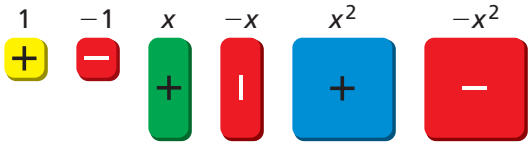
7.6 Factoring Polynomials Using the GCF

Essential Question How can you use common factors to write a polynomial in factored form?

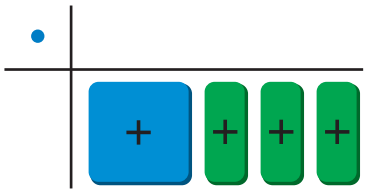
1 ACTIVITY: Finding Monomial Factors

Work with a partner. Six different algebra tiles are shown below.

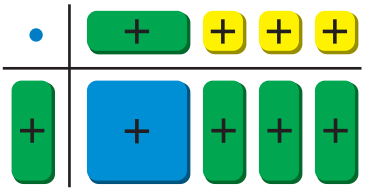


Sample:

Step 1: Look at the rectangular array for $x^2 + 3x$.



Step 2: Use algebra tiles to label the dimensions of the rectangle.



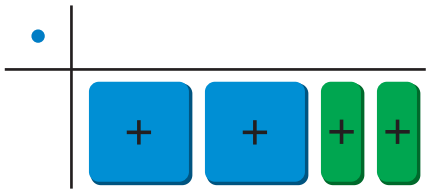
Step 3: Write the polynomial in factored form by finding the dimensions of the rectangle.

width, length

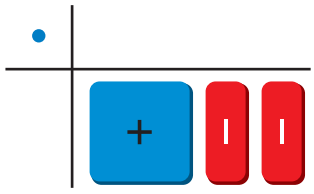
$$\text{Area} = x^2 + 3x = x(x + 3)$$

Use algebra tiles to write each polynomial in factored form.

a.

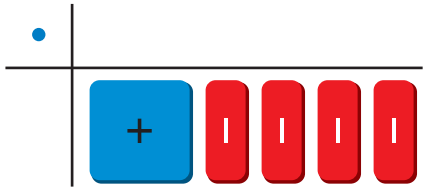


b.



c.

d.



Common Core

Polynomial Equations

In this lesson, you will

- factor polynomials using the greatest common factor.
- solve polynomial equations by factoring.

Learning Standards
A.REI.4b
A.SSE.3a

2 ACTIVITY: Finding Monomial Factors

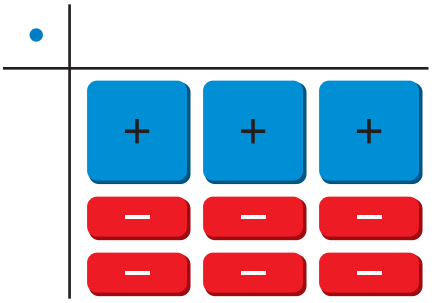
Math Practice 4

Interpret Results

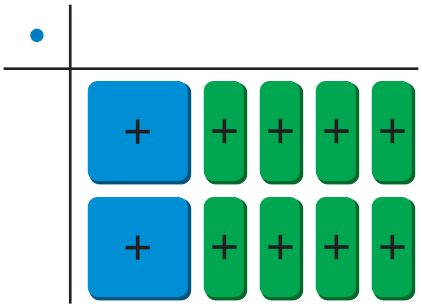
What does your answer represent? How can you make sure your answer makes sense?

Work with a partner. Use algebra tiles to write each polynomial in factored form.

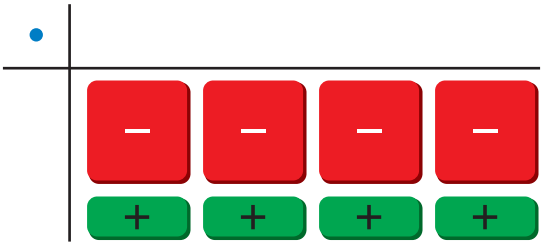
a.



b.



c.



3 ACTIVITY: Finding Monomial Factors

Work with a partner. Use algebra tiles to model each polynomial as a rectangular array. Then write the polynomial in factored form by finding the dimensions of the rectangle.

a. $3x^2 - 9x$ **b.** $7x + 14x^2$ **c.** $-2x^2 + 6x$

What Is Your Answer?

