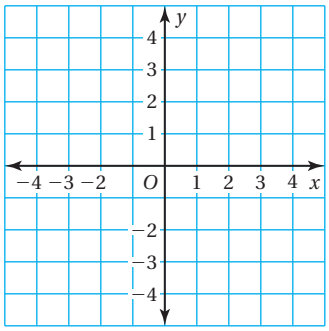
3.5 Graphing Linear Inequalities in Two Variables

Essential Question How can you use a coordinate plane to solve problems involving linear inequalities?

1 ACTIVITY: Graphing Inequalities

Work with a partner.



a. Graph $y = x + 1$ in a coordinate plane.

b. Choose three points that lie above the graph of $y = x + 1$. Substitute the values of x and y of each point in the inequality $y > x + 1$. If the substitutions result in true statements, plot the points on the graph.

c. Choose three points that lie below the graph of $y = x + 1$. Substitute the values of x and y of each point in the inequality $y > x + 1$. If the substitutions result in true statements, plot the points on the graph.

d. To graph $y > x + 1$, would you choose points above or below $y = x + 1$?

e. Choose a point that lies on the graph of $y = x + 1$. Substitute the values of x and y in the inequality $y > x + 1$. What do you notice? Do you think the graph of $y > x + 1$ includes the points that lie on the graph of $y = x + 1$? Explain your reasoning.

f. Explain how you could change the inequality so that it includes the points that lie on the graph of $y = x + 1$.

2 ACTIVITY: Writing and Graphing Inequalities

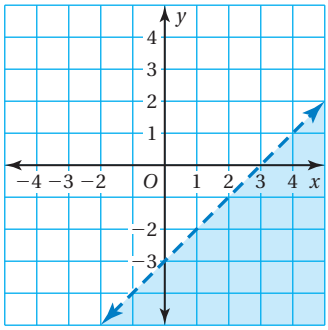
COMMON CORE

Graphing Inequalities

In this lesson, you will

- graph linear inequalities in two variables.

Learning Standard A.REI.12



Work with a partner. The graph of a linear inequality in two variables shows all the solutions of the inequality in a coordinate plane. An ordered pair (x, y) is a solution of an inequality if the inequality is true when the values of x and y are substituted in the inequality.

a. Write an equation for the graph of the dashed blue line.

b. The solutions of an inequality are represented by the shaded region. In words, describe the solutions of the inequality.

c. Write an inequality for the graph. Which inequality symbol did you use? Explain your reasoning.

3 EXAMPLE: Using a Graphing Calculator

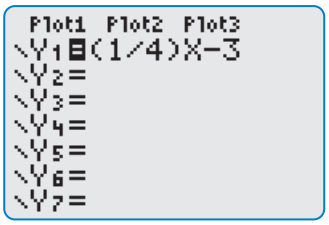
Math Practice 5

Recognize Usefulness of Tools

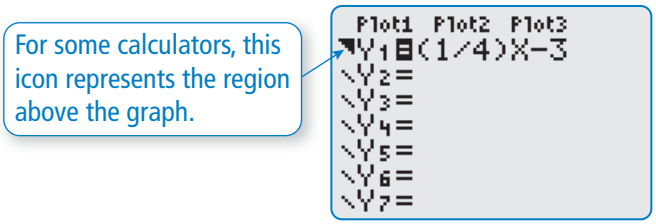
When do you think it would be useful to use a graphing calculator?

Use a graphing calculator to graph $y \geq \frac{1}{4}x - 3$.

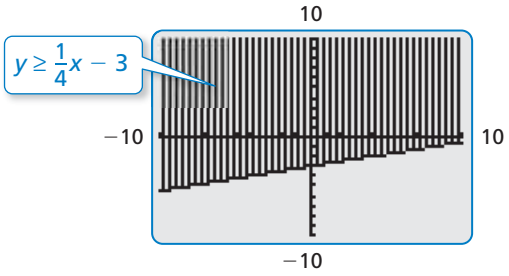
a. Enter the equation $y = \frac{1}{4}x - 3$ into your calculator.



b. The inequality contains the symbol ≥. So, the region to be shaded is above the graph of $y = \frac{1}{4}x - 3$. Adjust your graphing calculator so that the region above the graph will be shaded.



c. Graph $y \geq \frac{1}{4}x - 3$ on your calculator.



