

5.4 Function Notation

Essential Question How can you use function notation to represent a function?

By naming a function f , you can write the function using **function notation**.

$$f(x) = 2x - 3 \quad \text{Function notation}$$

This is read as “ f of x equals $2x$ minus 3.” The notation $f(x)$ is another name for y . When function notation is used, the parentheses do not imply multiplication. You can use letters other than f to name a function. The letters g , h , j , and k are often used to name functions.



1 ACTIVITY: Matching Functions with Their Graphs

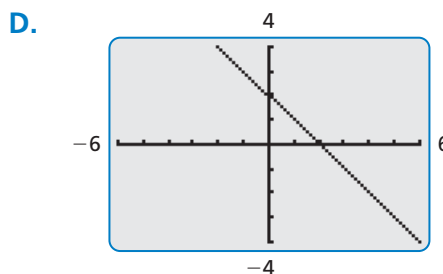
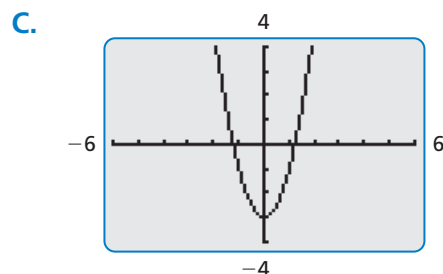
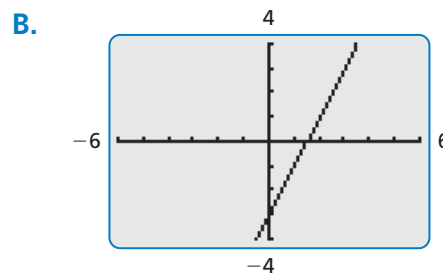
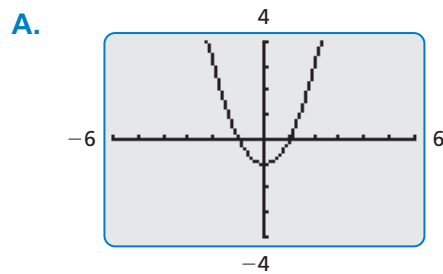
Work with a partner. Match each function with its graph.

a. $f(x) = 2x - 3$

b. $g(x) = -x + 2$

c. $h(x) = x^2 - 1$

d. $j(x) = 2x^2 - 3$



Functions

In this lesson, you will

- evaluate, solve, and graph functions written in function notation.
- compare graphs of linear functions.

Learning Standards

F.BF.3

F.IF.1

F.IF.2

F.IF.7b

2 ACTIVITY: Evaluating a Function

Math Practice 8

Evaluate Results

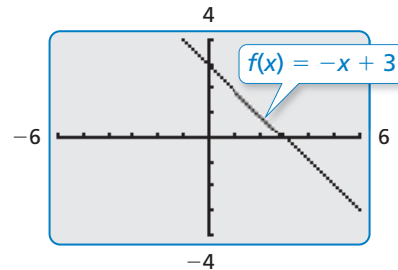
Does your answer seem reasonable?
How can you check your answer?

Work with a partner. Consider the function

$$f(x) = -x + 3.$$

Locate the points $(x, f(x))$ on the graph. Explain how you found each point.

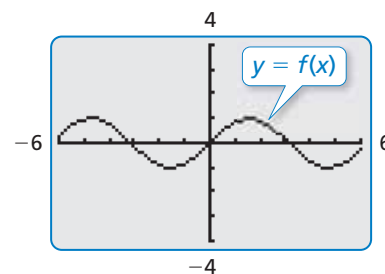
- $(-1, f(-1))$
- $(0, f(0))$
- $(1, f(1))$
- $(2, f(2))$



3 ACTIVITY: Comparing Graphs of Functions

Work with a partner. The graph of a function from trigonometry is shown at the right. Use the graph to sketch the graph of each function. Explain your reasoning.

- $g(x) = f(x) + 2$
- $g(x) = f(x) + 1$
- $g(x) = f(x) - 1$
- $g(x) = f(x) - 2$



What Is Your Answer?

- IN YOUR OWN WORDS** How can you use function notation to represent a function? How are standard notation and function notation similar? How are they different?

Standard Notation

$$y = 2x + 5$$

Function Notation

$$f(x) = 2x + 5$$

- Use what you discovered in Activity 3 to write a general observation that compares the graphs of

$$y = f(x) \quad \text{and} \quad y = f(x) + c.$$

Practice

Use what you learned about function notation to complete Exercises 4–6 on page 229.

