

6.1 Properties of Square Roots

Essential Question

How can you multiply and divide square roots?

Recall that when you multiply a number by itself, you square the number.

Symbol for squaring is 2nd power.

$$4^2 = 4 \cdot 4$$

$$= 16$$

4 squared is 16.

To “undo” this, take the square root of the number.

Symbol for square root is a radical sign.

$$\sqrt{16} = \sqrt{4^2} = 4$$

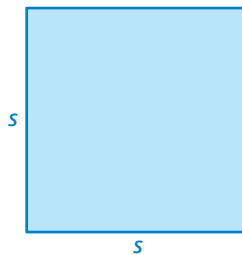
The square root of 16 is 4.

1 ACTIVITY: Finding Square Roots

Work with a partner. Use a square root symbol to write the side length of the square. Then find the square root. Check your answer by multiplying.

a. Sample: $s = \sqrt{81} = 9$ ft

Area = 81 ft^2

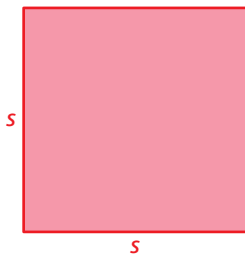


∴ The side length of the square is 9 feet.

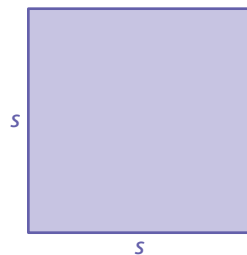
Check

$$\begin{array}{r} 9 \\ \times 9 \\ \hline 81 \end{array} \checkmark$$

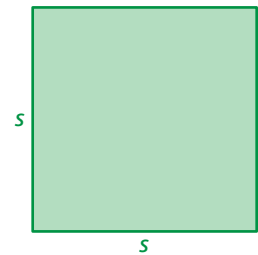
b. Area = 121 yd^2



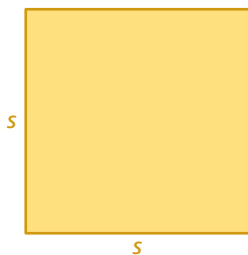
c. Area = 324 cm^2



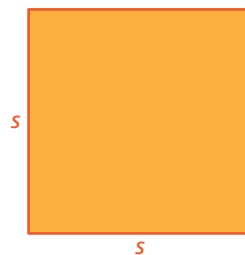
d. Area = 361 mi^2



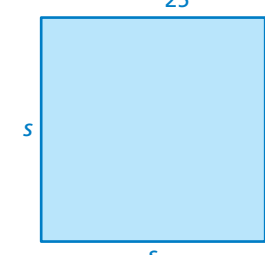
e. Area = 2.89 in.^2



f. Area = 6.25 m^2



g. Area = $\frac{16}{25} \text{ ft}^2$



Square Roots

In this lesson, you will

- simplify and evaluate square roots.
- simplify radical expressions.

Preparing for Standard N.RN.3

2 ACTIVITY: Operations with Square Roots

Math Practice 3

Analyze Conjectures

How do you know if your conclusion is accurate? How can you explain your conclusion?

Work with a partner. When you have an expression that involves two operations, you need to know whether you obtain the same result regardless of the order in which you perform the operations. In each of the following, compare the results obtained by the two orders. What can you conclude?



a. Square Roots and Addition

Is $\sqrt{36} + \sqrt{64}$ equal to $\sqrt{36 + 64}$?

In general, is $\sqrt{a} + \sqrt{b}$ equal to $\sqrt{a + b}$?

Explain your reasoning.



b. Square Roots and Multiplication

Is $\sqrt{4} \cdot \sqrt{9}$ equal to $\sqrt{4 \cdot 9}$?

In general, is $\sqrt{a} \cdot \sqrt{b}$ equal to $\sqrt{a \cdot b}$?

Explain your reasoning.

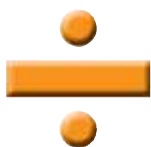


c. Square Roots and Subtraction

Is $\sqrt{64} - \sqrt{36}$ equal to $\sqrt{64 - 36}$?

In general, is $\sqrt{a} - \sqrt{b}$ equal to $\sqrt{a - b}$?

Explain your reasoning.



d. Square Roots and Division

Is $\frac{\sqrt{100}}{\sqrt{4}}$ equal to $\sqrt{\frac{100}{4}}$?

In general, is $\frac{\sqrt{a}}{\sqrt{b}}$ equal to $\sqrt{\frac{a}{b}}$?

Explain your reasoning.

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you multiply and divide square roots?

Write a rule for:

- The product of square roots
- The quotient of square roots

Practice

Use what you learned about square roots to complete Exercises 3–5 on page 264.



