



Angles and Triangles

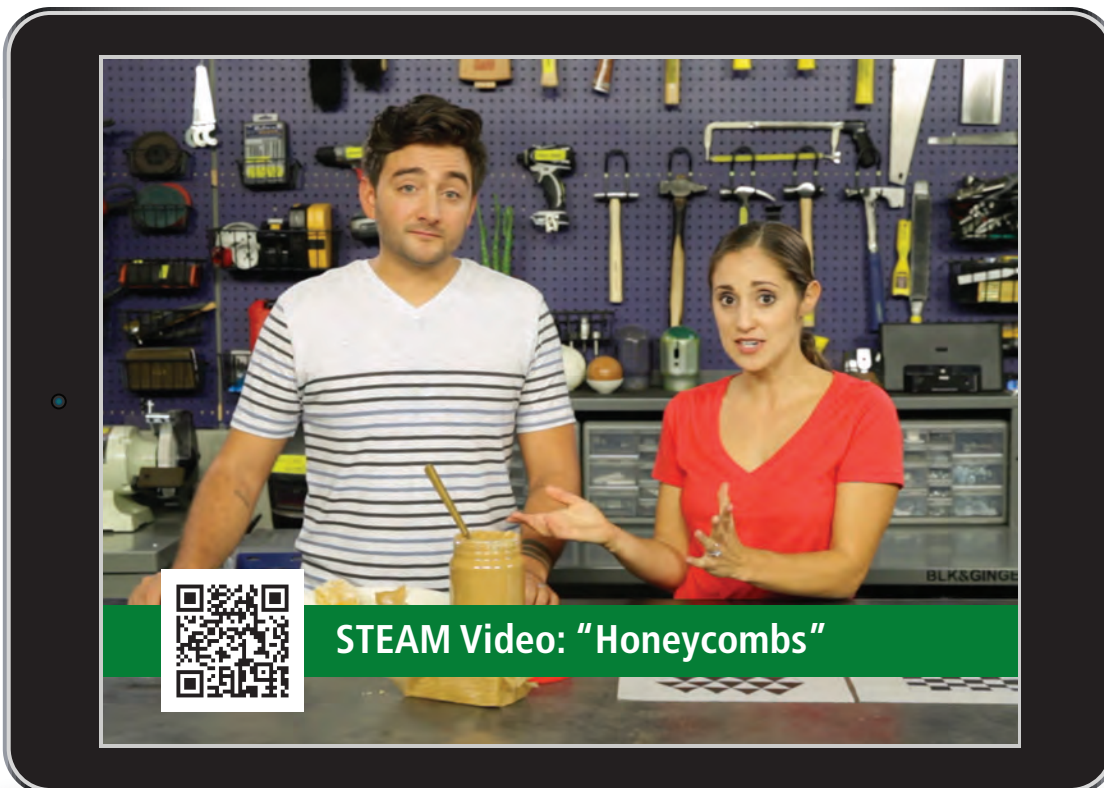
- 3.1** Parallel Lines and Transversals
- 3.2** Angles of Triangles
- 3.3** Angles of Polygons
- 3.4** Using Similar Triangles

Chapter Learning Target:

Understand angles.

Chapter Success Criteria:

- I can identify angle relationships.
- I can find angle measurements.
- I can compare angles.
- I can apply angle relationships to solve real-life problems.



STEAM Video

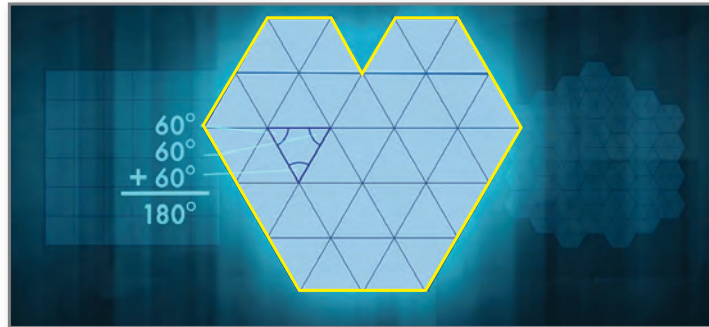


Honeycombs

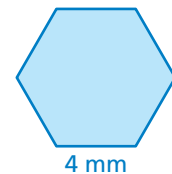
Each cell in a honeycomb is in the shape of a regular hexagon. Why might bees use this shape?

Watch the STEAM Video “Honeycombs.” Then answer the following questions.

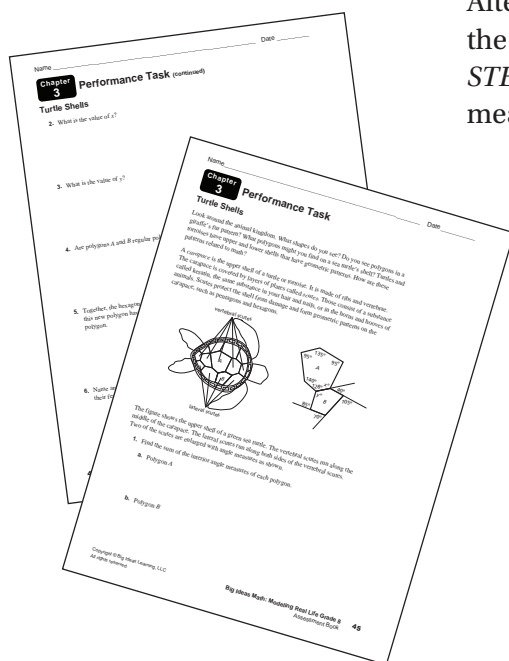
1. Enid and Tony show regular tilings made out of squares, equilateral triangles, and regular hexagons. What is the sum of the interior angle measures of the tiling made from equilateral triangles, outlined below in yellow?



2. The cells in a honeycomb use a tiling pattern of the regular hexagon shown. A cell is 10 millimeters deep. About how much honey can one cell hold? Explain.

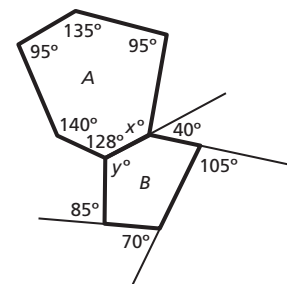
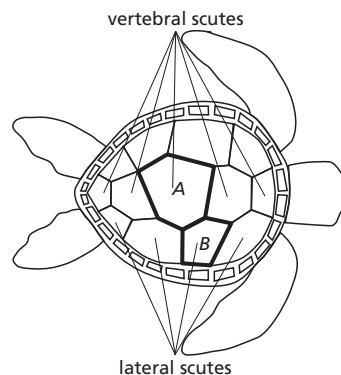


Performance Task



Turtle Shells

After completing this chapter, you will be able to use the concepts you learned to answer the questions in the STEAM Video Performance Task. You will be given angle measures of shapes seen on a turtle shell.



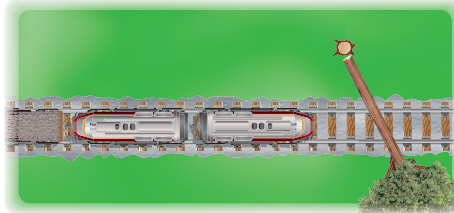
You will be asked to find angle sums and missing angle measures. What other animals have features that resemble geometric shapes?

Getting Ready for Chapter

3

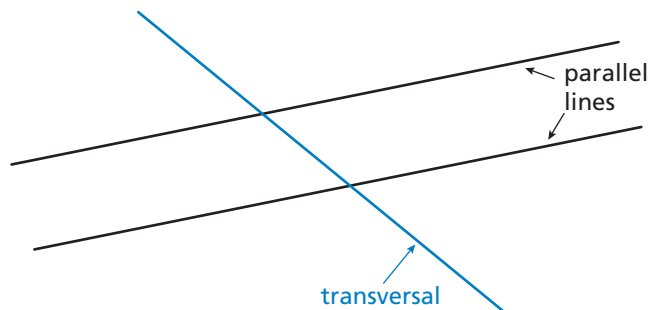
Chapter Exploration

When an object is **transverse**, it is lying or extending across something. In the drawing, the fallen tree lying across the railroad track is transverse to the track.



1. Work with a partner.

- Discuss what it means for two lines to be parallel. Decide on a strategy for drawing parallel lines. Then draw two parallel lines.
- Draw a third line that intersects the parallel lines. This line is called a *transversal*.



- a. How many angles are formed by the parallel lines and the transversal? Label each angle.
- b. Which of these angles have equal measures? Explain your reasoning.

Vocabulary

The following vocabulary terms are defined in this chapter. Think about what the terms might mean and record your thoughts.

transversal
interior angles of a polygon

exterior angles of a polygon
regular polygon

3.1 Parallel Lines and Transversals

Learning Target: Find missing angle measures created by the intersections of lines.

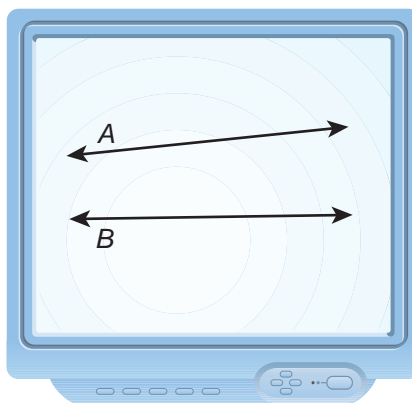
Success Criteria:

- I can identify congruent angles when a transversal intersects parallel lines.
- I can find angle measures when a transversal intersects parallel lines.

EXPLORATION 1

Exploring Intersections of Lines

Work with a partner. Use geometry software and the lines A and B shown.



Math Practice

Use Clear Definitions

What does it mean for two lines to be parallel? How does this help you answer the question in part (a)?

- Are line A and line B parallel? Explain your reasoning.
- Draw a line C that intersects both line A and line B . What do you notice about the measures of the angles that are created?
- Rotate line A or line B until the angles created by the intersection of line A and line C are congruent to the angles created by the intersection of line B and line C . What do you notice about line A and line B ?
- Rotate line C to create different angle measures. Are the angles that were congruent in part (c) still congruent?
- Make a conjecture about the measures of the angles created when a line intersects two parallel lines.

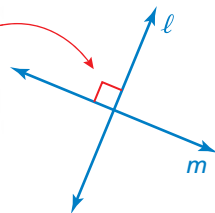
3.1 Lesson

Key Vocabulary

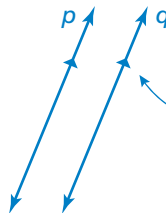
transversal, p. 104
 interior angles, p. 105
 exterior angles, p. 105

Lines in the same plane that do not intersect are called *parallel lines*.
 Lines that intersect at right angles are called *perpendicular lines*.

Indicates lines ℓ and m are perpendicular.



Indicates lines p and q are parallel.

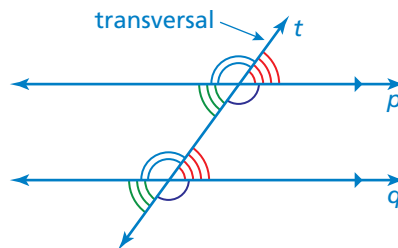


A line that intersects two or more lines is called a **transversal**. When parallel lines are cut by a transversal, several pairs of congruent angles are formed.

Key Idea

Corresponding Angles

When a transversal intersects parallel lines, corresponding angles are congruent.

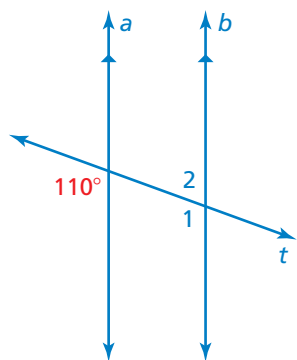


Corresponding angles

Corresponding angles lie on the same side of the transversal in corresponding positions.

EXAMPLE 1

Finding Angle Measures



Use the figure to find the measures of (a) $\angle 1$ and (b) $\angle 2$.

- a. $\angle 1$ and the 110° angle are corresponding angles formed by a transversal intersecting parallel lines. The angles are congruent.

► So, the measure of $\angle 1$ is 110° .

