3.3 Solving Inequalities Using Multiplication or Division

Essential Question How can you use multiplication or division to solve an inequality?

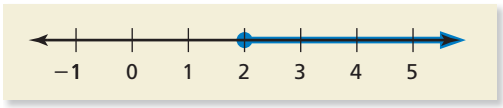
1 ACTIVITY: Using a Table to Solve an Inequality

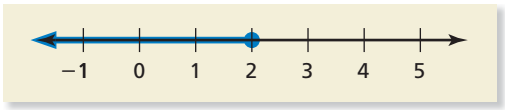
Work with a partner.

- **Copy and complete the table.**
- **Decide which graph represents the solution of the inequality.**
- **Write the solution of the inequality.**

a. $3x \leq 6$

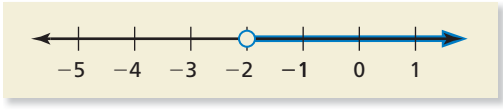
x	-1	0	1	2	3	4	5
$3x$							
$3x \stackrel{?}{\leq} 6$							

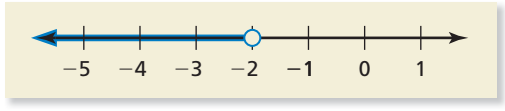




b. $-2x > 4$

x	-5	-4	-3	-2	-1	0	1
$-2x$							
$-2x \stackrel{?}{>} 4$							





Common Core

Solving Inequalities

In this lesson, you will

- solve inequalities using multiplication or division.
- solve real-life problems.

Learning Standards
A.CED.1
A.CED.3
A.REI.3

2 ACTIVITY: Writing a Rule

Work with a partner. Use a table to solve each inequality.

a. $3x > 3$ **b.** $4x \leq 4$ **c.** $-2x \geq 6$ **d.** $-5x < 10$

Write a rule that describes how to solve inequalities like those in Activity 1. Then use your rule to solve each of the four inequalities above.

3 ACTIVITY: Using a Table to Solve an Inequality

Math Practice 7

Look for Patterns

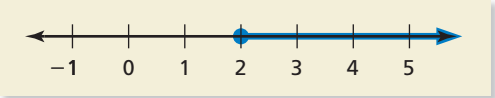
How do the patterns help you complete this activity?

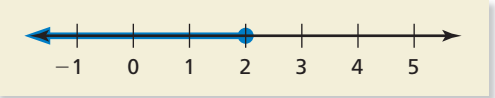
Work with a partner.

- **Copy and complete the table.**
- **Decide which graph represents the solution of the inequality.**
- **Write the solution of the inequality.**

a. $\frac{x}{2} \geq 1$

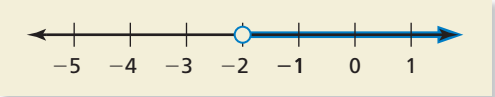
x	-1	0	1	2	3	4	5
$\frac{x}{2}$							
$\frac{x}{2} \stackrel{?}{\geq} 1$							

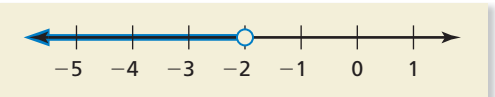




b. $\frac{x}{-3} < \frac{2}{3}$

x	-5	-4	-3	-2	-1	0	1
$\frac{x}{-3}$							
$\frac{x}{-3} \stackrel{?}{<} \frac{2}{3}$							





4 ACTIVITY: Writing a Rule

Work with a partner. Use a table to solve each inequality.

a. $\frac{x}{4} \geq 1$ **b.** $\frac{x}{2} < \frac{3}{2}$ **c.** $\frac{x}{-2} > 2$ **d.** $\frac{x}{-5} \leq \frac{1}{5}$

Write a rule that describes how to solve inequalities like those in Activity 3. Then use your rule to solve each of the four inequalities above.

What Is Your Answer?

