

Essential Question Can a system of linear equations have no solution? Can a system of linear equations have many solutions?

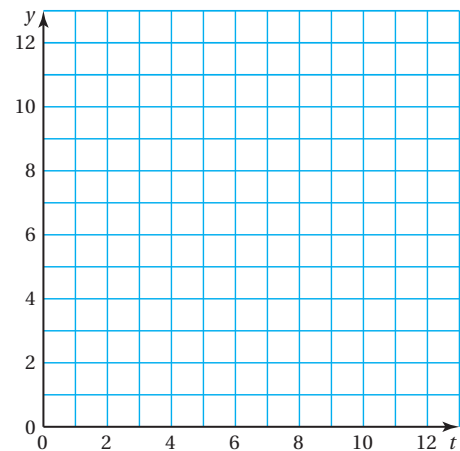
1 ACTIVITY: Writing a System of Linear Equations

Work with a partner. Your cousin is 3 years older than you. Your ages can be represented by two linear equations.

$$y = t \quad \text{Your age}$$

$$y = t + 3 \quad \text{Your cousin's age}$$

- Graph both equations in the same coordinate plane.
- What is the vertical distance between the two graphs? What does this distance represent?
- Do the two graphs intersect? If not, what does this mean in terms of your age and your cousin's age?



2 ACTIVITY: Using a Table to Solve a System

Work with a partner. You invest \$500 for equipment to make dog backpacks. Each backpack costs you \$15 for materials. You sell each backpack for \$15.



- Copy and complete the table for your cost C and your revenue R .

x	0	1	2	3	4	5	6	7	8	9	10
C											
R											

- When will your company break even? What is wrong?



COMMON
CORE

Systems of Equations

In this lesson, you will

- solve systems of linear equations having no solution or infinitely many solutions.

Learning Standards

8.EE.8a
8.EE.8b
8.EE.8c
A.CED.3
A.REI.6

3 ACTIVITY: Using a Graph to Solve a Puzzle

Math Practice 4

Analyze Relationships

What do you know about the graphs of the two equations? How does this relate to the number of solutions?

Work with a partner. Let x and y be two numbers. Here are two clues about the values of x and y .

Words

Clue 1: y is 4 more than twice the value of x .

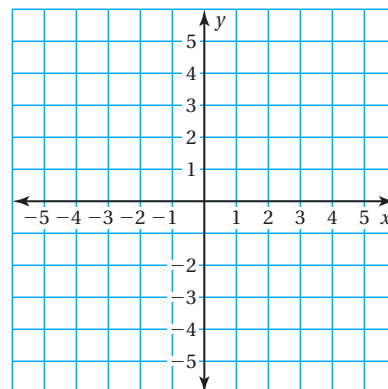
Clue 2: The difference of $3y$ and $6x$ is 12.

Equation

$$y = 2x + 4$$

$$3y - 6x = 12$$

- Graph both equations in the same coordinate plane.
- Do the two lines intersect? Explain.
- What is the solution of the puzzle?
- Use the equation $y = 2x + 4$ to complete the table.



x	0	1	2	3	4	5	6	7	8	9	10
y											

- Does each solution in the table satisfy *both* clues?
- What can you conclude? How many solutions does the puzzle have? How can you describe them?

What Is Your Answer?

- IN YOUR OWN WORDS** Can a system of linear equations have no solution? Can a system of linear equations have many solutions? Give examples to support your answers.

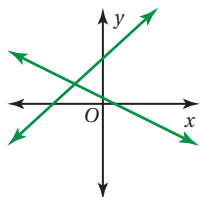
Practice

Use what you learned about special systems of linear equations to complete Exercises 3 and 4 on page 180.

Key Idea

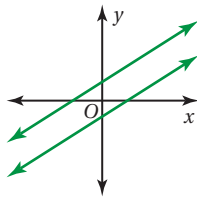
Solutions of Systems of Linear Equations

A system of linear equations can have *one solution*, *no solution*, or *infinitely many solutions*.



