



2

Transformations

- 2.1** Translations
- 2.2** Reflections
- 2.3** Rotations
- 2.4** Congruent Figures
- 2.5** Dilations
- 2.6** Similar Figures
- 2.7** Perimeters and Areas of Similar Figures

Chapter Learning Target:

Understand transformations.

Chapter Success Criteria:

- I can identify a translation.
- I can describe a transformation.
- I can describe a sequence of rigid motions between two congruent figures.
- I can solve real-life problems involving transformations.



STEAM Video: "Shadow Puppets"

STEAM Video



Shadow Puppets

Some puppets are controlled using strings or wires. How else can a puppet be controlled?

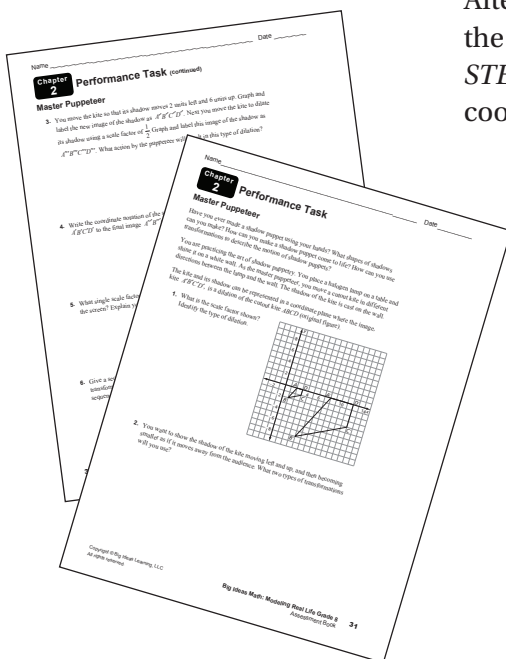
Watch the STEAM Video “Shadow Puppets.” Then answer the following questions.

1. Tory and Robert are using a light source to display puppets on a screen. Tory wants to show the pig jumping from the floor to the window. Should she use a *translation*, *reflection*, *rotation*, or *dilation*? Explain.



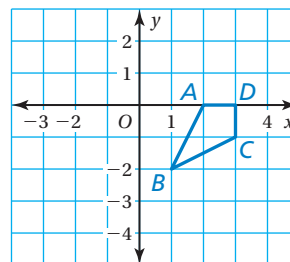
2. How can Tory show the pig getting smaller as it jumps out the window?

Performance Task



Master Puppeteer

After completing this chapter, you will be able to use the concepts you learned to answer the questions in the *STEAM Video Performance Task*. You will be given the coordinates of a kite being used by a puppeteer.

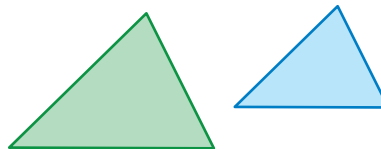
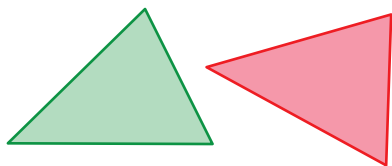


You will be asked to identify transformations for given movements of the kite. When might a puppeteer want to use a reflection?

Getting Ready for Chapter 2

2

Chapter Exploration

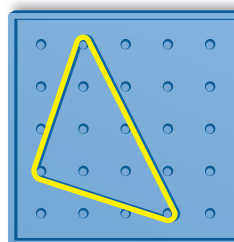


Congruent (same size and shape)

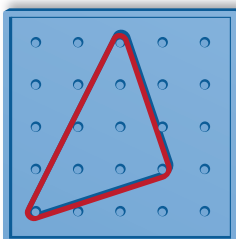
Not Congruent (same shape but not same size)

1. **Work with a partner. Form each triangle on a geoboard.**

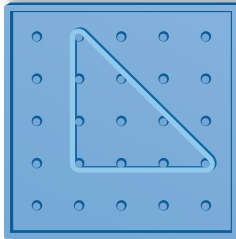
- Which of the triangles are congruent to the triangle at the right?
- Measure the sides of each triangle with a ruler. Record your results in a table.
- Write a conclusion about the side lengths of triangles that are congruent.



