

4.5 Systems of Linear Inequalities

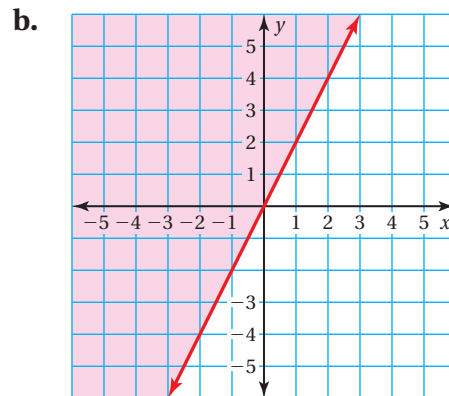
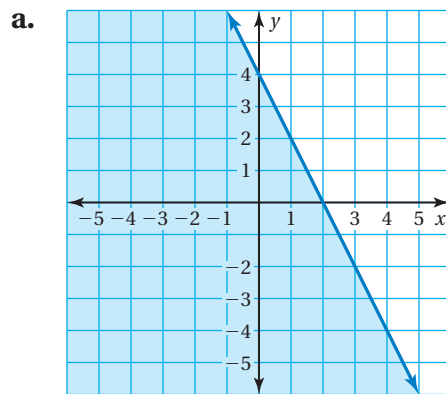
Essential Question How can you sketch the graph of a system of linear inequalities?

1 ACTIVITY: Graphing Linear Inequalities

Work with a partner. Match the linear inequality with its graph.

$$2x + y \leq 4 \quad \text{Inequality 1}$$

$$2x - y \leq 0 \quad \text{Inequality 2}$$



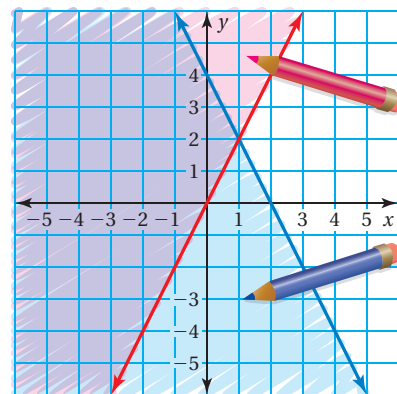
2 ACTIVITY: Graphing a System of Linear Inequalities

Work with a partner. Consider the system of linear inequalities given in Activity 1.

$$2x + y \leq 4 \quad \text{Inequality 1}$$

$$2x - y \leq 0 \quad \text{Inequality 2}$$

Use colored pencils to shade the solutions of the two linear inequalities. When you graph both inequalities in the same coordinate plane, what do you get?



Describe each of the shaded regions in the graph at the right. What does the unshaded region represent?



Systems of Inequalities

In this lesson, you will

- write and graph systems of linear inequalities in two variables.
- solve real-life problems.

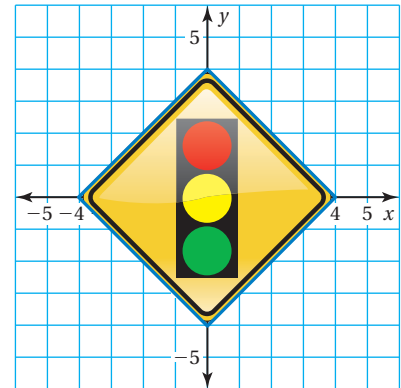
Learning Standards

A.CED.3

A.REI.12

3 ACTIVITY: Writing a System of Linear Inequalities

Work with a partner. Write a system of 4 linear inequalities whose solution is the traffic sign at the right.



(1) $x + y \leq$

(2) $x + y \geq$

(3) $x - y \leq$

(4) $x - y \geq$

4 ACTIVITY: Representing a State by a Linear System

Math Practice 6

Use Clear Definitions

What is a system of linear inequalities? How can this definition help you identify the states?

Two states can be represented as the graph of a system of linear inequalities. Identify the two states. Explain your reasoning.



