Essential Question What are the patterns in the special products $(a + b)(a - b)$, $(a + b)^2$, and $(a - b)^2$?

1 **ACTIVITY: Finding a Sum and Difference Pattern**
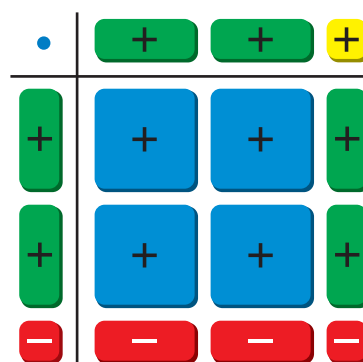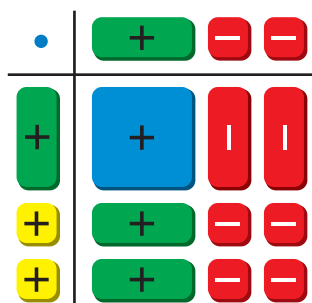
Work with a partner. Six different algebra tiles are shown below.

$$1 \quad -1 \quad x \quad -x \quad x^2 \quad -x^2$$
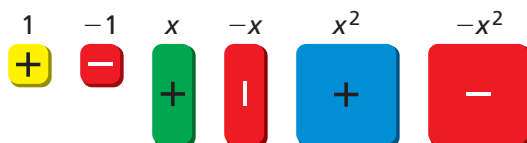
Write the product of the two binomials shown by the algebra tiles.

a. $(x + 2)(x - 2) = $ _____

b. $(2x - 1)(2x + 1) = $ _____

2 **ACTIVITY: Describing a Sum and Difference Pattern**

Work with a partner.

a. Describe the pattern for the special product: $(a + b)(a - b)$.

b. Use the pattern you described to find each product. Check your answers using algebra tiles.

i. $(x + 3)(x - 3)$ **ii.** $(x - 4)(x + 4)$ **iii.** $(3x + 1)(3x - 1)$

iv. $(3y + 4)(3y - 4)$ **v.** $(2x - 5)(2x + 5)$ **vi.** $(z + 1)(z - 1)$

COMMON CORE

Polynomials
In this lesson, you will
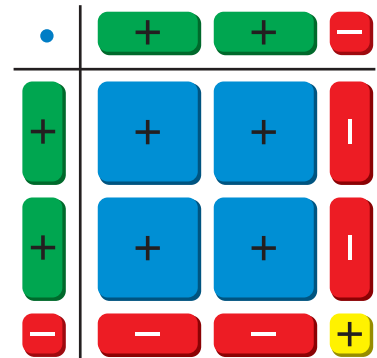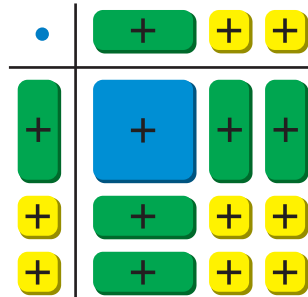• use patterns to multiply polynomials.
Learning Standard
A.APR.1

3 ACTIVITY: Finding the Square of a Binomial Pattern

Write the product of the two binomials shown by the algebra tiles.

a. $(x + 2)^2 = $ [____]

b. $(2x - 1)^2 = $ [____]

4 ACTIVITY: Describing the Square of a Binomial Pattern

Math Practice 8

Find General Methods

What did the products of the binomials in the previous activity have in common? How does this help in describing the pattern?

Work with a partner.

a. Describe the pattern for the special product: $(a + b)^2$.

b. Describe the pattern for the special product: $(a - b)^2$.

c. Use the patterns you described to find each product. Check your answers using algebra tiles.

    i. $(x + 3)^2$
      ii. $(x - 2)^2$
      iii. $(3x + 1)^2$

    iv. $(3y + 4)^2$
      v. $(2x - 5)^2$
      vi. $(z + 1)^2$

What Is Your Answer?

5. **IN YOUR OWN WORDS** What are the patterns in the special products $(a + b)(a - b)$, $(a + b)^2$, and $(a - b)^2$? Use the results of Activities 2 and 4 to write formulas for these special products.

Practice ▶ Use what you learned about the patterns in special products to complete Exercises 3–5 on page 352.

Some pairs of binomials show patterns when multiplied. You can use these patterns to multiply other similar pairs of binomials.

Key Idea

Study Tip

Because multiplication is commutative, the pattern also applies to $(a - b)(a + b)$.

Sum and Difference Pattern

Algebra

$$(a + b)(a - b) = a^2 - b^2$$

Example

$$(x + 3)(x - 3) = x^2 - 3^2$$
$$= x^2 - 9$$

EXAMPLE 1 Using the Sum and Difference Pattern

Find each product.

a. $(x + 7)(x - 7)$

$$(a + b)(a - b) = a^2 - b^2 \qquad \text{Sum and Difference Pattern}$$
$$(x + 7)(x - 7) = x^2 - 7^2 \qquad \text{Use pattern.}$$
$$= x^2 - 49 \qquad \text{Simplify.}$$

Check

Use the FOIL Method.

