6.2 Properties of Exponents

Essential Question How can you use inductive reasoning to observe patterns and write general rules involving properties of exponents?

1 ACTIVITY: Writing a Rule for Products of Powers

Work with a partner. Write the product of the two powers as a single power. Then, write a *general rule* for finding the product of two powers with the same base.

a. Sample: $(3^4)(3^3) = (3 \cdot 3 \cdot 3 \cdot 3)(3 \cdot 3 \cdot 3) = 3^7$

b. $(2^2)(2^3) =$ _____

c. $(4^1)(4^5) =$ _____

d. $(5^3)(5^5) =$ _____

e. $(x^2)(x^6) =$ _____

2 ACTIVITY: Writing a Rule for Quotients of Powers

Work with a partner. Write the quotient of the two powers as a single power. Then, write a *general rule* for finding the quotient of two powers with the same base.

a. Sample: $\frac{3^4}{3^2} = \frac{3 \cdot 3 \cdot \cancel{3} \cdot \cancel{3}}{\cancel{3} \cdot \cancel{3}} = 3^2$

b. $\frac{4^3}{4^2} =$ _____

c. $\frac{2^5}{2^2} =$ _____

d. $\frac{x^6}{x^3} =$ _____

e. $\frac{3^4}{3^4} =$ _____

COMMON CORE

Exponents

In this lesson, you will

- simplify expressions using the properties of exponents.

Learning Standard N.RN.2

3 ACTIVITY: Writing a Rule for Powers of Powers

Work with a partner. Write the expression as a single power. Then, write a *general rule* for finding a power of a power.

a. Sample: $(3^2)^3 = (3 \cdot 3)(3 \cdot 3)(3 \cdot 3) = 3^6$

b. $(2^2)^4 =$ _____

c. $(7^3)^2 =$ _____

d. $(y^3)^3 =$ _____

e. $(x^4)^2 =$ _____

4 ACTIVITY: Writing a Rule for Powers of Products

Math Practice 7

View as Components

What are the different parts of the expressions? How does this help you rewrite the product?

Work with a partner. Write the expression as the product of two powers. Then, write a *general rule* for finding a power of a product.

a. Sample: $(2 \cdot 3)^3 = (2 \cdot 3)(2 \cdot 3)(2 \cdot 3) = (2^3)(3^3)$

b. $(2 \cdot 5)^2 =$ _____

c. $(5 \cdot 4)^3 =$ _____

d. $(6a)^4 =$ _____

e. $(3x)^2 =$ _____

5 ACTIVITY: Writing a Rule for Powers of Quotients

Work with a partner. Write the expression as the quotient of two powers. Then, write a *general rule* for finding a power of a quotient.

a. Sample: $\left(\frac{3}{2}\right)^4 = \frac{3}{2} \cdot \frac{3}{2} \cdot \frac{3}{2} \cdot \frac{3}{2} = \frac{3 \cdot 3 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 2} = \frac{3^4}{2^4}$

b. $\left(\frac{2}{3}\right)^2 =$ _____

c. $\left(\frac{4}{3}\right)^3 =$ _____

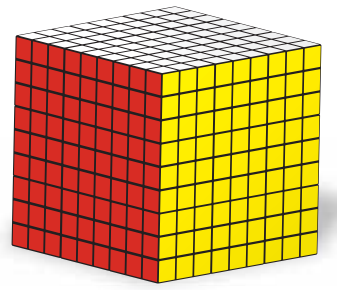
d. $\left(\frac{x}{2}\right)^3 =$ _____

e. $\left(\frac{a}{b}\right)^4 =$ _____

What Is Your Answer?

6. IN YOUR OWN WORDS How can you use inductive reasoning to observe patterns and write general rules involving properties of exponents?

7. There are 3^3 small cubes in the cube below. Write an expression for the number of small cubes in the large cube at the right.



Practice Use what you learned about exponents to complete Exercises 6–11 on page 273.

6.2 Lesson

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Key Ideas

Product of Powers Property

Words To multiply powers with the same base, add their exponents.

