Extension 6.4 Solving Exponential Equations

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To solve an exponential equation of the form $b^x = b^y$ when $b > 0$ and $b \neq 1$, solve the equation $x = y$.

EXAMPLE 1 Solving Exponential Equations

a. Solve $5^x = 125$.

$5^x = 125$ Write the equation.

$5^x = 5^3$ Rewrite 125 as 5^3.

$x = 3$ Equate the exponents.

Check

$4^x = 2^{x-3}$

$4^{-3} \stackrel{?}{=} 2^{-3-3}$

$\frac{1}{4^3} \stackrel{?}{=} \frac{1}{2^6}$

$\frac{1}{64} = \frac{1}{64}$ ✓

b. Solve $4^x = 2^{x-3}$.

$4^x = 2^{x-3}$ Write the equation.

$(2^2)^x = 2^{x-3}$ Rewrite 4 as 2^2.

$2^{2x} = 2^{x-3}$ Power of a Power Property

$2x = x - 3$ Equate the exponents.

$x = -3$ Solve for x.

Check

$9^{x+2} = 27^x$

$9^{4+2} \stackrel{?}{=} 27^4$

$531{,}441 = 531{,}441$ ✓

c. Solve $9^{x+2} = 27^x$.

$9^{x+2} = 27^x$ Write the equation.

$(3^2)^{x+2} = (3^3)^x$ Rewrite 9 as 3^2 and 27 as 3^3.

$3^{2x+4} = 3^{3x}$ Power of a Power Property

$2x + 4 = 3x$ Equate the exponents.

$4 = x$ Solve for x.

Practice

Solve the equation. Check your solution, if possible.

1. $3^x = 81$
2. $2^x = 32$
3. $\frac{1}{16} = 4^x$
4. $10^x = 10^{x+1}$
5. $\left(\frac{1}{5}\right)^x = \left(\frac{1}{5}\right)^{3x}$
6. $6^{x-5} = 36^x$
7. $100^{5x+2} = 1000^{4x-1}$
8. $32^{1-x} = 8^{2x-2}$
9. $\left(\frac{1}{8}\right)^{x-5} = 4^x$

10. **NUMBER SENSE** Explain how you can use mental math to solve the equation $8^{x-4} = 1$.

11. **REASONING** Why does this method for solving $b^x = b^y$ not work when $b = 1$? Give an example to justify your answer.

EXAMPLE 2 Solving an Equation by Graphing

COMMON CORE

Exponential Functions

In this extension, you will
- solve exponential equations algebraically and graphically.

Learning Standards
A.REI.3
A.REI.11
F.BF.3
F.IF.7e
F.LE.1a
F.LE.2

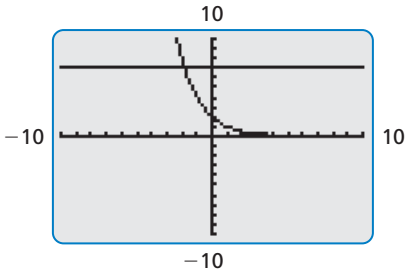
Use a graphing calculator to solve $\left(\frac{1}{2}\right)^{x-1} = 7$.

Step 1: Write a system of equations using each side of the equation.

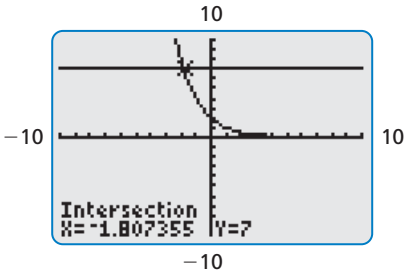
$y = \left(\frac{1}{2}\right)^{x-1}$ Equation 1

$y = 7$ Equation 2

Step 2: Enter the equations into your calculator. Then graph the equations in a standard viewing window.



Step 3: Use the *intersect* feature to find the point of intersection. It is at about $(-1.81, 7)$.



∴ So, the solution is $x \approx -1.81$.

Check: Check the solution algebraically.

$\left(\frac{1}{2}\right)^{x-1} = 7$ Write the equation.

$\left(\frac{1}{2}\right)^{-1.81-1} \stackrel{?}{=} 7$ Substitute −1.81 for x.

$7.01 \approx 7$ ✓ Use a calculator.

Practice

Use a graphing calculator to solve the equation.

12. $4^{x+3} = 6$

13. $2^x = 1.8$

14. $4 = 8^x$

15. $\left(\frac{3}{4}\right)^{x+2} = 10$

16. $2^{-x-3} = 3^{x+1}$

17. $5^x = -4^{x+4}$