Some graphing calculators always use a solid line when graphing inequalities. In this case, you will have to decide whether the line should be dashed or solid.

What Is Your Answer?

4. Use a graphing calculator to graph each inequality in a standard viewing window.

a. $y > x + 5$ **b.** $y \leq -\frac{1}{2}x + 1$ **c.** $y \geq -x - 4$

5. **IN YOUR OWN WORDS** How can you use a coordinate plane to solve problems involving linear inequalities? Give an example of a real-life problem that can be represented by an inequality in two variables.

Practice Use what you learned about writing and graphing inequalities to complete Exercises 8–10 on page 141.

3.5 Lesson

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Key Vocabulary
linear inequality in two variables, *p. 138*
solution of a linear inequality, *p. 138*
graph of a linear inequality, *p. 138*
half-planes, *p. 138*

A **linear inequality in two variables** x and y can be written as

$$ax + by < c \qquad ax + by \leq c \qquad ax + by > c \qquad ax + by \geq c$$

where a, b, and c are real numbers. A **solution of a linear inequality** in two variables is an ordered pair (x, y) that makes the inequality true.

EXAMPLE 1 Checking Solutions of a Linear Inequality

Tell whether the ordered pair is a solution of the inequality.

a. $2x + y < -3$; $(-1, 9)$

$2x + y < -3$ — Write the inequality.

$2(-1) + 9 \stackrel{?}{<} -3$ — Substitute -1 for x and 9 for y.

$7 \not< -3$ ✗ — Simplify. 7 is *not* less than -3.

∴ So, $(-1, 9)$ is *not* a solution of the inequality.

b. $x - 3y \geq 8$; $(2, -2)$

$x - 3y \geq 8$ — Write the inequality.

$2 - 3(-2) \stackrel{?}{\geq} 8$ — Substitute 2 for x and -2 for y.

$8 \geq 8$ ✓ — Simplify. 8 is equal to 8.

∴ So, $(2, -2)$ is a solution of the inequality.

On Your Own

Now You're Ready
Exercises 11–18

Tell whether the ordered pair is a solution of the inequality.

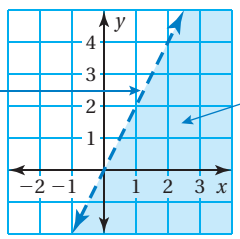
1. $x + y > 0$; $(-2, 2)$
2. $4x - y \geq 5$; $(0, 0)$
3. $5x - 2y \leq -1$; $(-4, -1)$
4. $-2x - 3y < 15$; $(5, -7)$

Reading

A dashed boundary line means that points on the line are *not* solutions. A solid boundary line means that points on the line are solutions.

The **graph of a linear inequality** in two variables shows all of the solutions of the inequality in a coordinate plane.

All solutions of $y < 2x$ lie on one side of the *boundary line* $y = 2x$.



The boundary line divides the coordinate plane into two **half-planes**. The shaded half-plane is the graph of $y < 2x$.

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Key Idea

Graphing a Linear Inequality in Two Variables

Step 1 Graph the boundary line for the inequality. Use a dashed line for $<$ or $>$. Use a solid line for $\leq$ or $\geq$.

Step 2 Test a point that is not on the boundary line to determine if it is a solution of the inequality.

Step 3 If the test point is a solution, shade the half-plane that contains the point. If the test point is *not* a solution, shade the half-plane that does *not* contain the point.

It is convenient to use the origin as a test point because it is easily substituted. However, you must choose a different test point if the origin is on the boundary line.

EXAMPLE 2 Graphing Linear Inequalities in One Variable

a. Graph $y \leq 2$ in a coordinate plane.

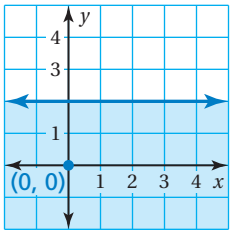
Step 1: Graph $y = 2$. Use a solid line because the inequality symbol is $\leq$.