- b. $\angle 1$ and $\angle 2$ are supplementary.

$$\angle 1 + \angle 2 = 180^\circ$$

Definition of supplementary angles

$$110^\circ + \angle 2 = 180^\circ$$

Substitute 110° for $\angle 1$.

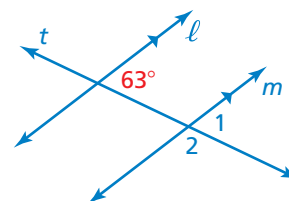
$$\angle 2 = 70^\circ$$

Subtract 110° from each side.

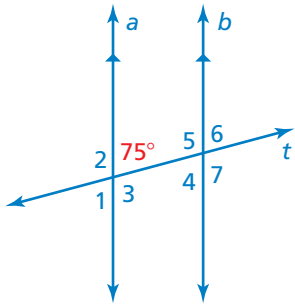
► So, the measure of $\angle 2$ is 70° .

Try It Use the figure to find the measure of the angle. Explain your reasoning.

- $\angle 1$
- $\angle 2$



EXAMPLE 2 Using Corresponding Angles



Use the figure to find the measures of the numbered angles.

∠1: ∠1 and the 75° angle are vertical angles. They are congruent.

So, the measure of ∠1 is 75°.

∠2 and ∠3: The 75° angle is supplementary to both ∠2 and ∠3.

$$75^\circ + \angle 2 = 180^\circ \quad \text{Definition of supplementary angles}$$

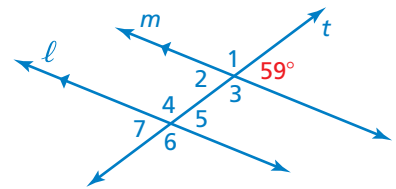
$$\angle 2 = 105^\circ \quad \text{Subtract } 75^\circ \text{ from each side.}$$

So, the measures of ∠2 and ∠3 are 105°.

∠4, ∠5, ∠6, and ∠7: Corresponding angles are congruent because they are formed by a transversal intersecting parallel lines. So, the measures of ∠4 and ∠6 are 75°, and the measures of ∠5 and ∠7 are 105°.

Try It

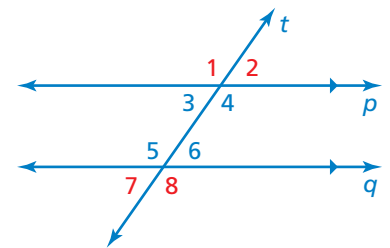
3. Use the figure to find the measures of the numbered angles.



When two parallel lines are cut by a transversal, four **interior angles** are formed on the inside of the parallel lines and four **exterior angles** are formed on the outside of the parallel lines.

∠3, ∠4, ∠5, and ∠6 are interior angles.

∠1, ∠2, ∠7, and ∠8 are exterior angles.

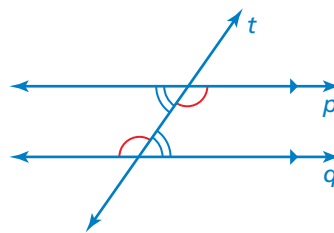


Key Ideas

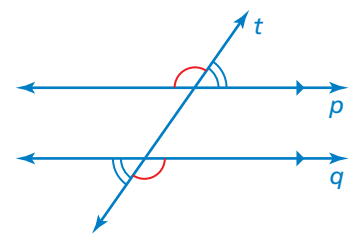
Alternate interior angles and alternate exterior angles lie on opposite sides of the transversal.

Alternate Interior Angles and Alternate Exterior Angles

When a transversal intersects parallel lines, alternate interior angles are congruent and alternate exterior angles are congruent.



Alternate interior angles



Alternate exterior angles

EXAMPLE 3

Identifying Angle Relationships

The photo shows a portion of an airport. Describe the relationship between each pair of angles.



a. $\angle 3$ and $\angle 6$

$\angle 3$ and $\angle 6$ are alternate exterior angles formed by a transversal intersecting parallel lines.

▶ So, $\angle 3$ is congruent to $\angle 6$.

b. $\angle 2$ and $\angle 7$

$\angle 2$ and $\angle 7$ are alternate interior angles formed by a transversal intersecting parallel lines.

▶ So, $\angle 2$ is congruent to $\angle 7$.

Try It In Example 3, the measure of $\angle 4$ is 84° . Find the measure of the angle. Explain your reasoning.

4. $\angle 3$

5. $\angle 5$

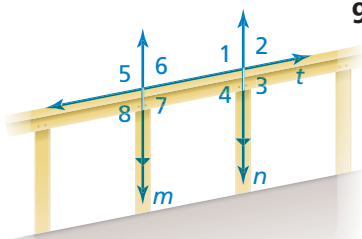
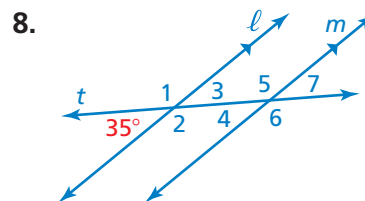
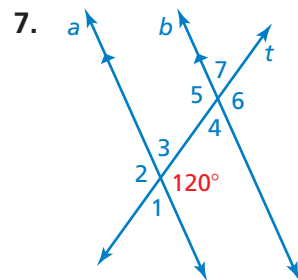
6. $\angle 6$



Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

FINDING ANGLE MEASURES Use the figure to find the measures of the numbered angles.



9. **WHICH ONE DOESN'T BELONG?** Which angle measure does *not* belong with the other three? Explain your reasoning.

the measure of $\angle 2$

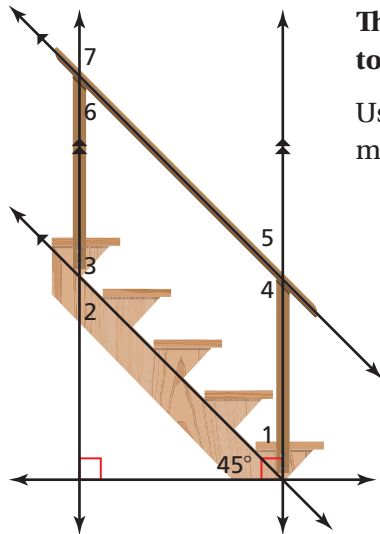
the measure of $\angle 5$

the measure of $\angle 6$

the measure of $\angle 8$

EXAMPLE 4

Modeling Real Life



The stairs have a 45° incline. At what angles do you need to attach a rail to two parallel posts so that the rail is parallel to the incline of the steps?

Use angle relationships to find the measures of $\angle 4$, $\angle 5$, $\angle 6$, and $\angle 7$ that make the rail parallel to the incline of the steps.

$\angle 1$: The 45° angle is complementary to $\angle 1$.

$$45^\circ + \angle 1 = 90^\circ$$

Definition of complementary angles

$$\angle 1 = 45^\circ$$

Subtract 45° from each side.

$\angle 5$: $\angle 1$ and $\angle 5$ are congruent because they are corresponding angles formed by a transversal intersecting parallel lines.

So, the measure of $\angle 5$ is 45° .

$\angle 4$: $\angle 4$ and $\angle 5$ are supplementary.

$$\angle 4 + \angle 5 = 180^\circ$$

Definition of supplementary angles

$$\angle 4 + 45^\circ = 180^\circ$$

Substitute 45° for $\angle 5$.

$$\angle 4 = 135^\circ$$

Subtract 45° from each side.

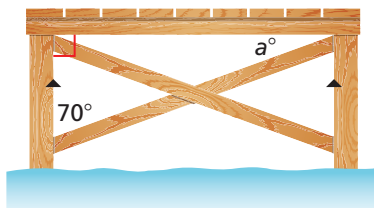
$\angle 6$ and $\angle 7$: Using alternate interior angles, the measure of $\angle 6$ is 45° and the measure of $\angle 7$ is 135° .

You need to attach the rail so that the measures of $\angle 5$ and $\angle 6$ are 45° and the measures of $\angle 4$ and $\angle 7$ are 135° .



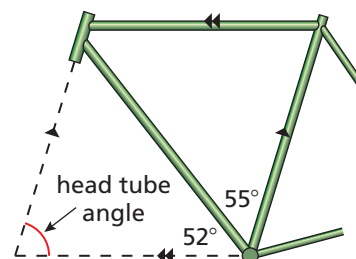
Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.



10. A cross section of a pier is shown. Find the value of a . Justify your answer.

11. The *head tube angle* of a bike determines how easy the bike is to steer. A bike frame with angle approximations is shown. What is the head tube angle of the bike?



3.1 Practice

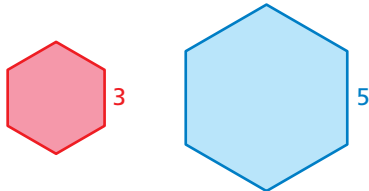


Go to BigIdeasMath.com to get HELP with solving the exercises.

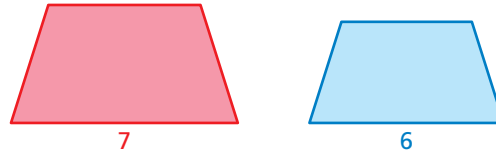
► Review & Refresh

Find the values of the ratios (red to blue) of the perimeters and areas of the similar figures.

1.



2.



Evaluate the expression.

3. $4 + 3^2$

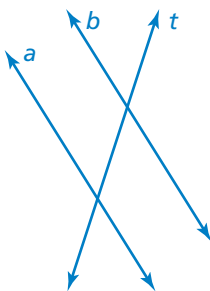
4. $5(2)^2 - 6$

5. $11 + (-7)^2 - 9$

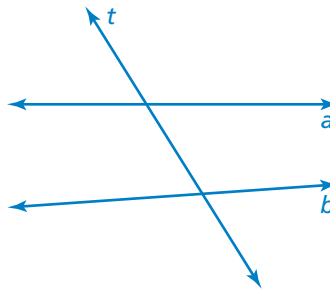
► Concepts, Skills, & Problem Solving

EXPLORING INTERSECTIONS OF LINES Use a protractor to determine whether lines a and b are parallel. (See Exploration 1, p. 103.)

6.

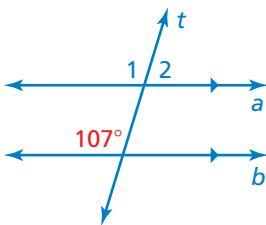


7.

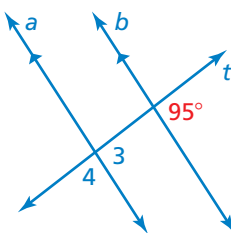


FINDING ANGLE MEASURES Use the figure to find the measures of the numbered angles. Explain your reasoning.

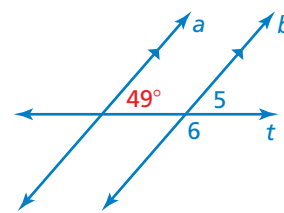
8.



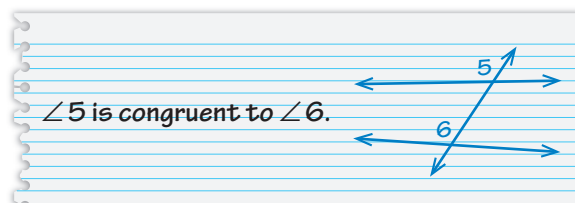
9.

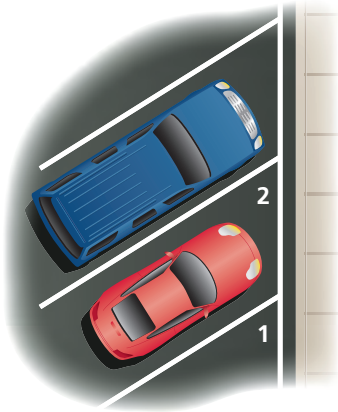


10.



