^2 + 5x - 14 = 0$ **32.** $n^2 - 5n = 24$

33. ERROR ANALYSIS Describe and correct the error in solving the equation.

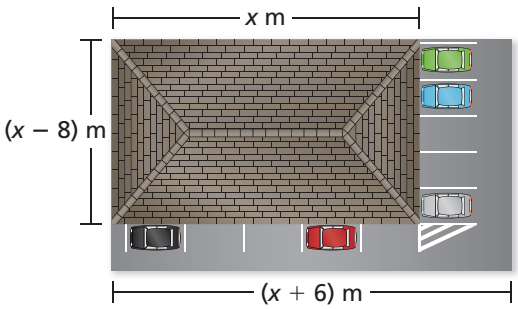
$$x^2 - 2x - 15 = 20$$
$$(x - 5)(x + 3) = 20$$
$$x - 5 = 20 \quad or \quad x + 3 = 20$$
$$x = 25 \quad or \quad x = 17$$

34. DENTIST A dentist's office and parking lot are on a rectangular piece of land. The area (in square meters) of the land can be represented by $x^2 + x - 30$.

a. Write a binomial that represents the width of the land.

b. Write an expression that represents the area of the parking lot.

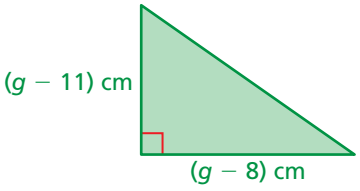
c. Evaluate the expressions in parts (a) and (b) when $x = 20$.



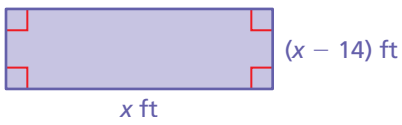
Find the dimensions of the polygon with the given area.

35. Area = 44 square feet

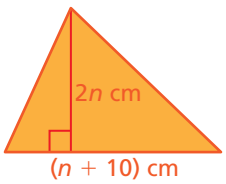
36. Area = 35 square centimeters



37. Area = 120 square feet



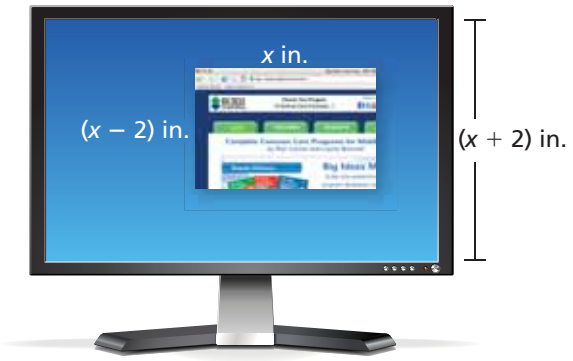
38. Area = 75 square centimeters

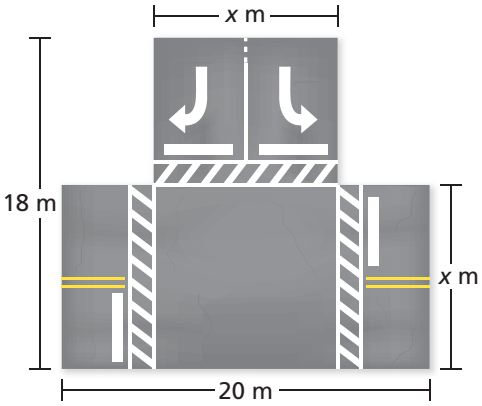


39. COMPUTER A web browser is open on your computer screen.

a. The area of the browser is 24 square inches. Find the value of x.

b. The browser covers $\frac{3}{13}$ of the screen. What are the dimensions of the screen?





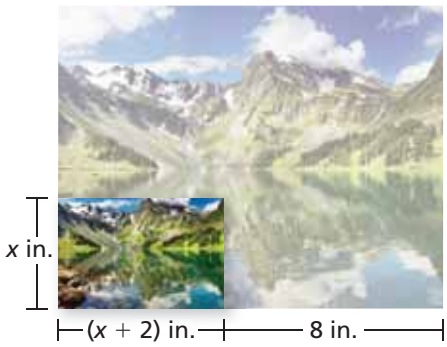
40. LOGIC Road construction workers are paving the area shown.

a. Write an expression that represents the area being paved.

b. The area being paved is 280 square meters. Write and solve an equation to find x.

c. The equation in part (b) has two solutions. Explain why one of the solutions is not reasonable.

41. PHOTOGRAPHY You enlarge a photograph on a computer. The area (in square inches) of the enlarged photograph can be represented by $x^2 + 17x + 70$.



a. Write binomials that represent the length and width of the enlarged photograph.

b. How many inches greater is the length of the enlarged photograph than the width? Explain.

c. The area of the enlarged photograph is 154 square inches. Find the dimensions of each photograph.

42. Number Sense Find all of the integer values of b for which the trinomial $x^2 + bx - 12$ has two binomial factors of the form $(x + p)$ and $(x + q)$.

Fair Game Review What you learned in previous grades & lessons

Factor the polynomial. *(Section 7.6)*

43. $2y - 18$

44. $7n^2 + 23n$

45. $8z^3 + 28z^2$

46. MULTIPLE CHOICE Which expression is *not* equivalent to $\sqrt{\frac{9}{4}}$? *(Section 6.1)*

Ⓐ $\frac{3}{2}$ Ⓑ $\sqrt{2.25}$ Ⓒ $2\sqrt{3}$ Ⓓ $3\sqrt{\frac{1}{4}}$