Step 2: Test (0, 0).

$y \leq 2$ Write the inequality.

$0 \leq 2$ ✓ Substitute.

Step 3: Because (0, 0) is a solution, shade the half-plane that contains (0, 0).



b. Graph $x > 1$ in a coordinate plane.

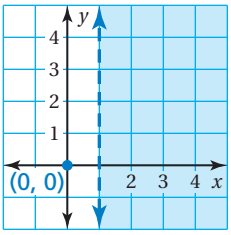
Step 1: Graph $x = 1$. Use a dashed line because the inequality symbol is $>$.

Step 2: Test (0, 0).

$x > 1$ Write the inequality.

$0 \not> 1$ ✗ Substitute.

Step 3: Because (0, 0) is *not* a solution, shade the half-plane that does *not* contain (0, 0).



On Your Own

Now You're Ready
Exercises 32–34

Graph the inequality in a coordinate plane.

5. $y > -1$

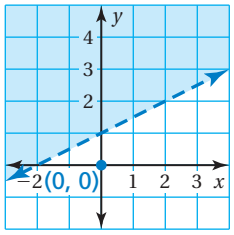
6. $y \geq -5$

7. $x \leq -4$

8. $3.5 > x$

EXAMPLE 3 Graphing Linear Inequalities in Two Variables

Graph $-x + 2y > 2$ in a coordinate plane.



Step 1: Graph $-x + 2y = 2$, or $y = \frac{1}{2}x + 1$.

Use a dashed line because the inequality symbol is >.

Step 2: Test (0, 0).

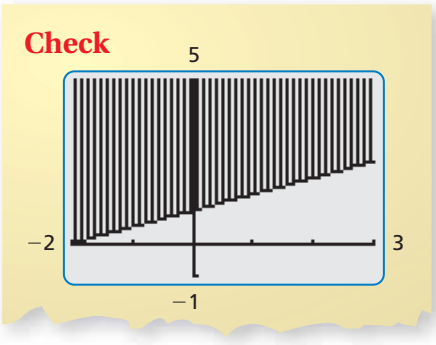
$-x + 2y > 2$ Write the inequality.

$-(0) + 2(0) \stackrel{?}{>} 2$ Substitute.

$0 \not> 2$ ✗ Simplify.

Step 3: Because (0, 0) is *not* a solution, shade the half-plane that does *not* contain (0, 0).

Check



EXAMPLE 4 Real-Life Application

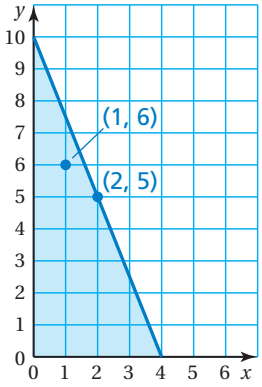
You can spend at most $10 on grapes and apples for a fruit salad. Grapes cost $2.50 per pound and apples cost $1 per pound. Write and graph an inequality for the amounts of grapes and apples you can buy. Identify and interpret two solutions of the inequality.

Words Cost per pound of grapes | times | Pounds of grapes | plus | Cost per pound of apples | times | Pounds of apples | is at most | Amount you can spend

Variables Let x be pounds of grapes and y be pounds of apples.

Inequality $2.50 \cdot x + 1 \cdot y \leq 10$

Step 1: Graph $2.5x + y = 10$, or $y = -2.5x + 10$. Use a solid line because the inequality symbol is ≤.



Step 2: Test (0, 0).

$2.5x + y \leq 10$ Write the inequality.

$2.5(0) + 0 \stackrel{?}{\leq} 10$ Substitute.

$0 \leq 10$ ✓ Simplify.

Step 3: Because (0, 0) is a solution, shade the half-plane that contains (0, 0).

Two possible solutions are (1, 6) and (2, 5). So, you can buy 1 pound of grapes and 6 pounds of apples, or 2 pounds of grapes and 5 pounds of apples.

On Your Own

Graph the inequality in a coordinate plane.

