Extension 6.7 Recursively Defined Sequences

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Key Vocabulary
recursive rule, *p. 312*

In Sections 5.6 and 6.7, you wrote *explicit* equations for sequences. Now, you will write *recursive* equations for sequences. A **recursive rule** gives the beginning term(s) of a sequence and an equation that indicates how any term a_n in the sequence relates to the previous term.

Key Idea

Recursive Equation for an Arithmetic Sequence

$a_n = a_{n-1} + d$, where d is the common difference.

Recursive Equation for a Geometric Sequence

$a_n = r \cdot a_{n-1}$, where r is the common ratio.

EXAMPLE 1 Writing Terms of Recursively Defined Sequences

Write the first six terms of each sequence. Then graph each sequence.

a. $a_1 = 2, a_n = a_{n-1} + 3$

$a_1 = 2$

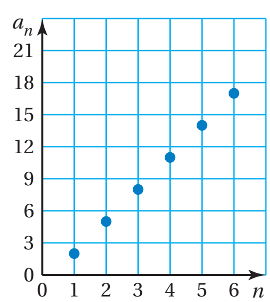
$a_2 = a_1 + 3 = 2 + 3 = 5$

$a_3 = a_2 + 3 = 5 + 3 = 8$

$a_4 = a_3 + 3 = 8 + 3 = 11$

$a_5 = a_4 + 3 = 11 + 3 = 14$

$a_6 = a_5 + 3 = 14 + 3 = 17$



b. $a_1 = 1, a_n = 3a_{n-1}$

$a_1 = 1$

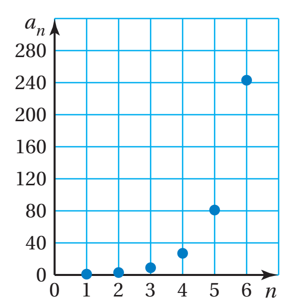
$a_2 = 3a_1 = 3(1) = 3$

$a_3 = 3a_2 = 3(3) = 9$

$a_4 = 3a_3 = 3(9) = 27$

$a_5 = 3a_4 = 3(27) = 81$

$a_6 = 3a_5 = 3(81) = 243$



Practice

Write the first six terms of the sequence. Then graph the sequence.

1. $a_1 = 0, a_n = a_{n-1} - 8$

2. $a_1 = -7.5, a_n = a_{n-1} + 2.5$

3. $a_1 = -36, a_n = \frac{1}{2}a_{n-1}$

4. $a_1 = 0.7, a_n = 10a_{n-1}$

EXAMPLE 2 Writing Recursive Rules

COMMON CORE

Recursive Sequences

In this extension, you will

- write the terms of recursively defined sequences.
- write recursive equations for sequences.

Learning Standards
F.BF.2
F.IF.3
F.LE.2

Write a recursive rule for each sequence.

a. $-30, -18, -6, 6, 18, \ldots$

Use a table to organize the terms and find the pattern.

Position	1	2	3	4	5
Term	-30	-18	-6	6	18

$+12 \quad +12 \quad +12 \quad +12$

The sequence is arithmetic with first term -30 and common difference 12.

$a_n = a_{n-1} + d$ — Recursive equation (arithmetic)

$a_n = a_{n-1} + 12$ — Substitute 12 for d.

So, a recursive rule for the sequence is $a_1 = -30$, $a_n = a_{n-1} + 12$.

b. $500, 100, 20, 4, 0.8, \ldots$

Use a table to organize the terms and find the pattern.

Position	1	2	3	4	5
Term	500	100	20	4	0.8

$\times \frac{1}{5} \quad \times \frac{1}{5} \quad \times \frac{1}{5} \quad \times \frac{1}{5}$

The sequence is geometric with first term 500 and common ratio $\frac{1}{5}$.

$a_n = r \cdot a_{n-1}$ — Recursive equation (geometric)

$a_n = \frac{1}{5} a_{n-1}$ — Substitute $\frac{1}{5}$ for r.

So, a recursive rule for the sequence is $a_1 = 500$, $a_n = \frac{1}{5} a_{n-1}$.

Practice

Write a recursive rule for the sequence.

5. $8, 3, -2, -7, -12, \ldots$

6. $1.3, 2.6, 3.9, 5.2, 6.5, \ldots$

7. $4, 20, 100, 500, 2500, \ldots$

8. $1600, -400, 100, -25, 6.25, \ldots$

9. **SUNFLOWERS** Write a recursive rule for the height of the sunflower over time.