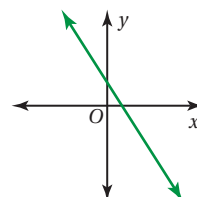
One solution

The lines intersect.



No solution

The lines are parallel.



Infinitely many solutions

The lines are the same.

EXAMPLE 1 Solving a System: No Solution

Solve the system.

$$y = 3x + 1 \quad \text{Equation 1}$$

$$y = 3x - 5 \quad \text{Equation 2}$$

Method 1: Solve by graphing.

Graph each equation.

The lines have the same slope and different y -intercepts. So, the lines are parallel.

Because parallel lines do not intersect, there is no point that is a solution of both equations.

∴ So, the system of linear equations has no solution.

Method 2: Solve by substitution.

Substitute $3x - 5$ for y in Equation 1.

$$y = 3x + 1 \quad \text{Equation 1}$$

$$3x - 5 = 3x + 1 \quad \text{Substitute } 3x - 5 \text{ for } y.$$

$$-5 \neq 1 \quad \text{Subtract } 3x \text{ from each side.}$$

∴ The equation $-5 = 1$ is never true. So, the system of linear equations has no solution.

On Your Own

Solve the system of linear equations. Check your solution.

1. $y = -x + 3$

2. $y = -5x - 2$

3. $x = 2y + 10$

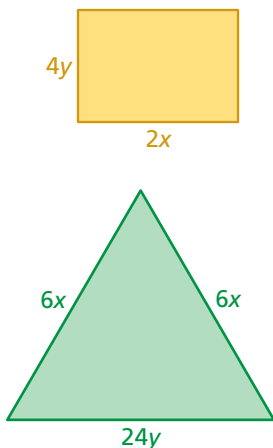
$y = -x + 5$

$5x + y = 0$

$2x + 3y = -1$

Now You're Ready
Exercises 8–10

EXAMPLE 2 Solving a System: Infinitely Many Solutions



The perimeter of the rectangle is 36 units. The perimeter of the triangle is 108 units. Write and solve a system of linear equations to find the values of x and y .

Perimeter of rectangle

$$2(2x) + 2(4y) = 36$$

$$4x + 8y = 36 \quad \text{Equation 1}$$

Perimeter of triangle

$$6x + 6x + 24y = 108$$

$$12x + 24y = 108 \quad \text{Equation 2}$$

The system is: $4x + 8y = 36$

$$12x + 24y = 108$$

Equation 1

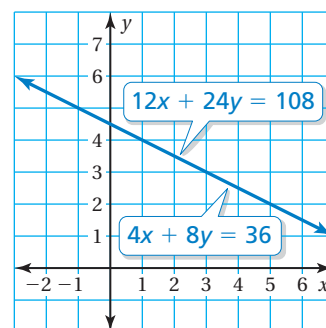
Equation 2

Method 1: Solve by graphing.

Graph each equation.

The lines have the same slope and the same y -intercept. So, the lines are the same.

Because the lines are the same, all the points on the line are solutions of both equations. So, the system of linear equations has infinitely many solutions.



Method 2: Solve by elimination.

Multiply Equation 1 by 3 and subtract the equations.

$$4x + 8y = 36$$

Multiply by 3.

$$12x + 24y = 108$$

Revised Equation 1

$$12x + 24y = 108$$

$$12x + 24y = 108$$

Equation 2

$$0 = 0$$

Subtract.

The equation $0 = 0$ is always true. So, the solutions are all the points on the line $4x + 8y = 36$. The system of linear equations has infinitely many solutions.

On Your Own

Solve the system of linear equations. Check your solution.

4. $x + y = 3$

5. $2x + y = 5$

6. $2x - 4y = 10$

$x - y = -3$

$4x + 2y = 0$

$-12x + 24y = -60$

7. **WHAT IF?** What happens to the solution in Example 2 if the perimeter of the rectangle is 54 units? Explain.

