Extension 5.4 Special Functions

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Lesson Tutorials
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Key Vocabulary
piecewise function, *p. 232*
step function, *p. 233*
absolute value function, *p. 234*

COMMON CORE

Functions

In this extension, you will
- graph piecewise, step, and absolute value functions.

Learning Standards
F.BF.3
F.IF.1
F.IF.2
F.IF.7b

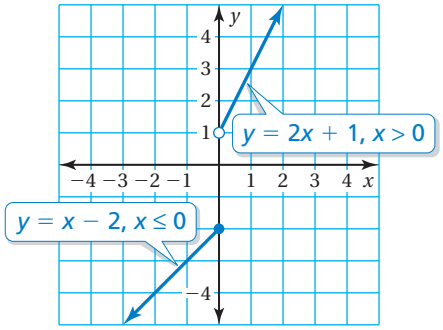
Key Idea

Piecewise Function

A **piecewise function** is a function defined by two or more equations. Each "piece" of the function applies to a different part of its domain. An example is shown below.

$$y = \begin{cases} x - 2, & \text{if } x \le 0 \\ 2x + 1, & \text{if } x > 0 \end{cases}$$

- The expression $x - 2$ gives the value of y when x is less than or equal to 0.
- The expression $2x + 1$ gives the value of y when x is greater than 0.

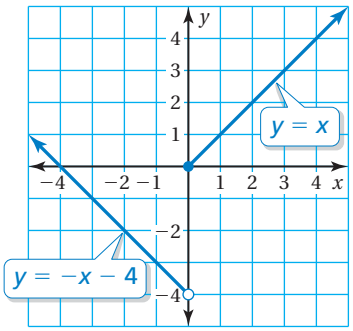


EXAMPLE 1 Graphing a Piecewise Function

Graph $y = \begin{cases} -x - 4, & \text{if } x < 0 \\ x, & \text{if } x \ge 0 \end{cases}$. Describe the domain and range.

Step 1: Graph $y = -x - 4$ for $x < 0$. Because x is not equal to 0, use an open circle at $(0, -4)$.

Step 2: Graph $y = x$ for $x \ge 0$. Because x is greater than or equal to 0, use a closed circle at $(0, 0)$.



∴ The domain is all real numbers. The range is $y > -4$.

Practice

Graph the function. Describe the domain and range.

1. $y = \begin{cases} x + 3, & \text{if } x \le 0 \\ -x, & \text{if } x > 0 \end{cases}$

2. $y = \begin{cases} x - 2, & \text{if } x < 0 \\ 4x, & \text{if } x \ge 0 \end{cases}$

3. $y = \begin{cases} -3x - 2, & \text{if } x \le 1 \\ x + 1, & \text{if } x > 1 \end{cases}$

4. $y = \begin{cases} 2x, & \text{if } x < -1 \\ -2x, & \text{if } x \ge -1 \end{cases}$

5. $y = \begin{cases} 1, & \text{if } x < -3 \\ x - 1, & \text{if } -3 \le x \le 3 \\ -2, & \text{if } x > 3 \end{cases}$

6. $y = \begin{cases} -x + 2, & \text{if } x \le -2 \\ 5, & \text{if } -2 < x < 1 \\ 3x, & \text{if } x \ge 1 \end{cases}$

7. **REASONING** Does $y = \begin{cases} 1 - x, & \text{if } x \le 0 \\ x - 1, & \text{if } x \ge -2 \end{cases}$ represent a function? Explain your reasoning.

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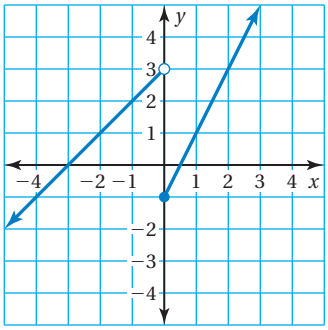
EXAMPLE 2 Writing a Piecewise Function

Write a piecewise function for the graph.