4. Consider the polynomial $4x^2 + 8x$.

a. What are the terms of the polynomial?

b. List all the factors that are common to both terms.

c. Of the common factors, which is the greatest? Explain your reasoning.

5. **IN YOUR OWN WORDS** How can you use common factors to write a polynomial in factored form?

Practice Use what you learned about factoring polynomials to complete Exercises 3–5 on page 366.

7.6 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Writing a polynomial as a product of factors is called *factoring*. When the terms of a polynomial have a common factor, you can factor the polynomial as shown below.

Key Idea

Factoring Polynomials Using the GCF

Step 1: Find the greatest common factor (GCF) of the terms.

Step 2: Use the Distributive Property to write the polynomial as a product of the GCF and its remaining factors.

EXAMPLE 1 Factoring Polynomials

Factor each polynomial.

a. $2x^2 + 18$

Step 1: Find the GCF of the terms.

$$2x^2 = 2 \cdot x \cdot x$$
$$18 = 2 \cdot 3 \cdot 3$$

The GCF is 2.

Step 2: Write the polynomial as a product of the GCF and its remaining factors.

$$2x^2 + 18 = 2(x^2) + 2(9) \quad \text{Factor out GCF.}$$
$$= 2(x^2 + 9) \quad \text{Distributive Property}$$

Study Tip

When you factor a polynomial, you *undo* the multiplication of its factors.

b. $15y^3 + 10y^2$

Step 1: Find the GCF of the terms.

$$15y^3 = 3 \cdot 5 \cdot y \cdot y \cdot y$$
$$10y^2 = 2 \cdot 5 \cdot y \cdot y$$

The GCF is $5 \cdot y \cdot y = 5y^2$.

Step 2: Write the polynomial as a product of the GCF and its remaining factors.

$$15y^3 + 10y^2 = 5y^2(3y) + 5y^2(2) \quad \text{Factor out GCF.}$$
$$= 5y^2(3y + 2) \quad \text{Distributive Property}$$

On Your Own

Now You're Ready
Exercises 6–11

Factor the polynomial.

1. $5z^2 + 30$ **2.** $3x^2 + 14x$ **3.** $8y^2 - 24y$

To solve an equation using the Zero-Product Property, you may need to first collect the terms on one side of the equation and then factor.

EXAMPLE 2 Solving an Equation by Factoring

Solve $4g^2 = -6g$.

$4g^2 = -6g$ — Write equation.

$4g^2 + 6g = 0$ — Add $6g$ to each side.

$2g(2g + 3) = 0$ — Factor the polynomial.

$2g = 0$ *or* $2g + 3 = 0$ — Use Zero-Product Property.

$g = 0$ *or* $g = -\frac{3}{2}$ — Solve for g.

∴ The solutions are $g = 0$ and $g = -\frac{3}{2}$.

On Your Own

Now You're Ready
Exercises 14–22

Solve the equation.

4. $3x^2 + 21x = 0$ **5.** $5z^2 = 5z$ **6.** $18y = 6y^2$

EXAMPLE 3 Real-Life Application

A female athlete tests her vertical jump by jumping straight into the air. Her height y (in feet) after t seconds can be modeled by $y = -16t^2 + 12t$. How many seconds is she in the air?

She is on the ground when $y = 0$. So, substitute 0 for y and solve for t.

$y = -16t^2 + 12t$ — Write equation.

$0 = -16t^2 + 12t$ — Substitute 0 for y.

$0 = 4t(-4t + 3)$ — Factor the polynomial.

$4t = 0$ *or* $-4t + 3 = 0$ — Use Zero-Product Property.

$t = 0$ *or* $t = 0.75$ — Solve for t.

She starts the jump at $t = 0$ and lands when $t = 0.75$.

∴ So, she is in the air for 0.75 second.

On Your Own

7. WHAT IF? The height of a male athlete testing his vertical jump can be modeled by $y = -16t^2 + 14t$. How many seconds is he in the air?

7.6 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **REASONING** What is the greatest common factor of $12y$ and $30y^2$?
2. **WRITING** Describe how to factor a polynomial using the greatest common factor.

Practice and Problem Solving

Use algebra tiles to factor the polynomial.