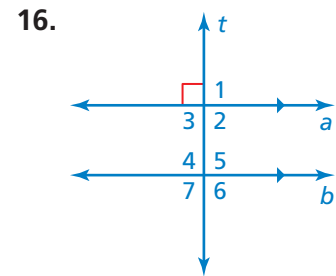
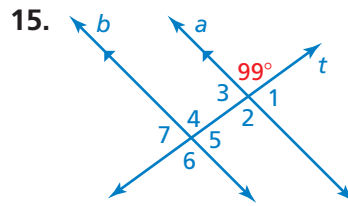
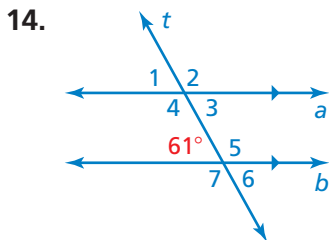
11. **YOU BE THE TEACHER** Your friend describes a relationship between the angles shown. Is your friend correct? Explain your reasoning.





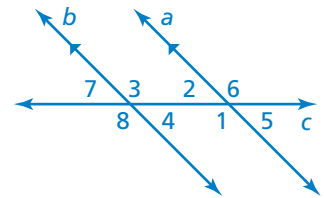
12. **MP PROBLEM SOLVING** The painted lines that separate parking spaces are parallel. The measure of $\angle 1$ is 60° . What is the measure of $\angle 2$? Explain.
13. **OPEN-ENDED** Describe two real-life situations that use parallel lines.

USING CORRESPONDING ANGLES Use the figure to find the measures of the numbered angles.

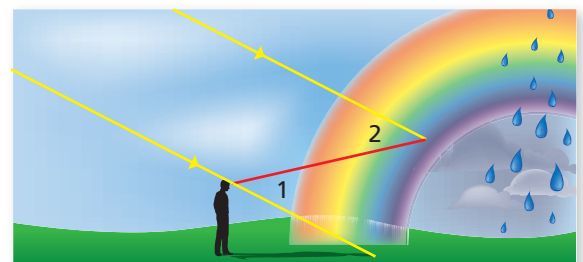


USING CORRESPONDING ANGLES Complete the statement. Explain your reasoning.

17. If the measure of $\angle 1 = 124^\circ$, then the measure of $\angle 4 = \square$.
18. If the measure of $\angle 2 = 48^\circ$, then the measure of $\angle 3 = \square$.
19. If the measure of $\angle 4 = 55^\circ$, then the measure of $\angle 2 = \square$.
20. If the measure of $\angle 6 = 120^\circ$, then the measure of $\angle 8 = \square$.
21. If the measure of $\angle 7 = 50.5^\circ$, then the measure of $\angle 6 = \square$.
22. If the measure of $\angle 3 = 118.7^\circ$, then the measure of $\angle 2 = \square$.

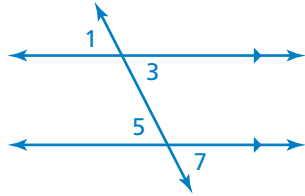


23. **MODELING REAL LIFE** A rainbow forms when sunlight reflects off raindrops at different angles. For blue light, the measure of $\angle 2$ is 40° . What is the measure of $\angle 1$?

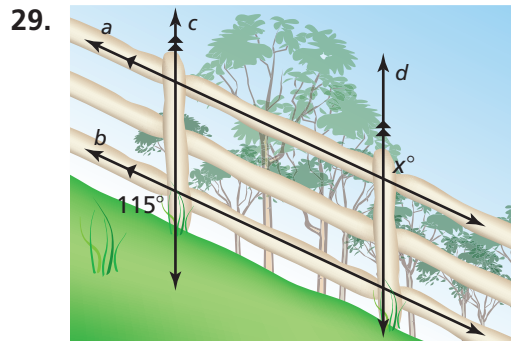
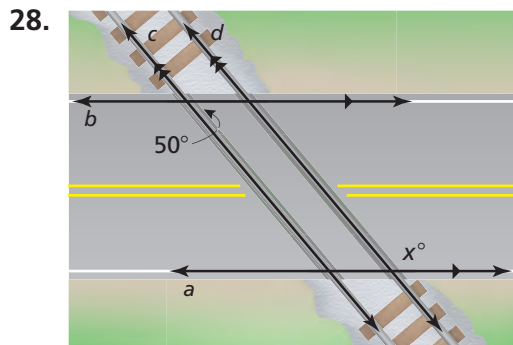


24. **MP REASONING** Is there a relationship between exterior angles that lie on the same side of a transversal? interior angles that lie on the same side of a transversal? Explain.
25. **MP REASONING** When a transversal is perpendicular to two parallel lines, all the angles formed measure 90° . Explain why.

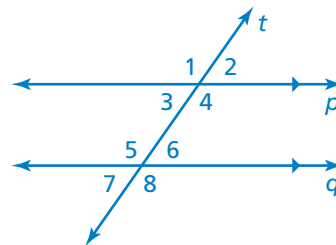
26. **MP REASONING** Two horizontal lines are cut by a transversal. What is the least number of angle measures you need to know to find the measure of every angle? Explain your reasoning.
27. **MP LOGIC** Describe two ways you can show that $\angle 1$ is congruent to $\angle 7$.



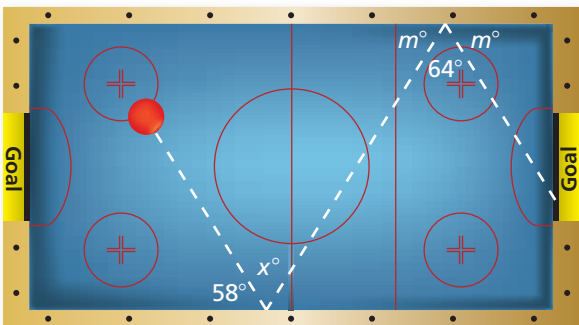
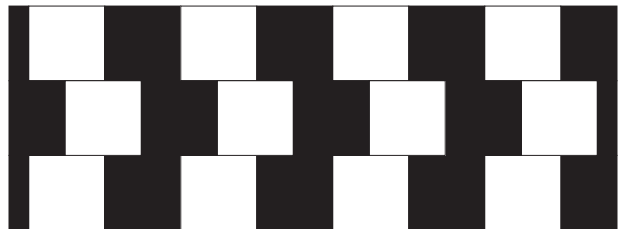
FINDING A VALUE Find the value of x .



30. **PROJECT** Trace line p and line t on a piece of paper. Label $\angle 1$. Move the paper so that $\angle 1$ aligns with $\angle 8$. Describe the transformations that you used to show that $\angle 1$ is congruent to $\angle 8$.



31. **OPEN-ENDED** Refer to the figure.
- Do the horizontal lines appear to be parallel? Explain.
 - Draw your own optical illusion using parallel lines.



32. **DIG DEEPER!** The figure shows the angles used to make a shot on an air hockey table.
- Find the value of x .
 - How does the angle the puck hits the edge of the table relate to the angle it leaves the edge of the table?

3.2 Angles of Triangles

Learning Target: Understand properties of interior and exterior angles of triangles.

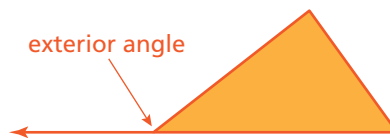
- Success Criteria:**
- I can use equations to find missing angle measures of triangles.
 - I can use interior and exterior angles of a triangle to solve real-life problems.

EXPLORATION 1

Exploring Interior and Exterior Angles of Triangles

Work with a partner.

- Draw several triangles using geometry software. What can you conclude about the sums of the angle measures?
- You can extend one side of a triangle to form an *exterior angle*, as shown.



Use geometry software to draw a triangle and an exterior angle. Compare the measure of the exterior angle with the measures of the interior angles. Repeat this process for several different triangles. What can you conclude?

EXPLORATION 2

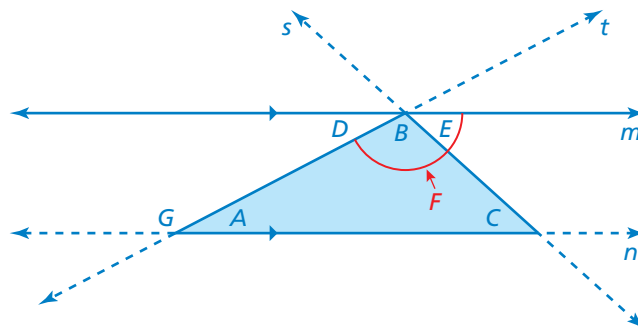
Using Parallel Lines and Transversals

Work with a partner. Describe what is shown in the figure below. Then use what you know about parallel lines and transversals to justify your conclusions in Exploration 1.

Math Practice

Look for Structure

Which angle labeled in the diagram is an exterior angle of $\triangle ABC$?



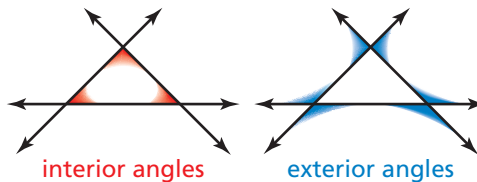
3.2 Lesson

Key Vocabulary

interior angles of a polygon, p. 112

exterior angles of a polygon, p. 112

The angles inside a polygon are called **interior angles**. When the sides of a polygon are extended, other angles are formed. The angles outside the polygon that are adjacent to the interior angles are called **exterior angles**.

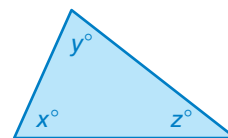


Key Idea

Interior Angle Measures of a Triangle

Words The sum of the interior angle measures of a triangle is 180° .

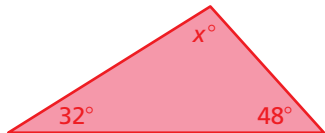
Algebra $x + y + z = 180$



EXAMPLE 1 Using Interior Angle Measures

Find the measures of the interior angles of each triangle.

a.



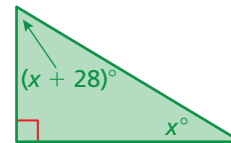
$$x + 32 + 48 = 180$$

$$x + 80 = 180$$

$$x = 100$$

So, the measures of the interior angles are 100° , 48° , and 32° .

b.



$$x + (x + 28) + 90 = 180$$

$$2x + 118 = 180$$

$$2x = 62$$

$$x = 31$$

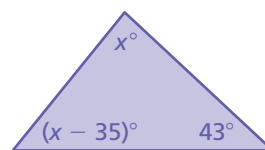
So, the measures of the interior angles are $(31 + 28)^\circ = 59^\circ$, 31° , and 90° .

Try It Find the measures of the interior angles of the triangle.

1.



2.

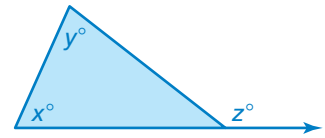


Key Idea

Exterior Angle Measures of a Triangle

Words The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

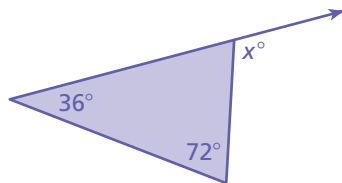
Algebra $z = x + y$



EXAMPLE 2 Finding Exterior Angle Measures

Find the measure of the exterior angle.

a.



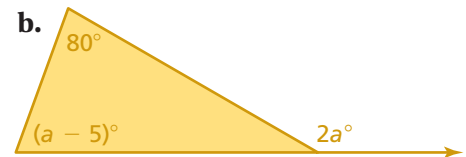
$$x = 36 + 72$$

$$x = 108$$



So, the measure of the exterior angle is 108° .

b.



$$2a = (a - 5) + 80$$

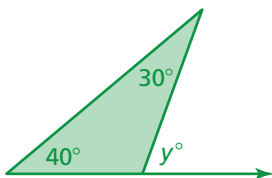
$$2a = a + 75$$

$$a = 75$$



So, the measure of the exterior angle is $2(75)^\circ = 150^\circ$.

Each vertex has a pair of congruent exterior angles. However, it is common to show only one exterior angle at each vertex.



Try It

3. Find the measure of the exterior angle of the triangle at the left.



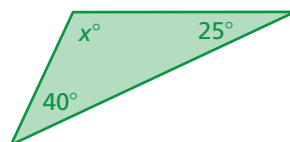
Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

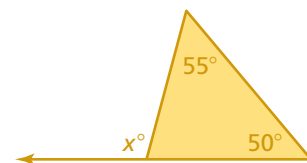
4. **VOCABULARY** How many exterior angles does a triangle have at each vertex? Explain.

