

Essential Question How can you solve a polynomial equation that is written in factored form?

Two polynomial equations are equivalent when they have the same solutions. For instance, the following equations are equivalent because the only solutions of each equation are $x = 1$ and $x = 2$.

Factored Form

$$(x - 1)(x - 2) = 0$$

Standard Form

$$x^2 - 3x + 2 = 0$$

Nonstandard Form

$$x^2 - 3x = -2$$

✓ Check this by substituting 1 and 2 for x in each equation.

1 ACTIVITY: Matching Equivalent Forms of an Equation

Work with a partner. Match each factored form of the equation with two other forms of equivalent equations. Notice that an equation is considered to be in factored form only when the product of the factors is equal to 0.

Factored Form

a. $(x - 1)(x - 3) = 0$

b. $(x - 2)(x - 3) = 0$

c. $(x + 1)(x - 2) = 0$

d. $(x - 1)(x + 2) = 0$

e. $(x + 1)(x - 3) = 0$

Standard Form

A. $x^2 - x - 2 = 0$

B. $x^2 + x - 2 = 0$

C. $x^2 - 4x + 3 = 0$

D. $x^2 - 5x + 6 = 0$

E. $x^2 - 2x - 3 = 0$

Nonstandard Form

1. $x^2 - 5x = -6$

2. $(x - 1)^2 = 4$

3. $x^2 - x = 2$

4. $x(x + 1) = 2$

5. $x^2 - 4x = -3$

2 ACTIVITY: Writing a Conjecture

Work with a partner. Substitute

1, 2, 3, 4, 5, and 6 for x

in each equation. Write a conjecture describing what you discovered.

a. $(x - 1)(x - 2) = 0$

b. $(x - 2)(x - 3) = 0$

c. $(x - 3)(x - 4) = 0$

d. $(x - 4)(x - 5) = 0$

e. $(x - 5)(x - 6) = 0$

f. $(x - 6)(x - 1) = 0$



COMMON
CORE

Polynomial Equations

In this lesson, you will

- solve polynomial equations in factored form.

Learning Standard
A.REI.4b

3 ACTIVITY: Special Properties of 0 and 1

Work with a partner. The numbers 0 and 1 have special properties that are shared by no other numbers. For each of the following, decide whether the property is true for 0, 1, both, or neither. Explain your reasoning.

- If you add \square to a number n , you get n .
- If the product of two numbers is \square , then one or both numbers are 0.
- The square of \square is equal to itself.
- If you multiply a number n by \square , you get n .
- If you multiply a number n by \square , you get 0.
- The opposite of \square is equal to itself.

4 ACTIVITY: Writing About Solving Equations

Math Practice 3

Use Definitions

What previous examples, information, and definitions can you use to reply to the student's comment?

Work with a partner. Imagine that you are part of a study group in your algebra class. One of the students in the group makes the following comment.

“I don't see why we spend so much time solving equations that are equal to zero. Why don't we spend more time solving equations that are equal to other numbers?”

Write an answer for this student.



What Is Your Answer?

- One of the properties in Activity 3 is called the Zero-Product Property. It is one of the most important properties in all of algebra. Which property is it? Explain how it is used in algebra and why it is so important.
- IN YOUR OWN WORDS** How can you solve a polynomial equation that is written in factored form?

Practice

Use what you learned about solving polynomial equations to complete Exercises 4–6 on page 360.

Key Vocabulary

factored form, p. 358
Zero-Product Property,
p. 358
root, p. 358

A polynomial is in **factored form** when it is written as a product of factors.

Standard form

$$x^2 + 2x$$

$$x^2 + 5x - 24$$

Factored form

$$x(x + 2)$$

$$(x - 3)(x + 8)$$

When one side of an equation is a polynomial in factored form and the other side is 0, use the **Zero-Product Property** to solve the polynomial equation. The solutions of a polynomial equation are also called **roots**.

Key Idea
Zero-Product Property

Words If the product of two real numbers is 0, then at least one of the numbers is 0.

Algebra If a and b are real numbers and $ab = 0$, then $a = 0$ or $b = 0$.

EXAMPLE 1 Solving Polynomial Equations

Solve each equation.

a. $x(x + 8) = 0$

$$x(x + 8) = 0$$

Write equation.

$$x = 0 \quad \text{or} \quad x + 8 = 0$$

Use Zero-Product Property.

$$x = -8$$

Solve for x .