Key Ideas

Product Property of Square Roots**Algebra**

$$\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}, \text{ where } x, y \geq 0$$

Numbers


$$\sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$$

Quotient Property of Square Roots**Algebra**

$$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}, \text{ where } x \geq 0 \text{ and } y > 0$$

Numbers

$$\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$$

EXAMPLE 1 Simplifying Square Roots**Remember**


A square root is simplified when the radicand has no perfect square factors other than 1.

$$\begin{aligned} \text{a. } \sqrt{150} &= \sqrt{25 \cdot 6} \\ &= \sqrt{25} \cdot \sqrt{6} \\ &= 5\sqrt{6} \end{aligned}$$

Factor using the greatest perfect square factor.

Product Property of Square Roots

Simplify.

$$\begin{aligned} \text{b. } \sqrt{\frac{15}{64}} &= \frac{\sqrt{15}}{\sqrt{64}} \\ &= \frac{\sqrt{15}}{8} \end{aligned}$$

Quotient Property of Square Roots

Simplify.

EXAMPLE 2 Evaluating Square RootsEvaluate $\sqrt{b^2 - 4ac}$ when $a = 2$, $b = -8$, and $c = 4$.

$$\begin{aligned} \sqrt{b^2 - 4ac} &= \sqrt{(-8)^2 - 4(2)(4)} \\ &= \sqrt{32} \\ &= \sqrt{16 \cdot 2} \\ &= \sqrt{16} \cdot \sqrt{2} \\ &= 4\sqrt{2} \end{aligned}$$

Substitute.

Simplify.

Factor.

Product Property of Square Roots

Simplify.

 **On Your Own**

Simplify the expression.

1. $\sqrt{\frac{23}{9}}$

2. $-\sqrt{80}$

3. $\sqrt{\frac{27}{100}}$

4. Evaluate $\sqrt{b^2 - 4ac}$ when $a = 2$, $b = -6$, and $c = -5$.



Now You're Ready
Exercises 6–17

EXAMPLE 3 Simplifying Radical Expressions

Simplify $\frac{6 + \sqrt{8}}{2}$.

$$\frac{6 + \sqrt{8}}{2} = \frac{6 + \sqrt{4 \cdot 2}}{2}$$

Factor the radicand.

$$= \frac{6 + \sqrt{4} \cdot \sqrt{2}}{2}$$

Product Property of Square Roots

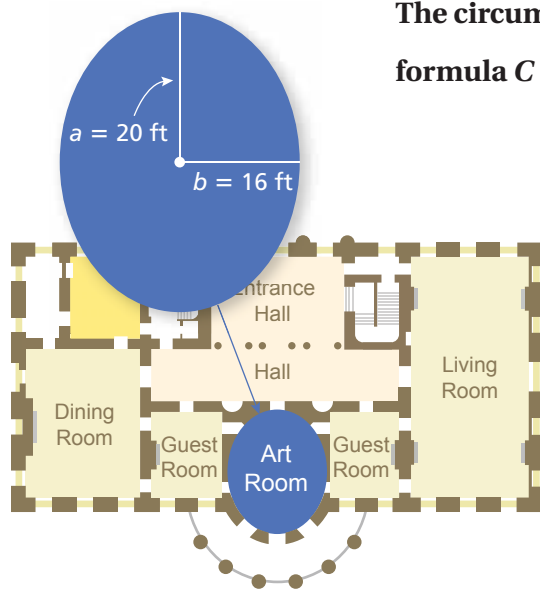
$$= \frac{6 + 2\sqrt{2}}{2}$$

Simplify.

$$= 3 + \sqrt{2}$$

Divide.

EXAMPLE 4 Real-Life Application



The circumference C of the art room in a mansion is given by the formula $C = 2\pi\sqrt{\frac{a^2 + b^2}{2}}$. Find the circumference of the room.

$$C = 2\pi\sqrt{\frac{a^2 + b^2}{2}}$$

Write formula.

$$= 2\pi\sqrt{\frac{20^2 + 16^2}{2}}$$

Substitute.

$$= 2\pi\sqrt{328}$$

Simplify.

$$= 2\pi\sqrt{4 \cdot 82}$$

Factor.

$$= 2\pi \cdot \sqrt{4} \cdot \sqrt{82}$$

Product Property of Square Roots

$$= 4\pi\sqrt{82}$$

Simplify.

❖ The circumference of the room is $4\pi\sqrt{82}$, or about 114 feet.

On Your Own

Simplify the expression.

