5.1 Domain and Range of a Function

Essential Question How can you find the domain and range of a function?

1 ACTIVITY: The Domain and Range of a Function

Work with a partner. In Activity 1 in Section 2.4, you completed the table shown below. The table shows the number of adult and child tickets sold for a school concert.

input →

output →

Number of Adult Tickets, x	0	1	2	3	4
Number of Child Tickets, y	8	6	4	2	0

The variables x and y are related by the linear equation $4x + 2y = 16$.

a. Write the equation in *function form* by solving for y.

b. The **domain** of a function is the set of all input values. Find the domain of the function.

Domain =

Why is $x = 5$ not in the domain of the function?

Why is $x = \dfrac{1}{2}$ not in the domain of the function?

c. The **range** of a function is the set of all output values. Find the range of the function.
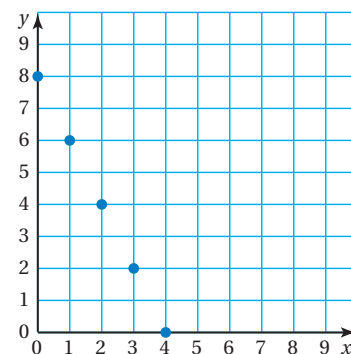
Range =

d. Functions can be described in many ways.

- by an equation
- by an input-output table
- in words
- by a graph
- as a set of ordered pairs

Use the graph to write the function as a set of ordered pairs.

$\left(\rule{0.6cm}{0.3cm} , \rule{0.6cm}{0.3cm} \right), \left(\rule{0.6cm}{0.3cm} , \rule{0.6cm}{0.3cm} \right), \left(\rule{0.6cm}{0.3cm} , \rule{0.6cm}{0.3cm} \right),$

$\left(\rule{0.6cm}{0.3cm} , \rule{0.6cm}{0.3cm} \right), \left(\rule{0.6cm}{0.3cm} , \rule{0.6cm}{0.3cm} \right)$

Math Practice 3

Use Definitions
What does the domain of a function represent? What does the range represent?
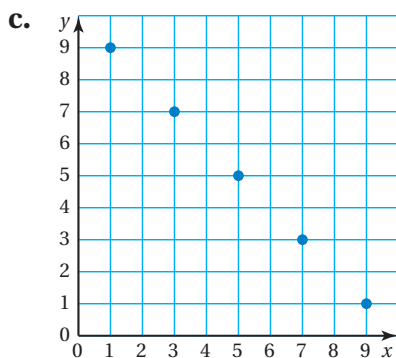
Work with a partner.

- Copy and complete each input-output table.
- Find the domain and range of the function represented by the table.
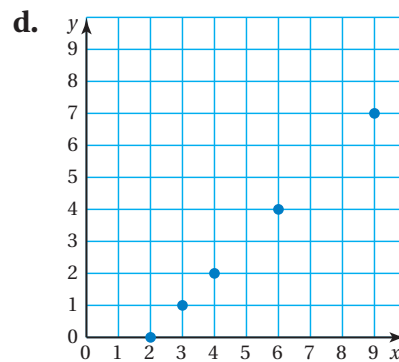
a. $y = -3x + 4$

x	-2	-1	0	1	2
y					

b. $y = \dfrac{1}{2}x - 6$

x	0	1	2	3	4
y					

c.

x					
y					

d.

x					
y					

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you find the domain and range of a function?

4. **The following are general rules for finding a person's foot length.**

   To find the length y (in inches) of a woman's foot, divide her shoe size x by 3 and add 7.

   To find the length y (in inches) of a man's foot, divide his shoe size x by 3 and add 7.3.

   a. Write an equation for one of the statements.

   b. Make an input-output table for the function in part (a). Use shoe sizes $5\dfrac{1}{2}$ to 12.

   c. Label the domain and range of the function on the table.