∴ The roots are $x = 0$ and $x = -8$.

b. $(x + 6)(x - 5) = 0$

$$(x + 6)(x - 5) = 0$$

Write equation.

$$x + 6 = 0 \quad \text{or} \quad x - 5 = 0$$

Use Zero-Product Property.

$$x = -6 \quad \text{or} \quad x = 5$$

Solve for x .

∴ The roots are $x = -6$ and $x = 5$.

Check

Substitute each solution in the original equation.

$$0(0 + 8) \stackrel{?}{=} 0$$

$$0(8) \stackrel{?}{=} 0$$

$$0 = 0 \quad \checkmark$$

$$-8(-8 + 8) \stackrel{?}{=} 0$$

$$-8(0) \stackrel{?}{=} 0$$

$$0 = 0 \quad \checkmark$$

On Your Own

Solve the equation.

1. $x(x - 1) = 0$

2. $3t(t + 2) = 0$

3. $(z - 4)(z - 6) = 0$

4. $(b + 7)^2 = 0$

Now You're Ready
Exercises 4–9

EXAMPLE 2 Solving a Polynomial Equation

What are the solutions of $(2a + 7)(2a - 7) = 0$?

- (A) -7 and 7 (B) $-\frac{7}{2}$ and $\frac{7}{2}$
(C) -2 and 2 (D) $-\frac{2}{7}$ and $\frac{2}{7}$

$$(2a + 7)(2a - 7) = 0$$

Write equation.

$$2a + 7 = 0 \quad \text{or} \quad 2a - 7 = 0$$

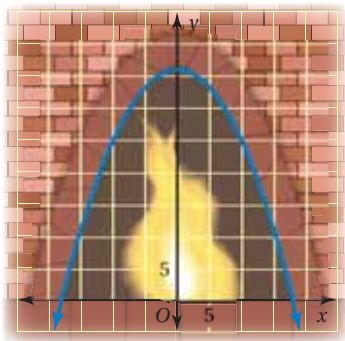
Use Zero-Product Property.

$$a = -\frac{7}{2} \quad \text{or} \quad a = \frac{7}{2}$$

Solve for a .

∴ The correct answer is (B).

EXAMPLE 3 Real-Life Application



The arch of a fireplace can be modeled by $y = -\frac{1}{9}(x + 18)(x - 18)$, where x and y are measured in inches. The x -axis represents the floor. Find the width of the arch at floor level.

Use the x -coordinates at floor level to find the width. At floor level, $y = 0$. So, substitute 0 for y and solve for x .

$$y = -\frac{1}{9}(x + 18)(x - 18)$$

Write equation.

$$0 = -\frac{1}{9}(x + 18)(x - 18)$$

Substitute 0 for y .

$$0 = (x + 18)(x - 18)$$

Multiply each side by -9 .

$$x + 18 = 0 \quad \text{or} \quad x - 18 = 0$$

Use Zero-Product Property.

$$x = -18 \quad \text{or} \quad x = 18$$

Solve for x .

The width is the distance between the x -coordinates, -18 and 18 .

∴ So, the width of the arch at floor level is $18 - (-18) = 36$ inches.

On Your Own

Solve the equation.

5. $(3p + 5)(3p - 5) = 0$

6. $(12 - 6x)^2 = 0$

7. The entrance to a mine shaft can be modeled by $y = -\frac{1}{2}(x + 4)(x - 4)$, where x and y are measured in feet. The x -axis represents the ground. Find the width of the entrance at ground level.

Now You're Ready
Exercises 10–15

Vocabulary and Concept Check

- REASONING** Is $x = 3$ a solution of $(x - 3)(x + 6) = 0$? Explain.
- WRITING** Describe how to solve $(x - 2)(x + 1) = 0$ using the Zero-Product Property.
- WHICH ONE DOESN'T BELONG?** Which statement does *not* belong with the other three? Explain your reasoning.

$$(n - 9)(n + 3)$$

$$(2k + 5)(k - 3)$$

$$(g + 2)^2$$

$$x^2 + 4x$$

Practice and Problem Solving

Solve the equation.

- | | | | | |
|---|---|--------------------------|--|--|
| 1 | 2 | 4. $x(x + 7) = 0$ | 5. $12t(t - 5) = 0$ | 6. $(s - 9)(s - 1) = 0$ |
| | | 7. $(q + 3)(q - 2) = 0$ | 8. $(h - 8)^2 = 0$ | 9. $(m + 4)^2 = 0$ |
| | | 10. $(5 - k)(5 + k) = 0$ | 11. $(3 - g)(7 - g) = 0$ | 12. $(3p + 6)^2 = 0$ |
| | | 13. $(4z - 12)^2 = 0$ | 14. $\left(\frac{1}{2}y + 4\right)(y - 8) = 0$ | 15. $\left(\frac{1}{3}d - 2\right)\left(\frac{1}{3}d + 2\right) = 0$ |

16. **ERROR ANALYSIS** Describe and correct the error in solving the equation.