$(3x - 1)(3x + 1)$

$= 9x^2 + 3x - 3x - 1$

$= 9x^2 - 1$ ✓

b. $(3x - 1)(3x + 1)$

$$(a - b)(a + b) = a^2 - b^2 \qquad \text{Sum and Difference Pattern}$$
$$(3x - 1)(3x + 1) = (3x)^2 - 1^2 \qquad \text{Use pattern.}$$
$$= 9x^2 - 1 \qquad \text{Simplify.}$$

On Your Own

Now You're Ready
Exercises 6–14

Find the product.

1. $(x - 4)(x + 4)$ **2.** $(b + 10)(b - 10)$ **3.** $(2g + 5)(2g - 5)$

Key Idea

Square of a Binomial Pattern

Algebra

$$(a + b)^2 = a^2 + 2ab + b^2$$

Example

$$(x + 3)^2 = x^2 + 2(x)(3) + 3^2$$
$$= x^2 + 6x + 9$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(x - 3)^2 = x^2 - 2(x)(3) + 3^2$$
$$= x^2 - 6x + 9$$

EXAMPLE **2** Using the Square of a Binomial Pattern

Find each product.

a. $(y + 1)^2$

$$(a + b)^2 = a^2 + 2ab + b^2 \qquad \text{Square of a Binomial Pattern}$$
$$(y + 1)^2 = y^2 + 2(y)(1) + 1^2 \qquad \text{Use pattern.}$$
$$= y^2 + 2y + 1 \qquad \text{Simplify.}$$

b. $(2z - 3)^2$

$$(a - b)^2 = a^2 - 2ab + b^2 \qquad \text{Square of a Binomial Pattern}$$
$$(2z - 3)^2 = (2z)^2 - 2(2z)(3) + 3^2 \qquad \text{Use pattern.}$$
$$= 4z^2 - 12z + 9 \qquad \text{Simplify.}$$

Check

Use the FOIL Method.

$$(2z - 3)^2 = (2z - 3)(2z - 3)$$
$$= 4z^2 - 6z - 6z + 9$$
$$= 4z^2 - 12z + 9 \checkmark$$

EXAMPLE **3** Real-Life Application

Each of two dogs has one black gene (B) and one white gene (W). The diagram shows the possible gene combinations of an offspring and the resulting colors.
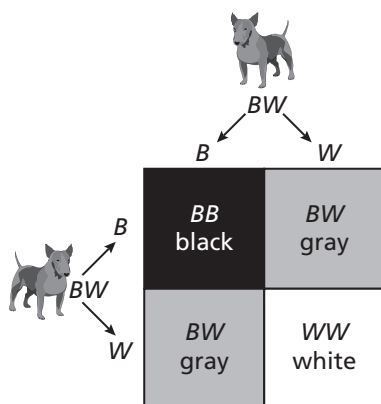
A diagram that models possible gene combinations in offspring is called a Punnett square.

a. What percent of the possible gene combinations result in black?

Use the diagram. One of the four possible gene combinations results in black.

So, $\frac{1}{4}$ or 25% of the possible gene combinations result in black.

b. The genetic makeup of an offspring can be modeled by $(0.5B + 0.5W)^2$. Use the square of a binomial pattern to model the possible gene combinations of an offspring.

$$(a + b)^2 = a^2 + 2ab + b^2 \qquad \text{Square of a Binomial Pattern}$$

$$(0.5B + 0.5W)^2 = (0.5B)^2 + 2(0.5B)(0.5W) + (0.5W)^2 \qquad \text{Use pattern.}$$

$$= 0.25B^2 + 0.5BW + 0.25W^2 \qquad \text{Simplify.}$$

25% BB (black)   50% BW (gray)   25% WW (white)

🔴 **On Your Own**

Now You're Ready
Exercises 16–24

Find the product.

4. $(w + 2)^2$ **5.** $(x - 7)^2$ **6.** $(3y - 1)^2$ **7.** $(5z + 4)^2$

Vocabulary and Concept Check

1. **OPEN-ENDED** Write two binomials whose product can be found using the sum and difference pattern.

2. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$$(x + 1)(x - 1) \qquad (3x + 2)(3x - 2) \qquad (x + 2)(x - 3) \qquad (2x + 5)(2x - 5)$$

Practice and Problem Solving

Use algebra tiles to find the product.

3. $(x + 6)(x - 6)$

4. $(3y - 2)(3y + 2)$

5. $(2z + 2)^2$

Find the product.

