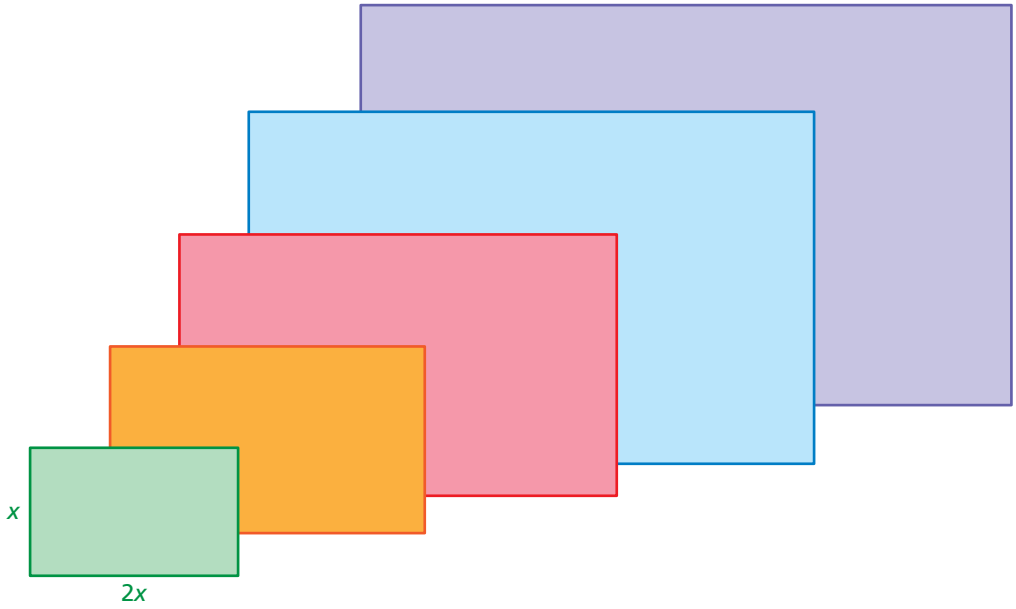
5.5 Comparing Linear and Nonlinear Functions

Essential Question How can you recognize when a pattern in real life is linear or nonlinear?

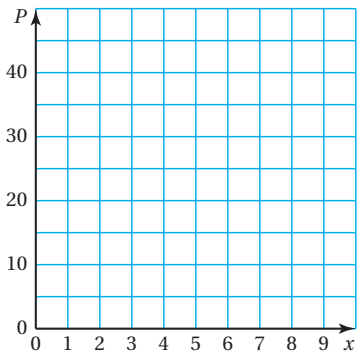
1 ACTIVITY: Finding Patterns for Similar Figures

Work with a partner. Copy and complete each table for the sequence of similar rectangles. Graph the data in each table. Decide whether each pattern is linear or nonlinear.



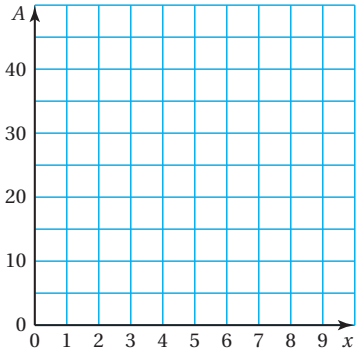
a. Perimeters of Similar Rectangles

x	1	2	3	4	5
P					



b. Areas of Similar Rectangles

x	1	2	3	4	5
A					



Common Core

Functions

In this lesson, you will
- identify linear and nonlinear functions from tables or graphs.

Learning Standards
8.F.3
F.LE.1b

2 ACTIVITY: Comparing Linear and Nonlinear Functions

Math Practice 4

Interpret Results

How do the graphs help you to answer the question? Does your answer make sense?

Work with a partner. The table shows the height h (in feet) of a falling object at t seconds.

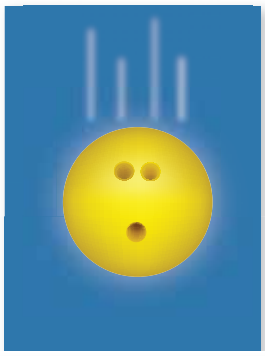
- **Graph the data in the table.**
- **Decide whether the graph is linear or nonlinear.**
- **Compare the two falling objects. Which one has an increasing speed?**