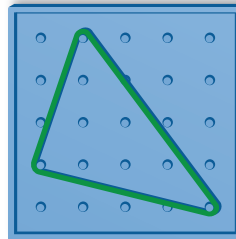
a.



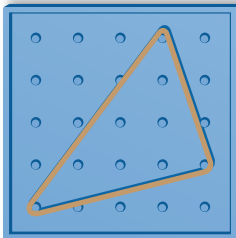
b.



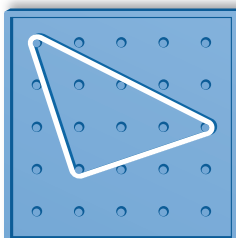
c.



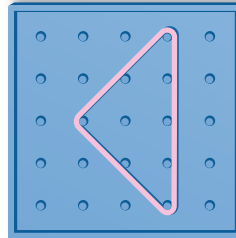
d.



e.



f.



Vocabulary

The following vocabulary terms are defined in this chapter. Think about what the terms might mean and record your thoughts.

translation
reflection

rotation
rigid motion

dilation
similar figures

2.1 Translations

Learning Target: Translate figures in the coordinate plane.

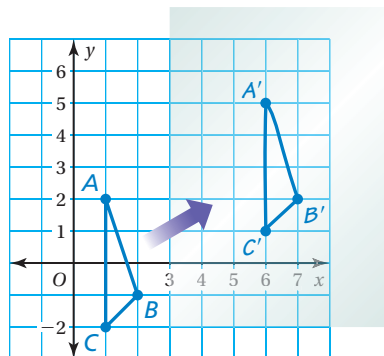
- Success Criteria:**
- I can identify a translation.
 - I can find the coordinates of a translated figure.
 - I can use coordinates to translate a figure.

EXPLORATION 1

Sliding Figures

Work with a partner.

- a. For each figure below, draw the figure in a coordinate plane. Then copy the figure onto a piece of transparent paper and slide the copy to a new location in the coordinate plane. Describe the location of the copy compared to the location of the original.
- point
 - line segment
 - line
 - triangle
 - rectangle



Math Practice

Recognize Usefulness of Tools

How does using transparencies help you compare each figure and its copy?

- b. When you slide figures, what do you notice about sides, angles, and parallel lines?

- c. Describe the location of each point below compared to the point $A(x, y)$.

$$B(x + 1, y + 2)$$

$$C(x - 3, y + 4)$$

$$D(x - 2, y + 3)$$

$$E(x + 4, y - 1)$$

- d. You copy a point with coordinates (x, y) and slide it horizontally a units and vertically b units. What are the coordinates of the copy?

2.1 Lesson

Key Vocabulary

transformation, p. 44
 image, p. 44
 translation, p. 44

A **transformation** changes a figure into another figure. The new figure is called the **image**.

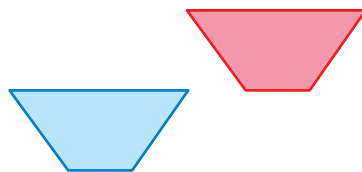
A **translation** is a transformation in which a figure *slides* but does not turn. Every point of the figure moves the same distance and in the same direction.



EXAMPLE 1 Identifying a Translation

Tell whether the blue figure is a translation of the red figure.

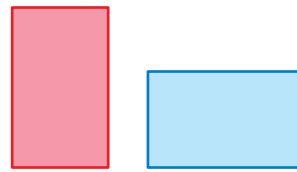
a.



The red figure *slides* to form the blue figure.

► So, the blue figure is a translation of the red figure.

b.

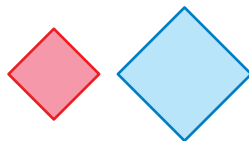


The red figure *turns* to form the blue figure.

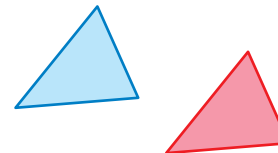
► So, the blue figure is *not* a translation of the red figure.

Try It Tell whether the blue figure is a translation of the red figure.

1.



2.

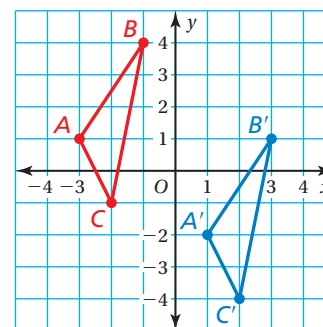


Key Idea

Translations in the Coordinate Plane

Words To translate a figure a units horizontally and b units vertically in a coordinate plane, add a to the x -coordinates and b to the y -coordinates of the vertices.

