

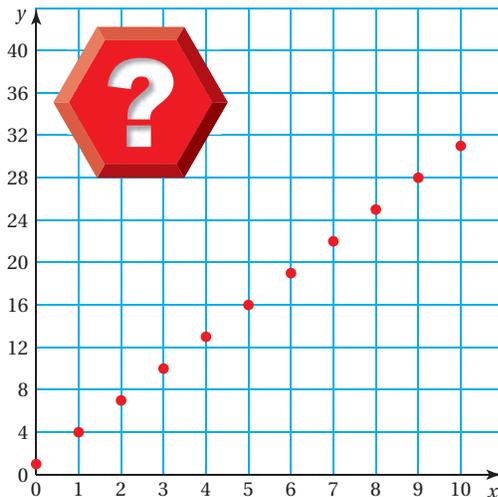
6.5 Exponential Growth

Essential Question What are the characteristics of exponential growth?

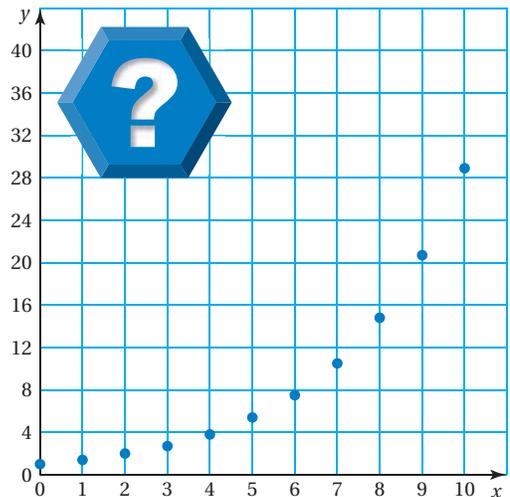
1 ACTIVITY: Comparing Types of Growth

Work with a partner. Describe the pattern of growth for each sequence and graph. How many of the patterns represent exponential growth? Explain your reasoning.

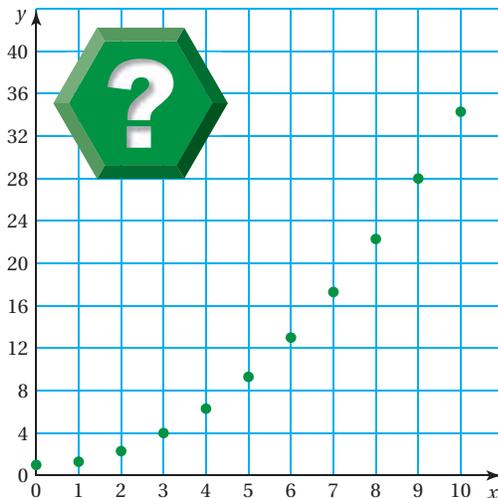
- a. 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31



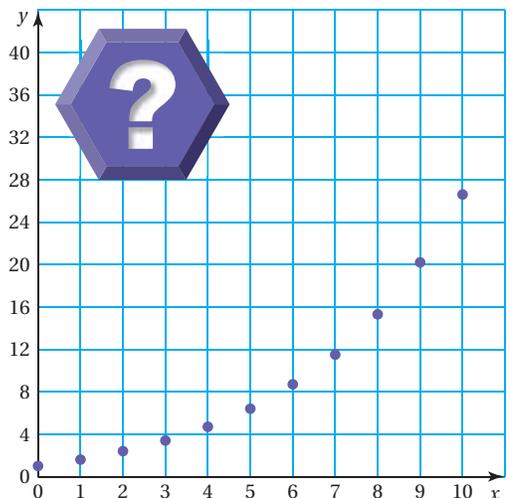
- b. 1.0, 1.4, 2.0, 2.7, 3.8, 5.4, 7.5, 10.5, 14.8, 20.7, 28.9



- c. 1.0, 1.3, 2.3, 4.0, 6.3, 9.3, 13.0, 17.3, 22.3, 28.0, 34.3



- d. 1.0, 1.6, 2.4, 3.4, 4.7, 6.4, 8.7, 11.5, 15.3, 20.2, 26.6



COMMON CORE

Exponential Functions

In this lesson, you will

- write, interpret, and graph exponential growth functions.