a. Falling parachute jumper

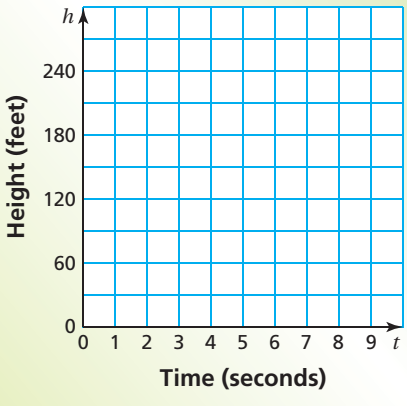
t	0	1	2	3	4
h	300	285	270	255	240

b. Falling bowling ball

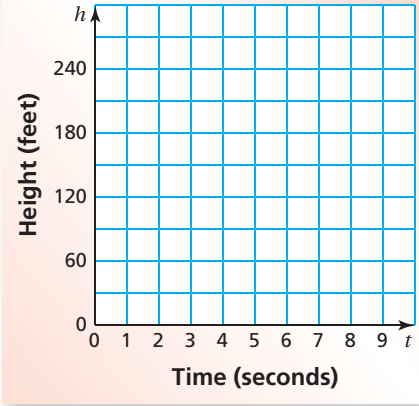
t	0	1	2	3	4
h	300	284	236	156	44



Parachute Jumper



Bowling Ball



What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you recognize when a pattern in real life is linear or nonlinear? Describe two real-life patterns: one that is linear and one that is nonlinear. Use patterns that are different from those described in Activities 1 and 2.

Practice Use what you learned about comparing linear and nonlinear functions to complete Exercises 3–6 on page 240.

5.5 Lesson

Check It Out
Lesson Tutorials
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Key Vocabulary
nonlinear function, *p. 238*

The graph of a linear function shows a constant rate of change. A **nonlinear function** does not have a constant rate of change. So, its graph is *not* a line.

EXAMPLE 1 Identifying Functions from Tables

Does the table represent a *linear* or *nonlinear* function? Explain.

a.

x	3	6	9	12
y	40	32	24	16

+3 +3 +3

−8 −8 −8

As x increases by 3, y decreases by 8. The rate of change is constant. So, the function is linear.

b.

x	1	3	5	7
y	2	11	33	88

+2 +2 +2

+9 +22 +55

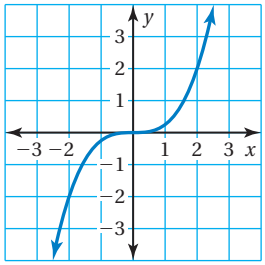
As x increases by 2, y increases by different amounts. The rate of change is *not* constant. So, the function is nonlinear.

Study Tip
A constant rate of change describes a quantity that changes by equal amounts over equal intervals.

EXAMPLE 2 Identifying Functions from Graphs

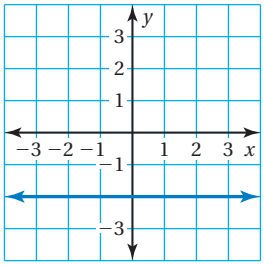
Does the graph represent a *linear* or *nonlinear* function? Explain.

a.



The graph is *not* a line. So, the function is nonlinear.

b.



The graph is a line. So, the function is linear.

On Your Own

Now You're Ready
Exercises 3–11

Does the table or graph represent a *linear* or *nonlinear* function? Explain.

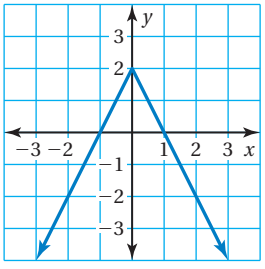
1.

x	y
0	25
7	20
14	15
21	10

2.

x	y
2	8
4	4
6	0
8	−4

3.



Multi-Language Glossary at BigIdeasMath.com

EXAMPLE 3 Identify a Function from an Equation

Which equation represents a *nonlinear* function?

(A) $y = 4.7$ (B) $y = \pi x$

(C) $y = \frac{4}{x}$ (D) $y = 4(x - 1)$

You can rewrite the equations $y = 4.7$, $y = \pi x$, and $y = 4(x - 1)$ in slope-intercept form. So, they are linear functions.

You cannot rewrite the equation $y = \frac{4}{x}$ in slope-intercept form. So, it is a nonlinear function.

∴ The correct answer is (C).

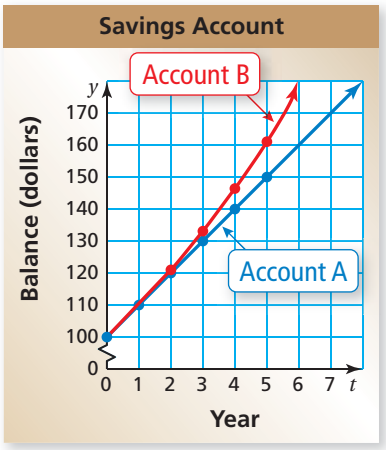
EXAMPLE 4 Real-Life Application

Study Tip

In Example 4, the *initial value* of each function is \$100.

Account A earns simple interest. Account B earns compound interest. The table shows the balances for 5 years. Graph the data and compare the graphs.

