6.4 Exponential Functions

Essential Question What are the characteristics of an exponential function?

1 ACTIVITY: Describing an Exponential Function

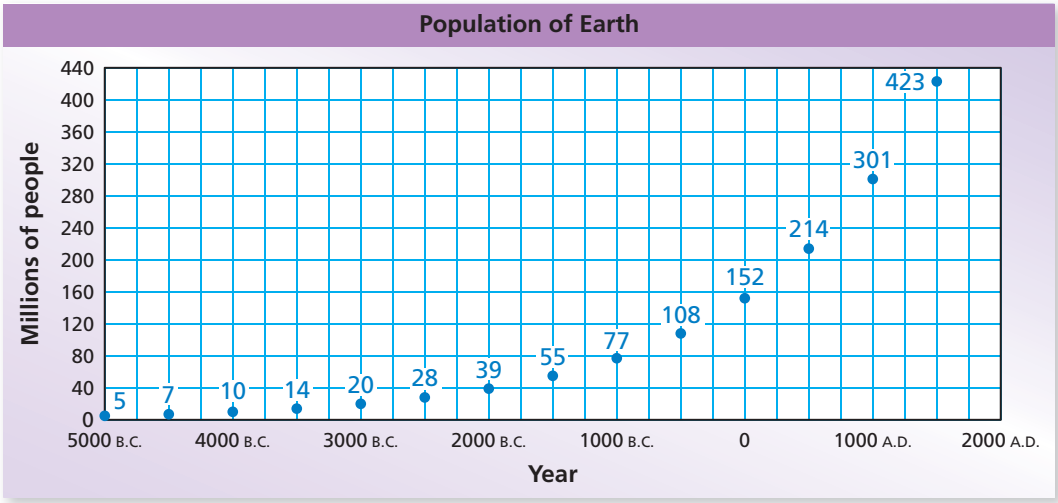
Work with a partner. The graph below shows estimates of the population of Earth from 5000 B.C. through 1500 A.D. at 500-year intervals.

a. Describe the pattern.

b. Did Earth's population increase by the same *amount* or the same *percent* for each 500-year period? Explain.

c. Assume the pattern continued. Estimate Earth's population in 2000.

d. Use the Internet to find Earth's population in 2000. Did the pattern continue? If not, why did the pattern change?



Common Core

Exponential Functions

In this lesson, you will
- identify, evaluate, and graph exponential functions.

Learning Standards
A.REI.3
A.REI.11
F.BF.3
F.IF.7e
F.LE.1a
F.LE.2

4000 B.C.
Civilization begins to develop in Mesopotamia.

3000 B.C.
Stonehenge is built in England.

2000 B.C.
Middle Kingdom in Egypt

2 ACTIVITY: Modeling an Exponential Function

Math Practice 6

Calculate Accurately

How can you check the accuracy of your answers?

Work with a partner. Use the following exponential function to complete the table. Compare the results with the data in Activity 1.

$$P = 152(1.406)^{t/500}$$

1 B.C.
Augustus Caesar controls most of the Mediterranean world. (Use $t = 0$ to approximate 1 B.C.)

1000 A.D.
Song Dynasty has about one-fifth of Earth's population.

Year	t	Population from Activity 1	P
5000 B.C.	−5000		
4500 B.C.	−4500		
4000 B.C.	−4000		
3500 B.C.	−3500		
3000 B.C.	−3000		
2500 B.C.	−2500		
2000 B.C.	−2000		
1500 B.C.	−1500		
1000 B.C.	−1000		
500 B.C.	−500		
1 B.C.	0		
500 A.D.	500		
1000 A.D.	1000		
1500 A.D.	1500		

What Is Your Answer?

3. **IN YOUR OWN WORDS** What are the characteristics of an exponential function?

4. Sketch the graph of each exponential function. Does the function match the characteristics you described in Question 3? Explain.

   a. $y = 2^x$ **b.** $y = 2(3)^x$ **c.** $y = 3(1.5)^x$

Practice Use what you learned about exponential functions to complete Exercises 4 and 5 on page 289.

6.4 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
exponential function, *p. 286*

A function of the form $y = ab^x$, where $a \neq 0$, $b \neq 1$, and $b > 0$ is an **exponential function.** The exponential function $y = ab^x$ is a nonlinear function that changes by equal factors over equal intervals.