9. $x + y \leq -4$ **10.** $x - 2y < 0$ **11.** $2x + 2y \geq 3$

Now You're Ready
Exercises 35–40

3.5 Exercises

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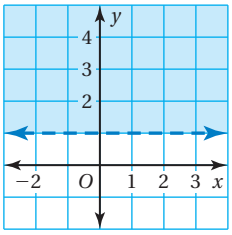
Vocabulary and Concept Check

1. **VOCABULARY** How can you tell whether an ordered pair is a solution of an inequality?
2. **OPEN-ENDED** Write an example of an inequality in two variables.
3. **WRITING** Compare the graph of a linear inequality in two variables with the graph of a linear equation in two variables.
4. **REASONING** Why do you only need to test one point when graphing a linear inequality?

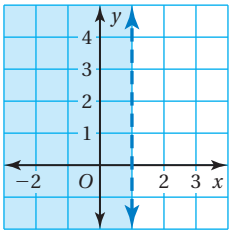
Match the inequality with its graph.

5. $x > -1$
6. $y > 1$
7. $x < 1$

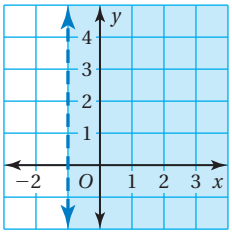
A.



B.



C.



Practice and Problem Solving

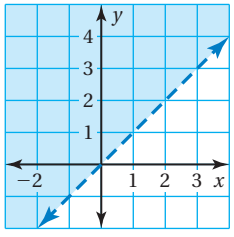
In words, describe the solutions of the inequality.

8. $y > x - 1$
9. $y \geq -x + 5$
10. $y < x - 2$

Tell whether the ordered pair is a solution of the inequality.

11. $x + y < 7$; $(6, -1)$
12. $2x - y \leq 0$; $(-2, -5)$
13. $x + 3y \geq -2$; $(-4, -2)$
14. $3x + 2y > -6$; $(0, 0)$
15. $-6x + 4y \leq 5$; $(3, -5)$
16. $3x - 5y \geq -8$; $(-1, 1)$
17. $-x - 6y > 12$; $(-8, 2)$
18. $-4x - 8y < -15$; $(-6, 3)$

Tell whether the ordered pair is a solution of the inequality whose graph is shown.



19. $(0, 4)$
20. $(0, 0)$
21. $(-1, -2)$
22. $(-1, 3)$
23. $(3, 3)$
24. $(-2, -1)$

25. **FABRIC** You can spend at most \$60 on lace. Cotton lace is \$2 per yard and linen lace is \$3 per yard. Write an inequality for the amounts of lace you can buy. Can you buy 12 yards of cotton lace and 15 yards of linen lace? Explain.

In Exercises 26–28, use the inequality $2x + y < -1$.

26. Write the equation of the boundary line in slope-intercept form.

27. Tell whether you would use a solid line or a dashed line to graph the boundary line. Then graph the boundary line.

28. Test the point (0, 0) in the inequality. Is the test point a solution? If so, shade the half-plane that contains the point. If not, shade the half-plane that does *not* contain the point.

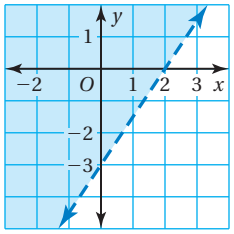
Match the inequality with its graph.

29. $3x - 2y \leq 6$

30. $3x - 2y < 6$

31. $3x - 2y \geq 6$

A.



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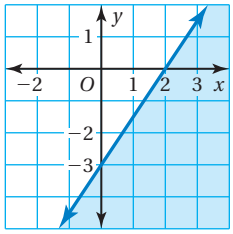

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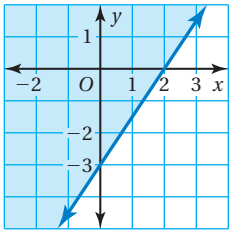


Graph the inequality in a coordinate plane.