Numbers $4^6 \cdot 4^3 = 4^{6+3} = 4^9$ **Algebra** $a^m \cdot a^n = a^{m+n}$

Quotient of Powers Property

Words To divide powers with the same base, subtract their exponents.

Numbers $\frac{4^6}{4^3} = 4^{6-3} = 4^3$ **Algebra** $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

Power of a Power Property

Words To find a power of a power, multiply the exponents.

Numbers $(4^6)^3 = 4^{6 \cdot 3} = 4^{18}$ **Algebra** $(a^m)^n = a^{mn}$

Remember

For any integer n and any nonzero integer a, $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$.

EXAMPLE 1 Using Properties of Exponents

Simplify. Write your answer using only positive exponents.

a. $3^2 \cdot 3^6 = 3^{2+6}$ Product of Powers Property

$= 3^8$ Simplify.

The base is 3. Add the exponents.

b. $\frac{(-4)^2}{(-4)^7} = (-4)^{2-7}$ Quotient of Powers Property

$= (-4)^{-5}$ Simplify.

$= \frac{1}{(-4)^5}$ Definition of negative exponent

The base is -4. Subtract the exponents.

c. $(z^4)^{-3} = z^{4 \cdot (-3)}$ Power of a Power Property

$= z^{-12}$ Simplify.

$= \frac{1}{z^{12}}$ Definition of negative exponent

The base is z. Multiply the exponents.

On Your Own

Now You're Ready
Exercises 12–17

Simplify. Write your answer using only positive exponents.

1. $10^4 \cdot 10^{-6}$
2. $x^9 \cdot x^{-9}$
3. $\frac{-5^8}{-5^4}$
4. $\frac{y^6}{y^7}$
5. $(6^{-2})^{-5}$
6. $(w^{12})^5$

Key Ideas

Power of a Product Property

Words To find a power of a product, find the power of each factor and multiply.

Numbers $(3 \cdot 2)^5 = 3^5 \cdot 2^5$ **Algebra** $(ab)^m = a^m b^m$

Power of a Quotient Property

Words To find a power of a quotient, find the power of the numerator and the power of the denominator and divide.

Numbers $\left(\frac{3}{2}\right)^5 = \frac{3^5}{2^5}$ **Algebra** $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, where $b \neq 0$

EXAMPLE 2 Using Properties of Exponents

Simplify. Write your answer using only positive exponents.

a. $(-1.5y)^2 = (-1.5)^2 \cdot y^2$ Power of a Product Property

$= 2.25y^2$ Simplify.

b. $\left(\frac{a}{-10}\right)^3 = \frac{a^3}{(-10)^3}$ Power of a Quotient Property

$= -\frac{a^3}{1000}$ Simplify.

c. $\left(\frac{2x}{3}\right)^{-5} = \frac{(2x)^{-5}}{3^{-5}}$ Power of a Quotient Property

$= \frac{3^5}{(2x)^5}$ Definition of negative exponent

$= \frac{3^5}{2^5 x^5}$ Power of a Product Property

$= \frac{243}{32x^5}$ Simplify.

On Your Own

Now You're Ready
Exercises 21–26

Simplify. Write your answer using only positive exponents.

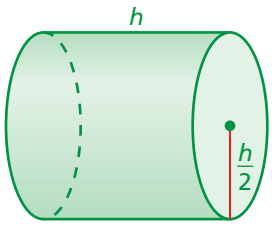
7. $(10y)^{-3}$

8. $\left(-\frac{4}{n}\right)^5$

9. $\left(\frac{1}{2k^2}\right)^5$

10. $\left(\frac{6c}{7}\right)^{-2}$

EXAMPLE 3 Simplifying an Expression



Which expression represents the volume of the cylinder?

(A) $\frac{h^2}{2}$ (B) $\frac{\pi h^2}{4}$ (C) $\frac{\pi h^3}{2}$ (D) $\frac{\pi h^3}{4}$

$V = \pi r^2 h$ Formula for volume of a cylinder

$= \pi\left(\frac{h}{2}\right)^2 (h)$ Substitute $\frac{h}{2}$ for r.