Learning Standards

A.SSE.1a

A.SSE.1b

F.IF.7e

2 ACTIVITY: Predicting a Future Event

Math Practice 1

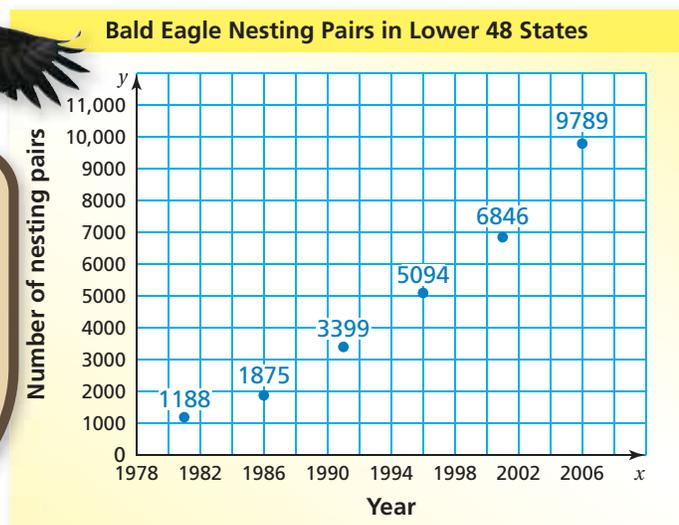
Consider Similar Problems

How can you use the results from the previous activity to help you solve this problem?

Work with a partner. It is estimated that in 1782 there were about 100,000 nesting pairs of bald eagles in the United States. By the 1960s, this number had dropped to about 500 nesting pairs. This decline was attributed to loss of habitat, loss of prey, hunting, and the use of the pesticide DDT.

The 1940 Bald Eagle Protection Act prohibited the trapping and killing of the birds. In 1967, the bald eagle was declared an endangered species in the United States. With protection, the nesting pair population began to increase, as shown in the graph. Finally, in 2007, the bald eagle was removed from the list of endangered and threatened species.

Describe the growth pattern shown in the graph. Is it exponential growth? Assume the pattern continues. When will the population return to the levels of the late 1700s? Explain your reasoning.



What Is Your Answer?

- IN YOUR OWN WORDS** What are the characteristics of exponential growth? How can you distinguish exponential growth from other growth patterns?
- Which of the following are examples of exponential growth? Explain.
 - Growth of the balance of a savings account
 - Speed of the moon in orbit around Earth
 - Height of a ball that is dropped from a height of 100 feet

Practice

Use what you learned about exponential growth to complete Exercises 3 and 4 on page 298.

Key Vocabulary

exponential growth,
p. 296
exponential growth
function, p. 296
compound interest,
p. 297

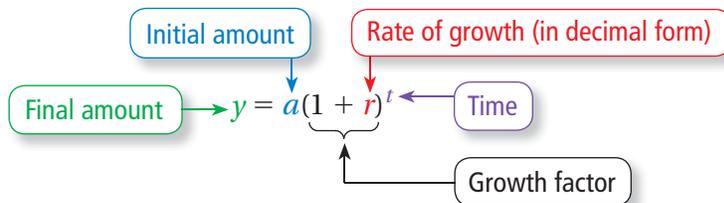
Study Tip

Notice that an exponential growth function is of the form $y = ab^x$, where b is replaced by $1 + r$ and x is replaced by t .

Exponential growth occurs when a quantity increases by the same factor over equal intervals of time.

Key Idea
Exponential Growth Functions

A function of the form $y = a(1 + r)^t$, where $a > 0$ and $r > 0$, is an **exponential growth function**.

**EXAMPLE 1 Using an Exponential Growth Function**

The function $y = 150,000(1.1)^t$ represents the attendance y at a music festival t years after 2010.

- a. By what percent does the festival attendance increase each year?
Use the growth factor $1 + r$ to find the rate of growth.

$$1 + r = 1.1$$

Write an equation.

$$r = 0.1$$

Subtract 1 from each side.