Key Vocabulary

function notation,
p. 226

In Section 5.3, you learned that you can write a linear function in the form $y = mx + b$. By naming a linear function f , you can also write the function using **function notation**.

$$f(x) = mx + b \quad \text{Function notation}$$

The notation $f(x)$ is another name for y . If f is a function and x is in its domain, then $f(x)$ represents the output of f corresponding to the input x . You can use letters other than f to name a function, such as g or h .

EXAMPLE 1 Evaluating a Function**Reading**

The notation $f(x)$ is read as "the value of f at x " or " f of x ." It does not mean " f times x ."

Evaluate $f(x) = -4x + 7$ when $x = 2$.

$f(x) = -4x + 7$	Write the function.
$f(2) = -4(2) + 7$	Substitute 2 for x .
$= -8 + 7$	Multiply.
$= -1$	Add.

∴ When $x = 2$, $f(x) = -1$.

On Your Own

Evaluate the function when $x = -4, 0$, and 3 .

1. $f(x) = 2x - 5$	2. $g(x) = -x - 1$
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Now You're Ready
Exercises 4–9

EXAMPLE 2 Solving for the Independent Variable

For $h(x) = \frac{2}{3}x - 5$, find the value of x for which $h(x) = -7$.

$h(x) = \frac{2}{3}x - 5$	Write the function.
$-7 = \frac{2}{3}x - 5$	Substitute -7 for $h(x)$.
$-2 = \frac{2}{3}x$	Add 5 to each side.
$-3 = x$	Multiply each side by $\frac{3}{2}$.

∴ When $x = -3$, $h(x) = -7$.

On Your Own

Find the value of x so that the function has the given value.

3. $f(x) = 6x + 9$; $f(x) = 21$	4. $g(x) = -\frac{1}{2}x + 3$; $g(x) = -1$
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Now You're Ready
Exercises 11–16

EXAMPLE 3 Graphing a Linear Function in Function Notation

Graph $f(x) = 2x + 5$.

Step 1: Make a table of values.

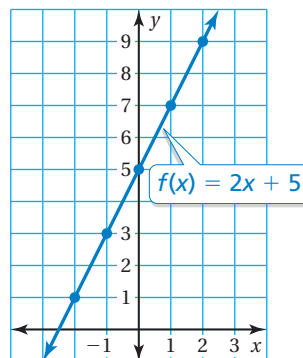
x	-2	-1	0	1	2
$f(x)$	1	3	5	7	9

Step 2: Plot the ordered pairs.

Step 3: Draw a line through the points.

Study Tip

The graph of $f(x)$ consists of the points $(x, f(x))$.



On Your Own

Graph the linear function.

Now You're Ready
Exercises 22–27

5. $f(x) = 3x - 2$

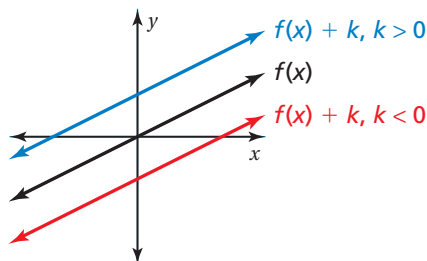
6. $g(x) = -x + 4$

7. $h(x) = -\frac{3}{4}x - 1$

Key Idea

Vertical Translations

The graph of $f(x) + k$ is a vertical translation of the graph of $f(x)$, where $k \neq 0$.

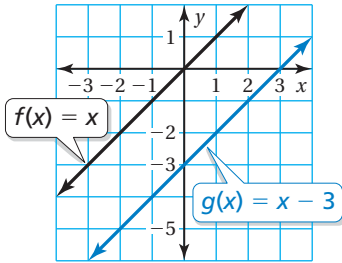


In vertical translations of graphs of linear functions, the graphs have the same slope but different y -intercepts.

EXAMPLE 4 Comparing Graphs of Linear Functions

Graph $g(x) = x - 3$. Compare the graph to the graph of $f(x) = x$.

Use the slope and y -intercept to graph the equations.



$$g(x) = x - 3$$

$$= 1x + (-3)$$

slope

y -intercept

$$f(x) = x$$

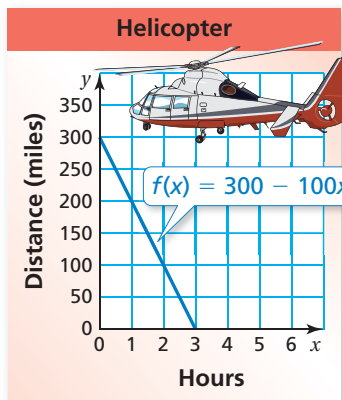
$$= 1x + 0$$

slope

y -intercept

∴ The graphs have the same slope but different y -intercepts. The graph of g is a translation 3 units down of the graph of f .

