

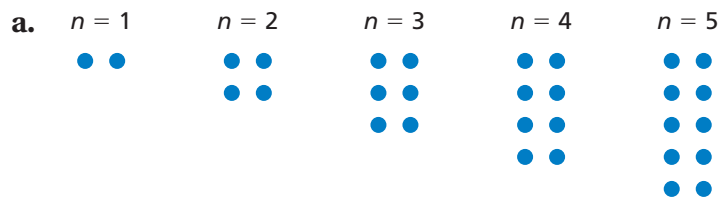
# 5.6 Arithmetic Sequences

**Essential Question** How are arithmetic sequences used to describe patterns?

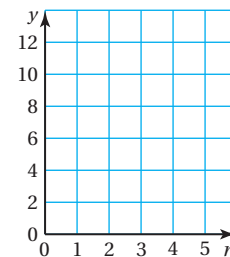
## 1 ACTIVITY: Describing a Pattern

Work with a partner.

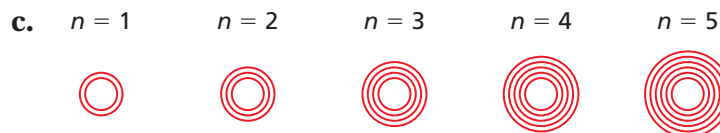
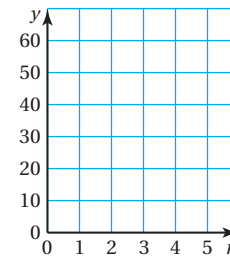
- Use the figures to complete the table.
- Plot the points in your completed table.
- Describe the pattern of the  $y$ -values.



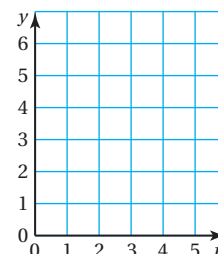
Number of Rows, $n$	1	2	3	4	5
Number of Dots, $y$					



Number of Stars, $n$	1	2	3	4	5
Number of Sides, $y$					



$n$	1	2	3	4	5
Number of Circles, $y$					



### Arithmetic Sequences

In this lesson, you will

- extend and graph arithmetic sequences.
- write equations for arithmetic sequences.
- solve real-life problems.

Learning Standards

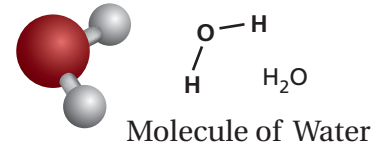
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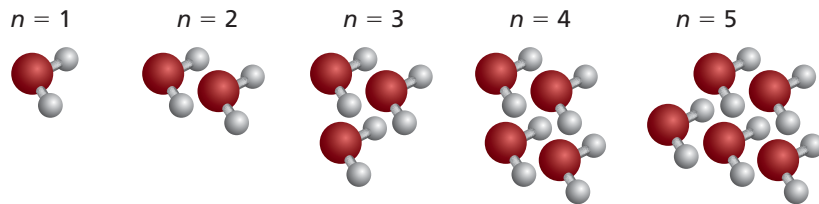
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## 2 ACTIVITY: Using a Pattern in Science to Predict

Work with a partner. In chemistry, water is called  $H_2O$  because each molecule of water has 2 hydrogen atoms and 1 oxygen atom.



- Use the figures to complete the table.
- Describe the pattern of the  $y$ -values.
- Use your pattern to predict the number of atoms in 23 molecules.



Number of Molecules, $n$	1	2	3	4	5
Number of Atoms, $y$					

## 3 ACTIVITY: Writing a Story

### Math Practice 1

**Make a Plan**  
What is your plan for writing your story?

Work with a partner.

- Describe the pattern in the table.
- Write and illustrate a story using the numbers in the table.
- Graph the data shown in the table.



Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
12	20	28	36	44	52	60	68	76	84	92	100

## What Is Your Answer?

4. **IN YOUR OWN WORDS** How are arithmetic sequences used to describe patterns? Give an example from real life.

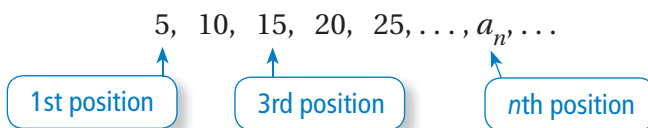
### Practice

Use what you learned about arithmetic sequences to complete Exercise 3 on page 247.