What Is Your Answer?

- IN YOUR OWN WORDS** How can you sketch the graph of a system of linear inequalities?
- When graphing a system of linear inequalities, which region represents the solution of the system? Do you think all systems have a solution? Explain.

Practice

Use what you learned about systems of linear inequalities to complete Exercises 7–9 on page 189.

Key Vocabulary

system of linear inequalities, p. 186
solution of a system of linear inequalities, p. 186
graph of a system of linear inequalities, p. 186

A **system of linear inequalities** is a set of two or more linear inequalities in the same variables. An example is shown below.

$$y < x + 2 \quad \text{Inequality 1}$$

$$y \geq 2x - 1 \quad \text{Inequality 2}$$

A **solution of a system of linear inequalities** in two variables is an ordered pair that is a solution of each inequality in the system.

EXAMPLE 1 Checking Solutions

Tell whether each ordered pair is a solution of the system.

$$y < 2x \quad \text{Inequality 1}$$

$$y \geq x + 1 \quad \text{Inequality 2}$$

a. (3, 5)

Inequality 1

$$y < 2x$$

$$5 \stackrel{?}{<} 2(3)$$

$$5 < 6 \quad \checkmark$$

Inequality 2

$$y \geq x + 1$$

$$5 \stackrel{?}{\geq} 3 + 1$$

$$5 \geq 4 \quad \checkmark$$

∴ (3, 5) is a solution of both inequalities. So, it is a solution of the system.

b. (-2, 0)

Inequality 1

$$y < 2x$$

$$0 \stackrel{?}{<} 2(-2)$$

$$0 \not< -4 \quad \times$$

Inequality 2

$$y \geq x + 1$$

$$0 \stackrel{?}{\geq} -2 + 1$$

$$0 \geq -1 \quad \checkmark$$

∴ (-2, 0) is not a solution of both inequalities. So, it is not a solution of the system.

On Your Own

Tell whether the ordered pair is a solution of the system of linear inequalities.

1. $y < 5$

$y > x - 4$; (-1, 5)

2. $y \leq -2x + 5$

$y < x + 3$; (0, -1)

Now You're Ready
Exercises 4–6

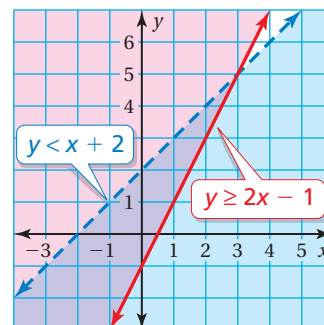
The **graph of a system of linear inequalities** is the graph of all of the solutions of the system.

Key Idea

Graphing a System of Linear Inequalities

Step 1 Graph each inequality in the same coordinate plane.

Step 2 Find the intersection of the half-planes. This intersection is the graph of the system.



EXAMPLE 2 Graphing a System of Linear Inequalities

Study Tip

For help with graphing linear inequalities, see Section 3.5.

Graph the system.

$$y \leq 3$$

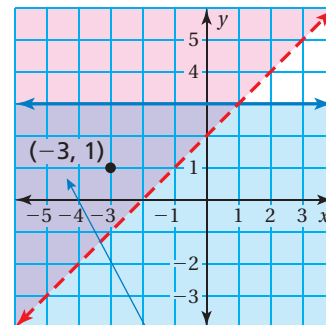
Inequality 1

$$y > x + 2$$

Inequality 2

Step 1: Graph each inequality.

Step 2: Find the intersection of the half-planes. One solution is $(-3, 1)$.



The solution is the purple shaded region.

Check

Verify that $(-3, 1)$ is a solution of each inequality.