FINDING ANGLE MEASURES Find the value of x .

5.



6.



EXAMPLE 3

Modeling Real Life

An airplane leaves Miami and travels around the Bermuda Triangle as shown in the diagram. What is the measure of the interior angle at Miami?

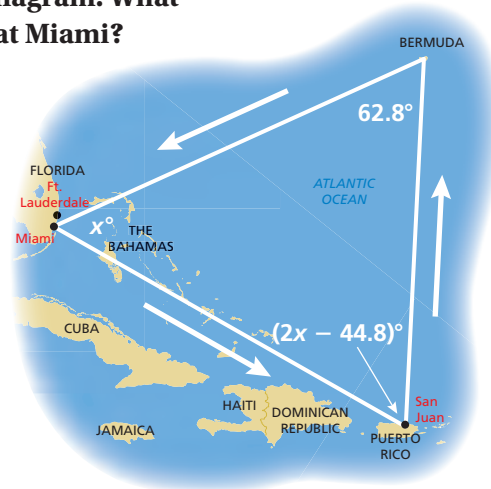
Understand the problem.

You are given expressions representing the interior angle measures of the Bermuda Triangle. You are asked to find the measure of the interior angle at Miami.

Make a plan.

Use what you know about interior angle measures of triangles to write and solve an equation for x .

Solve and check.



$$x + (2x - 44.8) + 62.8 = 180$$

$$3x + 18 = 180$$

$$3x = 162$$

$$x = 54$$

Write an equation.

Combine like terms.

Subtract 18 from each side.

Divide each side by 3.

Check

$$x + (2x - 44.8) + 62.8 = 180$$

$$54 + [2(54) - 44.8] + 62.8 \stackrel{?}{=} 180$$

$$54 + 63.2 + 62.8 \stackrel{?}{=} 180$$

$$180 = 180 \checkmark$$

So, the measure of the interior angle at Miami is 54° .

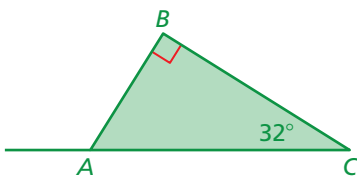


Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

7. The *Historic Triangle* in Virginia connects Jamestown, Williamsburg, and Yorktown. The interior angle at Williamsburg is 120° . The interior angle at Jamestown is twice the measure of the interior angle at Yorktown. Find the measures of the interior angles at Jamestown and Yorktown. Explain your reasoning.

8. A helicopter travels from point C to point A to perform a medical supply drop. The helicopter then needs to land at point B . How many degrees should the helicopter turn at point A to travel towards point B ? Justify your answer.



3.2 Practice

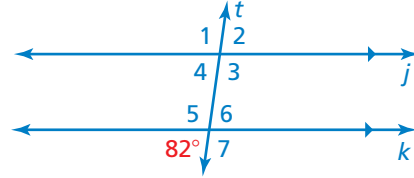


Go to BigIdeasMath.com to get HELP with solving the exercises.

► Review & Refresh

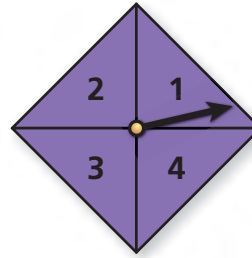
Use the figure to find the measure of the angle.
Explain your reasoning.

1. $\angle 2$
2. $\angle 6$
3. $\angle 4$
4. $\angle 1$



You spin the spinner shown.

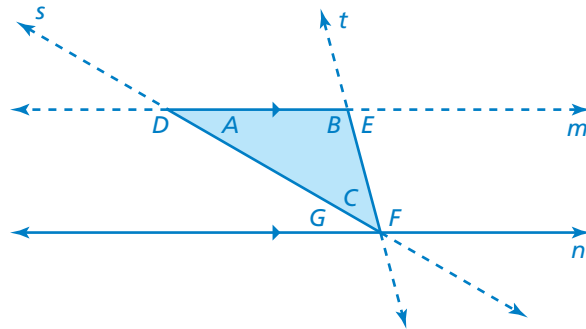
5. What are the favorable outcomes of spinning a number less than 4?
6. In how many ways can spinning an odd number occur?



► Concepts, Skills, & Problem Solving

USING PARALLEL LINES AND TRANSVERSALS Consider the figure below.
(See Exploration 2, p. 111.)

7. Use a protractor to find the measures of the labeled angles.
8. Is $\angle F$ an exterior angle of Triangle ABC ? Justify your answer.

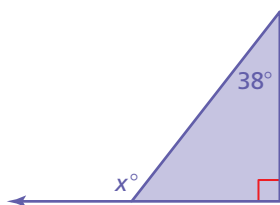


USING INTERIOR ANGLE MEASURES Find the measures of the interior angles of the triangle.

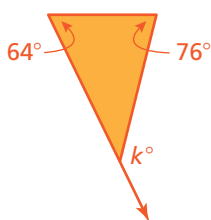
- 9.
- 10.
- 11.
- 12.
- 13.
- 14.

FINDING EXTERIOR ANGLE MEASURES Find the measure of the exterior angle.

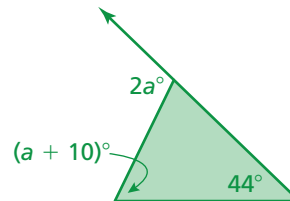
15.



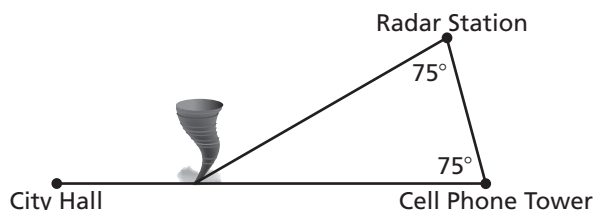
16.



17.



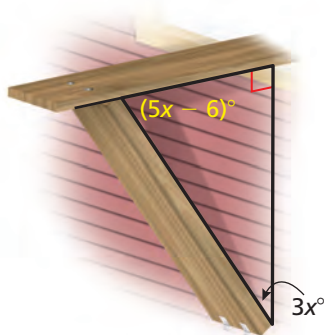
18. **MODELING REAL LIFE** A tornado is located between city hall and a cell phone tower and is heading towards the cell phone tower. By what angle does the tornado's direction need to change so that it passes over the radar station instead? Justify your answer.



19. **YOU BE THE TEACHER** Your friend finds the measure of the exterior angle shown. Is your friend correct? Explain your reasoning.

$(3x - 6) + x + 30 = 180$
 $4x + 24 = 180$
 $x = 39$

The exterior angle is $(3(39) - 6)^\circ = 111^\circ$.

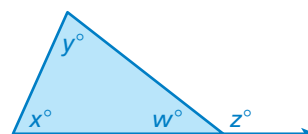


20. **MP REASONING** The ratio of the interior angle measures of a triangle is 2 : 3 : 5. What are the angle measures?
21. **MP PROBLEM SOLVING** The support for a window air-conditioning unit forms a triangle and an exterior angle. What is the measure of the exterior angle?
22. **MP REASONING** A triangle has an exterior angle with a measure of 120° . Can you determine the measures of the interior angles? Explain.

ANGLES OF TRIANGLES Determine whether the statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

23. Given three angle measures, you can construct a triangle.
24. The acute interior angles of a right triangle are complementary.
25. A triangle has more than one vertex with an acute exterior angle.

26. **DIG DEEPER!** Using the figure at the right, show that $z = x + y$. (Hint: Find two equations involving w .)



3.3 Angles of Polygons

Learning Target: Find interior angle measures of polygons.

- Success Criteria:**
- I can explain how to find the sum of the interior angle measures of a polygon.
 - I can use an equation to find an interior angle measure of a polygon.
 - I can find the interior angle measures of a regular polygon.

EXPLORATION 1

Exploring Interior Angles of Polygons

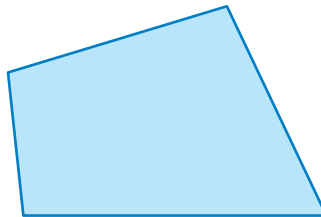
Work with a partner. In parts (a)–(f), use what you know about the interior angle measures of triangles to find the sum of the interior angle measures of each figure.

Math Practice

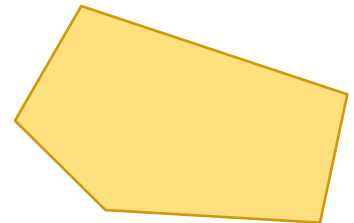
View as Components

How does dividing the figure into triangles help you find the sum of the interior angle measures?

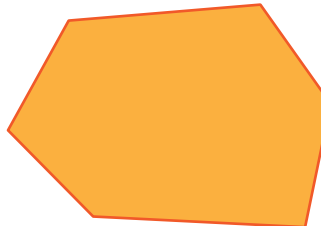
a.



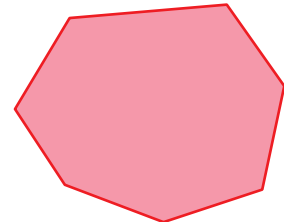
b.



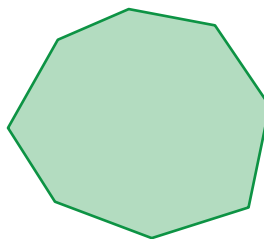
c.



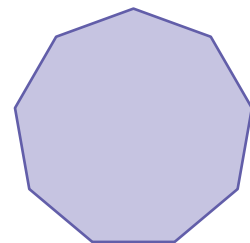
d.



e.



f.



- g. **REPEATED REASONING** Use your results in parts (a)–(f) to complete the table. Then write an equation that represents the sum S of the interior angle measures of a polygon with n sides.

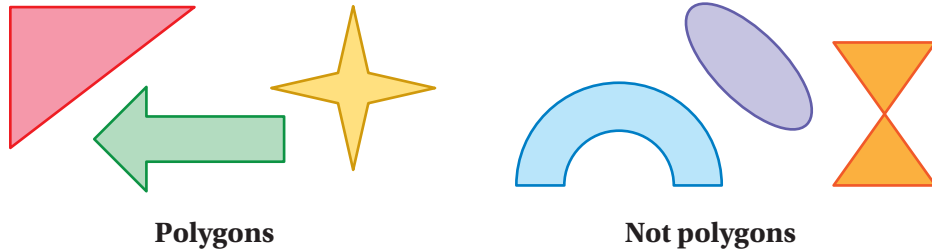
Number of Sides, n	3	4	5	6	7	8	9
Number of Triangles							
Interior Angle Sum, S							

3.3 Lesson

Key Vocabulary

regular polygon,
p. 120

A *polygon* is a closed plane figure made up of three or more line segments that intersect only at their endpoints.



Reading



For polygons whose names you have not learned, you can use the phrase "*n*-gon," where *n* is the number of sides. For example, a 15-gon is a polygon with 15 sides.

Key Idea

Interior Angle Measures of a Polygon

The sum *S* of the interior angle measures of a polygon with *n* sides is

$$S = (n - 2) \cdot 180^\circ.$$

EXAMPLE 1 Finding the Sum of Interior Angle Measures

Find the sum of the interior angle measures of the school crossing sign.

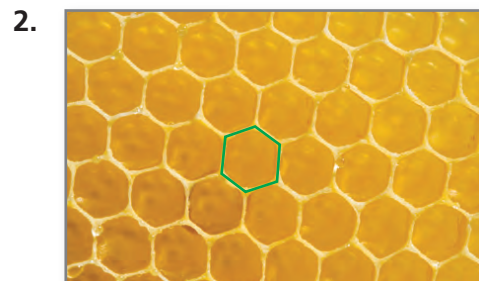
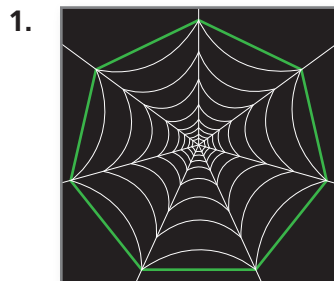
The sign is in the shape of a pentagon. It has 5 sides.

$$\begin{aligned} S &= (n - 2) \cdot 180^\circ && \text{Write the formula.} \\ &= (5 - 2) \cdot 180^\circ && \text{Substitute 5 for } n. \\ &= 3 \cdot 180^\circ && \text{Subtract.} \\ &= 540^\circ && \text{Multiply.} \end{aligned}$$

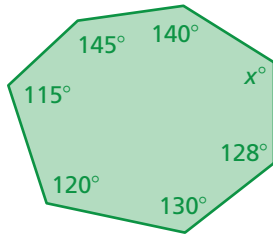


▶ The sum of the interior angle measures is 540° .