### Key Vocabulary

sequence, p. 244  
term, p. 244  
arithmetic sequence, p. 244  
common difference, p. 244

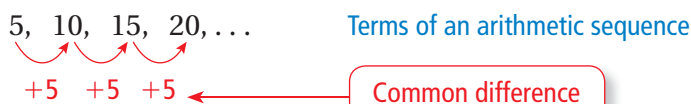
A **sequence** is an ordered list of numbers. Each number in a sequence is called a **term**. Each term  $a_n$  has a specific position  $n$  in the sequence.



## Key Idea

### Arithmetic Sequence

In an **arithmetic sequence**, the difference between consecutive terms is the same. This difference is called the **common difference**. Each term is found by adding the common difference to the previous term.

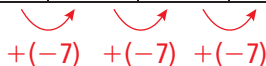


## EXAMPLE 1 Extending an Arithmetic Sequence

Write the next three terms of the arithmetic sequence  
 $-7, -14, -21, -28, \dots$

Use a table to organize the terms and find the pattern.

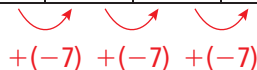
Position	1	2	3	4
Term	-7	-14	-21	-28



Each term is 7 less than the previous term. So, the common difference is  $-7$ .

Add  $-7$  to a term to find the next term.

Position	1	2	3	4	5	6	7
Term	-7	-14	-21	-28	-35	-42	-49



∴ The next three terms are  $-35, -42,$  and  $-49$ .

## On Your Own

Write the next three terms of the arithmetic sequence.

- $-12, 0, 12, 24, \dots$
- $0.2, 0.6, 1, 1.4, \dots$
- $4, 3\frac{3}{4}, 3\frac{1}{2}, 3\frac{1}{4}, \dots$

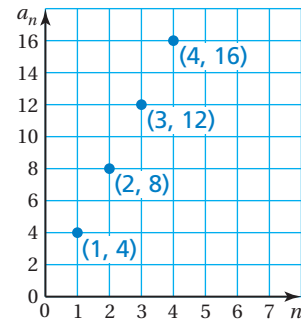
Now You're Ready  
Exercises 13–18

## EXAMPLE 2 Graphing an Arithmetic Sequence

Graph the arithmetic sequence 4, 8, 12, 16, ... What do you notice?

Make a table. Then plot the ordered pairs  $(n, a_n)$ .

Position, $n$	Term, $a_n$
1	4
2	8
3	12
4	16



∴ The points of the graph lie on a line.

### On Your Own

Now You're Ready  
Exercises 25–28

Write the next three terms of the arithmetic sequence. Then graph the sequence.

4. 3, 6, 9, 12, ...      5. 4, 2, 0, -2, ...      6. 1, 0.8, 0.6, 0.4, ...

Because consecutive terms of an arithmetic sequence have a common difference, the sequence has a constant rate of change. So, the points of any arithmetic sequence lie on a line. You can use the first term and the common difference to write a linear function that describes an arithmetic sequence.

Position, $n$	Term, $a_n$	Written using $a_1$ and $d$	Numbers
1	first term, $a_1$	$a_1$	4
2	second term, $a_2$	$a_1 + d$	$4 + 4 = 8$
3	third term, $a_3$	$a_1 + 2d$	$4 + 2(4) = 12$
4	fourth term, $a_4$	$a_1 + 3d$	$4 + 3(4) = 16$
∴	∴	∴	∴
$n$	$n$ th term, $a_n$	$a_1 + (n - 1)d$	$4 + (n - 1)(4)$

### Key Idea

#### Equation for an Arithmetic Sequence

Let  $a_n$  be the  $n$ th term of an arithmetic sequence with first term  $a_1$  and common difference  $d$ . The  $n$ th term is given by

$$a_n = a_1 + (n - 1)d.$$

### EXAMPLE 3 Writing an Equation for an Arithmetic Sequence

#### Study Tip

Notice that the equation in Example 3 is of the form  $y = mx + b$ , where  $y$  is replaced by  $a_n$  and  $x$  is replaced by  $n$ .

Write an equation for the  $n$ th term of the arithmetic sequence 14, 11, 8, 5, ... Then find  $a_{50}$ .

