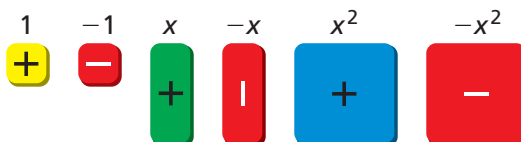


7.2 Adding and Subtracting Polynomials

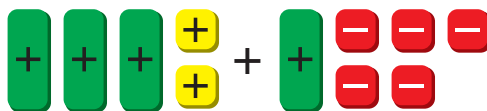
Essential Question How can you add polynomials? How can you subtract polynomials?


1 EXAMPLE: Adding Polynomials Using Algebra Tiles

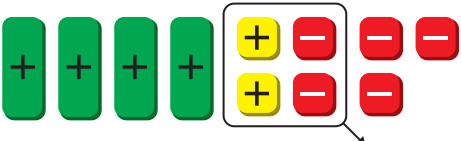
Work with a partner. Six different algebra tiles are shown at the right.




Write the polynomial addition steps shown by the algebra tiles.



Step 1:  Group like tiles.

Step 2:  Remove zero pairs.

Step 3:  Simplify.

2 ACTIVITY: Adding Polynomials Using Algebra Tiles

Use algebra tiles to find the sum of the polynomials.

- | | |
|---------------------------------------|---------------------------------------|
| a. $(x^2 + 2x - 1) + (2x^2 - 2x + 1)$ | b. $(4x + 3) + (x - 2)$ |
| c. $(x^2 + 2) + (3x^2 + 2x + 5)$ | d. $(2x^2 - 3x) + (x^2 - 2x + 4)$ |
| e. $(x^2 - 3x + 2) + (x^2 + 4x - 1)$ | f. $(4x - 3) + (2x + 1) + (-3x + 2)$ |
| g. $(-x^2 + 3x) + (2x^2 - 2x)$ | h. $(x^2 + 2x - 5) + (-x^2 - 2x + 5)$ |



Polynomials

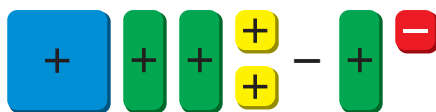
In this lesson, you will

- add and subtract polynomials.

Learning Standard
A.APR.1

3 EXAMPLE: Subtracting Polynomials Using Algebra Tiles

Write the polynomial subtraction steps shown by the algebra tiles.



Step 1:



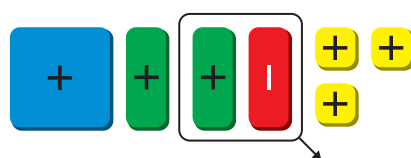
To subtract, add the opposite.

Step 2:



Group like tiles.

Step 3:



Remove zero pairs.

Step 4:



Simplify.

Math Practice 7

View as Components

How can you use algebra tiles to represent the sums and differences of polynomials?

4 ACTIVITY: Subtracting Polynomials Using Algebra Tiles

Use algebra tiles to find the difference of the polynomials.

- a. $(x^2 + 2x - 1) - (2x^2 - 2x + 1)$ b. $(4x + 3) - (x - 2)$
 c. $(x^2 + 2) - (3x^2 + 2x + 5)$ d. $(2x^2 - 3x) - (x^2 - 2x + 4)$

What Is Your Answer?

5. **IN YOUR OWN WORDS** How can you add polynomials? Use the results of Activity 2 to summarize a procedure for adding polynomials without using algebra tiles.
6. **IN YOUR OWN WORDS** How can you subtract polynomials? Use the results of Activity 4 to summarize a procedure for subtracting polynomials without using algebra tiles.

Practice

Use what you learned about adding and subtracting polynomials to complete Exercises 3 and 4 on page 338.

You can add polynomials using a vertical or horizontal method to combine like terms.

EXAMPLE 1 Adding Polynomials

Find each sum.

a. $(3a^2 + 8) + (5a - 1)$

b. $(-x^2 + 5x + 4) + (3x^2 - 8x + 9)$

a. **Vertical method:** Align like terms vertically and add.

$$\begin{array}{r} 3a^2 \quad + 8 \\ + \quad 5a - 1 \\ \hline 3a^2 + 5a + 7 \end{array}$$

Leave a space for the missing term.

b. **Horizontal method:** Group like terms and simplify.

$$\begin{aligned} (-x^2 + 5x + 4) + (3x^2 - 8x + 9) &= (-x^2 + 3x^2) + [5x + (-8x)] + (4 + 9) \\ &= 2x^2 - 3x + 13 \end{aligned}$$

To subtract one polynomial from another polynomial, add the opposite.