Try It Find the sum of the interior angle measures of the green polygon.



EXAMPLE 2 Finding an Interior Angle Measure of a Polygon



Find the value of x .

Step 1: The polygon has 7 sides. Find the sum of the interior angle measures.

$$\begin{aligned} S &= (n - 2) \cdot 180^\circ \\ &= (7 - 2) \cdot 180^\circ \\ &= 900^\circ \end{aligned}$$

Write the formula.

Substitute 7 for n .

Simplify. The sum of the interior angle measures is 900° .

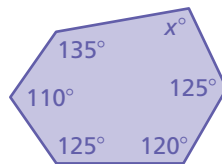
Step 2: Write and solve an equation.

$$\begin{aligned} 140 + 145 + 115 + 120 + 130 + 128 + x &= 900 \\ 778 + x &= 900 \\ x &= 122 \end{aligned}$$

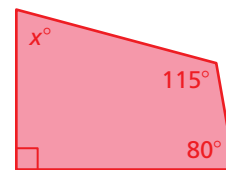
▶ The value of x is 122.

Try It Find the value of x .

3.



4.



Self-Assessment for Concepts & Skills

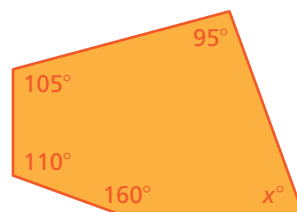
Solve each exercise. Then rate your understanding of the success criteria in your journal.

- WRITING** Explain how to find the sum of the interior measures of a polygon.
- FINDING THE SUM OF INTERIOR ANGLE MEASURES** Find the sum of the interior angle measures of the green polygon.

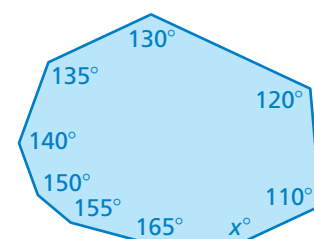


FINDING AN INTERIOR ANGLE MEASURE Find the value of x .

7.

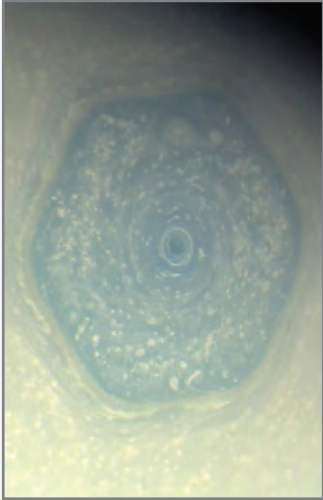


8.



In a **regular polygon**, all the sides are congruent, and all the interior angles are congruent.

EXAMPLE 3 Modeling Real Life



The hexagon is about 15,000 miles across. Approximately four Earths can fit inside it.

A cloud system discovered on Saturn is in the approximate shape of a regular hexagon. Find the measure of each interior angle of the hexagon.

A hexagon has 6 sides. Use the formula to find the sum of the interior angle measures.

$$S = (n - 2) \cdot 180^\circ \quad \text{Write the formula.}$$

$$= (6 - 2) \cdot 180^\circ \quad \text{Substitute 6 for } n.$$

$$= 720^\circ \quad \text{Simplify. The sum of the interior angle measures is } 720^\circ.$$

In a regular polygon, each interior angle is congruent. So, divide the sum of the interior angle measures by the number of interior angles, 6.

$$720^\circ \div 6 = 120^\circ$$

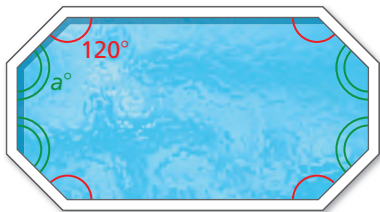
► The measure of each interior angle is 120° .



Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

9. A company installs an octagonal swimming pool.



- a. Find the value of a for the pool shown at the left.

- b. The company installs a different pool that is also in the shape of an octagon. The second pool has twice the length and one-third the width of the first pool. Are the sums of the interior angles of the pools different? Justify your answer.

10. **DIG DEEPER!** A Bronze Star Medal is shown.

- a. How many interior angles are there?
b. What is the sum of the interior angle measures?



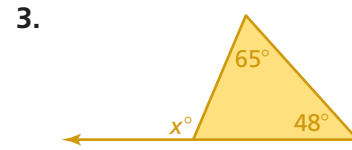
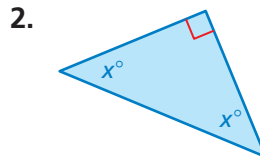
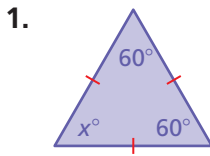
3.3 Practice



Go to BigIdeasMath.com to get HELP with solving the exercises.

► Review & Refresh

Find the value of x .



Solve the proportion.

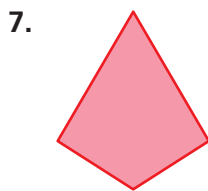
4. $\frac{x}{12} = \frac{3}{4}$

5. $\frac{14}{21} = \frac{x}{3}$

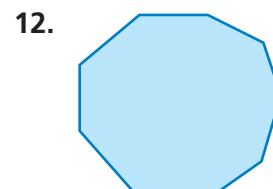
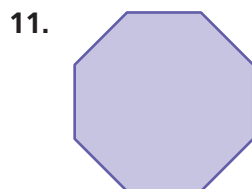
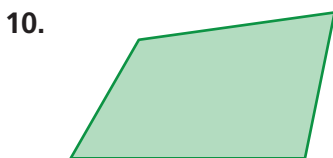
6. $\frac{9}{x} = \frac{6}{2}$

► Concepts, Skills, & Problem Solving

EXPLORING INTERIOR ANGLES OF POLYGONS Use triangles to find the sum of the interior angle measures of the polygon. (See Exploration 1, p. 117.)



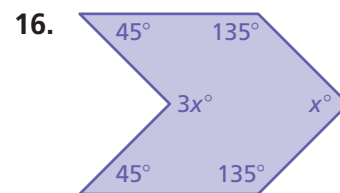
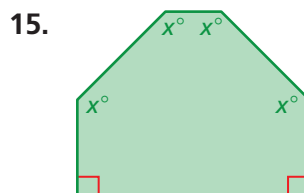
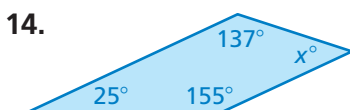
FINDING THE SUM OF INTERIOR ANGLE MEASURES Find the sum of the interior angle measures of the polygon.



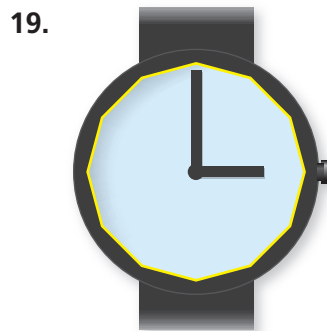
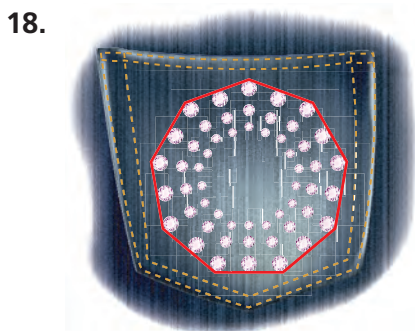
13. **YOU BE THE TEACHER** Your friend finds the sum of the interior angle measures of a 13-gon. Is your friend correct? Explain your reasoning.

$$\begin{aligned} S &= n \cdot 180^\circ \\ &= 13 \cdot 180^\circ \\ &= 2340^\circ \end{aligned}$$

FINDING AN INTERIOR ANGLE MEASURE Find the value of x .



FINDING A MEASURE Find the measure of each interior angle of the regular polygon.



20. **YOU BE THE TEACHER** Your friend finds the measure of each interior angle of a regular 20-gon. Is your friend correct? Explain your reasoning.

$$\begin{aligned}
 S &= (n - 2) \cdot 180^\circ \\
 &= (20 - 2) \cdot 180^\circ \\
 &= 18 \cdot 180^\circ \\
 &= 3240^\circ \\
 3240^\circ \div 18 &= 180^\circ \\
 \text{The measure of each} \\
 \text{interior angle is } 180^\circ.
 \end{aligned}$$



21. **MODELING REAL LIFE** A fire hydrant bolt is in the shape of a regular pentagon.

- What is the measure of each interior angle?
- RESEARCH** Why are fire hydrants made this way?

22. **MP PROBLEM SOLVING** The interior angles of a regular polygon each measure 165° . How many sides does the polygon have?

23. **MP STRUCTURE** A molecule can be represented by a polygon with interior angles that each measure 120° . What polygon represents the molecule? Does the polygon have to be regular? Justify your answers.



24. **MP PROBLEM SOLVING** The border of a Susan B. Anthony dollar is in the shape of a regular polygon.

- How many sides does the polygon have?
- What is the measure of each interior angle of the border? Round your answer to the nearest degree.

25. **MP REASONING** The center of the stained glass window is in the shape of a regular polygon. What are the measures of the interior angles of the green triangle?



26. **GEOMETRY** Draw a pentagon that has two right interior angles, two 45° interior angles, and one 270° interior angle.

27. **DIG DEEPER!** The floor of a gazebo is in the shape of a heptagon, a seven-sided polygon. Four of the interior angles measure 135° . The other interior angles have equal measures. Find their measures.



3.4 Using Similar Triangles

Learning Target: Use similar triangles to find missing measures.

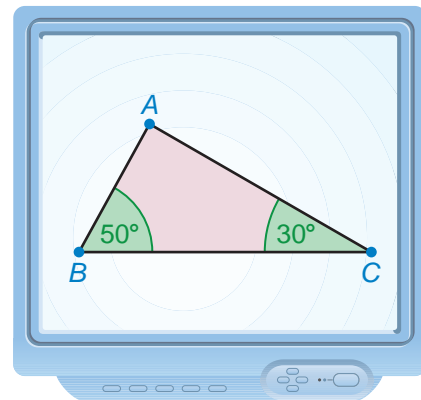
- Success Criteria:**
- I can use angle measures to determine whether triangles are similar.
 - I can use similar triangles to solve real-life problems.

EXPLORATION 1

Drawing Triangles Given Two Angle Measures

Work with a partner. Use geometry software.

- a. Draw a triangle that has a 50° angle and a 30° angle. Then draw a triangle that is either larger or smaller that has the same two angle measures. Are the triangles congruent? similar? Explain your reasoning.



- b. Choose any two angle measures whose sum is less than 180° . Repeat part (a) using the angle measures you chose.
- c. Compare your results in parts (a) and (b) with other pairs of students. Make a conjecture about two triangles that have two pairs of congruent angles.