5. IN YOUR OWN WORDS How can you use multiplication or division to solve an inequality?

Practice Use what you learned about solving inequalities using multiplication or division to complete Exercises 4–9 on page 121.

3.3 Lesson

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Key Idea

Remember
Multiplication and division are inverse operations.

Multiplication and Division Properties of Inequality (Case 1)

Words If you multiply or divide each side of an inequality by the same *positive* number, the inequality remains true.

Numbers

$-6 < 8$ $\qquad$ $6 > -8$

$2 \cdot (-6) < 2 \cdot 8$ $\qquad$ $\frac{6}{2} > \frac{-8}{2}$

$-12 < 16$ $\qquad$ $3 > -4$

Algebra

$\frac{x}{2} < -9$ $\qquad$ $4x > -12$

$2 \cdot \frac{x}{2} < 2 \cdot (-9)$ $\qquad$ $\frac{4x}{4} > \frac{-12}{4}$

$x < -18$ $\qquad$ $x > -3$

These properties are also true for $\leq$ and $\geq$.

EXAMPLE 1 Solving an Inequality Using Multiplication

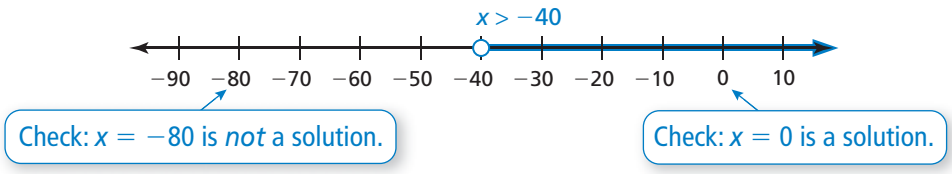
Solve $\frac{x}{8} > -5$. Graph the solution.

$\frac{x}{8} > -5$ Write the inequality.

Undo the division. → $8 \cdot \frac{x}{8} > 8 \cdot (-5)$ Multiply each side by 8.

$x > -40$ Simplify.

∴ The solution is $x > -40$.



On Your Own

Solve the inequality. Graph the solution.

1. $a \div 2 < 4$
2. $\frac{n}{7} \geq -1$
3. $-6.4 \geq \frac{w}{5}$

EXAMPLE 2 Solving an Inequality Using Division

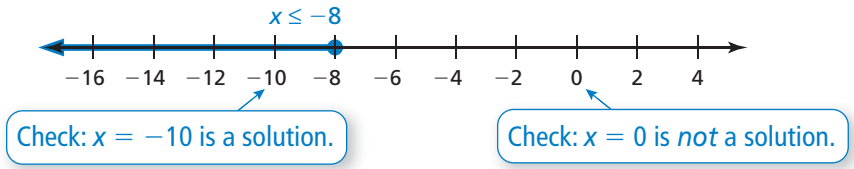
Solve $3x \leq -24$. Graph the solution.

$3x \leq -24$ — Write the inequality.

Undo the multiplication. → $\frac{3x}{3} \leq \frac{-24}{3}$ — Divide each side by 3.

$x \leq -8$ — Simplify.

∴ The solution is $x \leq -8$.



On Your Own

Now You're Ready
Exercises 10–18

Solve the inequality. Graph the solution.

4. $4b \geq 36$ **5.** $2k > -10$ **6.** $-18 > 1.5q$

Key Idea

Multiplication and Division Properties of Inequality (Case 2)

Words If you multiply or divide each side of an inequality by the same *negative* number, the direction of the inequality symbol must be reversed for the inequality to remain true.

Numbers

$-6 < 8$ $\qquad$ $6 > -8$

$(-2) \cdot (-6) > (-2) \cdot 8$ $\qquad$ $\frac{6}{-2} < \frac{-8}{-2}$

$12 > -16$ $\qquad$ $-3 < 4$

Algebra

$\frac{x}{-6} < 3$ $\qquad$ $-5x > 30$

$-6 \cdot \frac{x}{-6} > -6 \cdot 3$ $\qquad$ $\frac{-5x}{-5} < \frac{30}{-5}$

$x > -18$ $\qquad$ $x < -6$

These properties are also true for $\leq$ and $\geq$.