Practice

Use what you learned about the domain and range of a function to complete Exercise 3 on page 206.
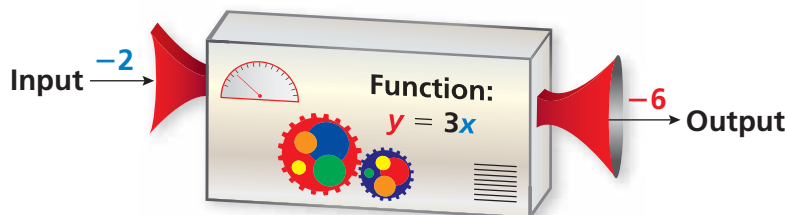
Key Vocabulary 🔊
function, *p. 204*
domain, *p. 204*
range, *p. 204*
independent variable,
  p. 204
dependent variable,
  p. 204

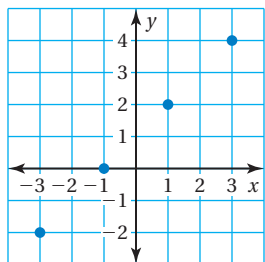## 🔑 Key Idea

Functions

A **function** is a relationship that pairs each *input* with exactly one *output*. The **domain** is the set of all possible input values. The **range** is the set of all possible output values.
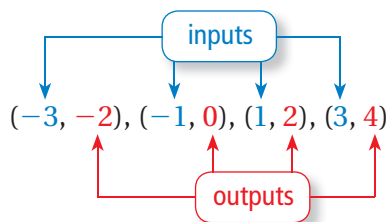
Input $\xrightarrow{-2}$ Function: $y = 3x$ $\xrightarrow{-6}$ Output

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EXAMPLE ① **Finding Domain and Range from a Graph**

Find the domain and range of the function represented by the graph.

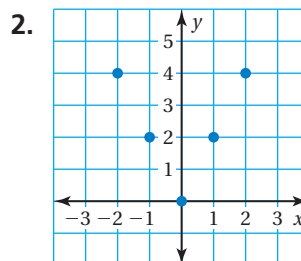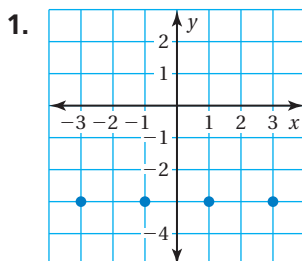Write the ordered pairs. Identify the inputs and outputs.

inputs

$(-3, -2), (-1, 0), (1, 2), (3, 4)$

outputs

∴ The domain is $-3, -1, 1,$ and 3. The range is $-2, 0, 2,$ and 4.

🔴 On Your Own

Now You're Ready
Exercises 4–6

Find the domain and range of the function represented by the graph.

1.

2.

When an equation represents a function, the variable that represents input values is the **independent variable** because it can be *any* value in the domain. The variable that represents output values is the **dependent variable** because it *depends* on the value of the independent variable.

🔊 Multi-Language Glossary at BigIdeasMath.com

EXAMPLE 2 **Finding the Range of a Function**

The function $y = -3x + 12$ gives the amount y (in fluid ounces) of juice remaining in a bottle after you take x gulps. **(a)** Identify the independent and dependent variables. **(b)** The domain is 0, 1, 2, 3, and 4. What is the range?

a. Because the amount y remaining depends on the number x of gulps, y is the dependent variable and x is the independent variable.

b. Make an input-output table to find the range.

:·: The range is 12, 9, 6, 3, and 0.

Input, x	$-3x + 12$	Output, y
0	$-3(0) + 12$	12
1	$-3(1) + 12$	9
2	$-3(2) + 12$	6
3	$-3(3) + 12$	3
4	$-3(4) + 12$	0

EXAMPLE 3 **Real-Life Application**

The table shows the percent y (in decimal form) of the moon that was visible at midnight x days after May 19, 2014. **(a)** Interpret the domain and range. **(b)** What percent of the moon was visible on May 21, 2014?

x	y
0	0.76
1	0.65
2	0.54
3	0.43
4	0.32

a. Zero days after May 19 is May 19. One day after May 19 is May 20. So, the domain of 0, 1, 2, 3, and 4 represents May 19, 20, 21, 22, and 23.

The range is 0.76, 0.65, 0.54, 0.43, and 0.32. These amounts are decreasing, so the moon was less visible each day.

b. May 21, 2014 corresponds to the input $x = 2$. When $x = 2$, $y = 0.54$. So, 0.54, or 54% of the moon was visible on May 21, 2014.

On Your Own

3. The function $y = -4x + 14$ gives the number y of avocados you have left after making x batches of guacamole.

   a. Identify the independent and dependent variables.

   b. The domain is 0, 1, 2, and 3. What is the range?