32. $y < 5$

33. $x \geq -3$

34. $x < 2$

35. $y \leq 3x - 1$

36. $-2x + y > -4$

37. $3x - 2y \geq 0$

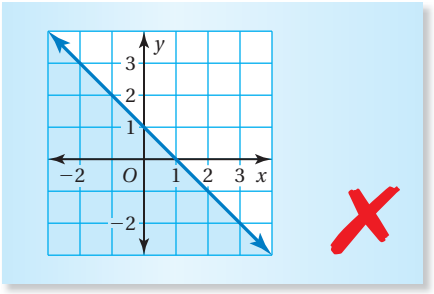
38. $5x - 2y \leq 6$

39. $2x - y < -3$

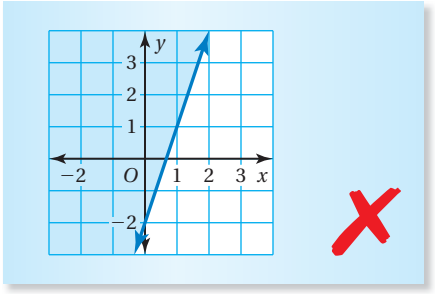
40. $-x + 4y > -2$

ERROR ANALYSIS **Describe and correct the error in graphing the inequality.**

41. $y < -x + 1$



42. $y \leq 3x - 2$



43. **CRITICAL THINKING** When graphing a linear inequality in two variables, why must you choose a test point that is *not* on the boundary line?

44. **MODELING** In order for the drama club to cover the expenses of producing a play, at least \$1500 worth of tickets must be sold.

a. Use a model to write an inequality that represents this situation.

b. Graph the inequality.

c. Eighty adults and 110 students attend the play. Does the drama club cover its expenses? Explain.

Tell whether the ordered pair is a solution of the inequality.

45. $y < \frac{1}{3}x + \frac{1}{4}$; $(6, 2)$

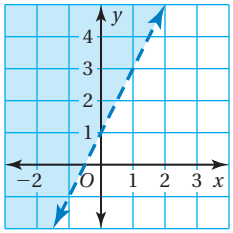
46. $2.5 - y \leq 1.8x$; $(0.5, 1.5)$

47. $0.2x + 1.6y \geq -1$; $(10, -2.2)$

48. $2x - \frac{2}{3}y > -5$; $\left(\frac{3}{4}, 4\right)$

Write an inequality that represents the graph.

49.



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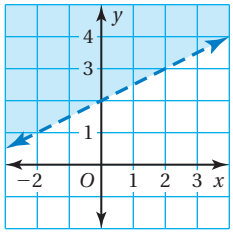

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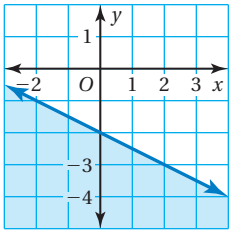


52. **REASONING** How many solutions does the inequality $2x + y \geq 5$ have?

53. **PROBLEM SOLVING** After buying your admission ticket, you have \$9 to spend at the movies. Arcade games cost \$0.75 per game and soft drinks cost \$2.25.

a. Write and graph an inequality that represents the numbers of arcade games you can play and soft drinks you can buy.

b. Identify and interpret two solutions of the inequality.

54. Critical Thinking Large boxes weigh 75 pounds and small boxes weigh 40 pounds.

a. Write and graph an inequality that represents the numbers of large and small boxes a 200-pound delivery person can take on the elevator.

b. Identify and interpret two solutions of the inequality that are on the boundary line.

c. Explain why the solutions in part (b) might not be practical in real life.

Fair Game Review What you learned in previous grades & lessons

Multiply. *(Skills Review Handbook)*

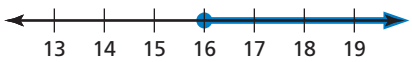
55. $4 \cdot 4 \cdot 4 \cdot 4$

56. $(-2) \cdot (-2) \cdot (-2)$

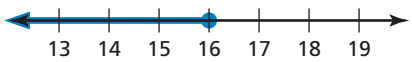
57. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

58. **MULTIPLE CHOICE** Which graph represents the solution of $-5(x - 9) \geq -35$? *(Section 3.4)*

A



B



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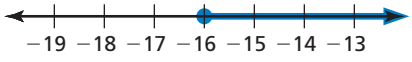

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