Inequality 1

$$y \leq 3$$

$$1 \leq 3 \quad \checkmark$$

Inequality 2

$$y > x + 2$$

$$1 > -3 + 2 \quad ?$$

$$1 > -1 \quad \checkmark$$

EXAMPLE 3 Graphing a System of Linear Inequalities: No Solution

Graph the system.

$$2x + y < -1$$

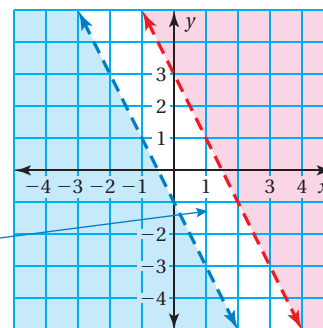
Inequality 1

$$2x + y > 3$$

Inequality 2

Step 1: Graph each inequality.

Step 2: Find the intersection of the half-planes.



The lines are parallel and the half-planes do not intersect.

∴ So, the system has no solution.

On Your Own

Graph the system of linear inequalities.

3. $y \geq -x + 4$

4. $y > 2x - 3$

5. $-2x + y < 4$

$x + y \leq 0$

$y \geq \frac{1}{2}x + 1$

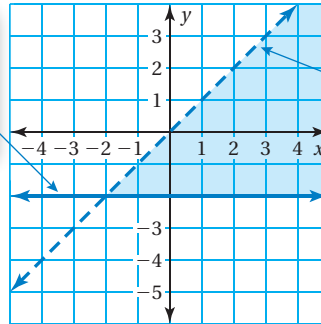
$2x + y > 4$

Now You're Ready
Exercises 7–15

EXAMPLE 4 Writing a System of Linear Inequalities

Write a system of linear inequalities represented by the graph.

The horizontal boundary line passes through $(0, -2)$. So, an equation of the line is $y = -2$.



The slope of the other boundary line is 1 and the y-intercept is 0. So, an equation of the line is $y = x$.

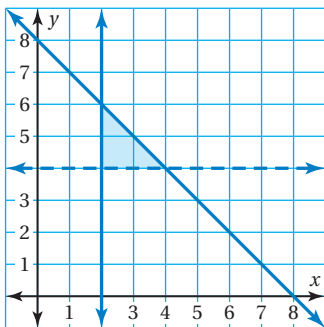
The shaded region is *above* the *solid* boundary line, so the inequality is $y \geq -2$.

The shaded region is *below* the *dashed* boundary line, so the inequality is $y < x$.

∴ The system is $y \geq -2$ and $y < x$.

EXAMPLE 5 Real-Life Application

You have at most 8 hours to spend at the mall and at the beach. You want to spend at least 2 hours at the mall and more than 4 hours at the beach. Write and graph a system that represents the situation. How much time could you spend at each location?



Use the constraints to write a system of linear inequalities. Let x be the number of hours at the mall and let y be the number of hours at the beach.

$$\begin{array}{ll} x + y \leq 8 & \text{at most 8 hours at the mall and at the beach} \\ x \geq 2 & \text{at least 2 hours at the mall} \\ y > 4 & \text{more than 4 hours at the beach} \end{array}$$

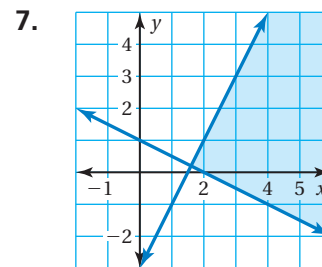
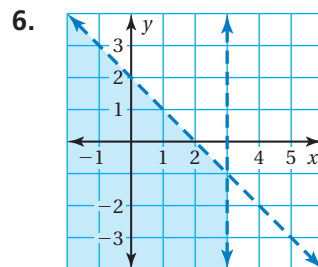
Graph the system. One ordered pair in the solution region is $(2.5, 5)$.

∴ So, you could spend 2.5 hours at the mall and 5 hours at the beach.

On Your Own

Write a system of linear inequalities represented by the graph.