Positive values of a and b represent translations up and right. Negative values of a and b represent translations down and left.



Algebra $(x, y) \rightarrow (x + a, y + b)$

In a translation, the original figure and its image are identical.

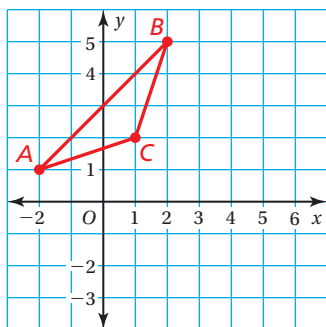
Reading

A' is read "A prime."
 Use *prime* symbols when naming an image.

$A \rightarrow A'$
 $B \rightarrow B'$
 $C \rightarrow C'$

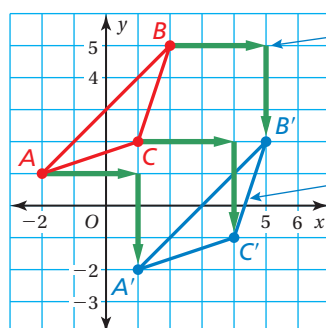
EXAMPLE 2

Translating a Figure in the Coordinate Plane



Translate the red triangle 3 units right and 3 units down. What are the coordinates of the image?

Method 1: Use a coordinate plane. Move each vertex 3 units right and 3 units down.



Move each vertex 3 units right and 3 units down.

Connect the vertices. Label as A' , B' , and C' .

▶ The coordinates of the image are $A'(1, -2)$, $B'(5, 2)$, and $C'(4, -1)$.

Method 2: Use coordinates. Add 3 to the x -coordinates of the vertices and add -3 , or subtract 3, from the y -coordinates of the vertices.

$$A(-2, 1) \rightarrow A'(-2 + 3, 1 - 3) \rightarrow A'(1, -2)$$

$$B(2, 5) \rightarrow B'(2 + 3, 5 - 3) \rightarrow B'(5, 2)$$

$$C(1, 2) \rightarrow C'(1 + 3, 2 - 3) \rightarrow C'(4, -1)$$

▶ The coordinates of the image are $A'(1, -2)$, $B'(5, 2)$, and $C'(4, -1)$.

Try It

3. **WHAT IF?** The red triangle is translated 4 units left and 2 units up. What are the coordinates of the image?

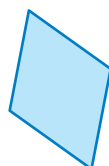


Self-Assessment for Concepts & Skills

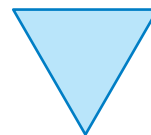
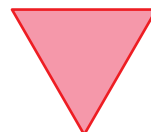
Solve each exercise. Then rate your understanding of the success criteria in your journal.

IDENTIFYING A TRANSLATION Tell whether the blue figure is a translation of the red figure.

4.



5.



6. **TRANSLATING A FIGURE** The vertices of a triangle are $A(-2, -2)$, $B(0, 2)$, and $C(3, 0)$. Translate the triangle 1 unit left and 2 units up. What are the coordinates of the image?

EXAMPLE 3

Modeling Real Life

A landscaper represents a park using a coordinate plane. He draws a square with vertices $A(1, -2)$, $B(3, -2)$, $C(3, -4)$, and $D(1, -4)$ to represent the location of a new fountain. City officials want to move the fountain 4 units left and 6 units up. Find the coordinates of the image. Then draw the original figure and the image in a coordinate plane.

Understand the problem.

You are given the coordinates for the vertices of a fountain. You are asked to find the coordinates after a translation 4 units left and 6 units up, and then graph the original figure and its image in a coordinate plane.

Make a plan.

Use the coordinates of the original figure to calculate the coordinates of the image after the translation. Then graph each figure in a coordinate plane.

Solve and check.

To find the coordinates of the image, subtract 4 from each x -coordinate and add 6 to each y -coordinate.

