Extension 3.4 Solving Compound Inequalities

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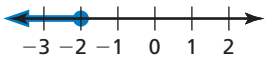
Key Vocabulary
compound inequality, *p. 132*
absolute value inequality, *p. 134*

A **compound inequality** is an inequality formed by joining two inequalities with the word "and" or the word "or."

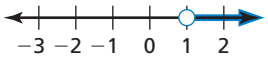
Solutions of a compound inequality with "and" consist of numbers that are solutions of both inequalities.

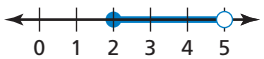
Solutions of a compound inequality with "or" consist of numbers that are solutions of at least one of the inequalities.

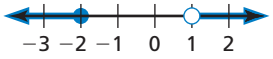
$x \geq 2$ 

$y \leq -2$ 

$x < 5$ 

$y > 1$ 

$2 \leq x$ *and* $x < 5$
$2 \leq x < 5$ 

$y \leq -2$ *or* $y > 1$ 

EXAMPLE 1 Writing and Graphing Compound Inequalities

Write each word sentence as an inequality. Graph the inequality.

a. A number x is greater than -8 and less than or equal to 4.

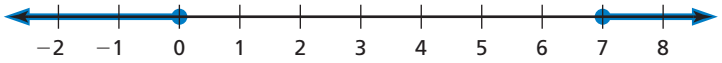
A number x is greater than -8 and less than or equal to 4.

$x > -8$ *and* $x \leq 4$

b. A number y is at most 0 or at least 7.

A number y is at most 0 or at least 7.

$y \leq 0$ *or* $y \geq 7$



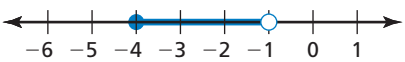
Study Tip

A compound inequality with "and" can be written as a single inequality. For example, you can write $x > -8$ and $x \leq 4$ as $-8 < x \leq 4$.

Practice

In Exercises 1–4, write the word sentence as an inequality. Graph the inequality.

1. A number k is more than 3 and less than 9.
2. A number n is greater than or equal to 6 and no more than 11.
3. A number w is fewer than -10 or no less than -6.
4. A number z is less than or equal to -5 or more than 4.
5. Write an inequality to describe the graph.



6. The world's longest human life span is 122 years. Write and graph a compound inequality that describes the ages of all humans.

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You can solve compound inequalities by solving two inequalities separately. When a compound inequality with "and" is written as a single inequality, you can solve the inequality by performing the same operation on each expression.

EXAMPLE 2 Solving a Compound Inequality with "And"

Solve $-3 < -2x + 1 \leq 9$. Graph the solution.

$-3 < -2x + 1 \leq 9$ Write the inequality.

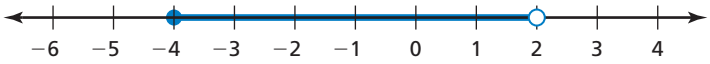
$\underline{-1} \quad \underline{-1} \quad \underline{-1}$ Subtract 1 from each expression.

$-4 < -2x \leq 8$ Simplify.

$\frac{-4}{-2} > \frac{-2x}{-2} \geq \frac{8}{-2}$ Divide each expression by -2. Reverse the inequality symbols.

$2 > x \geq -4$ Simplify.

∴ The solution is $-4 \leq x < 2$.



Study Tip

You can also solve the inequality in Example 2 by solving the inequalities

$-3 < -2x + 1$

and

$-2x + 1 \leq 9$

separately.

EXAMPLE 3 Solving a Compound Inequality with "Or"

Common Core

Solving Inequalities

In this extension, you will

- write, solve, and graph compound inequalities.
- write, solve, and graph absolute value inequalities.

Applying Standards
A.CED.1
A.CED.3
A.REI.3

Solve $3x - 5 < -8$ *or* $2x - 1 > 5$. Graph the solution.

$3x - 5 < -8$ *or* $2x - 1 > 5$ Write the inequality.

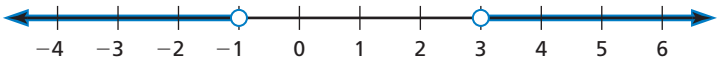
$\underline{+5} \quad \underline{+5}$ $\underline{+1} \quad \underline{+1}$ Addition Property of Inequality

$3x < -3$ *or* $2x > 6$ Simplify.

$\frac{3x}{3} < \frac{-3}{3}$ *or* $\frac{2x}{2} > \frac{6}{2}$ Division Property of Inequality

$x < -1$ *or* $x > 3$ Simplify.

∴ The solution is $x < -1$ *or* $x > 3$.



Practice

Solve the inequality. Graph the solution.

