Essential Question How can the slope of a line be used to describe the line?
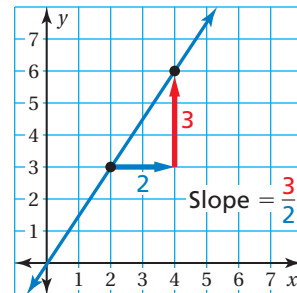
Slope is the rate of change between any two points on a line. It is the measure of the *steepness* of the line.

To find the slope of a line, find the ratio of the change in *y* (vertical change) to the change in *x* (horizontal change).

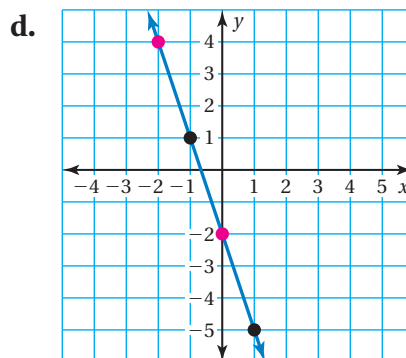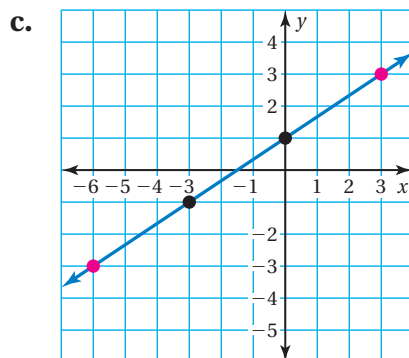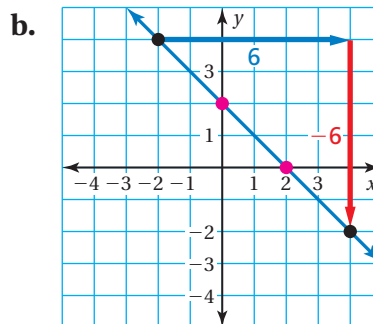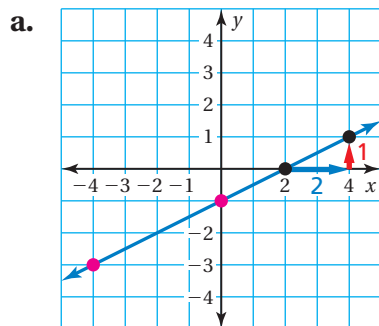$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$

Slope $= \dfrac{3}{2}$

1 ACTIVITY: Finding the Slope of a Line

Work with a partner. Find the slope of each line using two methods.

Method 1: Use the two black points. ●

Method 2: Use the two pink points. ●

Do you get the same slope using each method? Why do you think this happens?

a.

b.

c.

d.

2 ACTIVITY: Drawing Lines with Given Slopes

Work with a partner.

- Draw a line through the black point using the given slope.
- Draw a line through the pink point using the given slope.
- What do you notice about the two lines?

a. Slope $= \dfrac{3}{4}$
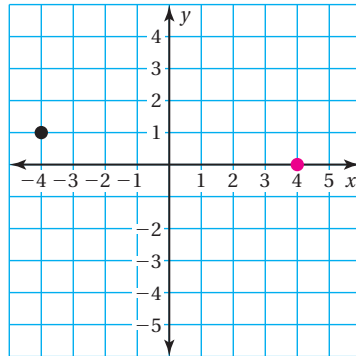
b. Slope $= -\dfrac{4}{3}$

3 ACTIVITY: Drawing Lines with Given Slopes

Work with a partner.
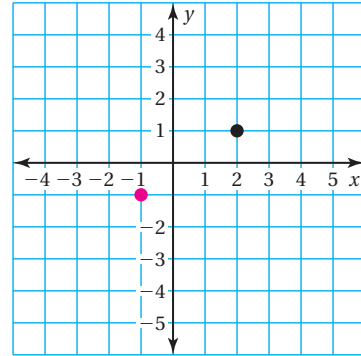
- Examine the lines drawn through the black points in parts (a) and (b) of Activity 2. Draw these two lines in the same coordinate plane.
- Describe the angle formed by the two lines. What do you notice about the product of the slopes of the two lines?

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can the slope of a line be used to describe the line?

5. Based on your results in Activity 2, make a conjecture about two different nonvertical lines in the same plane that have the same slope.

6. **REPEATED REASONING** Repeat Activity 3 for the lines drawn through the pink points in Activity 2. Based on your results, make a conjecture about two lines in the same plane whose slopes have a product of -1.