Now You're Ready
Exercises 11–13


Vocabulary and Concept Check

- WRITING** Describe the difference between the graph of a system of linear equations that has *no solution* and the graph of a system of linear equations that has *infinitely many solutions*.
- REASONING** When solving a system of linear equations algebraically, how do you know when the system has *no solution*? *infinitely many solutions*?


Practice and Problem Solving

Let x and y be two numbers. Find the solution of the puzzle.

3. y is $\frac{1}{3}$ more than 4 times the value of x .

The difference of $3y$ and $12x$ is 1.

4. $\frac{1}{2}$ of x plus 3 is equal to y .

x is 6 more than twice the value of y .

Without graphing, determine whether the system of linear equations has *one solution*, *infinitely many solutions*, or *no solution*. Explain your reasoning.

5. $y = 5x - 9$

$y = 5x + 9$

6. $y = 6x + 2$

$y = 3x + 1$

7. $y = 8x - 2$

$y - 8x = -2$

Solve the system of linear equations. Check your solution.

1 8. $y = 2x - 2$

$y = 2x + 9$

9. $y = 3x + 1$

$-x + 2y = -3$

10. $y = \frac{\pi}{3}x + \pi$

$-\pi x + 3y = -6\pi$

2 11. $y = -\frac{1}{6}x + 5$

$x + 6y = 30$

12. $\frac{1}{3}x + y = 1$

$2x + 6y = 6$

13. $-2x + y = 1.3$

$2(0.5x - y) = 4.6$

14. **ERROR ANALYSIS** Describe and correct the error in solving the system of linear equations.



$y = -2x + 4$

$y = -2x + 6$

The lines have the same slope so there are infinitely many solutions.



15. **PIG RACE** In a pig race, your pig gets a head start of 3 feet and is running at a rate of 2 feet per second. Your friend's pig is also running at a rate of 2 feet per second. A system of linear equations that represents this situation is $y = 2x + 3$ and $y = 2x$. Will your friend's pig catch up to your pig? Explain.

16. **REASONING** One equation in a system of linear equations has a slope of -3 . The other equation has a slope of 4 . How many solutions does the system have? Explain.
17. **LOGIC** How can you use the slopes and y -intercepts of equations in a system of linear equations to determine whether the system has *one solution*, *infinitely many solutions*, or *no solution*? Explain your reasoning.

$$4x + 8y = 64$$

$$8x + 16y = 128$$



18. **MONEY** You and a friend both work two different jobs. The system of linear equations represents the total earnings for x hours worked at the first job and y hours worked at the second job. Your friend earns twice as much as you.
- One week, both of you work 4 hours at the first job. How many hours do you and your friend work at the second job?
 - Both of you work the same number of hours at the second job. Compare the number of hours you each work at the first job.

19. **DOWNLOADS** You download a digital album for \$10. Then you and your friend download the same number of individual songs for \$0.99 each. Write a system of linear equations that represents this situation. Will you and your friend spend the same amount of money? Explain.

20. **REASONING** Does the system shown *always*, *sometimes*, or *never* have no solution when $a = b$? $a \geq b$? $a < b$? Explain your reasoning.

$$y = ax + 1$$

$$y = bx + 4$$

21. **SKIING** The table shows the number of lift tickets and ski rentals sold to two different groups. Is it possible to determine how much each lift ticket costs? Justify your answer.

Group	1	2
Number of Lift Tickets	36	24
Number of Ski Rentals	18	12
Total Cost (dollars)	684	456

22. **Precision** Find the values of a and b so the system shown has the solution $(2, 3)$. Does the system have any other solutions? Explain.

$$12x - 2by = 12$$

$$3ax - by = 6$$



Fair Game Review

what you learned in previous grades & lessons

Graph the inequality in a coordinate plane. (Section 3.5)

23. $3x + y \geq 6$ 24. $-3x - 4y \geq 4$ 25. $-4x + 3y < -12$
26. **MULTIPLE CHOICE** What is the solution of $-2(y + 5) \leq 16$? (Section 3.4)
- (A) $y \leq -13$ (B) $y \geq -13$ (C) $y \leq -3$ (D) $y \geq -3$