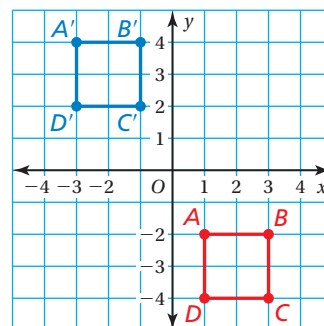
$$(x, y) \longrightarrow (x - 4, y + 6)$$

$$A(1, -2) \longrightarrow A'(1 - 4, -2 + 6) \longrightarrow A'(-3, 4)$$

$$B(3, -2) \longrightarrow B'(3 - 4, -2 + 6) \longrightarrow B'(-1, 4)$$

$$C(3, -4) \longrightarrow C'(3 - 4, -4 + 6) \longrightarrow C'(-1, 2)$$

$$D(1, -4) \longrightarrow D'(1 - 4, -4 + 6) \longrightarrow D'(-3, 2)$$



Check Counting grid lines in the graph shows that each vertex of the image is translated 4 units left and 6 units up. ✓

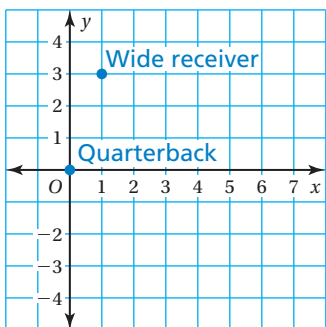
The coordinates of the image are $A'(-3, 4)$, $B'(-1, 4)$, $C'(-1, 2)$, and $D'(-3, 2)$.



Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

- A neighborhood planner uses a coordinate plane to design a new neighborhood. The coordinates $A(1, -1)$, $B(1, -2)$, and $C(2, -1)$ represent House A, House B, and House C. The planner decides to place a playground centered at the origin, and moves the houses to make space. House A is now located at $A'(3, -4)$. What are the new coordinates of House B and House C when each house is moved using the same translation? Justify your answer.
- The locations of a quarterback and a wide receiver on a football field are represented in a coordinate plane. The quarterback throws the football to the point $(6, -2)$. Use a translation to describe a path the wide receiver can take to catch the pass.



2.1 Practice



Go to BigIdeasMath.com to get HELP with solving the exercises.

► Review & Refresh

Solve the equation for y .

1. $6x + y = 12$

2. $9 = x + 3y$

3. $\frac{1}{3}x + 2y = 8$

4. You put \$550 in an account that earns 4.4% simple interest per year. How much interest do you earn in 6 months?

A. \$1.21

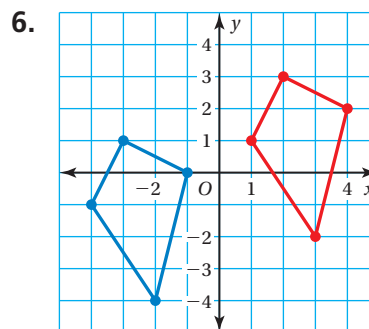
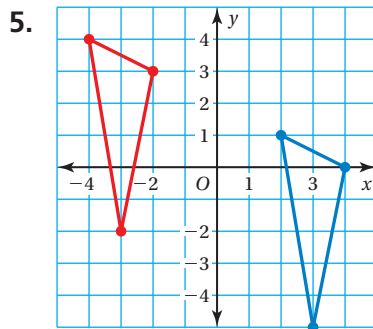
B. \$12.10

C. \$121.00

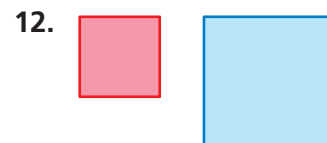
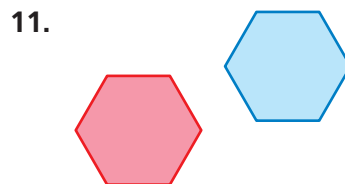
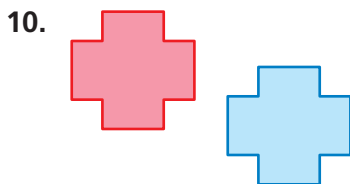
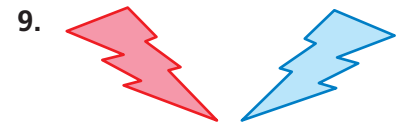
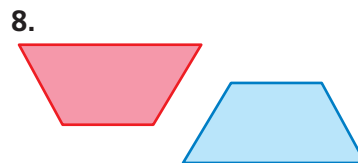
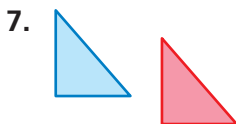
D. \$145.20

► Concepts, Skills, & Problem Solving

DESCRIBING RELATIONSHIPS For each figure, describe the location of the blue figure relative to the location of the red figure. (See Exploration 1, p. 43.)