Practice

Use what you learned about the slope of a line to complete Exercises 4–6 on page 53.

Key Vocabulary
slope, *p. 50*
rise, *p. 50*
run, *p. 50*

🔑 Key Idea

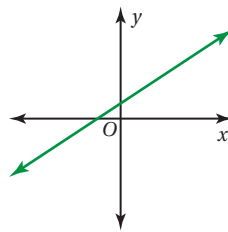**Slope**

The **slope** of a line is a ratio of the change in y (the **rise**) to the change in x (the **run**) between any two points, (x_1, y_1) and (x_2, y_2), on the line.
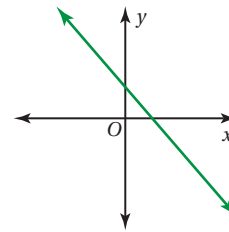
$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Rise $= y_2 - y_1$
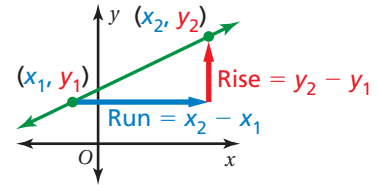*Run* $= x_2 - x_1$

Positive slope

Negative slope

The line rises from left to right.   The line falls from left to right.

Reading

In the slope formula, x_1 is read as "x sub one" and y_2 is read as "y sub two". The numbers 1 and 2 in x_1 and y_2 are called *subscripts*.

EXAMPLE 1   Finding the Slope of a Line

Describe the slope of the line. Then find the slope.

a.

(3, 4)

5

(−3, −1) 6

b.

(−1, 1)

2

−3

(1, −2)
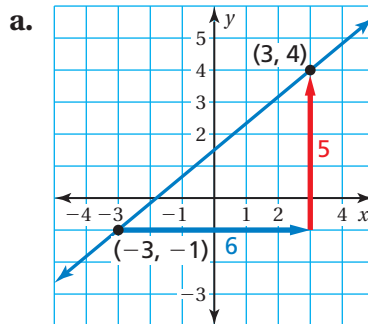
The line rises from left to right. So, the slope is positive. Let $(x_1, y_1) = (-3, -1)$ and $(x_2, y_2) = (3, 4)$.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4 - (-1)}{3 - (-3)}$$

$$= \frac{5}{6}$$
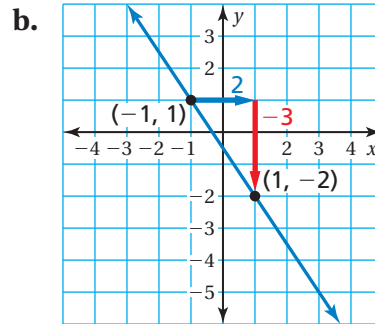
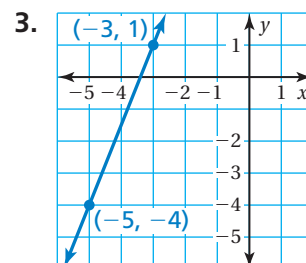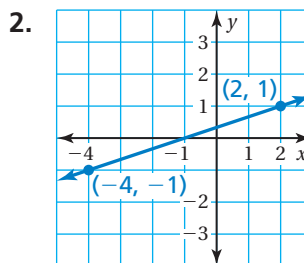The line falls from left to right. So, the slope is negative. Let $(x_1, y_1) = (-1, 1)$ and $(x_2, y_2) = (1, -2)$.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-2 - 1}{1 - (-1)}$$

$$= \frac{-3}{2}, \text{ or } -\frac{3}{2}$$

Study Tip

When finding slope, you can label either point as (x_1, y_1) and the other point as (x_2, y_2).

🔊 Multi-Language Glossary at BigIdeasMath.com

On Your Own

Find the slope of the line.

1.

2.

3.
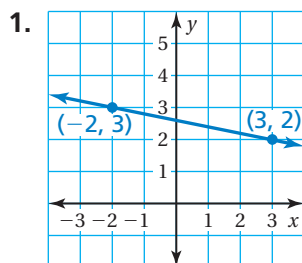
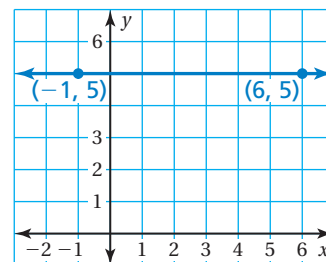**EXAMPLE 2   Finding the Slope of a Horizontal Line**

Find the slope of the line.