Common Error

A negative sign in an inequality does not necessarily mean you must reverse the inequality symbol.

Only reverse the inequality symbol when you multiply or divide both sides by a negative number.

EXAMPLE 3 Solving an Inequality Using Multiplication

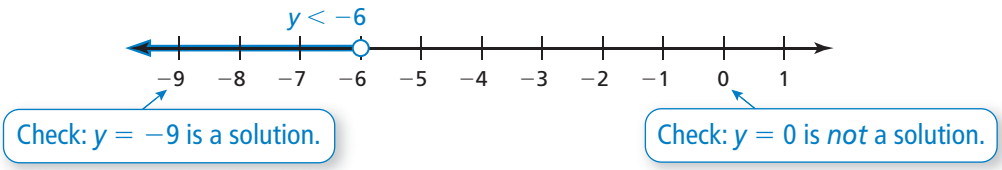
Solve $\frac{y}{-3} > 2$. Graph the solution.

$\frac{y}{-3} > 2$ — Write the inequality.

Undo the division. → $-3 \cdot \frac{y}{-3} < -3 \cdot 2$ — Multiply each side by -3. Reverse the inequality symbol.

$y < -6$ — Simplify.

∴ The solution is $y < -6$.



EXAMPLE 4 Solving an Inequality Using Division

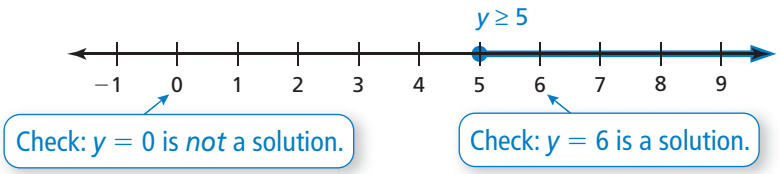
Solve $-7y \leq -35$. Graph the solution.

$-7y \leq -35$ — Write the inequality.

Undo the multiplication. → $\frac{-7y}{-7} \geq \frac{-35}{-7}$ — Divide each side by -7. Reverse the inequality symbol.

$y \geq 5$ — Simplify.

∴ The solution is $y \geq 5$.



On Your Own

Now You're Ready
Exercises 27–35

Solve the inequality. Graph the solution.

7. $\frac{p}{-4} < 7$

8. $\frac{x}{-5} \leq -5$

9. $1 \geq -\frac{1}{10}z$

10. $-9m > 63$

11. $-2r \geq -22$

12. $-0.4y \geq -12$

3.3 Exercises

Check It Out
Help with Homework
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Vocabulary and Concept Check

1. **VOCABULARY** Explain how to solve $\frac{x}{6} < -5$.
2. **WRITING** Explain how solving $2x < -8$ is different from solving $-2x < 8$.
3. **OPEN-ENDED** Write an inequality that is solved using the Division Property of Inequality where the inequality symbol needs to be reversed.

Practice and Problem Solving

Use a table to solve the inequality.

4. $4x < 4$
5. $-2x \leq 2$
6. $-5x > 15$
7. $\frac{x}{-3} \geq 1$
8. $\frac{x}{-2} > \frac{5}{2}$
9. $\frac{x}{4} \leq \frac{3}{8}$

Solve the inequality. Graph the solution.

10. $3n > 18$
11. $\frac{c}{4} \leq -9$
12. $1.2m < 12$
13. $-14 > x \div 2$
14. $\frac{w}{5} \geq -2.6$
15. $5 < 2.5k$
16. $4x \leq -\frac{3}{2}$
17. $2.6y \leq -10.4$
18. $10.2 > \frac{b}{3.4}$

19. **ERROR ANALYSIS** Describe and correct the error in solving the inequality.