Each "piece" of the function is linear.

When $x < 0$, the graph is the line given by $y = x + 3$.

When $x \geq 0$, the graph is the line given by $y = 2x - 1$.

So, a piecewise function for the graph is $f(x) = \begin{cases} x + 3, & \text{if } x < 0 \\ 2x - 1, & \text{if } x \geq 0 \end{cases}$.

Study Tip

The graph of a step function can look like a staircase.

A **step function** is a piecewise function defined by constant values over its domain. The graph of a step function consists of a series of line segments.

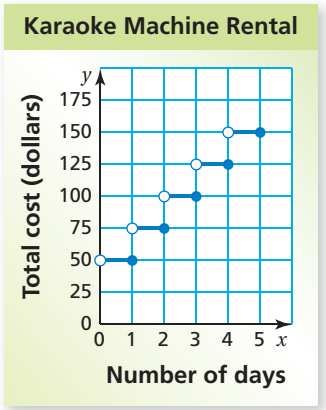
EXAMPLE 3 Graphing a Step Function

You rent a karaoke machine for 5 days. The rental company charges $50 for the first day and $25 for each additional day. Write and graph a step function that represents the relationship between the number of days x and the total cost of renting the karaoke machine.

Use a table to organize the information.

Time (days)	Total Cost
$0 < x \leq 1$	50
$1 < x \leq 2$	75
$2 < x \leq 3$	100
$3 < x \leq 4$	125
$4 < x \leq 5$	150

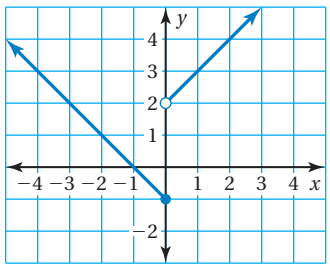
$$f(x) = \begin{cases} 50, & \text{if } 0 < x \leq 1 \\ 75, & \text{if } 1 < x \leq 2 \\ 100, & \text{if } 2 < x \leq 3 \\ 125, & \text{if } 3 < x \leq 4 \\ 150, & \text{if } 4 < x \leq 5 \end{cases}$$



Practice

Write a piecewise function for the graph.

8.



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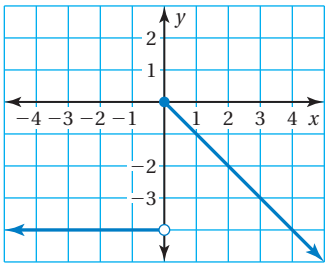

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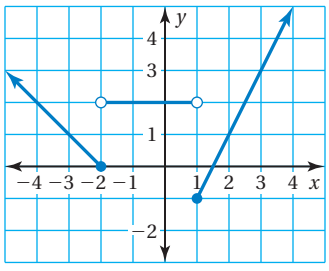


11. LANDSCAPING A landscaper rents a wood chipper for 4 days. The rental company charges $100 for the first day and $50 for each additional day. Write and graph a step function that represents the relationship between the number of days x and the total cost of renting the chipper.

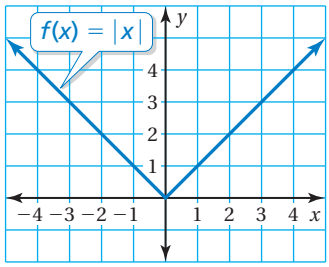
Key Idea

Absolute Value Function

An **absolute value function** has a V-shaped graph that opens up or down.

The most basic absolute value function is $f(x) = |x|$.

The absolute value of a number is always nonnegative. So, the range of $f(x) = |x|$ is $y \geq 0$.



Study Tip

The absolute value function $f(x) = |x|$ can be written as a piecewise function.

$$f(x) = \begin{cases} -x, & \text{if } x < 0 \\ 0, & \text{if } x = 0 \\ x, & \text{if } x > 0 \end{cases}$$

EXAMPLE 4 Graphing Absolute Value Functions

Graph each function. Compare the graph to the graph of $y = |x|$. Describe the domain and range.