There is no change in y. So, the change in y is 0.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5 - 5}{6 - (-1)}$$
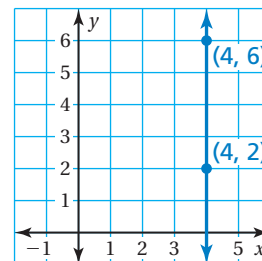
$$= \frac{0}{7}, \text{ or } 0$$

∴ The slope is 0.

EXAMPLE 3   Finding the Slope of a Vertical Line

Find the slope of the line.

There is no change in x. So, the change in x is 0.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{6 - 2}{4 - 4}$$

$$= \frac{4}{0} \quad ✗$$

∴ Because division by zero is undefined, the slope of the line is undefined.

On Your Own

Find the slope of the line through the given points.

4. $(1, -2), (7, -2)$ **5.** $(-3, -3), (-3, -5)$ **6.** $(0, 8), (0, 0)$

7. How do you know that the slope of every horizontal line is 0? How do you know that the slope of every vertical line is undefined?

EXAMPLE 4 **Finding Slope from a Table**

The points in the table lie on a line. How can you find the slope of the line from the table? What is the slope?

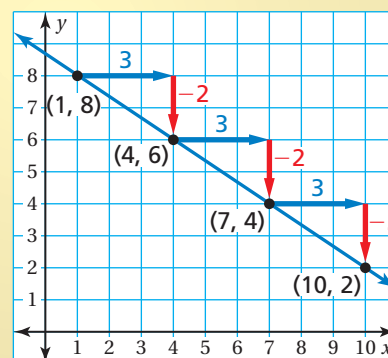| x	1	4	7	10
y	8	6	4	2

Choose any two points from the table and use the slope formula.

Use the points $(x_1, y_1) = (1, 8)$ and $(x_2, y_2) = (4, 6)$.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{6 - 8}{4 - 1}$$

$$= \frac{-2}{3}$$

Check

The slope is $-\dfrac{2}{3}$.

On Your Own

The points in the table lie on a line. How can you find the slope of the line from the table? What is the slope?
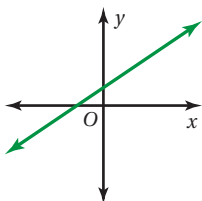
8.

x	1	3	5	7
y	2	5	8	11

9.

x	−3	−2	−1	0
y	6	4	2	0

Summary

Slope
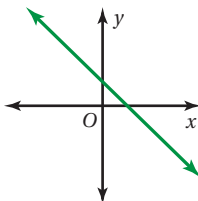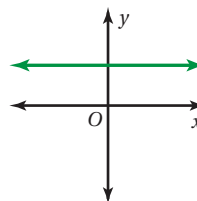
| *Positive slope*	*Negative slope*	*Slope of 0*	*Undefined slope*
The line rises from left to right.	The line falls from left to right.	The line is horizontal.	The line is vertical.
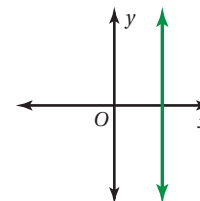
✓ Vocabulary and Concept Check

1. **CRITICAL THINKING** Refer to the graph.

   a. Which lines have positive slopes?

   b. Which line has the steepest slope?

   c. Do any of the lines have undefined slope? Explain.

2. **OPEN-ENDED** Describe a real-life situation in which you need to know the slope.

3. **REASONING** The slope of a line is 0. What do you know about the line?
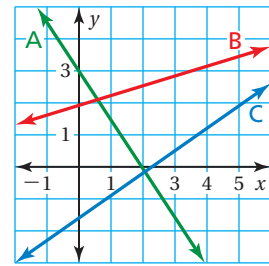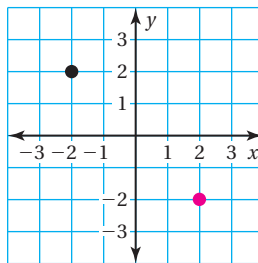
Practice and Problem Solving