EXAMPLE 5 Real-Life Application



The graph shows the number y of miles a helicopter is from its destination after x hours on its first flight. On its second flight, the helicopter travels at the same speed but 50 miles farther. Which statement is true about the graph of the function that represents the second flight compared to the graph of the function that represents the first flight?

- (A) The slope decreases.
- (B) The slope increases.
- (C) The graph is a translation 50 units down.
- (D) The graph is a translation 50 units up.

The helicopter travels at the same speed on both flights. So, the graphs have the same slope. You can eliminate choices A and B.

Because the helicopter travels 50 miles farther on the second flight, it is 50 miles farther from its destination when $x = 0$. So, the graph of the function that represents the second flight is a vertical translation 50 units up of the graph of the function that represents the first flight.

∴ The correct answer is (D).

On Your Own

Graph the function. Compare the graph to the graph of $f(x) = -2x$.

8. $g(x) = -2x + 3$

9. $h(x) = -2x - 5$

10. **WHAT IF?** In Example 5, the helicopter travels the same distance but 50 miles per hour faster on the second flight. How does the graph of the function that represents the second flight compare to the graph of the function that represents the first flight?

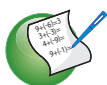
Now You're Ready
Exercises 29–31

5.4 Exercises



Vocabulary and Concept Check

- VOCABULARY** What is function notation? Give an example.
- VOCABULARY** Your height can be represented by a function $h(x)$ where x is your age. What does $h(13)$ represent?
- WRITING** What type of graph is given by $y = mx + b$? How does changing the value of b affect the graph?



Practice and Problem Solving

Evaluate the function when $x = -2, 0,$ and 5 .

- 4.** $f(x) = x + 6$ **5.** $g(x) = 3x - 2$ **6.** $h(x) = -2x + 9$
- 7.** $h(x) = -x - 7$ **8.** $g(x) = 6x - 3$ **9.** $f(x) = -5x + 2$
- 10. ERROR ANALYSIS** Describe and correct the error in evaluating the function $g(x) = 4x + 6$ when $x = -2$.



$$\begin{aligned}g(-2) &= 4(-2) + 6 \\-2g &= -8 + 6 \\-2g &= -2 \\g &= 1\end{aligned}$$

Find the value of x so that the function has the given value.

- 11.** $h(x) = -7x + 10$; $h(x) = 3$ **12.** $t(x) = -3x - 5$; $t(x) = 4$
- 13.** $n(x) = 4x + 15$; $n(x) = 7$ **14.** $p(x) = 6x - 12$; $p(x) = 18$
- 15.** $q(x) = \frac{1}{3}x - 2$; $q(x) = -4$ **16.** $r(x) = -\frac{4}{5}x + 7$; $r(x) = -5$
- 17. SUMMER JOB** You earn \$11 per hour working at a grocery store during the summer. The function $p(x) = 11x$ represents the amount you earn for working x hours.
 - You work 18 hours. How much do you earn?
 - How many hours do you have to work to earn \$275?
- 18. ORCHESTRA** A group of friends are buying tickets to the orchestra. Each ticket costs \$17.50 and one of the friends has a coupon for \$10. The function $C(x) = 17.5x - 10$ represents the total cost of buying x tickets.
 - How much does it cost to buy 5 tickets?
 - How many tickets can you buy with \$130.00?

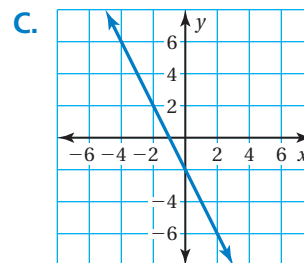
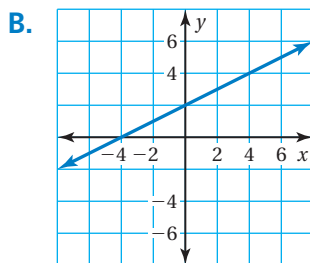
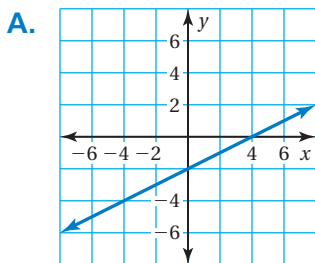


Match the function with its graph.

19. $f(x) = -2x - 2$

20. $g(x) = \frac{1}{2}x + 2$

21. $h(x) = \frac{1}{2}x - 2$



Graph the linear function.