The first term is 14 and the common difference is  $-3$ .

$$a_n = a_1 + (n - 1)d \quad \text{Equation for an arithmetic sequence}$$

$$a_n = 14 + (n - 1)(-3) \quad \text{Substitute 14 for } a_1 \text{ and } -3 \text{ for } d.$$

$$a_n = -3n + 17 \quad \text{Simplify.}$$

Use the equation to find the 50th term.

$$a_n = -3n + 17 \quad \text{Write the equation.}$$

$$a_{50} = -3(50) + 17 \quad \text{Substitute 50 for } n.$$

$$= -133 \quad \text{Simplify.}$$

### EXAMPLE 4 Real-Life Application



Online bidding for a purse increases \$5 for each bid after the \$60 initial bid.

Bid Number	1	2	3
Bid Amount	\$60	\$65	\$70

a. Write an equation for the  $n$ th term of the arithmetic sequence.

The first term is 60 and the common difference is 5.

$$a_n = a_1 + (n - 1)d \quad \text{Equation for an arithmetic sequence}$$

$$a_n = 60 + (n - 1)5 \quad \text{Substitute 60 for } a_1 \text{ and 5 for } d.$$

$$a_n = 5n + 55 \quad \text{Simplify.}$$

b. The winning bid is \$90. How many bids were there?

Use the equation to find the value of  $n$  for which  $a_n = 90$ .

$$a_n = 5n + 55 \quad \text{Write the equation.}$$

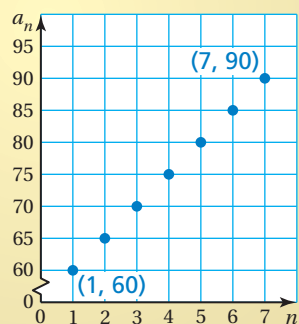
$$90 = 5n + 55 \quad \text{Substitute 90 for } a_n.$$

$$35 = 5n \quad \text{Subtract 55 from each side.}$$

$$7 = n \quad \text{Divide each side by 5.}$$

∴ There were 7 bids.

#### Check



#### On Your Own

Write an equation for the  $n$ th term of the arithmetic sequence. Then find  $a_{25}$ .

7. 4, 5, 6, 7, ...

8. 8, 16, 24, 32, ...

9.  $-2, -1, 0, 1, \dots$

10. **WHAT IF?** In Example 4, the winning bid is \$105. How many bids were there?

Now You're Ready  
Exercises 33–38

## Vocabulary and Concept Check

- VOCABULARY** How do you find the common difference of an arithmetic sequence?
- WRITING** How are the graphs of arithmetic sequences and linear functions similar? How are they different?

## Practice and Problem Solving

Use the figures to complete the table. Then describe the pattern of the y-values.



Number of Quarters, $n$	1	2	3	4
Number of Cents, $y$				

Write the next three terms of the arithmetic sequence.

- |                                           |                                             |                                                       |
|-------------------------------------------|---------------------------------------------|-------------------------------------------------------|
| 4. First term: 2<br>Common difference: 11 | 5. First term: 18<br>Common difference: 3.5 | 6. First term: 0<br>Common difference: $4\frac{1}{2}$ |
|-------------------------------------------|---------------------------------------------|-------------------------------------------------------|

Find the common difference of the arithmetic sequence.

- |                                               |                                |                             |
|-----------------------------------------------|--------------------------------|-----------------------------|
| 7. 5, 10, 15, 20, ...                         | 8. 16.1, 14.1, 12.1, 10.1, ... | 9. 100, 125, 150, 175, ...  |
| 10. $3, 3\frac{1}{2}, 4, 4\frac{1}{2}, \dots$ | 11. 6.5, 5, 3.5, 2, ...        | 12. 350, 500, 650, 800, ... |

Write the next three terms of the arithmetic sequence.

- |                         |                           |                                                                 |
|-------------------------|---------------------------|-----------------------------------------------------------------|
| 13. 10, 13, 16, 19, ... | 14. 1, 12, 23, 34, ...    | 15. 16, 21, 26, 31, ...                                         |
| 16. 60, 30, 0, -30, ... | 17. 1.3, 1, 0.7, 0.4, ... | 18. $\frac{5}{6}, \frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \dots$ |

19. **PATTERN** Write a sequence to represent the number of smiley faces in each group. Is the sequence arithmetic? Explain.



Determine whether the sequence is arithmetic. If so, find the common difference.