EXAMPLE 2 Subtracting Polynomials

Find each difference.

a. $(y^2 + 4y + 2) - (2y^2 - 5y - 3)$

b. $(5x^2 + 4x - 1) - (2x^2 - 6)$

a. Use the vertical method.

$$\begin{array}{r} (y^2 + 4y + 2) \\ - (2y^2 - 5y - 3) \\ \hline \end{array} \quad \begin{array}{l} \text{Add the opposite.} \\ + \quad -2y^2 + 5y + 3 \\ \hline -y^2 + 9y + 5 \end{array}$$

b. Use the horizontal method.

$$\begin{aligned} (5x^2 + 4x - 1) - (2x^2 - 6) &= (5x^2 + 4x - 1) + (-2x^2 + 6) \\ &= [5x^2 + (-2x^2)] + 4x + (-1 + 6) \\ &= 3x^2 + 4x + 5 \end{aligned}$$

Study Tip

You can think of finding the opposite of a polynomial as finding the opposite of each term's coefficient.

Now You're Ready

Exercises 5–10 and 12–17

On Your Own

Find the sum or difference.

1. $(b - 10) + (4b - 3)$

2. $(x^2 - x - 2) + (7x^2 - x)$

3. $(p^2 + p + 3) - (-4p^2 - p + 3)$

4. $(-k + 5) - (3k^2 - 6)$

EXAMPLE 3 Adding Polynomials

Which polynomial represents the sum of $x^2 - 2xy - y^2$ and $x^2 + xy + y^2$?

- (A) $-3xy$ (B) $-3xy - 2y^2$ (C) $2x^2 - xy$ (D) $2x^2 + 3xy + 2y^2$

Use the horizontal method to find the sum.

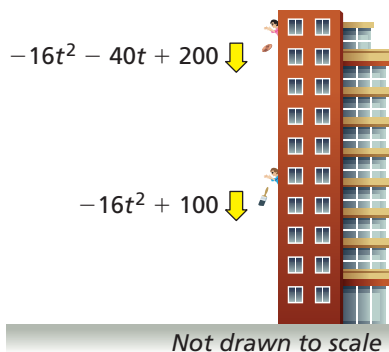
$$\begin{aligned}(x^2 - 2xy - y^2) + (x^2 + xy + y^2) &= (x^2 + x^2) + (-2xy + xy) + (-y^2 + y^2) \\ &= 2x^2 - xy\end{aligned}$$

∴ The correct answer is (C).

EXAMPLE 4 Real-Life Application

A penny is thrown straight downward from a height of 200 feet. At the same time, a paintbrush falls from a height of 100 feet. The polynomials represent the heights (in feet) of the objects after t seconds.

- a. Write a polynomial that represents the distance between the penny and the paintbrush after t seconds.



To find the distance between the objects after t seconds, subtract the polynomials.

<i>Penny</i>	<i>Paintbrush</i>
$(-16t^2 - 40t + 200)$	$(-16t^2 + 100)$
$(-16t^2 - 40t + 200) - (-16t^2 + 100)$	
$= (-16t^2 - 40t + 200) + (16t^2 - 100)$	
$= (-16t^2 + 16t^2) - 40t + [200 + (-100)]$	
$= -40t + 100$	

∴ The polynomial $-40t + 100$ represents the distance between the objects after t seconds.

- b. What is the distance between the objects after 2 seconds?

Find the value of $-40t + 100$ when $t = 2$.

$$\begin{aligned}-40t + 100 &= -40(2) + 100 && \text{Substitute 2 for } t. \\ &= 20 && \text{Simplify.}\end{aligned}$$

∴ After 2 seconds, the distance between the objects is 20 feet.

Study Tip

To check your answer, substitute 2 into the original polynomials and verify that the difference of the heights is 20.

On Your Own

- In Example 3, which polynomial represents the difference of the two polynomials?
- In Example 4, the polynomial $-16t^2 - 25t + 200$ represents the height of the penny after t seconds. What is the distance between the objects after 1 second?

Now You're Ready
Exercises 20–23


Vocabulary and Concept Check

- WRITING** How do you add $(4x^2 - 3 + 2y^3)$ and $(-6x^2 - 15)$ using a vertical method? a horizontal method?
- REASONING** Describe how subtracting polynomials is similar to subtracting integers.


Practice and Problem Solving

Use algebra tiles to find the sum or difference of the polynomials.

3. $(x^2 - 3x + 2) + (x^2 + 4x - 1)$

4. $(x^2 + 2x - 5) - (-x^2 - 2x + 5)$

Find the sum.