① 6. $(x + 2)(x - 2)$

7. $(g - 5)(g + 5)$

8. $(z - 8)(z + 8)$

9. $(b + 12)(b - 12)$

10. $(2x + 1)(2x - 1)$

11. $(3x - 4)(3x + 4)$

12. $(6x + 7)(6x - 7)$

13. $(9 - c)(9 + c)$

14. $(8 - 3m)(8 + 3m)$

15. **REASONING** Write two binomials whose product is $x^2 - 16$. Explain how you found your answer.

Find the product.

② 16. $(b - 2)^2$

17. $(y + 8)^2$

18. $(n + 6)^2$

19. $(d - 10)^2$

20. $(2f - 1)^2$

21. $(5p + 2)^2$

22. $(4b - 5)^2$

23. $(12 - x)^2$

24. $(4 + 7t)^2$

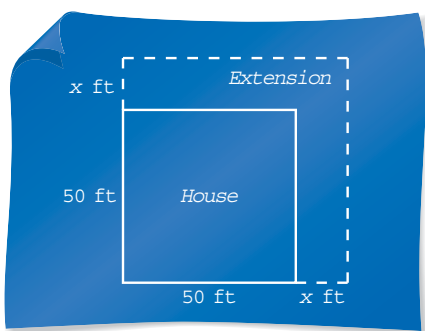**ERROR ANALYSIS** Describe and correct the error in finding the product.

25.
✗
$(k + 4)^2 = k^2 + 4^2$
$= k^2 + 16$

26.
✗
$(s + 5)(s - 5)$
$= s^2 + 2(s)(5) - 5^2$
$= s^2 + 10s - 25$

x ft — Extension

50 ft — House

50 ft   x ft

27. **CONSTRUCTION** A contractor extends a house on two sides.

a. The area of the first level of the house after the renovation is represented by $(x + 50)^2$. Find this product.

b. Use the polynomial in part (a) to find the area of the first level when $x = 15$. What is the area of the extension?

Write a polynomial that represents the area of the figure.
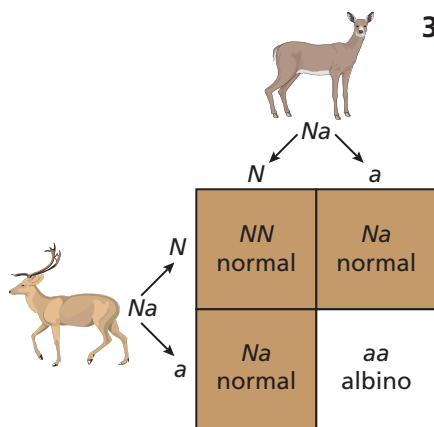
28.

29.
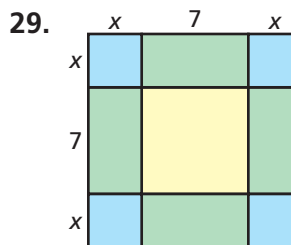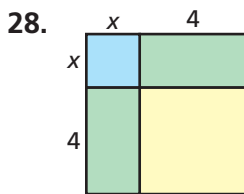
Find the product.

30. $(x^2 + 1)(x^2 - 1)$

31. $(x + y)(x - y)$

32. $(2x - y)^2$

33. **GENETICS** In deer, the gene N is for normal coloring and the gene a is for no coloring, or albino. Any gene combination with an N results in normal coloring. The diagram shows the possible gene combinations of an offspring and the resulting colors from parents that both have the gene combination Na.

    a. What percent of the possible gene combinations result in normal coloring?

    b. The genetic makeup of an offspring can be modeled by $(0.5N + 0.5a)^2$. Use the square of a binomial pattern to model the possible gene combinations of an offspring.

34. **VISION** Your iris controls the amount of light that enters your eye by changing the size of your pupil.

    a. Write a polynomial that represents the area of your pupil. Write your answer in terms of π.

    b. The width x of your iris decreases from 4 millimeters to 2 millimeters when you enter a dark room. How many times greater is the area of your pupil after entering the room than before entering the room? Explain.
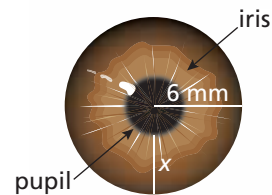
35. **Repeated Reasoning** Find $(x + 1)^3$ and $(x + 2)^3$. Find a pattern in the terms and use it to write a pattern for the cube of a binomial $(a + b)^3$.

Fair Game Review What you learned in previous grades & lessons

Find the product. *(Section 7.3)*

36. $(x + 4)(x + 9)$

37. $(y - 7)(y + 3)$

38. $(z - 10)(z - 1)$

39. **MULTIPLE CHOICE** What is the solution of the linear system? *(Section 4.2)*

    (A) $(-3, -1)$ (B) $(-3, 1)$

    (C) $(3, -1)$ (D) $(3, 1)$

$$y = 2x - 5$$
$$3x - 8y = 1$$