Year, t	Account A Balance	Account B Balance
0	\$100	\$100
1	\$110	\$110
2	\$120	\$121
3	\$130	\$133.10
4	\$140	\$146.41
5	\$150	\$161.05



Both graphs show that the balances are positive and increasing.

The balance of Account A has a constant rate of change of \$10. So, the function representing the balance of Account A is linear.

The balance of Account B increases by different amounts each year. Because the rate of change is not constant, the function representing the balance of Account B is nonlinear.

On Your Own

Now You're Ready
Exercises 12–14

Does the equation represent a *linear* or *nonlinear* function? Explain.

4. $y = x + 5$ **5.** $y = \frac{4x}{3}$ **6.** $y = 1 - x^2$

5.5 Exercises

Check It Out
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Vocabulary and Concept Check

1. **VOCABULARY** Describe how linear functions and nonlinear functions are different.

2. **WHICH ONE DOESN'T BELONG?** Which equation does *not* belong with the other three? Explain your reasoning.

$5y = 2x$ $\quad y = \frac{2}{5}x$ $\quad 10y = 4x$ $\quad 5xy = 2$

Practice and Problem Solving

Graph the data in the table. Decide whether the function is *linear* or *nonlinear*.

1

3.

x	0	1	2	3
y	4	8	12	16

4.

x	1	2	3	4
y	1	2	6	24

5.

x	6	5	4	3
y	21	15	10	6

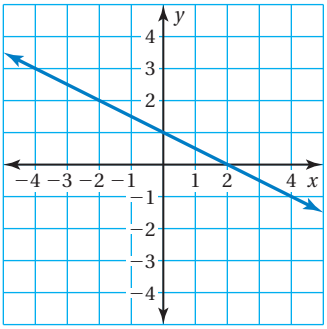
6.

x	−1	0	1	2
y	−7	−3	1	5

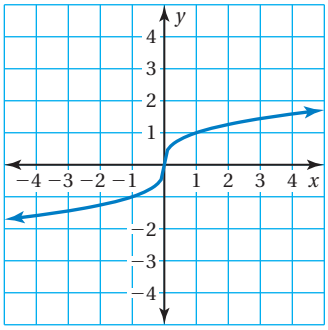
Does the table or graph represent a *linear* or *nonlinear* function? Explain.

2

7.



8.



9.

x	5	11	17	23
y	7	11	15	19

10.

x	−3	−1	1	3
y	9	1	1	9

11. **VOLUME** The table shows the volume V (in cubic feet) of a cube with a side length of x feet. Does the table represent a linear or nonlinear function? Explain.

Side Length, x	1	2	3	4	5	6	7	8
Volume, V	1	8	27	64	125	216	343	512

Does the equation represent a *linear* or *nonlinear* function? Explain.

12. $2x + 3y = 7$ **13.** $y + x = 4x + 5$ **14.** $y = \frac{8}{x^2}$

15. LIGHT The frequency y (in terahertz) of a light wave is a function of its wavelength x (in nanometers). Does the table represent a linear or nonlinear function? Explain.

Color	Red	Yellow	Green	Blue	Violet
Wavelength, x	660	595	530	465	400
Frequency, y	454	504	566	645	749

16. MODELING The table shows the cost y (in dollars) of x pounds of sunflower seeds.

Pounds, x	Cost, y
2	2.80
3	?
4	5.60

a. What is the missing y-value that makes the table represent a linear function?

b. Write a linear function that represents the cost y of x pounds of seeds.

c. What is the initial value of the function?

d. Does the function have a maximum value? Explain your reasoning.

17. TREES Tree A grows at a rate of 1.5 feet per year. The table shows the height h (in feet) of Tree B after x years.

Years, x	Height, h
0	0
2	3.2
5	8

a. Does the table represent a linear or nonlinear function? Explain.

b. Which tree is growing at a faster rate? Explain.

18. PRECISION The radius of the base of a cylinder is 3 feet. Is the volume of the cylinder a linear or nonlinear function of the height of the cylinder? Explain.

19. Number Sense The ordered pairs represent a function.

(0, 0), (1, 1), (2, 4), (3, 9), and (4, 16)

a. Graph the ordered pairs and describe the pattern. Is the function linear or nonlinear?

b. Write an equation that represents the function.

Fair Game Review *What you learned in previous grades & lessons*

Find the square root(s). *(Skills Review Handbook)*

20. $\sqrt{49}$ **21.** $-\sqrt{36}$ **22.** $\pm\sqrt{9}$

23. MULTIPLE CHOICE Which of the following equations has a slope of -2 and passes through the point (2, 3)? *(Section 2.6)*

(A) $y = -2x + 6$ (B) $y - 3 = -2(x + 2)$ (C) $y = -2x + 7$ (D) $y - 2 = -2(x - 3)$