Now You're Ready
Exercises 24–26



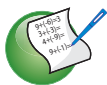
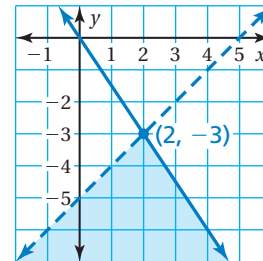
8. **WHAT IF?** In Example 5, you want to spend at least 3 hours at the mall. How does this change the system? Is $(2.5, 5)$ still a solution? Explain.

4.5 Exercises



Vocabulary and Concept Check

- VOCABULARY** How can you verify that an ordered pair is a solution of a system of linear inequalities?
- WRITING** How are solving systems of linear inequalities and systems of linear equations similar? How are they different?
- REASONING** Is the point shown a solution of the system of linear inequalities? Explain.



Practice and Problem Solving

Tell whether the ordered pair is a solution of the system of linear inequalities.

- | | | | |
|---|---------------------------------------|---|---|
| 1 | 4. $y < 4$ $y > x + 3$; $(-5, 2)$ | 5. $y > -2$ $y \leq x - 5$; $(1, -1)$ | 6. $y \leq x + 7$ $y \geq 2x + 3$; $(0, 0)$ |
|---|---------------------------------------|---|---|

Graph the system of linear inequalities.

- | | | | | |
|-----|---|-----------------------------------|---|---|
| 2 | 3 | 7. $y < -3$ $y \geq 5x$ | 8. $y > -x + 3$ $-2x + y \geq 0$ | 9. $x + y > 1$ $-x - y < -3$ |
| 10. | | $y < -2$ $y > 2$ | 11. $y \geq -5$ $y < 3x + 1$ | 12. $x + y > 4$ $y > \frac{3}{2}x - 9$ |
| 13. | | $-x + y < -1$ $-x - 1 \geq -y$ | 14. $2x + y \leq 5$ $y + 2 \geq -2x$ | 15. $-2x - 5y < 15$ $-4x > 10y + 60$ |

16. **MUFFINS** You can spend at most \$21 on fruit. Blueberries cost \$4 per pound and strawberries cost \$3 per pound. You need at least 3 pounds to make muffins.

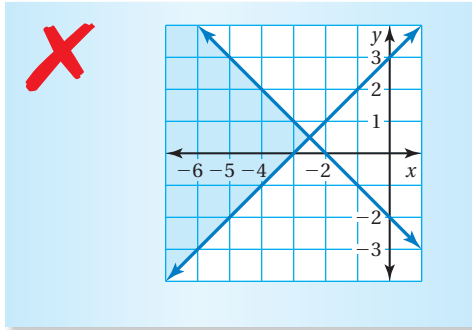
- Define the variables.
- Write a system of linear inequalities that represents this situation.
- Graph the system of linear inequalities.
- Is it possible to buy 4 pounds of blueberries and 1 pound of strawberries in this situation? Justify your answer.



ERROR ANALYSIS Describe and correct the error in graphing the system of linear inequalities.

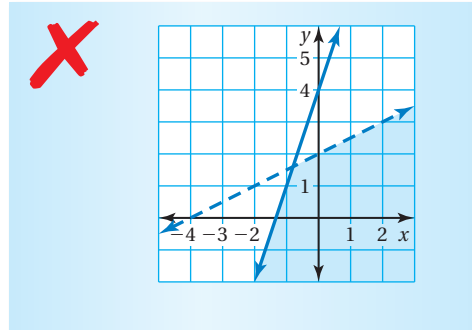
17. $y \geq x + 3$

$y < -x - 2$

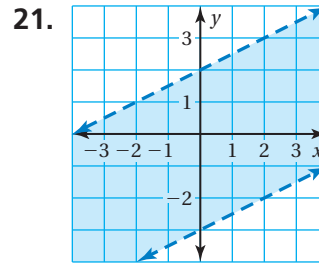
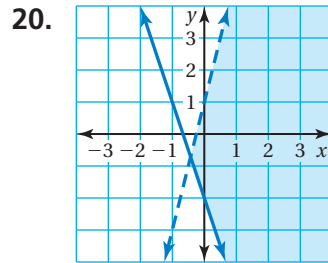
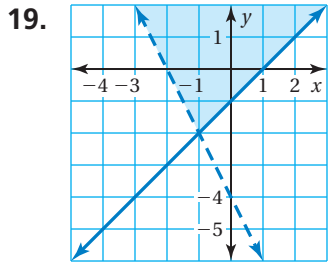


18. $y \leq 3x + 4$

$y > \frac{1}{2}x + 2$



Match the graph with the corresponding system of linear inequalities.