EXPLORATION 2

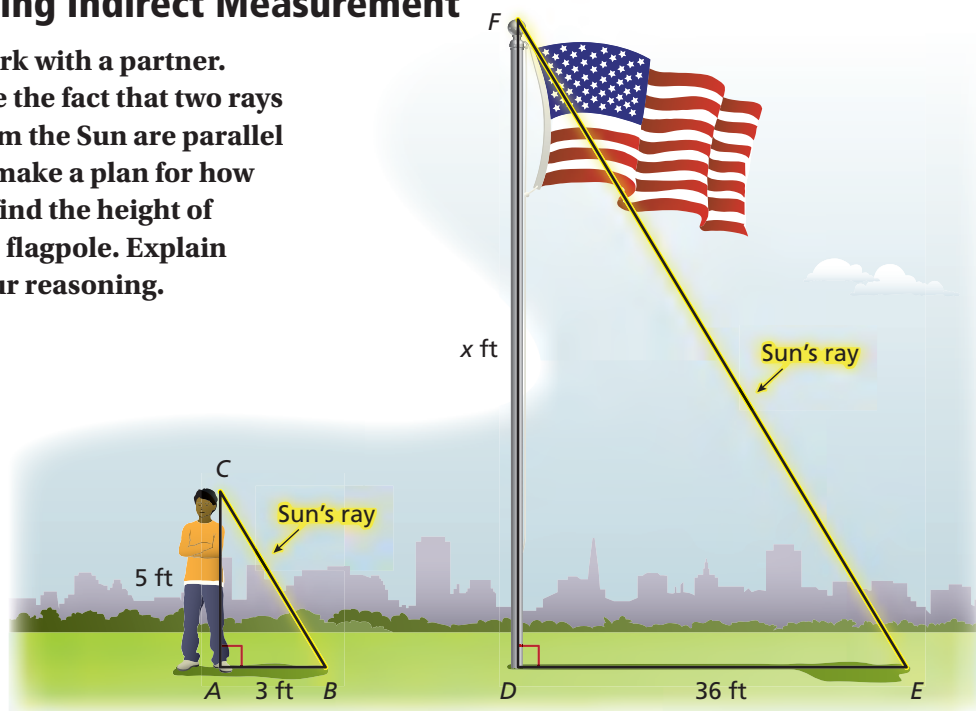
Using Indirect Measurement

Work with a partner.
Use the fact that two rays from the Sun are parallel to make a plan for how to find the height of the flagpole. Explain your reasoning.

Math Practice

Make Sense of Quantities

What do you know about the sides of similar triangles?



3.4 Lesson

Key Vocabulary

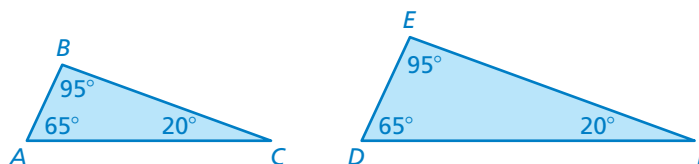
indirect measurement,
p. 126

Key Idea

Angles of Similar Triangles

Words When two angles in one triangle are congruent to two angles in another triangle, the third angles are also congruent and the triangles are similar.

Example

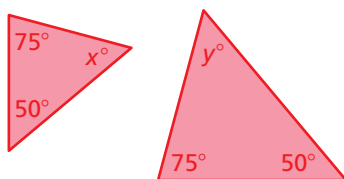


Triangle ABC is similar to Triangle DEF : $\triangle ABC \sim \triangle DEF$.

EXAMPLE 1 Identifying Similar Triangles

Tell whether the triangles are similar. Explain.

a.

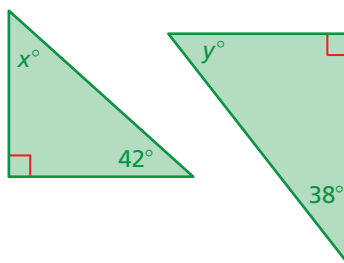


The triangles have two pairs of congruent angles.



So, the third angles are congruent, and the triangles are similar.

b.



Write and solve an equation to find x .

$$x + 90 + 42 = 180$$

$$x + 132 = 180$$

$$x = 48$$

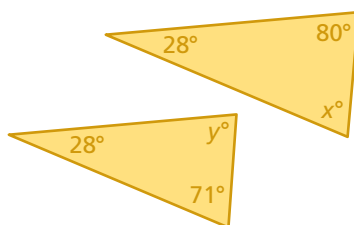
The triangles do not have two pairs of congruent angles.



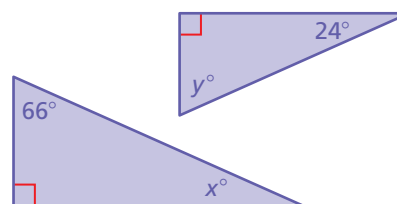
So, the triangles are not similar.

Try It Tell whether the triangles are similar. Explain.

1.



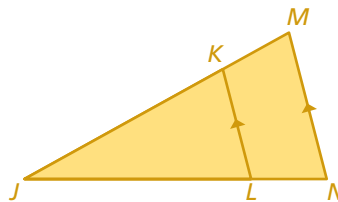
2.



EXAMPLE 2

Identifying Similar Triangles

Can you determine whether $\triangle JKL$ and $\triangle JMN$ are similar? Explain.



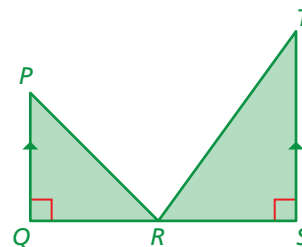
You can also use corresponding angles to show that $\angle JKL$ is congruent to $\angle M$.

Side KL and side MN are parallel, and each is intersected by side JN . So, $\angle JKL$ and $\angle M$ are congruent corresponding angles. Each triangle also shares $\angle J$.

▶ Because two angles in $\triangle JKL$ are congruent to two angles in $\triangle JMN$, the third angles are also congruent and the triangles are similar.

Try It

3. Can you determine whether $\triangle PQR$ and $\triangle TSR$ are similar? Explain.

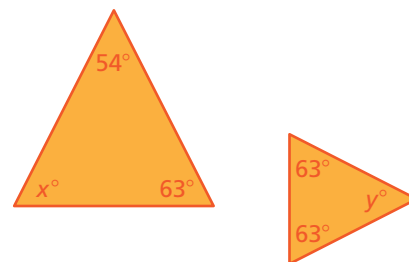


Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

4. IDENTIFYING SIMILAR TRIANGLES

Tell whether the triangles are similar. Explain.



5. DIFFERENT WORDS, SAME QUESTION

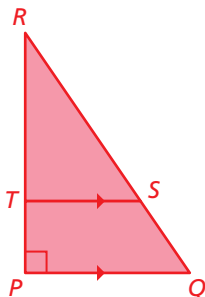
Which is different? Find “both” answers.

Are $\triangle PQR$ and $\triangle TSR$ similar?

Are $\triangle PQR$ and $\triangle TSR$ the same size and shape?

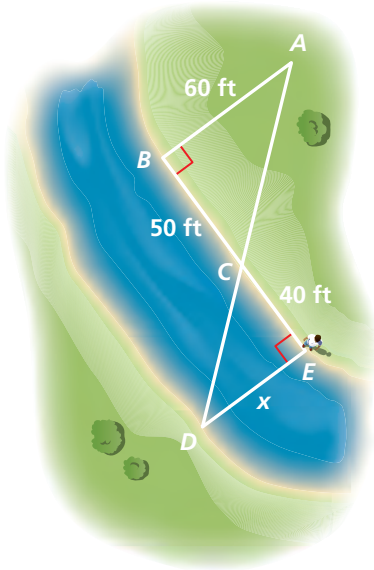
Is $\triangle PQR$ a dilation of $\triangle TSR$?

Is $\triangle PQR$ a scale drawing of $\triangle TSR$?



Indirect measurement uses similar figures to find a missing measure when it is difficult to find directly.

EXAMPLE 3 Modeling Real Life



You plan to cross a river and want to know how far it is to the other side. You take measurements on your side of the river and make the drawing shown. What is the distance x across the river?

Notice that $\angle B$ and $\angle E$ are right angles, so they are congruent. $\angle ACB$ and $\angle DCE$ are vertical angles, so they are congruent. Because two angles in $\triangle ABC$ are congruent to two angles in $\triangle DEC$, the third angles are also congruent and the triangles are similar.

Ratios of corresponding side lengths in similar triangles are equivalent. So, the ratios $x : 60$ and $40 : 50$ are equivalent. Write and solve a proportion to find x .

$$\frac{x}{60} = \frac{40}{50}$$

Write a proportion.

$$60 \cdot \frac{x}{60} = 60 \cdot \frac{40}{50}$$

Multiplication Property of Equality

$$x = 48$$

Simplify.

► So, the distance across the river is 48 feet.

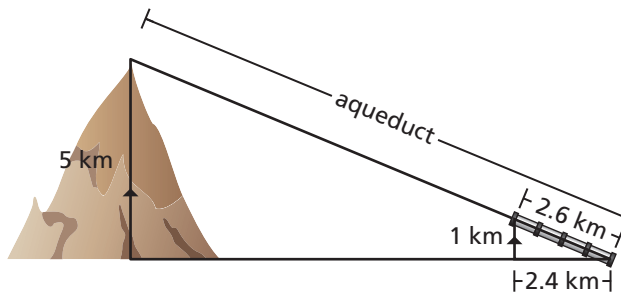


Self-Assessment for Problem Solving

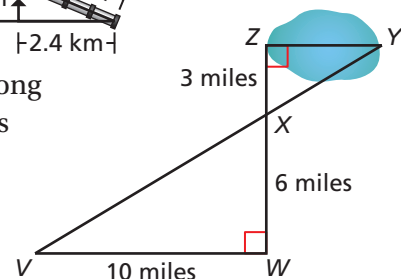
Solve each exercise. Then rate your understanding of the success criteria in your journal.



6. **DIG DEEPER!** Engineers plan to construct an aqueduct to transport water from the top of a ridge to farmland. A portion of the project is complete. Find the length of the entire aqueduct.



7. You want to go on a swamp tour. How long does it take a swamp vehicle that travels at 3.2 miles per hour to travel across the swamp, from point Z to point Y ? Justify your answer.



3.4 Practice



Go to BigIdeasMath.com to get HELP with solving the exercises.

► Review & Refresh

Find the measure of each interior angle of the regular polygon.

1. octagon 2. decagon 3. 18-gon

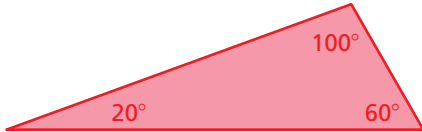
Solve the equation. Check your solution.

4. $3.5 + y = -1$ 5. $9x = 54$ 6. $-4 = \frac{2}{7}p$

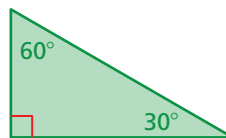
► Concepts, Skills, & Problem Solving

CREATING SIMILAR TRIANGLES Draw a triangle that is either larger or smaller than the one given and has two of the same angle measures. Explain why the new triangle is similar to the original triangle. (See Exploration 1, p. 123.)

7.

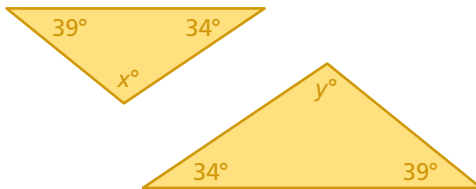


8.

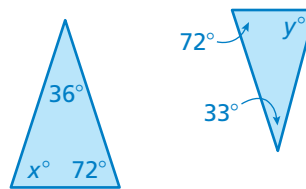


IDENTIFYING SIMILAR TRIANGLES Tell whether the triangles are similar. Explain.

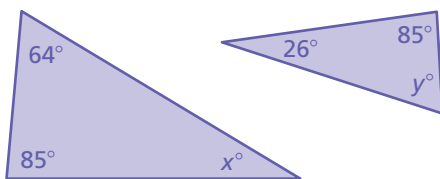
9.



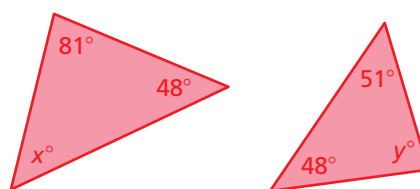
10.



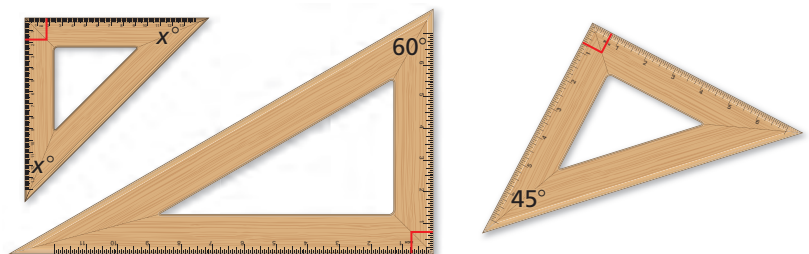
11.