3. $4x + 8$
4. $2x^2 + 4x$
5. $x^2 - 4x$

Factor the polynomial.

1 6. $5z^2 + 45z$
7. $8m^2 + 4m$
8. $3y^3 - 9y^2$
9. $20x^3 + 30x^2$
10. $4w^3 - 8w + 12$
11. $5t^2 + 20t + 50$

12. **ERROR ANALYSIS** Describe and correct the error in factoring the polynomial.

$$2x^2 + 2x = 2(x^2) + 2(x)$$
$$= 2(x^2 + x)$$

13. **INTEREST** You deposit \$100 in a savings account that earns simple interest. The balance of the account can be represented by $100 + 100rt$, where r is the annual interest rate and t is the time in years. Factor the polynomial.

Solve the equation.

2 14. $2q + 10 = 0$
15. $10x + 15 = 0$
16. $4p^2 - p = 0$
17. $6m^2 + 12m = 0$
18. $3n^2 = 9n$
19. $4r^2 = -28r$
20. $4a^3 = 44a^2$
21. $6k^3 + 39k^2 = 0$
22. $2y^2 = 2\pi y$

23. **ERROR ANALYSIS** Describe and correct the error in solving the equation.

$$3x^2 = 15x$$
$$3x^2 - 15x = 0$$
$$3x(x - 15) = 0$$
$$3x = 0 \text{ or } x - 15 = 0$$
$$x = 0 \text{ or } x = 15$$

The roots are $x = 0$ and $x = 15$.

24. **AGES** Your brother is y years old. Your older cousins are $2y^2$ and $6y$ years old. The difference between your cousins' ages is zero. Your brother is older than 1 year old. How old is he?

Solve the equation.

25. $5b^2 - 20b = b^2$

26. $5n^2 + 40n = 5n$

27. $2s^3 + 15s^2 = 3s^2$

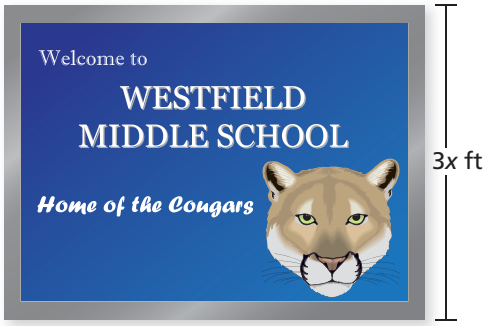
28. $8g^3 - 2g^2 = 2g^3 - 5g^2$

29. **OPEN-ENDED** Write a binomial whose terms have a GCF of $3x$.

30. **SCHOOL SIGN** The area (in square feet) of the school sign can be represented by $15x^2 - 6x$.

a. Write an expression that represents the length of the sign.

b. Describe two ways to find the area of the sign when $x = 2$.



31. **DOLPHIN** A dolphin jumps straight into the air during a performance. The dolphin's height y (in feet) after t seconds can be modeled by $y = -16t^2 + 24t$.

a. How many seconds is the dolphin in the air?

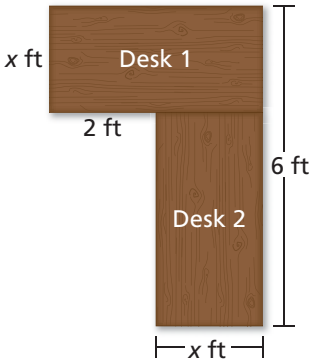
b. The dolphin reaches its maximum height after 0.75 second. What is the maximum height of the jump?

32. Modeling Your teacher's work station is made up of two identical desks arranged as shown.

a. Write an equation in terms of x that relates the area of Desk 1 to the area of Desk 2.

b. What is the value of x?

c. Find the area of the top of your teacher's work station.



Fair Game Review What you learned in previous grades & lessons

Find the product. *(Section 7.3)*

33. $(y + 4)(y + 6)$

34. $(m - 2)(m - 9)$

35. $(2k + 1)(2k - 3)$

36. **MULTIPLE CHOICE** An African elephant weighs 5,200,000 grams. Write this number in scientific notation. *(Skills Review Handbook)*

A 0.52×10^{-7} g

B 5.2×10^{-6} g

C 52×10^{5} g

D 5.2×10^{6} g