EXAMPLE 1 Identifying Functions

Does each table represent a *linear* or an *exponential* function? Explain.

a.

x	0	1	2	3
y	2	4	6	8

(+1, +1, +1 over the x-row; +2, +2, +2 over the y-row)

As x increases by 1, y increases by 2. The rate of change is constant. So, the function is linear.

b.

x	y
0	4
1	8
2	16
3	32

(+1, +1, +1 down the x-column; ×2, ×2, ×2 down the y-column)

As x increases by 1, y is multiplied by 2. So, the function is exponential.

EXAMPLE 2 Evaluating Exponential Functions

Evaluate each function for the given value of x.

a. $y = -2(5)^x$; $x = 3$

$y = -2(5)^x$ Write the function.

$= -2(5)^3$ Substitute for x.

$= -2(125)$ Evaluate the power.

$= -250$ Multiply.

b. $y = 3(0.5)^x$; $x = -2$

$y = 3(0.5)^x$

$= 3(0.5)^{-2}$

$= 3(4)$

$= 12$

On Your Own

Now You're Ready
Exercises 6–15

Does the table represent a *linear* or an *exponential* function? Explain.

1.

x	0	1	2	3
y	8	4	2	1

2.

x	y
−4	1
0	0
4	−1
8	−2

Evaluate the function when $x = -2, 0$, and $\frac{1}{2}$.

3. $y = 2(9)^x$

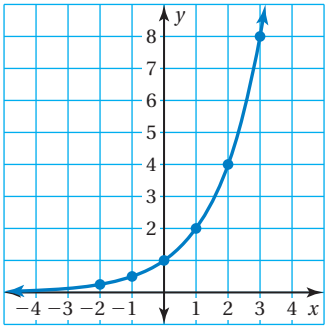
4. $y = 1.5(2)^x$

EXAMPLE 3 Graphing an Exponential Function

Graph $y = 2^x$. Describe the domain and range.

Step 1: Make a table of values.

x	-2	-1	0	1	2	3
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



Step 2: Plot the ordered pairs.

Step 3: Draw a smooth curve through the points.

From the graph, you can see that the domain is all real numbers and the range is all positive real numbers.

Study Tip

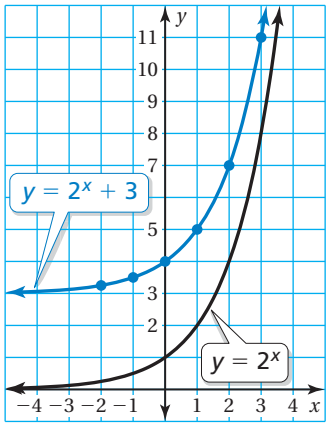
In Example 3, you can substitute any value for x. So, the domain is all real numbers.

EXAMPLE 4 Graphing a Vertical Translation

Graph $y = 2^x + 3$. Describe the domain and range. Compare the graph to the graph of $y = 2^x$.

Step 1: Make a table of values.

x	-2	-1	0	1	2	3
y	$\frac{13}{4}$	$\frac{7}{2}$	4	5	7	11



Step 2: Plot the ordered pairs.

Step 3: Draw a smooth curve through the points.

From the graph, you can see that the domain is all real numbers and the range is all real numbers greater than 3. The graph of $y = 2^x + 3$ is a translation 3 units up of the graph of $y = 2^x$.

Remember

In Section 5.4, you learned that the graph of $f(x) + k$ is a vertical translation of the graph of $f(x)$.

On Your Own

Graph the function. Describe the domain and range.

5. $y = 3^x$ **6.** $y = \left(\frac{1}{2}\right)^x$ **7.** $y = -2\left(\frac{1}{4}\right)^x$

8. Graph $y = \left(\frac{1}{2}\right)^x - 2$. Describe the domain and range. Compare the graph to the graph of $y = \left(\frac{1}{2}\right)^x$.

Now You're Ready
Exercises 21–23 and 27–29

Study Tip

To find the y-intercept of the graph of $y = ab^x$, substitute 0 for x.

$y = ab^0$

$y = a(1)$

$y = a$

So, the y-intercept is a.

For an exponential function of the form $y = ab^x$, the y-values change by a factor of b as x increases by 1. Also notice that a is the y-intercept.

		+1	+1	+1	+1
x	0	1	2	3	4
y	2	10	50	250	1250
		×5	×5	×5	×5

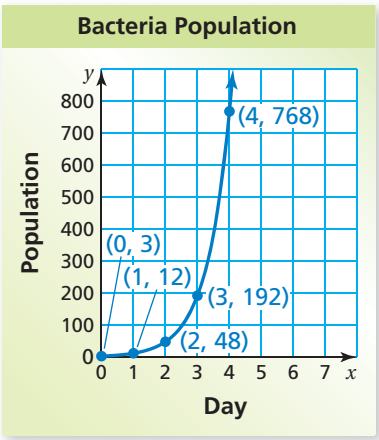
$y = 2(5)^x$

EXAMPLE 5 Real-Life Application

The graph represents a bacteria population y after x days.

a. Write an exponential function that represents the population.