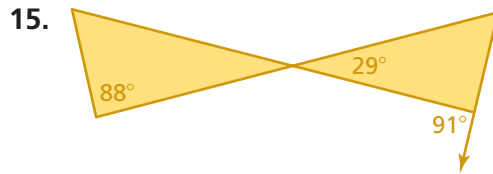
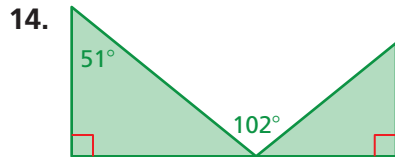
12.



13. **GEOMETRY** Which of the rulers are similar in shape? Explain.

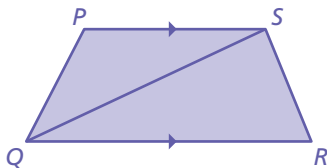


MP STRUCTURE Tell whether the triangles are similar. Explain.

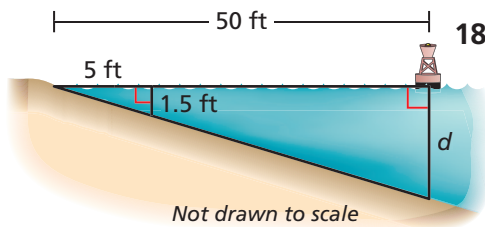
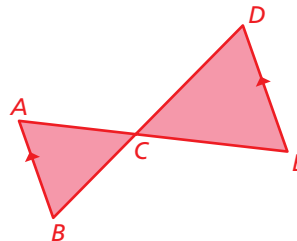


IDENTIFYING SIMILAR TRIANGLES Can you determine whether the triangles are similar? Explain.

16. $\triangle PQS$ and $\triangle RQS$

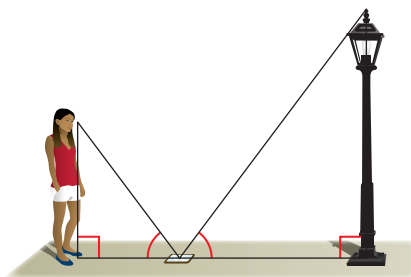


17. $\triangle ABC$ and $\triangle EDC$



18. **MP PROBLEM SOLVING** A water sample must be taken from water at least 20 feet deep. Find the depth of the water 50 feet from shore. Is this an appropriate location for a water sample?

19. **MODELING REAL LIFE** A map shows the number of steps you must take to get to a treasure. However, the map is old, and the last dimension is unreadable. Explain why the triangles are similar. How many steps do you take from the pyramids to the treasure?

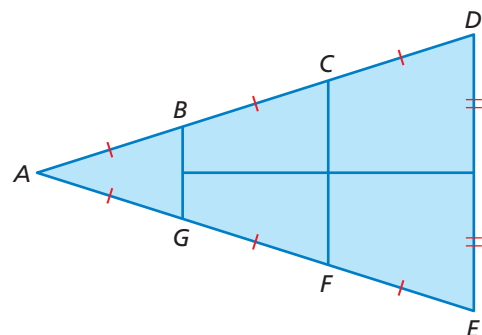


20. **MP PROBLEM SOLVING** A person who is 6 feet tall casts a 3-foot-long shadow. A nearby pine tree casts a 15-foot-long shadow. What is the height h of the pine tree?

21. **OPEN-ENDED** You place a mirror on the ground 6 feet from the lamppost. You move back 3 feet and see the top of the lamppost in the mirror. What is the height of the lamppost?

22. **DIG DEEPER!** In each of two right triangles, one angle measure is two times another angle measure. Can you determine that the triangles are similar? Explain your reasoning.

23. **GEOMETRY** In the diagram, \overline{BG} , \overline{CF} , and \overline{DE} are parallel. The length of \overline{BD} is 6.32 feet, and the length of \overline{DE} is 6 feet. Name all pairs of similar triangles in the diagram. Then find the lengths of \overline{BG} and \overline{CF} .



3

Connecting Concepts

Using the Problem-Solving Plan

1. A dog park is divided into sections for large and small dogs. The ratio of the perimeter of the small dog section to the perimeter of the entire dog park is $7 : 12$. Find the area of each section.

Understand the problem.

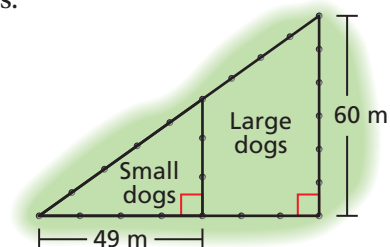
You know two dimensions of a dog park and the ratio of the perimeter of the small dog section to the perimeter of the entire park. You are asked to find the area of each section.

Make a plan.

Verify that the small triangle and the large triangle are similar. Then use the ratio of the perimeters to find the base or the height of each triangle and calculate the areas.

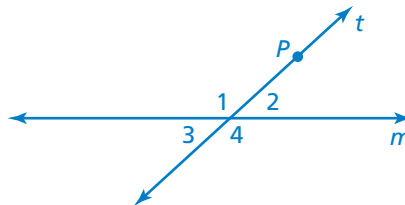
Solve and check.

Use the plan to solve the problem. Then check your solution.



2. You rotate lines m and t 180° about point P . The image of line m is parallel to the original line. Use the diagram to show that when a transversal intersects parallel lines, each of the following pairs of angles are congruent. Explain your reasoning.

- a. alternate interior angles
- b. alternate exterior angles
- c. corresponding angles



Performance Task



Turtle Shells

At the beginning of this chapter, you watched a STEAM Video called "Honeycombs." You are now ready to complete the performance task related to this video, available at BigIdeasMath.com. Be sure to use the problem-solving plan as you work through the performance task.



3

Chapter Review



Go to BigIdeasMath.com to download blank graphic organizers.

► Review Vocabulary

Write the definition and give an example of each vocabulary term.

transversal, p. 104

interior angles, p. 105

exterior angles, p. 105

interior angles of a polygon,
p. 112

exterior angles of a polygon,
p. 112

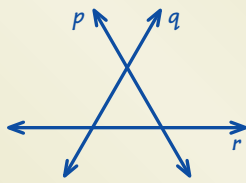
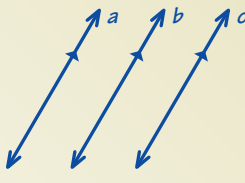
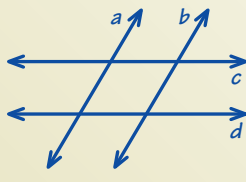
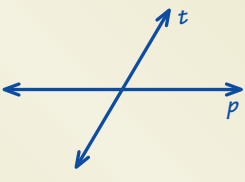
regular polygon, p. 120

indirect measurement, p. 126

► Graphic Organizers

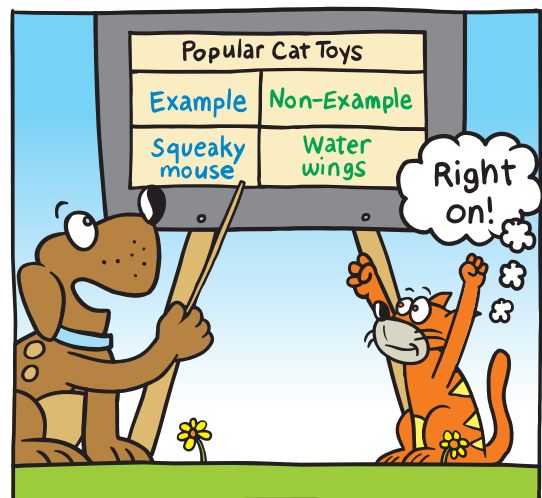
You can use an **Example and Non-Example Chart** to list examples and non-examples of a concept. Here is an Example and Non-Example Chart for *transversals*.

Transversals

Examples	Non-Examples
 <p>line <i>p</i>, line <i>q</i>, line <i>r</i></p>	 <p>line <i>a</i>, line <i>b</i>, line <i>c</i></p>
 <p>line <i>a</i>, line <i>b</i>, line <i>c</i>, line <i>d</i></p>	 <p>line <i>p</i>, line <i>t</i></p>

Choose and complete a graphic organizer to help you study the concept.

- interior angles formed by parallel lines and a transversal
- exterior angles formed by parallel lines and a transversal
- interior angles of a triangle
- exterior angles of a triangle
- polygons
- similar triangles



"What do you think of my **Example & Non-Example Chart** for popular cat toys?"

▶ Chapter Self-Assessment

As you complete the exercises, use the scale below to rate your understanding of the success criteria in your journal.

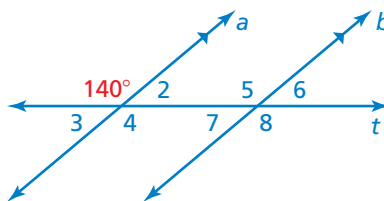


3.1 Parallel Lines and Transversals (pp. 103–110)

Learning Target: Find missing angle measures created by the intersections of lines.

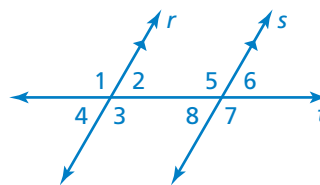
Use the figure to find the measure of the angle. Explain your reasoning.

- $\angle 8$
- $\angle 5$
- $\angle 7$
- $\angle 2$
- $\angle 6$

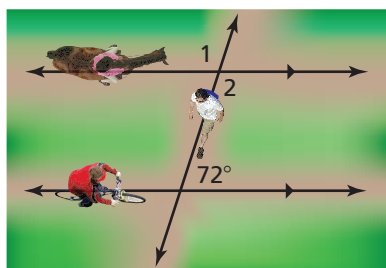


Complete the statement. Explain your reasoning.

- If the measure of $\angle 1 = 123^\circ$, then the measure of $\angle 7 = \square$.
- If the measure of $\angle 2 = 58^\circ$, then the measure of $\angle 5 = \square$.
- If the measure of $\angle 5 = 119^\circ$, then the measure of $\angle 3 = \square$.
- If the measure of $\angle 4 = 60^\circ$, then the measure of $\angle 6 = \square$.



- In Exercises 6–9, describe the relationship between $\angle 2$ and $\angle 8$.



- In a park, a bike path and a horse riding path are parallel. In one part of the park, a hiking trail intersects the two paths. Find the measures of $\angle 1$ and $\angle 2$. Explain your reasoning.

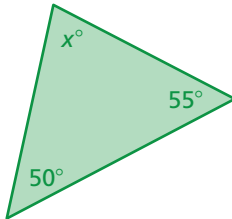


3.2 Angles of Triangles (pp. 111–116)

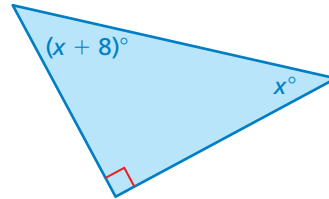
Learning Target: Understand properties of interior and exterior angles of triangles.

Find the measures of the interior angles of the triangle.

12.

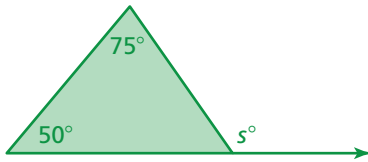


13.

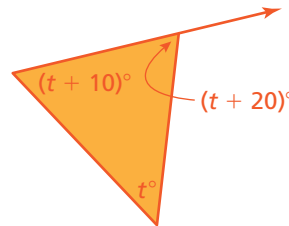


Find the measure of the exterior angle.

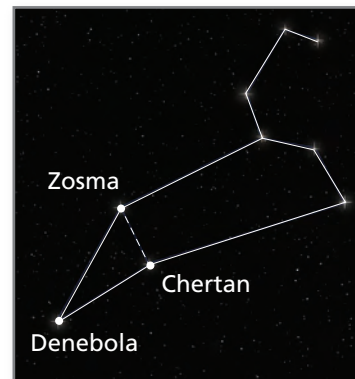
14.