5. $\frac{8 + \sqrt{32}}{2}$

6. $\frac{-1 - \sqrt{27}}{4}$

7. $\frac{2 - \sqrt{28}}{2(3)}$

8. Use the formula in Example 4 to find the circumference of an ellipse in which $a = 14$ feet and $b = 6$ feet.

Now You're Ready
Exercises 21–26

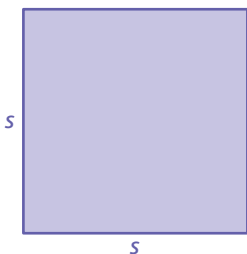
Vocabulary and Concept Check

- WRITING** How do you know when the square root of a positive integer is simplified?
- WRITING** How is the Product Property of Square Roots similar to the Quotient Property of Square Roots?

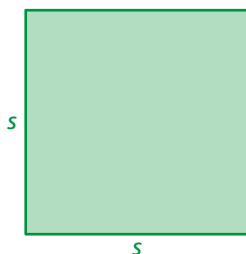
Practice and Problem Solving

Find the dimensions of the square. Check your answer.

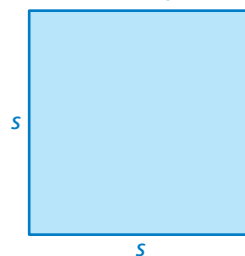
3. Area = 64 ft^2



4. Area = 144 in.^2



5. Area = $\frac{9}{16} \text{ cm}^2$



Simplify the expression.

1 6. $\sqrt{18}$

7. $-\sqrt{200}$

8. $\sqrt{12}$

9. $\sqrt{48}$

10. $\sqrt{125}$

11. $-\sqrt{\frac{23}{64}}$

12. $-\sqrt{\frac{65}{121}}$

13. $\sqrt{\frac{18}{49}}$

14. $\sqrt{\frac{25}{36}}$

Evaluate the expression when $x = -2$, $y = 8$, and $z = \frac{1}{2}$.

2 15. $\sqrt{x^2 + yz}$

16. $\sqrt{2x^2 + y^2}$

17. $\sqrt{y - 44xz}$



18. **ERROR ANALYSIS** Describe and correct the error in simplifying the expression.

X $\sqrt{\frac{20}{9}} = \frac{\sqrt{20}}{\sqrt{9}} = \frac{\sqrt{20}}{3}$

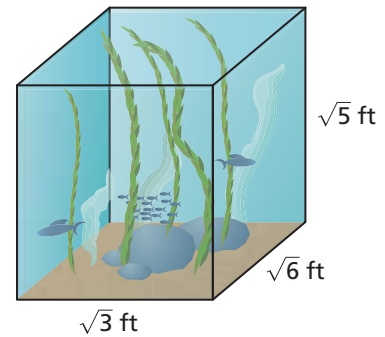
19. **ELECTRICITY** The electric current I (in amperes) an appliance uses is given by the formula $I = \sqrt{\frac{P}{R}}$, where P is the power (in watts) and R is the resistance (in ohms). Find the current an appliance uses when the power is 147 watts and the resistance is 4 ohms.

20. **BASEBALL** You drop a baseball from a height of 56 feet. Use the expression $\sqrt{\frac{h}{16}}$, where h is the height (in feet), to find the time (in seconds) it takes the baseball to hit the ground.

Simplify the expression.

21. $\frac{6 + \sqrt{44}}{2}$ 22. $\frac{-7 - \sqrt{98}}{7}$ 23. $\frac{10 + \sqrt{300}}{5}$
 24. $\frac{-3 - \sqrt{80}}{6}$ 25. $\frac{2 + \sqrt{28}}{4}$ 26. $\frac{-4 + \sqrt{32}}{-2(5)}$

27. **VOLUME** A pet store installs a new aquarium in your teacher's classroom. What is the volume of the aquarium?

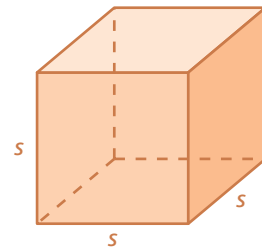


28. **BILLBOARD** What is the area of the rectangular billboard?

Simplify the expression. Assume all variables are positive.

29. $\sqrt{42x^2y^2}$ 30. $\sqrt{25y^2z}$ 31. $\sqrt{18x^3y^2z}$

32. **Modeling** Write an equation that represents the side length s of a cube as a function of the surface area A of the cube. Find the side length when the surface area is 72 square feet.



Fair Game Review what you learned in previous grades & lessons

Evaluate the expression. (*Skills Review Handbook*)

33. 3^5 34. 2^4 35. 5^3
 36. **MULTIPLE CHOICE** Which value is equivalent to $6(0.2)^3$? (*Skills Review Handbook*)
 (A) 0.008 (B) 0.048 (C) 1.728 (D) 3.6