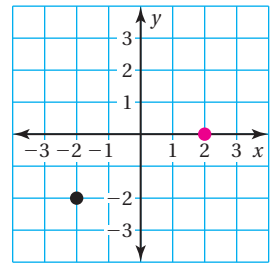**Draw a line through each point using the given slope. What do you notice about the two lines?**
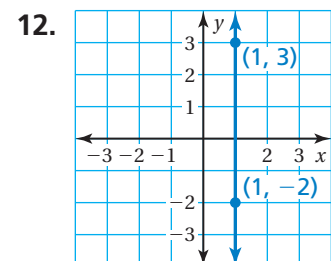
4. Slope $= 1$
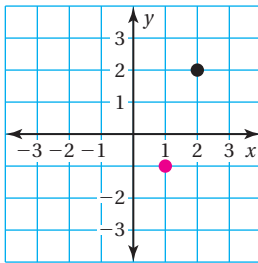
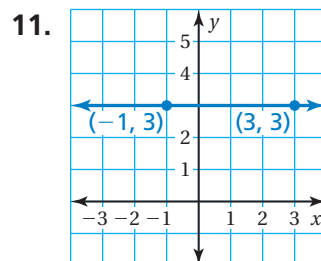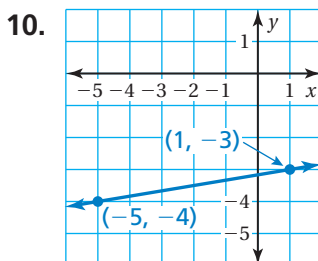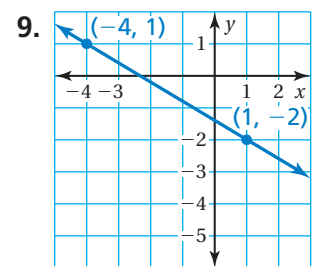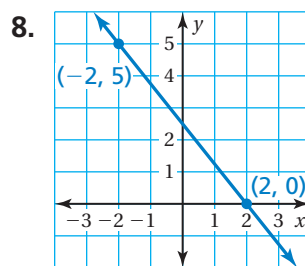5. Slope $= -3$

6. Slope $= \dfrac{1}{4}$

Find the slope of the line.
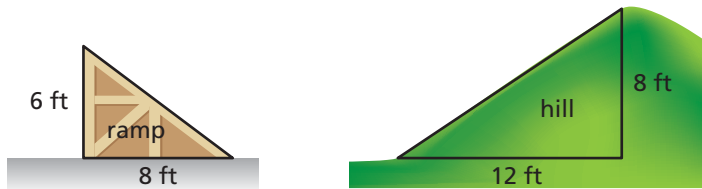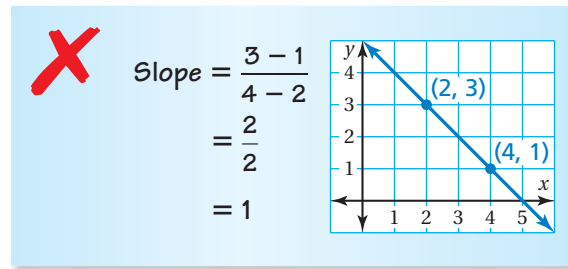
① 7.

8.

9.

10.

11.

12.
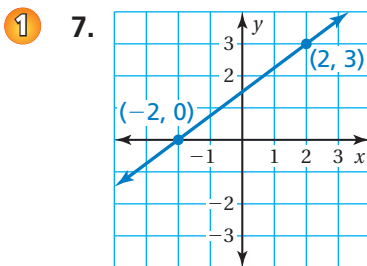
Find the slope of the line through the given points.

2 3 13. $(4, -1), (-2, -1)$ **14.** $(5, -3), (5, 8)$ **15.** $(-7, 0), (-7, -6)$

16. $(-3, 1), (-1, 5)$ **17.** $(10, 4), (4, 15)$ **18.** $(-3, 6), (2, 6)$

19. ERROR ANALYSIS Describe and correct the error in finding the slope of the line.

20. CRITICAL THINKING Is it more difficult to walk up the ramp or the hill? Explain.

$$\text{Slope} = \frac{3 - 1}{4 - 2}$$
$$= \frac{2}{2}$$
$$= 1$$

6 ft — ramp — 8 ft

hill — 8 ft — 12 ft

The points in the table lie on a line. How can you find the slope of the line from the table? What is the slope?
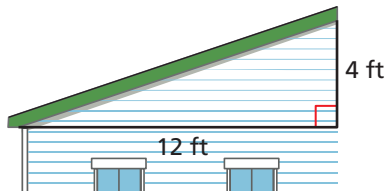
4 21.

x	1	3	5	7
y	2	10	18	26

22.

x	-3	2	7	12
y	0	2	4	6

23.

x	-6	-2	2	6
y	8	5	2	-1

24.

x	-8	-2	4	10
y	8	1	-6	-13

12 ft — 4 ft

25. PITCH Carpenters refer to the slope of a roof as the *pitch* of the roof. Find the pitch of the roof.

26. PROJECT The guidelines for a wheelchair ramp suggest that the ratio of the rise to the run be no greater than $1:12$.