IDENTIFYING A TRANSLATION Tell whether the blue figure is a translation of the red figure.



TRANSLATING A FIGURE The vertices of a triangle are $L(0, 1)$, $M(1, -2)$, and $N(-2, 1)$. Draw the figure and its image after the translation.

13. 1 unit left and 6 units up

14. 5 units right

15. $(x + 2, y + 3)$

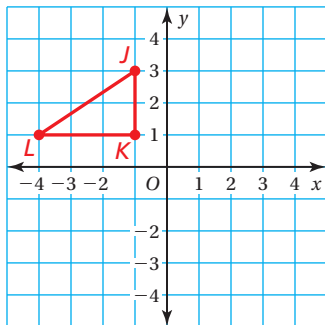
16. $(x - 3, y - 4)$

17. **YOU BE THE TEACHER** Your friend translates point A 2 units down and 1 unit right. Is your friend correct? Explain your reasoning.

$A(3, 1) \rightarrow A'(3 - 2, 1 + 1) \rightarrow A'(1, 2)$
The translated point is $A'(1, 2)$.

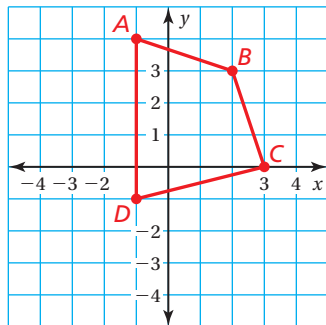
18. TRANSLATING A FIGURE

Translate the triangle 4 units right and 3 units down. What are the coordinates of the image?



19. TRANSLATING A FIGURE

Translate the figure 2 units left and 4 units down. What are the coordinates of the image?



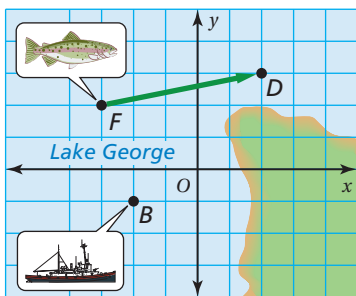
DESCRIBING A TRANSLATION Describe the translation of the point to its image.

20. $(3, -2) \rightarrow (1, 0)$

21. $(-8, -4) \rightarrow (-3, 5)$

22. **MP REASONING** You can click and drag an icon on a computer's desktop. Is this an example of a translation? Explain.

23. **MODELING REAL LIFE** The proposed location for a new oil platform is represented in a coordinate plane by a rectangle with vertices $A(1, -3)$, $B(1, 4)$, $C(4, 4)$, and $D(4, -3)$. An inspector recommends moving the oil platform 4 units right and 2 units down. Find the coordinates of the image. Then draw the original figure and the image in the coordinate plane.



24. **MP PROBLEM SOLVING** A school of fish translates from point F to point D .

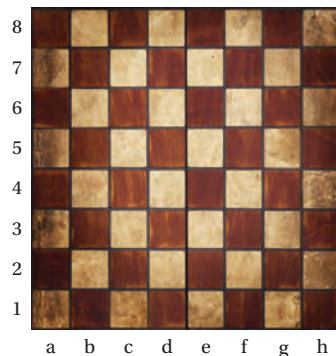
- Describe the translation of the school of fish.
- Can the fishing boat make the same translation? Explain.
- Describe a translation the fishing boat could make to get to point D .

25. **MP REASONING** The vertices of a triangle are $A(0, -3)$, $B(2, -1)$, and $C(3, -3)$. You translate the triangle 5 units right and 2 units down. Then you translate the image 3 units left and 8 units down. Is the original triangle identical to the final image? Explain your reasoning.

26. **DIG DEEPER!** In chess, a knight can move only in an L-shaped pattern:

- *two* vertical squares, then *one* horizontal square;
- *two* horizontal squares, then *one* vertical square;
- *one* vertical square, then *two* horizontal squares; or
- *one* horizontal square, then *two* vertical squares.