4. The table shows the percent y (in decimal form) of the moon that was visible at midnight x days after March 24, 2015.

x	0	1	2	3	4
y	0.19	0.29	0.39	0.49	0.59

   a. Interpret the domain and range.

   b. What percent of the moon was visible on March 28, 2015?

✔️ Vocabulary and Concept Check

1. **VOCABULARY** How are independent variables and dependent variables different?

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

Find the range of the function represented by the table.	Find the inputs of the function represented by the table.

Find the x-values of the function represented by $(2, 7)$, $(4, 5)$, and $(6, -1)$.	Find the domain of the function represented by $(2, 7)$, $(4, 5)$, and $(6, -1)$.

| x	2	4	6
y	7	5	-1

📝 Practice and Problem Solving

3. The number of earrings and headbands you can buy with $24 is represented by the equation $8x + 4y = 24$. The table shows the numbers of earrings and headbands.

   a. Write the equation in function form.
   b. Find the domain and range.
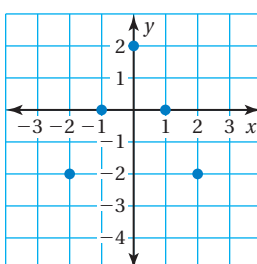   c. Why is $x = 6$ not in the domain of the function?

Earrings, x	0	1	2	3
Headbands, y	6	4	2	0

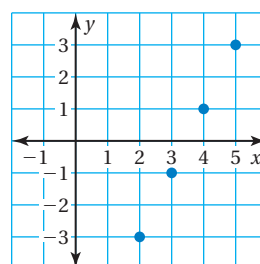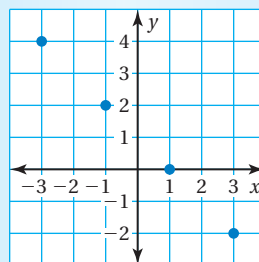**Find the domain and range of the function represented by the graph.**

① 4.

5.

6.
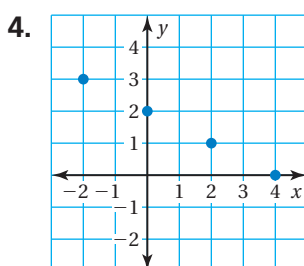
7. **ERROR ANALYSIS** Describe and correct the error in finding the domain and range of the function represented by the graph.

The domain is -2, 0, 2, and 4.

The range is -3, -1, 1, 3.

② 8. **PARKING METER** The number of quarters you put into a parking meter affects the amount of time on the meter. Identify the independent and dependent variables.

Copy and complete the input-output table for the function. Then find the domain and range of the function represented by the table.

9. $y = 6x + 2$

x	−1	0	1	2
y				

10. $y = -\dfrac{1}{4}x - 2$

x	0	4	8	12
y				

11. $y = 1.5x + 3$

x	−1	0	1	2
y				

12. VAULTING In the sport of vaulting, a vaulter performs a routine while on a moving horse. For each round x of competition, the vaulter receives a score y from 1 to 10.

    a. Find the domain and range of the function represented by the table.

    b. Interpret the domain and range.

    c. What is the mean score of the vaulter?

x	y
1	6.856
2	7.923
3	8.135

13. MANATEE A manatee eats the equivalent of about 12% of its body weight each day.

    a. Write an equation that represents the amount y (in pounds) of food a manatee eats each day for its weight x. Identify the independent variable and the dependent variable.

    b. Make an input-output table for the equation in part (a). Use the inputs 150, 300, 450, 600, 750, and 900.

    c. Find the domain and range of the function represented by the table.

    d. The weights of three manatees are 300 pounds, 750 pounds, and 1050 pounds. What is the total amount of food that these three manatees eat in a day? in a week?

14. **Precision** Describe the domain and range of the function.

    a. $y = |x|$ **b.** $y = -|x|$ **c.** $y = |x| - 6$ **d.** $y = -|x| + 4$

Fair Game Review What you learned in previous grades & lessons

Graph the linear equation. *(Section 2.1)*

15. $y = 2x + 8$ **16.** $5x + 6y = 12$ **17.** $-x - 3y = 2$ **18.** $y = 7x - 5$

19. MULTIPLE CHOICE The minimum number of people needed for a group rate at an amusement park is 8. Which inequality represents the number of people needed to get the group rate? *(Section 3.1)*

    A $x \le 8$ **B** $x > 8$ **C** $x < 8$ **D** $x \ge 8$