1 month: 2 feet
2 months: 3.5 feet
3 months: 5 feet
4 months: 6.5 feet

EXAMPLE 3 Translating Recursive Rules into Explicit Equations

Write an explicit equation for each recursive rule.

a. $a_1 = 25, a_n = a_{n-1} - 10$

The recursive rule represents an arithmetic sequence with first term 25 and common difference −10.

$a_n = a_1 + (n - 1)d$ — Equation for an arithmetic sequence

$a_n = 25 + (n - 1)(-10)$ — Substitute 25 for a_1 and −10 for d.

$a_n = -10n + 35$ — Simplify.

b. $a_1 = 19.6, a_n = -0.5a_{n-1}$

The recursive rule represents a geometric sequence with first term 19.6 and common ratio −0.5.

$a_n = a_1 r^{n-1}$ — Equation for a geometric sequence

$a_n = 19.6(-0.5)^{n-1}$ — Substitute 19.6 for a_1 and −0.5 for r.

EXAMPLE 4 Translating Explicit Equations into Recursive Rules

Write a recursive rule for each explicit equation.

a. $a_n = -2n + 3$

The explicit equation represents an arithmetic sequence with first term $-2(1) + 3 = 1$ and common difference −2.

$a_n = a_{n-1} + d$ — Recursive equation (arithmetic)

$a_n = a_{n-1} + (-2)$ — Substitute −2 for d.

So, a recursive rule for the sequence is $a_1 = 1, a_n = a_{n-1} - 2$.

b. $a_n = -3(2)^{n-1}$

The explicit equation represents a geometric sequence with first term −3 and common ratio 2.

$a_n = r \cdot a_{n-1}$ — Recursive equation (geometric)

$a_n = 2a_{n-1}$ — Substitute 2 for r.

So, a recursive rule for the sequence is $a_1 = -3, a_n = 2a_{n-1}$.

Practice

Write an explicit equation for the recursive rule.

10. $a_1 = -45, a_n = a_{n-1} + 20$

11. $a_1 = 13, a_n = -3a_{n-1}$

Write a recursive rule for the explicit equation.

12. $a_n = -n + 1$

13. $a_n = -2.5(2)^{n-1}$

You can write recursive rules for sequences that are neither arithmetic nor geometric. One way is to look for patterns in the sums of consecutive terms.

EXAMPLE 5 Writing Recursive Rules for Other Sequences

Write a recursive rule for the sequence 1, 1, 2, 3, 5, 8, ... Then write the next 3 terms of the sequence.

The sequence in Example 5 is called the *Fibonacci sequence.* This pattern is naturally occurring in many objects, such as flowers.

The sequence does not have a common difference or a common ratio. Find the sums of consecutive terms.

$a_1 + a_2 = 1 + 1 = 2$ 2 is the third term.

$a_2 + a_3 = 1 + 2 = 3$ 3 is the fourth term.

$a_3 + a_4 = 2 + 3 = 5$ 5 is the fifth term.

$a_4 + a_5 = 3 + 5 = 8$ 8 is the sixth term.

So, a recursive equation for the sequence is $a_n = a_{n-2} + a_{n-1}$. Use the equation to find the next three terms.

$a_7 = a_5 + a_6 = 5 + 8 = 13$

$a_8 = a_6 + a_7 = 8 + 13 = 21$

$a_9 = a_7 + a_8 = 13 + 21 = 34$

A recursive rule for the sequence is $a_1 = 1$, $a_2 = 1$, $a_n = a_{n-2} + a_{n-1}$. The next three terms are 13, 21, and 34.

Practice

Write a recursive rule for the sequence. Then write the next 3 terms of the sequence.

14. 5, 6, 11, 17, 28, . . .

15. −3, −4, −7, −11, −18, . . .

16. 1, 1, 0, −1, −1, 0, 1, 1, . . .

17. 4, 3, 1, 2, −1, 3, −4, . . .

Use a pattern in the products of consecutive terms to write a recursive rule for the sequence. Then write the next 2 terms of the sequence.

18. 2, 3, 6, 18, 108, . . .

19. −2, 2.5, −5, −12.5, 62.5, . . .

20. GEOMETRY Consider squares 1–6 in the diagram.

a. Write a sequence in which each term a_n is the side length of square n.

b. What is the name of this sequence? What is the next term of this sequence?

c. Use the term in part (b) to add another square to the diagram and extend the spiral.