Use the graph to make a table of values.



		+1	+1	+1	+1
x	0	1	2	3	4
y	3	12	48	192	768
		×4	×4	×4	×4

The y-intercept is 3 and the y-values increase by a factor of 4 as x increases by 1.

∴ So, the population can be modeled by $y = 3(4)^x$.

Study Tip

For help with rational exponents, see Section 6.3.

b. Find the population after 12 hours and after 5 days.

12 hours $= \frac{1}{2}$ day

Population after 12 hours		*Population after 5 days*
$y = 3(4)^x$	Write the function.	$y = 3(4)^x$
$= 3(4)^{1/2}$	Substitute for x.	$= 3(4)^5$
$= 3(2)$	Evaluate the power.	$= 3(1024)$
$= 6$	Multiply.	$= 3072$

∴ There are 6 bacteria after 12 hours and 3072 bacteria after 5 days.

On Your Own

Now You're Ready
Exercises 36–39

9. A bacteria population y after x days can be represented by an exponential function whose graph passes through (0, 100) and (1, 200).

a. Write a function that represents the population.

b. Find the population after 6 days. Does this bacteria population grow faster than the bacteria population in Example 5? Explain.

6.4 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **VOCABULARY** Describe how linear and exponential functions change over equal intervals.
2. **OPEN-ENDED** Sketch an increasing exponential function whose graph has a y-intercept of 2.
3. **WHICH ONE DOESN'T BELONG?** Which equation does *not* belong with the other three? Explain your reasoning.

$y = 3^x$ $f(x) = 2(4)^x$ $f(x) = (-3)^x$ $y = 5(3)^x$

Practice and Problem Solving

Sketch the graph of the exponential function.

4. $y = 4^x$
5. $y = 2(2)^x$

Does the table represent a *linear* or an *exponential* function? Explain.

1

6.

x	y
0	−2
1	0
2	2
3	4

7.

x	y
1	6
2	12
3	24
4	48

8.

x	−1	0	1	2
y	0.25	1	4	16

9.

x	−3	0	3	6
y	10	1	−8	−17

Evaluate the function for the given value of x.

2

10. $y = 3^x;\ x = 2$
11. $f(x) = 3(2)^x;\ x = -1$
12. $y = -4(5)^x;\ x = 2$
13. $f(x) = 0.5^x;\ x = -3$
14. $f(x) = \frac{1}{3}(6)^x;\ x = 3$
15. $y = \frac{1}{4}(4)^x;\ x = \frac{3}{2}$

16. **ERROR ANALYSIS** Describe and correct the error in evaluating the function.

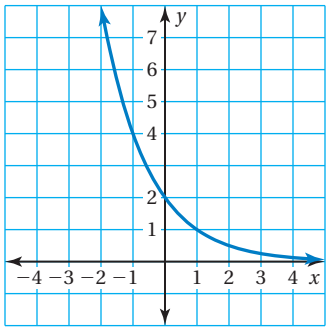
$$g(x) = 6(0.5)^x;\ x = -2$$
$$g(-2) = 6(0.5)^{-2}$$
$$= 3^{-2}$$
$$= \frac{1}{9}$$

17. **CALCULATOR** You graph an exponential function on a calculator. You zoom in repeatedly at 25% of the screen size. The function $y = 0.25^x$ represents the percent (in decimal form) of the original screen display that you see, where x is the number of times you zoom in. You zoom in twice. What percent of the original screen do you see?

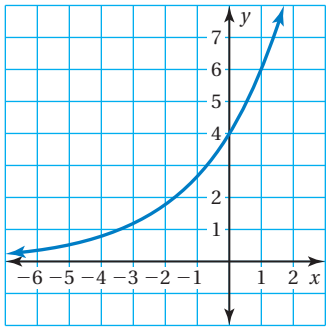
Match the function with its graph.

18. $f(x) = -3(4)^x$ **19.** $y = 2(0.5)^x$ **20.** $y = 4(1.5)^x$

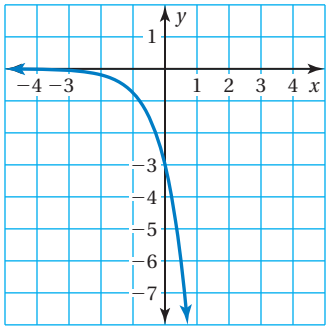
A.