So, the festival attendance increases by 10% each year.

- b. How many people will attend the festival in 2014? Round your answer to the nearest ten thousand.

The value $t = 4$ represents 2014.

$$y = 150,000(1.1)^t$$

Write exponential growth function.

$$= 150,000(1.1)^4$$

Substitute 4 for t .

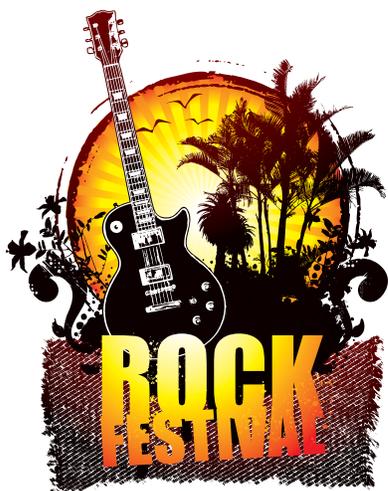
$$= 219,615$$

Use a calculator.

About 220,000 people will attend the festival in 2014.

On Your Own

- The function $y = 500,000(1.15)^t$ represents the number y of members of a website t years after 2010.
 - By what percent does the website membership increase each year?
 - How many members will there be in 2016? Round your answer to the nearest hundred thousand.



Now You're Ready
Exercises 5–10

Key Idea

Compound Interest

Compound interest is interest earned on the principal *and* on previously earned interest. The balance y of an account earning compound interest is

$$y = P \left(1 + \frac{r}{n} \right)^{nt}$$

P = principal (initial amount)
 r = annual interest rate (in decimal form)
 t = time (in years)
 n = number of times interest is compounded per year

EXAMPLE 2 Writing a Function

Study Tip

For interest compounded yearly, you can substitute 1 for n in the formula to get $y = P(1 + r)^t$.

You deposit \$100 in a savings account that earns 5% annual interest compounded yearly. Write a function for the balance after t years.

$$y = P \left(1 + \frac{r}{n} \right)^{nt} \quad \text{Write compound interest formula.}$$

$$y = 100 \left(1 + \frac{0.05}{1} \right)^{(1)t} \quad \text{Substitute 100 for } P, 0.05 \text{ for } r, \text{ and 1 for } n.$$

$$y = 100(1.05)^t \quad \text{Simplify.}$$

EXAMPLE 3 Real-Life Application

The table shows the balance of a money market account over time.

Year, t	Balance
0	\$100
1	\$110
2	\$121
3	\$133.10
4	\$146.41
5	\$161.05

a. Write a function for the balance after t years.

From the table, you know the balance increases 10% each year.

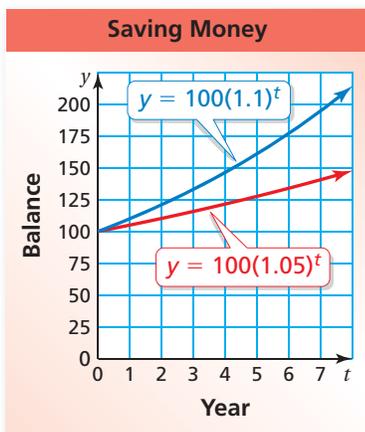
$$y = a(1 + r)^t \quad \text{Write exponential growth function.}$$

$$y = 100(1 + 0.1)^t \quad \text{Substitute 100 for } a \text{ and } 0.1 \text{ for } r.$$

$$y = 100(1.1)^t \quad \text{Simplify.}$$

b. Graph the functions from part (a) and Example 2 in the same coordinate plane. Compare the account balances.

The money market account earns 10% interest each year and the savings account earns 5% interest each year. So, the balance of the money market account increases faster.



On Your Own

- You deposit \$500 in a savings account that earns 4% annual interest compounded yearly. Write and graph a function that represents the balance y (in dollars) after t years.


Vocabulary and Concept Check

- VOCABULARY** When does the exponential function $y = a(1 + r)^t$ represent an exponential growth function?
- VOCABULARY** The population of a city grows by 3% each year. What is the growth factor?