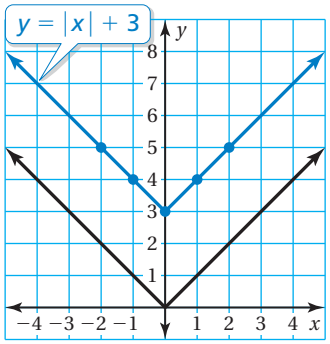
a. $y = |x| + 3$

Step 1: Make a table of values.

x	−2	−1	0	1	2
y	5	4	3	4	5

Step 2: Plot the ordered pairs.

Step 3: Draw the V-shaped graph.



The graph of $y = |x| + 3$ is a translation 3 units up of the graph of $y = |x|$. The domain is all real numbers. The range is $y \geq 3$.

Study Tip

The function $y = |x| + 3$ can be written as a piecewise function.

$$f(x) = \begin{cases} -x + 3, & \text{if } x < 0 \\ x + 3, & \text{if } x \geq 0 \end{cases}$$

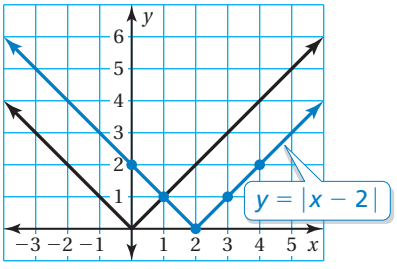
b. $y = |x - 2|$

Step 1: Make a table of values.

x	0	1	2	3	4
y	2	1	0	1	2

Step 2: Plot the ordered pairs.

Step 3: Draw the V-shaped graph.



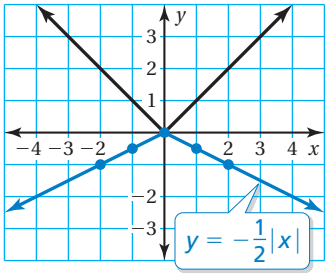
The graph of $y = |x - 2|$ is a translation 2 units to the right of the graph of $y = |x|$. The domain is all real numbers. The range is $y \geq 0$.

EXAMPLE 5 Graphing Absolute Value Functions

Graph $y = -\frac{1}{2}|x|$. Compare the graph to the graph of $y = |x|$. Describe the domain and range.

Step 1: Make a table of values.

x	-2	-1	0	1	2
y	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1



Step 2: Plot the ordered pairs.

Step 3: Draw the V-shaped graph.

The graph of $y = -\frac{1}{2}|x|$ opens down and is wider than the graph of $y = |x|$. The domain is all real numbers. The range is $y \leq 0$.

Practice

Graph the function. Compare the graph to the graph of $y = |x|$. Describe the domain and range.

12. $y = |x| - 1$

13. $y = |x| + 5$

14. $y = |x + 4|$

15. $y = |x - 3|$

16. $y = \frac{1}{4}|x|$

17. $y = -3|x|$

18. $y = |x + 1| - 2$

19. $y = -|x - 5| + 1$

20. $y = 4|x| - 4$

Write an equation for the given translation of $y = |x|$.

21. 7 units down

22. 10 units left

23. 1 unit down and 5 units right

24. 4 units up and 6 units left

25. REASONING Explain how the graph of each function compares to the graph of $y = |x|$ for positive and negative values of k, h, and a.

a. $y = |x| + k$

b. $y = |x - h|$

c. $y = a|x|$

Solve each equation using a graph. Check your solution.

26. $|x - 1| = 3$

27. $|x + 2| - 6 = -1$

28. $2|x + 7| = 4$

29. STRUCTURE Rewrite the function $y = |x + 4|$ using piecewise notation.

30. STRUCTURE Graph $y = \begin{cases} -x + 5, & \text{if } x \leq 0 \\ |x|, & \text{if } x > 0 \end{cases}$. Describe the domain and range.