B.



C.



Graph the function. Describe the domain and range.

21. $y = 9^x$ **22.** $f(x) = -7^x$ **23.** $f(x) = 4\left(\frac{1}{4}\right)^x$

24. LOGIC Describe the graph of $y = a(2)^x$ when a is (a) positive and (b) negative. (c) How does the graph change as a changes?

25. NUMBER SENSE Consider the graph of $f(x) = 2(b)^x$. How do the graphs differ when $b > 1$ and $0 < b < 1$?

26. COYOTES A population y of coyotes in a national park triples every 20 years. The function $y = 15(3)^x$ represents the population, where x is the number of 20-year periods.

a. Graph the function. Describe the domain and range.

b. Find and interpret the y-intercept.

c. How many coyotes are in the national park after 20 years?

Graph the function. Describe the domain and range. Compare the graph to the graph of $y = 3^x$.

27. $y = 3^x - 1$ **28.** $y = 3^x + 3$ **29.** $y = 3^x - \frac{1}{2}$

30. REASONING Graph the function $f(x) = -2^x$. Then graph $g(x) = -2^x - 3$.

a. Describe the domain and range of each function.

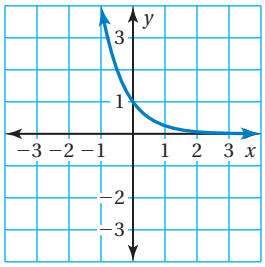
b. Find the y-intercept of the graph of each function.

c. How are the y-intercept, domain, and range affected by the translation?

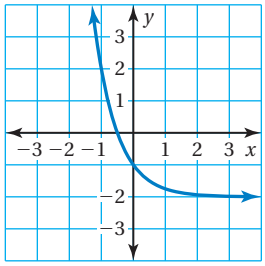
31. REASONING When does an exponential function intersect the x-axis? Give an example to justify your answer.

Given $g(x) = 0.25^x - 1$, find the value of k so that the graph is $g(x) + k$.

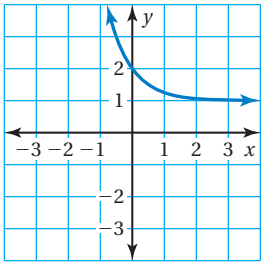
32.



33.



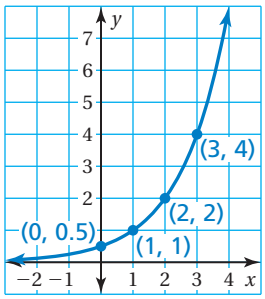
34.



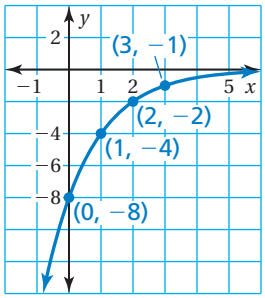
35. REASONING Graph $g(x) = 4^{x+2}$. Compare the graph to the graph of $f(x) = 4^x$.

Write an exponential function represented by the graph or table.

36.



37.



38.

x	0	1	2	3
y	2	8	32	128

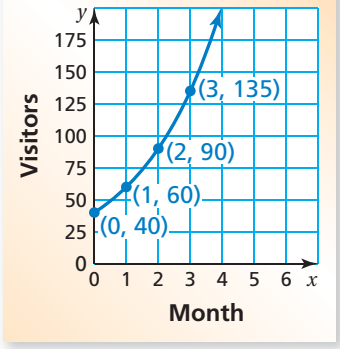
39.

x	0	1	2	3
y	−3	−15	−75	−375

40. ART GALLERY The graph represents the number y of visitors to a new art gallery after x months.

a. Write an exponential function that represents this situation.

b. Approximate the number of visitors after 5 months.



41. SALES A sales report shows that 3300 gas grills were purchased from a chain of hardware stores last year. The store expects grill sales to increase 6% each year. About how many grills does the store expect to sell in year 6? Use an equation to justify your answer.

42. Structure The graph of g is a translation 4 units up and 3 units right of the graph of $f(x) = 2^x$. Write an equation for g.

Fair Game Review *What you learned in previous grades & lessons*

Write the percent as a decimal. *(Skills Review Handbook)*

43. 23%

44. 3%

45. 150%

46. MULTIPLE CHOICE Which of the following is equivalent to 100(0.95)? *(Skills Review Handbook)*

Ⓐ 0.95 Ⓑ 9.5 Ⓒ 95 Ⓓ 950