3 22. $f(x) = 4x + 1$

23. $g(x) = -2x - 5$

24. $h(x) = -\frac{1}{2}x - 3$

25. $f(x) = \frac{3}{5}x + 2$

26. $g(x) = 7x - 4$

27. $h(x) = -6x + 3$

28. **ATMOSPHERIC TEMPERATURE** Under normal conditions, the atmospheric temperature drops 3.5°F per 1000 feet of altitude up to 40,000 feet. When the outside temperature is 80°F , the atmospheric temperature can be modeled by $t(x) = -3.5x + 80$, where x is the altitude in thousands of feet.



- Graph the function and identify its domain and range.
- Find and interpret the value of x so that $t(x) = -25$.

Graph the function. Compare the graph to the graph of $f(x) = 3x$.

4 29. $g(x) = 3x + 2$

30. $n(x) = 3x - 7$

31. $v(x) = 3x - \frac{7}{2}$

32. **DECK** The function $C(x) = 25x + 50$ represents the labor cost for Jones Remodeling to build a deck, where x is the number of hours. Sample labor costs from their main competitor, Premiere Remodeling, are shown in the table.

Hours	Cost
2	\$130
4	\$160
6	\$190

- Which cost function has the greater rate of change? What does the rate of change represent?
- The graph of which cost function has the greater y -intercept? Interpret the y -intercept.
- The job is estimated to take 8 hours. Which company would you hire? Explain your reasoning.

Graph the functions $f(x)$ and $g(x)$ in the same coordinate plane. Use the graph to solve $f(x) = g(x)$.

33. $f(x) = x - 2$

$g(x) = 4x - 8$

34. $f(x) = -\frac{1}{5}x - 3$

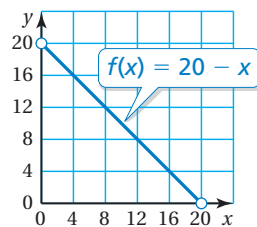
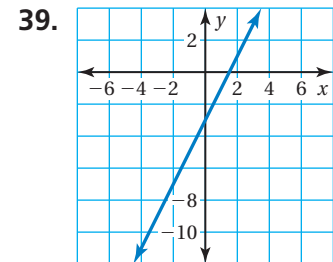
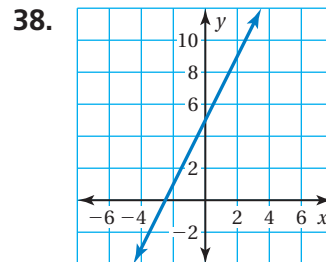
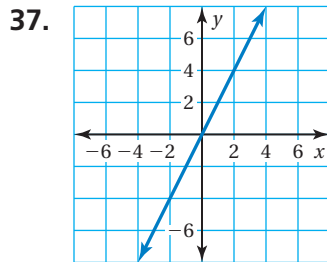
$g(x) = 2x + 8$

35. $f(x) = \frac{2}{3}x - 7$

$g(x) = -x + 3$

36. **CHOOSE TOOLS** What tool would you use to solve $f(x) = g(x)$ when $f(x) = 2.5x + 17$ and $g(x) = 0.8x$? Explain. Then solve $f(x) = g(x)$.

Given $f(x) = 2x + 1$, find the value of k so that the graph is $f(x) + k$.



40. **PERIMETER** The graph shows the relationship between the width y and length x of a rectangle in inches. A second rectangle has a perimeter that is 10 inches less than the perimeter of the first rectangle. How does the graph relating the width and length of the second rectangle compare to the graph shown?

41. **CRITICAL THINKING** The graph of $y = x + 4$ is a translation 4 units up of the graph of $y = x$. How can you obtain the graph of $y = x + 4$ from the graph of $y = x$ using a horizontal translation?

42. **Structure** Given that $f(x) = 3x - 5$ and $g(x) = 4x$, write a function that represents $f(g(x))$ and a function that represents $g(f(x))$.



Fair Game Review What you learned in previous grades & lessons

Write in slope-intercept form an equation of the line that passes through the given points. (Section 2.6)

43. (0, 0), (4, 4)

44. (-4, 9), (1, -1)

45. (-2, 1), (3, 1)

46. **MULTIPLE CHOICE** You buy a pair of gardening gloves for \$2.25 and x packets of seeds for \$0.88 each. Which equation represents the total cost y ?

(Skills Review Handbook)

(A) $y = 0.88x - 2.25$

(B) $y = 0.88x + 2.25$

(C) $y = 2.25x - 0.88$

(D) $y = 2.25x + 0.88$