$= \pi\left(\frac{h^2}{2^2}\right)(h)$ Power of a Quotient Property

$= \frac{\pi h^3}{4}$ Simplify.

∴ The correct answer is (D).

EXAMPLE 4 Real-Life Application

A jellyfish emits about 1.25×10^8 particles of light, or photons, in 6.25×10^{-4} second. How many photons does the jellyfish emit each second? Write your answer in scientific notation and in standard form.

Divide to find the unit rate.

$\frac{1.25 \times 10^8}{6.25 \times 10^{-4}}$ ← photons, ← seconds — Write the rate.

$= \frac{1.25}{6.25} \times \frac{10^8}{10^{-4}}$ Rewrite.

$= 0.2 \times 10^{12}$ Simplify.

$= 2 \times 10^{11}$ Write in scientific notation.

∴ The jellyfish emits 2×10^{11}, or 200,000,000,000 photons per second.

Remember

A number is written in scientific notation when it is of the form $a \times 10^b$, where $1 \leq a < 10$ and b is an integer.

On Your Own

11. In Example 3, which expression represents the area of a base of the cylinder?

12. It takes the Sun about 2.3×10^8 years to orbit the center of the Milky Way. It takes Pluto about 2.5×10^2 years to orbit the Sun. How many times does Pluto orbit the Sun while the Sun completes one orbit around the Milky Way? Write your answer in scientific notation.

6.2 Exercises

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Vocabulary and Concept Check

MATCHING **Match the property with its example.**

1. Quotient of Powers Property
2. Power of a Power Property
3. Power of a Quotient Property
4. Power of a Product Property

A. $(4^5)^2 = 4^{5 \cdot 2}$ **B.** $\left(\frac{5}{2}\right)^4 = \frac{5^4}{2^4}$ **C.** $(5 \cdot 2)^4 = 5^4 \cdot 2^4$ **D.** $\frac{4^5}{4^2} = 4^{5-2}$

5. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

Simplify $3^3 \cdot 3^6$. | Simplify 3^{3+6}. | Simplify 3^{6-3}. | Simplify $3^6 \cdot 3^3$.

Practice and Problem Solving

Simplify the expression.

6. $(n^4)(n^3)$
7. $\frac{x^5}{x^3}$
8. $(c^5)^3$
9. $(4b)^3$
10. $\left(\frac{k}{3}\right)^5$
11. $\frac{(2a)^6}{a^2}$

Simplify. Write your answer using only positive exponents.

12. $8^{-2} \cdot 8^7$
13. $b^4 \cdot b^7$
14. $\frac{12^7}{12^2}$
15. $\frac{d^5}{d^8}$
16. $(5^5)^4$
17. $(x^3)^{-2}$

ERROR ANALYSIS **Describe and correct the error in simplifying the expression.**

18. ✗
$$x^5 \cdot x^{-2} = x^{5 \cdot (-2)}$$
$$= x^{-10}$$
$$= \frac{1}{x^{10}}$$

19. ✗
$$(m^3)^4 = m^{3+4}$$
$$= m^7$$

20. **MICROSCOPE** A microscope magnifies an object 10^5 times. The length of an object is 10^2 nanometers. What is its magnified length?

Simplify. Write your answer using only positive exponents.

2 3 **21.** $(6.2y)^2$

22. $\left(\frac{w}{4}\right)^4$

23. $\left(-\frac{6}{d}\right)^{-2}$

24. $(7p)^{-3}$

25. $(-5x)^5$

26. $\left(\frac{3n^3}{4}\right)^2$

27. ERROR ANALYSIS Describe and correct the error in simplifying the expression.

$$\left(\frac{x^3}{3}\right)^2 = \frac{(x^3)^2}{3} = \frac{x^6}{3}$$

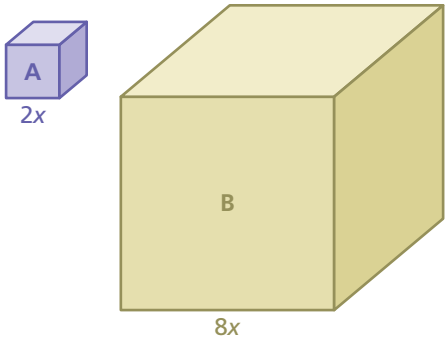
28. OPEN-ENDED Use the properties of exponents to write three expressions equivalent to x^8.