7. $4 < x - 5 < 7$

8. $-1 \leq 2x + 3 < 7$

9. $15 > -3x + 9 \geq 0$

10. $4x + 1 \leq -11$ *or* $3x - 4 \geq 5$

11. $-2x - 7 < 5$ *or* $-5x + 6 \geq 41$

Study Tip

When an absolute value expression is on the left side of an inequality, use an "and" statement for < and ≤, and an "or" statement for > and ≥.

An **absolute value inequality** is an inequality that contains an absolute value expression. For example, $|x| < 2$ and $|x| > 2$ are absolute value inequalities.

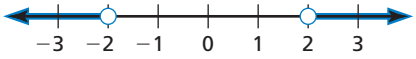
The distance between x and 0 is less than 2.

$|x| < 2$

The graph of $|x| < 2$ is $x > -2$ *and* $x < 2$.

The distance between x and 0 is greater than 2.

$|x| > 2$



The graph of $|x| > 2$ is $x < -2$ *or* $x > 2$.

You can solve these types of inequalities by solving a compound inequality.

Key Idea

Solving Absolute Value Inequalities

To solve $|ax + b| < c$ for $c > 0$, solve the compound inequality

$$ax + b > -c \quad \textit{and} \quad ax + b < c.$$

To solve $|ax + b| > c$ for $c > 0$, solve the compound inequality

$$ax + b < -c \quad \textit{or} \quad ax + b > c.$$

In the inequalities above, you can replace < with ≤ and > with ≥.

EXAMPLE 4 Solving Absolute Value Inequalities

a. Solve $|x + 7| \leq 2$. Graph the solution.

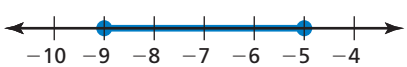
Use $|x + 7| \leq 2$ to write a compound inequality. Then solve.

$x + 7 \geq -2$ *and* $x + 7 \leq 2$ — Write compound inequality.

$-7 \quad -7 \qquad -7 \quad -7$ — Subtract 7 from each side.

$x \geq -9$ *and* $x \leq -5$ — Simplify.

∴ The solution is $x \geq -9$ *and* $x \leq -5$.



b. Solve $|8x - 11| < 0$.

The absolute value of an expression must be greater than or equal to 0. The expression $|8x - 11|$ cannot be less than 0.

∴ So, the inequality has no solution.

EXAMPLE 5 Solving an Absolute Value Inequality

Solve $4|2x - 5| + 1 > 29$.

$4|2x - 5| + 1 > 29$ Write the inequality.

$|2x - 5| > 7$ Isolate the absolute value expression.

Use $|2x - 5| > 7$ to write a compond inequality. Then solve.

$2x - 5 < -7$ *or* $2x - 5 > 7$ Write compound inequality.

$+5 \quad +5$ $+5 \quad +5$ Add 5 to each side.

$2x < -2$ *or* $2x > 12$ Simplify.

$\frac{2x}{2} < \frac{-2}{2}$ *or* $\frac{2x}{2} > \frac{12}{2}$ Divide each side by 2.

$x < -1$ *or* $x > 6$ Simplify.

EXAMPLE 6 Real-Life Application

In a poll, 47% of voters say they plan to reelect the mayor. The poll has a margin of error of ±2 percentage points. Write and solve an absolute value inequality to find the least and greatest percents of voters who plan to reelect the mayor.

Words | Actual percent of voters | minus | percent of voters in poll | | is less than or equal to | the margin of error.

Variable Let x represent the actual percent of voters who plan on reelecting the mayor.

Inequality $|x - 47| \leq 2$

$x - 47 \geq -2$ *and* $x - 47 \leq 2$ Write compound inequality.

$+47 \quad +47$ $+47 \quad +47$ Add 47 to each side.

$x \geq 45$ *and* $x \leq 49$ Simplify.

∴ The least percent of voters who plan to reelect the mayor is 45%. The greatest percent of voters who plan to reelect the mayor is 49%.

Practice

Solve the inequality. Graph the solution, if possible.

12. $|x - 3| \geq 4$

13. $|x + 7| < 1$

14. $11 \geq |4x - 5|$

15. $|8x - 9| < 0$

16. $3|2x + 5| - 8 \geq 19$

17. $-2|x - 10| + 1 > -7$

18. **NUMBER SENSE** What is the solution of $|4x - 2| \geq -6$? Explain.

19. **MODELING** In Example 6, 44% of the voters say they plan to reelect the mayor. The poll has a margin of error of ±3 percentage points. Use a model to write and solve an absolute value inequality to find the least and greatest percents of voters who plan to reelect the mayor.