    a. CHOOSE TOOLS Find a wheelchair ramp in your school or neighborhood. Measure its slope. Does the ramp follow the guidelines?

    b. Design a wheelchair ramp that provides access to a building with a front door that is 2.5 feet higher than the sidewalk. Illustrate your design.
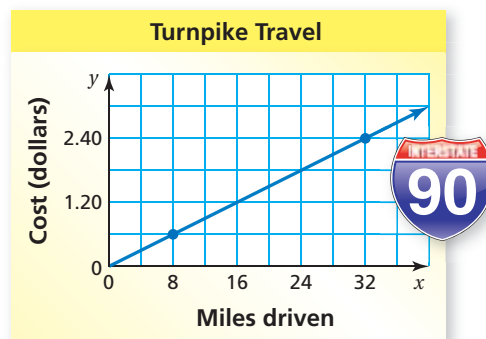
Use an equation to find the value of k so that the line that passes through the given points has the given slope.

27. $(1, 3), (5, k)$; slope $= 2$ **28.** $(-2, k), (2, 0)$; slope $= -1$

29. $(-4, k), (6, -7)$; slope $= -\dfrac{1}{5}$ **30.** $(4, -4), (k, -1)$; slope $= \dfrac{3}{4}$

31. TURNPIKE TRAVEL The graph shows the cost of traveling by car on a turnpike.

    a. Find the slope of the line.

    b. Explain the meaning of the slope as a rate of change.

Turnpike Travel

32. BOAT RAMP Which is steeper: the boat ramp or a road with a 12% grade? Explain. (*Note:* Road grade is the vertical increase divided by the horizontal distance.)


6 ft

36 ft
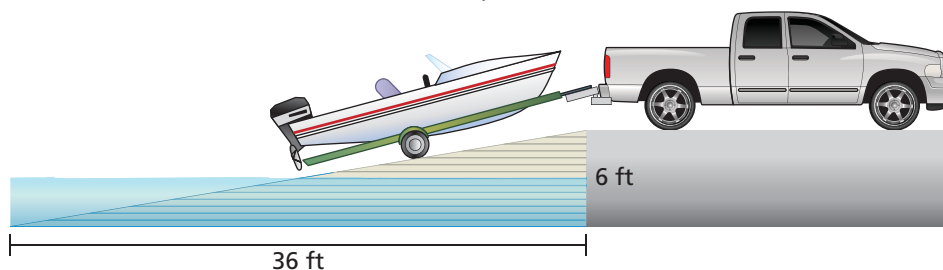
33. REASONING Do the points $A(-2, -1)$, $B(1, 5)$, and $C(4, 11)$ lie on the same line? Without using a graph, how do you know?

34. BUSINESS A small business earns a profit of $6500 in January and $17,500 in May. What is the rate of change in profit for this time period?

35. STRUCTURE Choose two points in the coordinate plane. Use the slope formula to find the slope of the line that passes through the two points. Then find the slope using the formula $\dfrac{y_1 - y_2}{x_1 - x_2}$. Are your results the same? Explain.

36. **Critical Thinking** The top and bottom of the slide are level with the ground, which has a slope of 0.

    a. What is the slope of the main portion of the slide?

    b. How does the slope change if the bottom of the slide is only 12 inches above the ground? Is the slide steeper? Explain.

1 ft

8 ft

1 ft

18 in.

12 ft
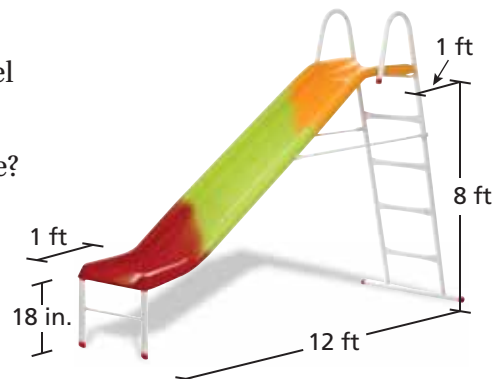
 Fair Game Review What you learned in previous grades & lessons

Graph the linear equation. *(Section 2.1)*

37. $y = -\dfrac{1}{2}x$

38. $y = 3x - \dfrac{3}{4}$

39. $y = -\dfrac{x}{3} - \dfrac{3}{2}$

40. MULTIPLE CHOICE What is the prime factorization of 84? *(Skills Review Handbook)*

    (**A**) $2 \times 3 \times 7$ (**B**) $2^2 \times 3 \times 7$ (**C**) $2 \times 3^2 \times 7$ (**D**) $2^2 \times 21$