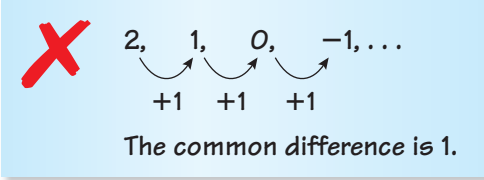
20. 13, 26, 39, 52, ...

21. 5, 9, 14, 20, ...

22. 6, 12, 24, 48, ...

23. 69, 75, 81, 87, ...

24. **ERROR ANALYSIS** Describe and correct the error in finding the common difference of the arithmetic sequence.



The common difference is 1.

Write the next three terms of the arithmetic sequence. Then graph the sequence.

25. 7, 6.4, 5.8, 5.2, ...

26. -15, 0, 15, 30, ...

27.  $\frac{1}{2}, \frac{5}{8}, \frac{3}{4}, \frac{7}{8}, \dots$

28. -1, -3, -5, -7, ...



29. **NUMBER SENSE** The first term of an arithmetic sequence is 3. The common difference of the sequence is 1.5 times the first term. Write the next three terms of the sequence. Then graph the sequence.

30. **DOMINOES** The first row of a dominoes display has 10 dominoes. Each row after the first has two more dominoes than the row before it. Write the first five terms of the sequence that represents the number of dominoes in each row. Then graph the sequence.

31. **ZOO** A zoo charges \$8 per person for admission.

- Copy and complete the table.
- Do the costs in your table show an arithmetic sequence? If so, graph the sequence.
- What is the cost for one person to visit the zoo six times?
- An annual family pass costs \$130. How many times does a family of five have to visit the zoo for the annual pass to be the better deal? Explain.

Number of Visits in One Year	Cost
1	\$8
2	
3	
4	

32. **REPEATED REASONING** Firewood is stacked in a pile. The bottom row has 20 logs and the top row has 14 logs. Each row has one more log than the row above it. How many logs are in the pile?

Write an equation for the  $n$ th term of the arithmetic sequence. Then find  $a_{10}$ .

33. -5, -4, -3, -2, ...

34. -3, -6, -9, -12, ...

35.  $\frac{1}{2}, 1, 1\frac{1}{2}, 2, \dots$

36. 10, 11, 12, 13, ...

37. -10, -20, -30, -40, ...

38.  $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \dots$

39. **MOVIE REVENUE** A movie earns \$100 million the first week it is released. The movie earns \$20 million less each additional week. Write an equation for the  $n$ th term of the arithmetic sequence.

40. **REASONING** Are the terms of an arithmetic sequence independent or dependent? Explain your reasoning.

41. **SPEED** On a highway, you take 3 seconds to increase your speed from 32 to 35 miles per hour. Your speed increases the same amount each second.

- Write the first four terms of the sequence that represents your speed each second.
- Write an equation that describes the arithmetic sequence.
- The speed limit is 65 miles per hour. What is the domain of the function?

42. **OPEN-ENDED** Write the first four terms of two different arithmetic sequences with a common difference of  $-3$ . Write an equation for the  $n$ th term of each sequence.



43. **REASONING** Is the domain of an arithmetic sequence discrete or continuous? Describe the types of numbers in the domain.

44. **EARTH DAY** You and a group of friends take turns planting 2 trees each at a campsite. After the first person plants 2 trees, there are 12 trees at the campsite.

- Write an equation for the  $n$ th term of the sequence.
- What do you notice about the slope given by the equation and the common difference of the sequence?
- After 8 more people plant trees, how many trees are at the campsite?

45. **Critical Thinking** The number of births in a country each minute after midnight January 1st can be estimated by the sequence in the table.

- Write an equation for the  $n$ th term of the sequence.
- Is the domain discrete or continuous?
- Explain how to use your function to estimate the number of births in a day.

<b>Minutes after Midnight January 1st</b>	1	2	3	4
<b>Babies Born</b>	5	10	15	20



### Fair Game Review What you learned in previous grades & lessons

Solve the system of linear equations by graphing. (Section 4.1)

46.  $y = 2x$

$y = 3x + 2$

47.  $y = -2x + 6$

$y = \frac{1}{4}x - 3$

48.  $y + x = 0$

$y + 2 = -\frac{1}{2}x$

49. **MULTIPLE CHOICE** What expression is equivalent to  $4^5$ ? (Skills Review Handbook)

(A)  $4 \cdot 5$

(B)  $4 \cdot 4 \cdot 4 \cdot 4$

(C)  $5^4$

(D)  $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$