1 5. $(5y + 4) + (-2y + 6)$

6. $(3g^2 - g) + (3g^2 - 8g + 4)$


7. $(2n^2 - 5n - 6) + (-n^2 - 3n + 11)$

8. $(-3p^2 + 5p - 2) + (-p^2 - 8p - 15)$

9. $(-a^3 + 4a - 3) + (5a^3 - a)$

10. $(-s^2 - \frac{2}{9}s + 1) + (-\frac{5}{9}s - 4)$

11. **ERROR ANALYSIS** Describe and correct the error in finding the sum of the polynomials.



$$\begin{array}{r} -5x^2 + 1 \\ + 2x - 8 \\ \hline -3x - 7 \end{array}$$

Find the difference.

2 12. $(d^2 - 9) - (3d - 1)$

13. $(k^2 - 7k + 2) - (k^2 - 12)$


14. $(x^2 - 4x + 9) - (3x^2 - 6x - 7)$

15. $(-r - 10) - (-4r^2 + r + 7)$

16. $(t^4 - t^2 + t) - (-9t^2 + 7t - 12)$

17. $(\frac{1}{6}q^2 + \frac{2}{3}) - (\frac{1}{12}q^2 - \frac{1}{3})$

18. **ERROR ANALYSIS** Describe and correct the error in finding the difference of the polynomials.



$$\begin{aligned} (x^2 - 5x) - (-3x^2 + 2x) &= (x^2 - 5x) + (3x^2 + 2x) \\ &= (x^2 + 3x^2) + (-5x + 2x) \\ &= 4x^2 - 3x \end{aligned}$$



19. **COST** The cost (in dollars) of making b bracelets is represented by $4 + 5b$. The cost (in dollars) of making b necklaces is $8b + 6$. Write a polynomial that represents how much more it costs to make b necklaces than b bracelets.

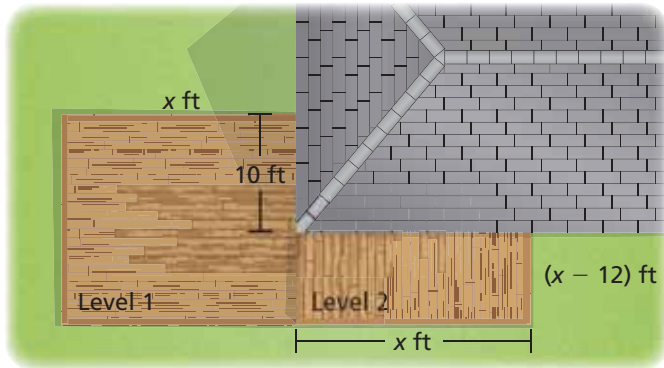
Find the sum or difference.

20. $(c^2 - 6d^2) + (c^2 - 2cd + 2d^2)$

22. $(2s^2 - 5st - t^2) - (s^2 + 7st - t^2)$

21. $(-x^2 + 9xy) - (x^2 + 6xy - 8y^2)$

23. $(a^2 - 3ab + 2b^2) + (-4a^2 + 5ab - b^2)$



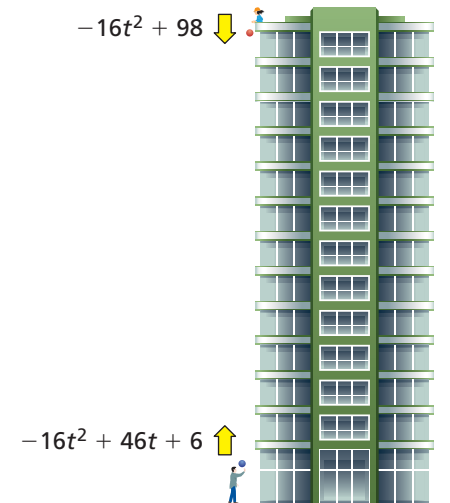
24. **MODELING** You are building a multi-level deck.

- Write a polynomial that represents the area of each level.
- Write a polynomial that represents the total area of the deck.
- What is the total area of the deck when $x = 20$?
- A gallon of deck sealant covers 400 square feet. How many gallons of sealant do you need to cover the deck once? Explain.

25. **Problem Solving** You drop a ball from a height of 98 feet.

At the same time, your friend throws a ball upward. The polynomials represent the heights (in feet) of the balls after t seconds.

- Write a polynomial that represents the distance between your ball and your friend's ball after t seconds.
- What is the distance between the balls after 1.5 seconds?
- After how many seconds are the balls at the same height? How far are they from the ground? Explain your reasoning.



Not drawn to scale



Fair Game Review What you learned in previous grades & lessons

Simplify the expression. (*Skills Review Handbook*)

26. $2(x - 1) + 3(x + 2)$

27. $(4y - 3) - 2(y - 5)$

28. $-5(2w + 1) - 3(-4w + 2)$

29. **MULTIPLE CHOICE** Which inequality is represented by the graph? (*Section 3.1*)



(A) $x < -2$

(B) $x > -2$

(C) $x \leq -2$

(D) $x \geq -2$