15.



16. What is the measure of each interior angle of an equilateral triangle? Explain.
17. You draw the Leo constellation. You notice that the three stars Denebola, Zosma, and Chertan form a triangle. In your drawing, you find the measure of the interior angle at Denebola is 30° and the measure of the interior angle of the triangle at Zosma is 56° . What is the measure of the interior angle of the triangle at Chertan?

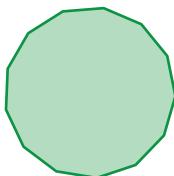


3.3 Angles of Polygons (pp. 117–122)

Learning Target: Find interior angle measures of polygons.

Find the sum of the interior angle measures of the polygon.

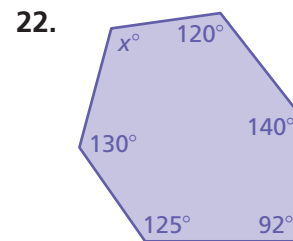
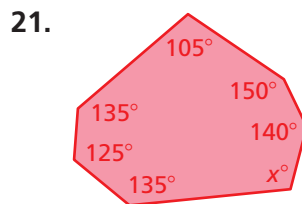
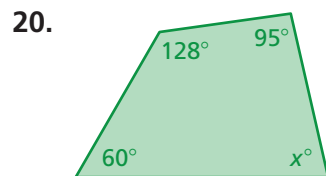
18.



19.



Find the value of x .



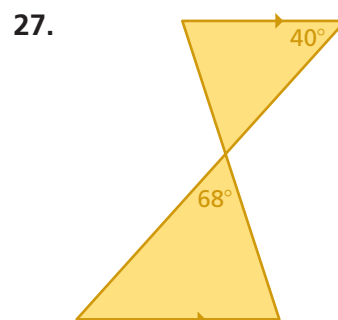
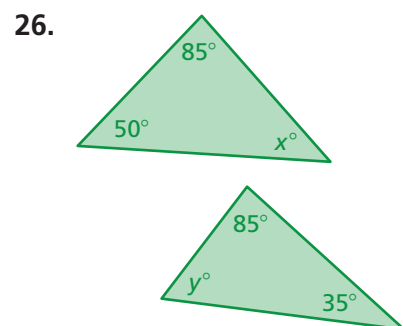
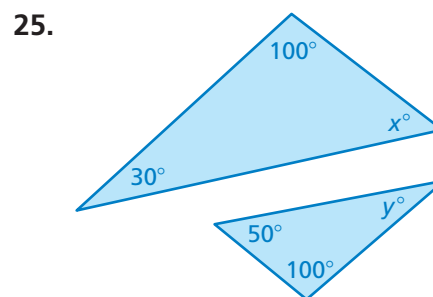
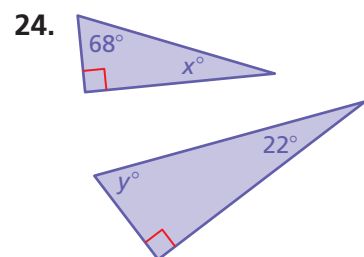
23. Find the measure of each interior angle of the regular polygon.



3.4 Using Similar Triangles (pp. 123–128)

Learning Target: Use similar triangles to find missing measures.

Tell whether the triangles are similar. Explain.



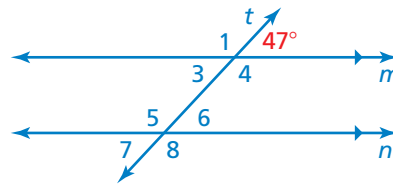
28. A person who is 5 feet tall casts a shadow that is 4 feet long. A nearby building casts a shadow that is 24 feet long. What is the height of the building?

3

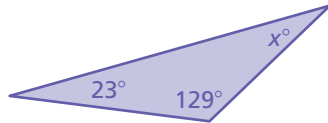
Practice Test

Use the figure to find the measure of the angle. Explain your reasoning.

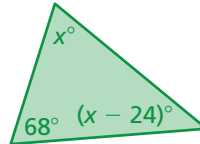
1. $\angle 7$
2. $\angle 6$
3. $\angle 4$
4. $\angle 5$



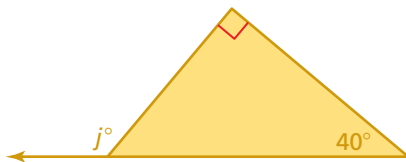
5. Find the value of x .



6. Find the measures of the interior angles.



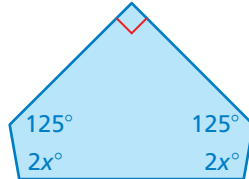
7. Find the measure of the exterior angle.



8. Find the sum of the interior angle measures of the border of the coin.



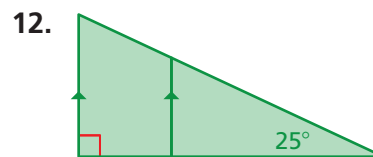
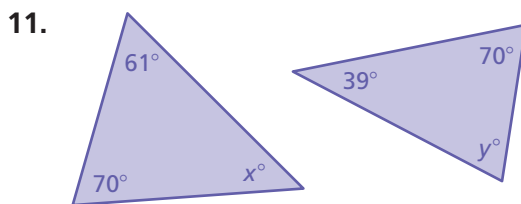
9. Find the value of x .



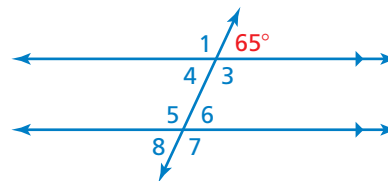
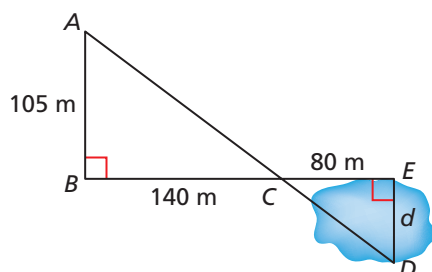
10. Find the measure of each interior angle of the regular polygon.



Tell whether the triangles are similar. Explain.



13. Describe two ways you can find the measure of $\angle 5$.



14. You swim 3.6 kilometers per hour. How long (in minutes) will it take you to swim the distance d across the pond?

3

Cumulative Practice

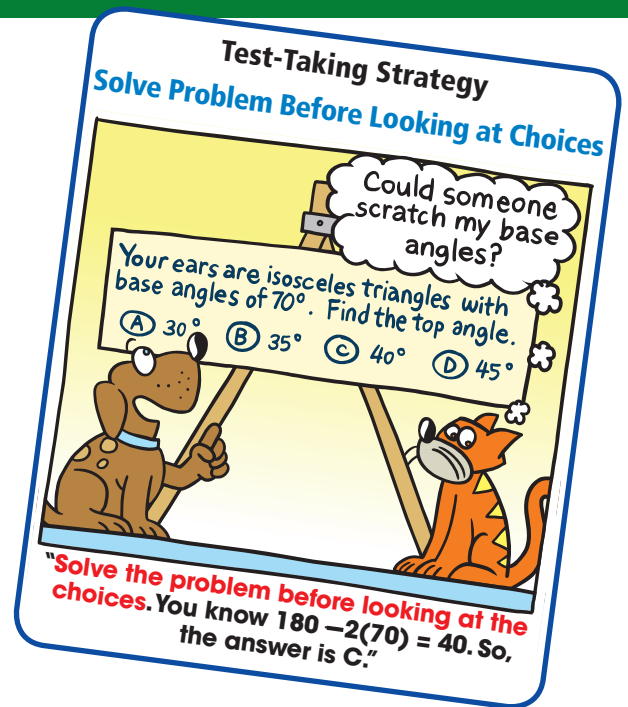
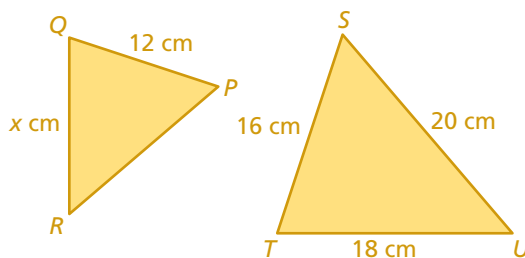


- The border of a Canadian one-dollar coin is shaped like an 11-sided regular polygon. The shape was chosen to help visually impaired people identify the coin. How many degrees are in each interior angle along the border? Round your answer to the nearest degree.
- A public utility charges its residential customers for natural gas based on the number of therms used each month. The formula shows how the monthly cost C in dollars is related to the number t of therms used.

$$C = 11 + 1.6t$$

Solve this formula for t .

- $t = \frac{C}{12.6}$
- $t = \frac{C}{1.6} - 11$
- What is the value of x ?
 - -10
 - 2
 - $2\frac{1}{2}$
 - 10
- In the figures, $\triangle PQR$ is similar to $\triangle STU$. What is the value of x ?
 - 9.6
 - $10\frac{2}{3}$
 - 13.5
 - 15



5. What is the value of x ?



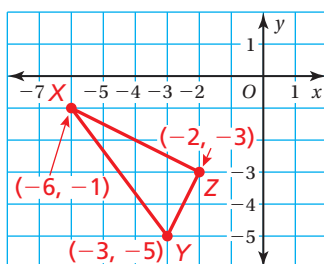
6. Your friend was solving an equation in the box shown.

$$\begin{aligned} -\frac{2}{5}(10x - 15) &= -30 \\ 10x - 15 &= -30\left(-\frac{2}{5}\right) \\ 10x - 15 &= 12 \\ 10x - 15 + 15 &= 12 + 15 \\ 10x &= 27 \\ \frac{10x}{10} &= \frac{27}{10} \\ x &= \frac{27}{10} \end{aligned}$$

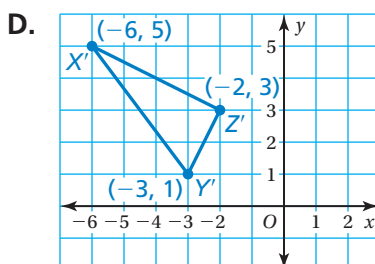
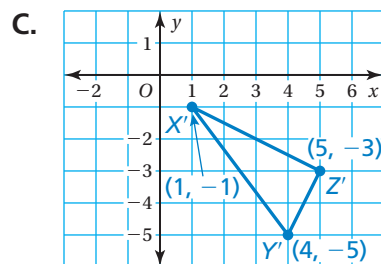
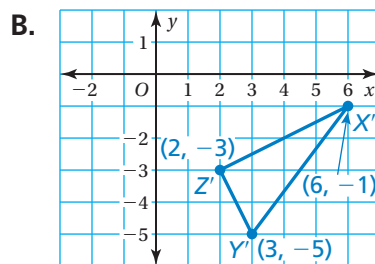
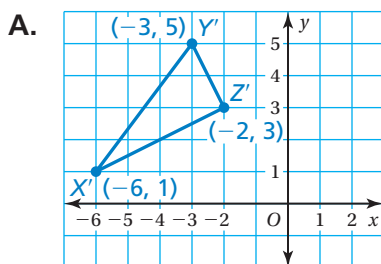
What should your friend do to correct the error that she made?

- F. Multiply both sides by $-\frac{5}{2}$ instead of $-\frac{2}{5}$.
- G. Multiply both sides by $\frac{2}{5}$ instead of $-\frac{2}{5}$.
- H. Distribute $-\frac{2}{5}$ to get $-4x - 6$.
- I. Add 15 to -30 .

7. In the coordinate plane below, $\triangle XYZ$ is plotted and its vertices are labeled.



Which of the following shows $\triangle X'Y'Z'$, the image of $\triangle XYZ$ after it is reflected in the y -axis?



8. The sum S of the interior angle measures of a polygon with n sides can be found by using a formula.



Part A Write the formula.

Part B A quadrilateral has angles measuring 100° , 90° , and 90° . Find the measure of its fourth angle. Show your work and explain your reasoning.

Part C The sum of the measures of the angles of the pentagon shown is 540° . Divide the pentagon into triangles to show why this must be true. Show your work and explain your reasoning.