A. $y < 4x + 1$

$y \geq -3x - 2$

B. $-x + y \geq -1$

$2x + y > -4$

C. $-\frac{1}{2}x + y < 2$

$-2x + 4y > -12$

22. **REASONING** Describe the intersection of the half-planes of the system shown.

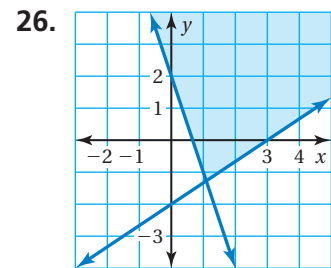
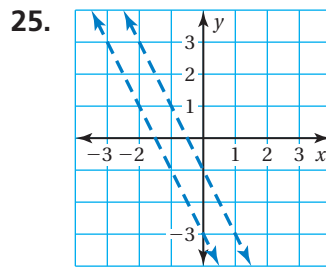
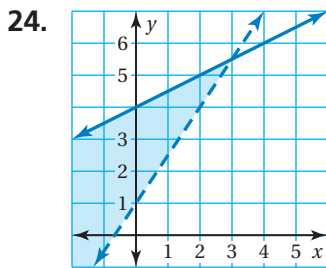
$x - y \leq 4$
 $x - y \geq 4$



23. **JOBS** You earn \$12 per hour working as a manager at a grocery store. You also coach a soccer team for \$10 per hour. You need to earn at least \$110 per week, but you do not want to work more than 20 hours per week.

- a. Write and graph a system of linear inequalities that represents this situation.
- b. Identify and interpret one solution of the system.

4 Write a system of linear inequalities represented by the graph.



Graph the system of linear inequalities.

27. $y > 1$

$x \geq 2$

$y > x - 1$

28. $y \leq 5x - 6$

$y > 0.5x - 4$

$y < -x + 7$

29. $-4x + 2y < 12$

$6x + y \leq 9$

$-9x + 3y \geq -15$

30. **STRUCTURE** Write a system of linear inequalities that is equivalent to $|y| < x$ where $x > 0$. Graph the system.

31. **REPEATED REASONING** One inequality in a system is $-4x + 2y > 6$. Write another inequality so the system has (a) *no solution* and (b) *infinitely many solutions*.

32. **AMUSEMENT PARK** You have at most 8 hours to spend at an amusement park. You want to spend less than 3 hours playing games and at least 4 hours on rides. How much time can you spend on each activity?

33. **ROAD TRIP** On a road trip, you drive about 70 miles per hour and your friend drives about 60 miles per hour. The plan is to drive less than 15 hours and at least 600 miles each day. Your friend will drive more hours than you. Identify and interpret one solution of this situation.

34. **Geometry** The following points are the vertices of a triangle.

$(2, 5), (6, -3), (-2, -3)$

a. Write a system of linear inequalities that represents the triangle.

b. Find the area of the triangle.



Fair Game Review what you learned in previous grades & lessons

Evaluate the expression when $a = -2$, $b = 3$, and $c = -1$. (*Skills Review Handbook*)

35. $4a - bc$

36. $ab + c^2$

37. $-3c - ac$

38. **MULTIPLE CHOICE** What is the solution of $2(x - 4) = -(-x + 3)$? (*Section 1.3*)

(A) $x = -5$

(B) $x = 2$

(C) $x = 5$

(D) $x = 7$