Write a series of translations to move the knight from g8 to g5.



2.2 Reflections

Learning Target: Reflect figures in the coordinate plane.

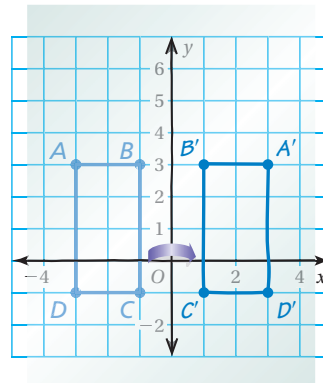
- Success Criteria:**
- I can identify a reflection.
 - I can find the coordinates of a figure reflected in an axis.
 - I can use coordinates to reflect a figure in the x - or y -axis.

EXPLORATION 1

Reflecting Figures

Work with a partner.

- a. For each figure below, draw the figure in the coordinate plane. Then copy the axes and the figure onto a piece of transparent paper. Flip the transparent paper and align the origin and the axes with the coordinate plane. For each pair of figures, describe the line of symmetry.
- point
 - line segment
 - line
 - triangle
 - rectangle



- b. When you reflect figures, what do you notice about sides, angles, and parallel lines?
- c. Describe the relationship between each point below and the point $A(4, 7)$ in terms of reflections.
- $B(-4, 7)$ $C(4, -7)$ $D(-4, -7)$
- d. A point with coordinates (x, y) is reflected in the x -axis. What are the coordinates of the image?
- e. Repeat part (d) when the point is reflected in the y -axis.

Math Practice

Look for Structure

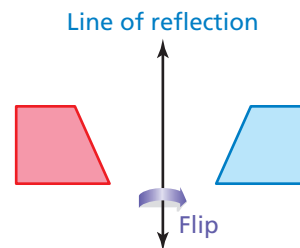
How can you show that the image of a figure reflected in the coordinate plane is identical to the original figure?

2.2 Lesson

Key Vocabulary

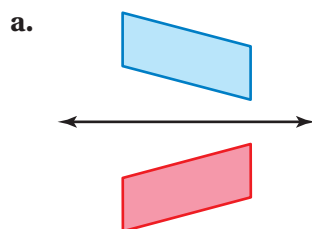
reflection, p. 50
line of reflection,
p. 50

A **reflection**, or *flip*, is a transformation in which a figure is reflected in a line called the **line of reflection**. A reflection creates a mirror image of the original figure.



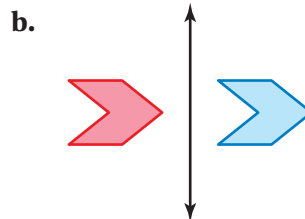
EXAMPLE 1 Identifying Reflections

Tell whether the blue figure is a reflection of the red figure.



The red figure can be *flipped* to form the blue figure.

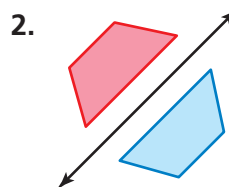
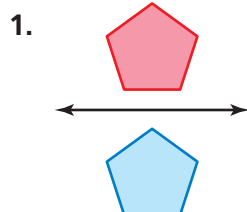
▶ So, the blue figure is a reflection of the red figure.



If the red figure were *flipped*, it would point to the left.

▶ So, the blue figure is *not* a reflection of the red figure.

Try It Tell whether the blue figure is a reflection of the red figure.



Key Idea

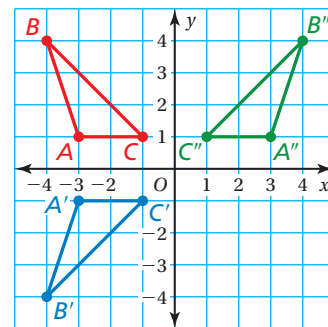
Reflections in the Coordinate Plane

Words To reflect a figure in the x -axis, take the opposite of the y -coordinate.

To reflect a figure in the y -axis, take the opposite of the x -coordinate.

Algebra Reflection in x -axis: $(x, y) \rightarrow (x, -y)$

Reflection in y -axis: $(x, y) \rightarrow (-x, y)$



In a reflection, the original figure and its image are identical.