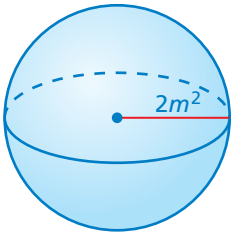
29. REASONING Are the expressions $(a^4)^2$ and a^{4^2} equivalent? Explain your reasoning.

30. GEOMETRY Consider Cube A and Cube B.

a. Which property of exponents should you use to find the volume of each cube?

b. How can you use the Power of a Quotient Property to find how many times greater the volume of Cube B is than the volume of Cube A?





31. SPHERE The volume V of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius. What is the volume of the sphere in terms of m and π?

32. PROBABILITY The probability of rolling a 6 on a number cube is $\frac{1}{6}$. The probability of rolling a 6 twice in a row is $\left(\frac{1}{6}\right)^2 = \frac{1}{36}$.

a. Write an expression that represents the probability of rolling a 6 n times in a row.

b. What is the probability of rolling a 6 five times in a row?

c. What is the probability of flipping heads on a coin five times in a row?

Evaluate the expression. Write your answer in scientific notation.

33. $(3.4 \times 10^2)(1.5 \times 10^{-5})$ **34.** $(6.1 \times 10^{-3})(8 \times 10^9)$ **35.** $(4.8 \times 10^{-4})(7.2 \times 10^{-6})$

36. $\dfrac{(3 \times 10^3)}{(4 \times 10^5)}$ **37.** $\dfrac{(6.4 \times 10^{-7})}{(1.6 \times 10^{-5})}$ **38.** $\dfrac{(3.9 \times 10^{-5})}{(7.8 \times 10^{-8})}$

Simplify. Write your answer using only positive exponents.

39. $(6x^2y^{-4})^{-3}$ **40.** $\dfrac{(2m)^{-2}n^5}{-m^4n^{-3}}$ **41.** $\dfrac{15b^{-3}c^4}{(6b^{-4}c^{-5})^2}$

42. REASONING Write $8x^3y^3$ as the power of a product.

43. COMPUTER CHIP The area of a rectangular computer chip is $112a^3b^2$ square microns. The width is $8ab$ microns. What is the length?

44. PROBLEM SOLVING The speed of light is approximately 3×10^5 kilometers per second. The table shows the average distance each planet is from the Sun. How long does it take sunlight to reach Earth? Jupiter? Neptune?

Planet	Average Distance from the Sun (km)
Mercury	5.8×10^7
Venus	1.1×10^8
Earth	1.5×10^8
Mars	2.3×10^8
Jupiter	7.8×10^8
Saturn	1.4×10^9
Uranus	2.9×10^9
Neptune	4.5×10^9

45. RICHTER SCALE The Richter Scale is used to compare the intensities of earthquakes. An increase of 1 in magnitude on the Richter Scale represents a tenfold increase in intensity. An earthquake registers 7.4 on the Richter Scale and is followed by an aftershock that is 1000 times less intense. What is the magnitude of the aftershock?

46. Precision Find x and y when $\dfrac{k^{2x}}{k^y} = k^{13}$ and $(k^x k^{2y})^2 = k^{28}$. Explain how you found your answer.

Fair Game Review What you learned in previous grades & lessons

Simplify the expression. *(Section 6.1)*

47. $\sqrt{48}$ **48.** $\sqrt{\dfrac{70}{36}}$ **49.** $\sqrt{\dfrac{180}{121}}$

50. MULTIPLE CHOICE Which of the following is the solution of $\dfrac{x}{3} < -6$? *(Section 3.3)*

(A) $x > -2$ (B) $x < -2$ (C) $x > -18$ (D) $x < -18$