✗
$$\frac{x}{2} < -5$$
$$2 \cdot \frac{x}{2} > 2 \cdot (-5)$$
$$x > -10$$

Write the word sentence as an inequality. Then solve the inequality.

20. The quotient of a number and 3 is at most 4.
21. A number divided by 8 is less than -2.
22. Four times a number is at least -12.
23. The product of 5 and a number is greater than 20.

24. **CAMERA** You earn \$9.50 per hour at your summer job. Write and solve an inequality that represents the number of hours you need to work in order to buy a digital camera that costs \$247.

25. COPIES You have \$3.65 to make copies. Write and solve an inequality that represents the number of copies you can make.

26. SPEED LIMIT The maximum speed limit for a school bus is 55 miles per hour. Write and solve an inequality that represents the number of hours it takes to travel 165 miles in a school bus.

Solve the inequality. Graph the solution.

27. $-2n \leq 10$

28. $-5w > 30$

29. $\frac{h}{-6} \geq 7$

30. $-8 < -\frac{1}{3}x$

31. $-2y < -11$

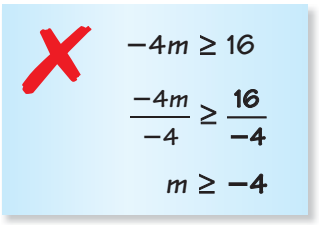
32. $-7d \geq 56$

33. $2.4 > -\frac{m}{5}$

34. $\frac{k}{-0.5} \leq 18$

35. $-2.5 > \frac{b}{-1.6}$

36. ERROR ANALYSIS Describe and correct the error in solving the inequality.



37. CRITICAL THINKING Are all numbers greater than zero solutions of $-x > 0$? Explain.

38. TRUCKING In many states, the maximum height (including freight) of a vehicle is 13.5 feet.

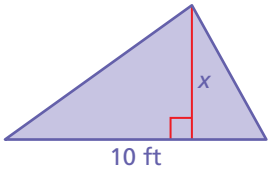
a. Five crates are stacked vertically on the bed of the truck. Is this legal? Explain.

b. Write and solve an inequality to justify your answer to part (a).

Write and solve an inequality that represents the value of x.

39. Area ≥ 102 cm^2

40. Area < 30 ft^2



41. **TRIP** You and three friends are planning a trip. You want to keep the cost below $80 per person. Write and solve an inequality that represents the total cost of the trip.

42. **PRECISION** Explain why the direction of the inequality symbol must be reversed when multiplying or dividing by the same negative number.

43. **PROJECT** Choose two musical artists to research.

a. Use the Internet or a magazine to complete the table.

b. Find and compare the average number of copies sold per month for each CD. Which CD do you consider to be the most successful? Explain.

c. Assume each CD continues to sell at the average rate. Write and solve an inequality that represents the number of months it will take for the total number of copies sold to exceed twice the current number sold.

Artist	Name of CD	Release Date	Current Number of Copies Sold
1.			
2.			

Structure **Describe all numbers that satisfy *both* inequalities. Include a graph with your description.**

44. $3m > -12$ and $2m < 12$

45. $\frac{n}{2} \geq -3$ and $\frac{n}{-4} \geq 1$

46. $2x \geq -4$ and $2x \geq 4$

47. $\frac{m}{-4} > -5$ and $\frac{m}{4} < 10$

Fair Game Review What you learned in previous grades & lessons

Solve the equation. *(Section 1.2)*

48. $-4w + 5 = -11$

49. $4(x - 3) = 21$

50. $\frac{v}{6} - 7 = 4$

51. $\frac{m + 300}{4} = 96$

52. **MULTIPLE CHOICE** Which of the following is *not* a solution of $p - 3.9 \geq 0.8$? *(Section 3.2)*

(A) $p = -4.5$ (B) $p = 4.7$ (C) $p = 4.75$ (D) $p = 5$