X

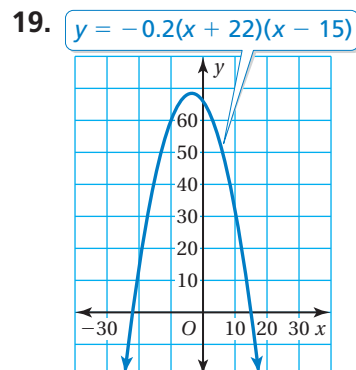
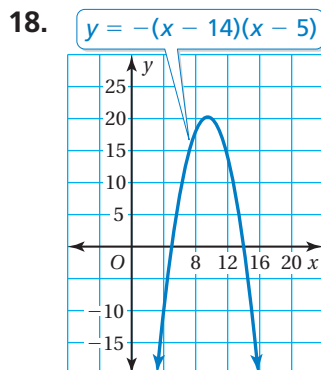
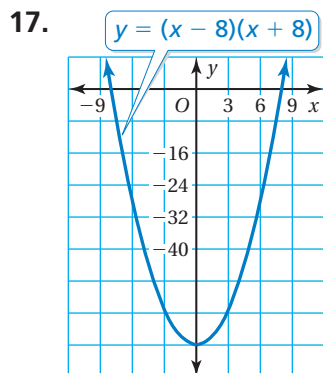
$$6x(x + 5) = 0$$

$$x + 5 = 0$$

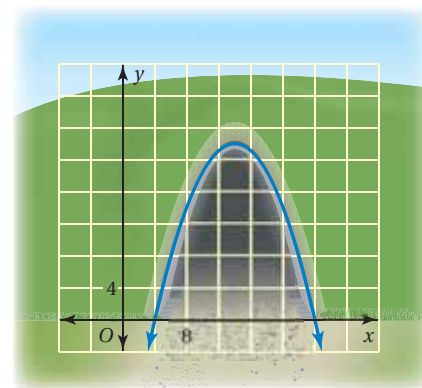
$$x = -5$$

The root is $x = -5$.

Find the x -coordinates of the points where the graph crosses the x -axis.

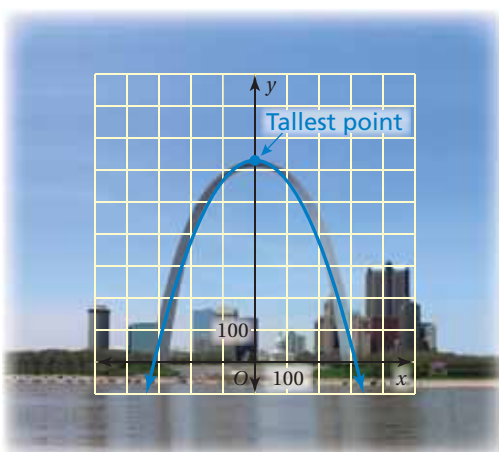


20. **CHOOSE TOOLS** The entrance of a tunnel can be modeled by $y = -\frac{11}{50}(x - 4)(x - 24)$, where x and y are measured in feet. The x -axis represents the ground. Find the width of the tunnel at ground level.



Solve the equation.

21. $5z(z + 2)(z - 1) = 0$
22. $w(w - 6)^2 = 0$
23. $(r - 4)(r + 4)(r + 8) = 0$
24. $(2p + 3)(2p - 3)(p + 7) = 0$



25. **GATEWAY ARCH** The Gateway Arch in St. Louis can be modeled by $y = -\frac{2}{315}(x + 315)(x - 315)$, where x and y are measured in feet. The x -axis represents the ground.
- Find the width of the arch at ground level.
 - How tall is the arch?

26. **Algebra** Find the values of x in terms of y that are solutions of the equation.
- $(x + y)(2x - y) = 0$
 - $(x^2 - y^2)(4x + 16y) = 0$



Fair Game Review What you learned in previous grades & lessons

Find the greatest common factor of the numbers. (*Skills Review Handbook*)

27. 21 and 63
28. 12 and 27
29. 30, 75, and 90

30. **MULTIPLE CHOICE** What is the slope of the line?
(*Section 2.2*)

- (A) -3
- (B) $-\frac{1}{3}$
- (C) $\frac{1}{3}$
- (D) 3