Practice and Problem Solving

Describe the pattern of growth for the sequence.

- 1.0, 1.2, 1.4, 1.7, 2.1, 2.5, 3.0, 3.6,
- 4.3, 5.2, 6.2
- 1, 7, 13, 19, 25, 31, 37, 43,
49, 55, 61

Identify the initial amount a and the rate of growth r (as a percent) of the exponential function. Evaluate the function when $t = 5$. Round your answer to the nearest tenth.

5. $y = 25(1.2)^t$
6. $f(t) = 12(1.05)^t$
7. $d(t) = 1500(1.074)^t$
8. $y = 175(1.028)^t$
9. $g(t) = 6.7(2)^t$
10. $h(t) = 1.8^t$

Write and graph a function that represents the situation.

11. You deposit \$800 in an account that earns 7% annual interest compounded yearly.
12. Your \$35,000 annual salary increases by 4% each year.
13. A population of 210,000 increases by 12.5% each year.
14. Sales of \$10,000 increase by 70% each year.
- ERROR ANALYSIS** The growth rate of a bacteria culture is 150% each hour. Initially, there are 10 bacteria. Describe and correct the error in finding the number of bacteria in the culture after 8 hours.

X $b(t) = 10(1.5)^t$
 $b(8) = 10(1.5)^8 \approx 256.3$
 After 8 hours, there are about 256 bacteria in the culture.



- INVESTMENT** The function $y = 7500(1.08)^t$ represents the value y of an investment after t years.
 - What is the initial investment?
 - What is the value of the investment after 6 years?
- POPULATION** The population of a city has been increasing by 2% annually. In 2000, the population was 315,000. Predict the population of the city in 2020. Round your answer to the nearest thousand.

Write a function that represents the situation. Find the balance in the account after the given time period.

18. \$2000 deposit that earns 5% annual interest compounded quarterly; 5 years
19. \$6200 deposit that earns 8.4% annual interest compounded monthly; 18 months
20. **NUMBER SENSE** During a flu epidemic, the number of sick people triples every week. What is the growth rate as a percent? Explain your reasoning.
21. **SAVINGS** You deposit \$9000 in a savings account that earns 3.6% annual interest compounded monthly. You also save \$40 per month in a safe at home. Write a function $C(t) = b(t) + h(t)$, where $b(t)$ represents the balance of your savings account and $h(t)$ represents the amount in your safe after t years. What does $C(t)$ represent?
22. **REASONING** The number of concert tickets sold doubles every hour. After 12 hours, all of the tickets are sold. After how many hours are about one-fourth of the tickets sold? Explain your reasoning.
23. **YOU BE THE TEACHER** The balance of a savings account can be modeled by the function $b(t) = 5000(1.024)^t$, where t is the time in years. To model the monthly balance, a student writes

$$b(t) = 5000(1.024)^t = 5000(1.024)^{\left(\frac{1}{12} \cdot 12\right)^t} = 5000\left(1.024^{\frac{1}{12}}\right)^{12t} \approx 5000(1.002)^{12t}.$$

Is the student correct? Explain your reasoning.



24. **Critical Thinking** Gordon Moore stated that the number of transistors that can be placed on an integrated circuit will double every 2 years. This trend is known as Moore's Law. In 1978, the Intel® 8086 held 29,000 transistors on an integrated circuit.
 - a. Write a function that represents Moore's Law, where t is the number of years since 1978.
 - b. How many transistors could be placed on an integrated circuit in 2015?



Fair Game Review What you learned in previous grades & lessons

Simplify the expression. (Section 6.2)

25. $\left(\frac{2}{3}\right)^2$

26. $\left(\frac{1}{4}\right)^3$

27. $\left(\frac{3}{5}\right)^4$

28. **MULTIPLE CHOICE** The domain of the function $y = 4x - 3$ is 1, 4, 7, 10, and 13. Which number is *not* in the range of the function? (Section 5.1)

(A) 1

(B) 10

(C) 13

(D) 25