EXAMPLE 2

Reflecting Figures

The vertices of a triangle are $A(1, 1)$, $B(1, 4)$, and $C(3, 4)$. Draw the figure and its reflection in (a) the x -axis and (b) the y -axis. What are the coordinates of the image?

Another Method

Take the opposite of each y -coordinate. The x -coordinates do not change.

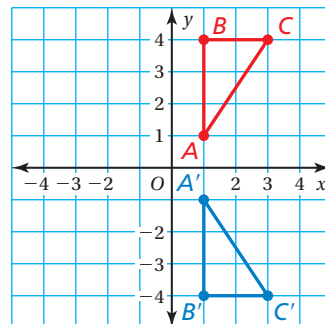
$$A(1, 1) \rightarrow A'(1, -1)$$

$$B(1, 4) \rightarrow B'(1, -4)$$

$$C(3, 4) \rightarrow C'(3, -4) \quad \checkmark$$

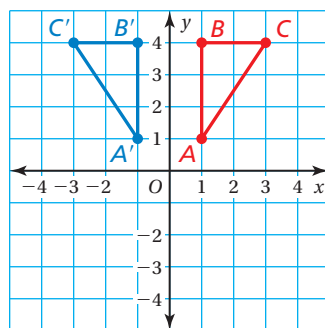
- a. Point A is 1 unit above the x -axis, so plot A' 1 unit below the x -axis. Points B and C are 4 units above the x -axis, so plot B' and C' 4 units below the x -axis.

► The coordinates of the image are $A'(1, -1)$, $B'(1, -4)$, and $C'(3, -4)$.



- b. Points A and B are 1 unit to the right of the y -axis, so plot A' and B' 1 unit to the left of the y -axis. Point C is 3 units to the right of the y -axis, so plot C' 3 units to the left of the y -axis.

► The coordinates of the image are $A'(-1, 1)$, $B'(-1, 4)$, and $C'(-3, 4)$.



Try It

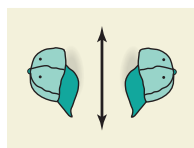
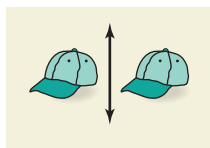
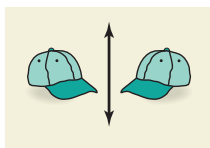
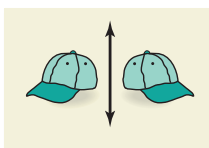
3. The vertices of a rectangle are $A(-4, -3)$, $B(-4, -1)$, $C(-1, -1)$, and $D(-1, -3)$. Draw the figure and its reflection in (a) the x -axis and (b) the y -axis.



Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

4. **REFLECTING A FIGURE** The vertices of a triangle are $J(-3, -5)$, $K(-2, 2)$, and $L(1, -4)$. Draw the figure and its reflection in (a) the x -axis and (b) the y -axis.
5. **WHICH ONE DOESN'T BELONG?** Which transformation does *not* belong with the other three? Explain your reasoning.



EXAMPLE 3 Modeling Real Life



A graphic artist designs a T-shirt using a pentagon with vertices $P(0, 0)$, $Q(-2, 0)$, $R(-1, 3)$, $S(-4, 3)$, and $T(0, 7)$. The artist reflects the pentagon in the y -axis to create the design. Find the coordinates of the reflected image. Then draw the design in the coordinate plane.

The pentagon is reflected in the y -axis. To find the coordinates of the reflected image, take the opposite of each x -coordinate. The y -coordinates do not change.

$$(x, y) \longrightarrow (-x, y)$$

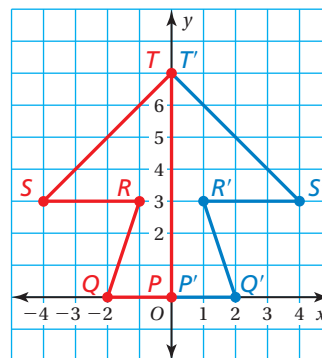
$$P(0, 0) \longrightarrow P'(0, 0)$$

$$Q(-2, 0) \longrightarrow Q'(2, 0)$$

$$R(-1, 3) \longrightarrow R'(1, 3)$$

$$S(-4, 3) \longrightarrow S'(4, 3)$$

$$T(0, 7) \longrightarrow T'(0, 7)$$

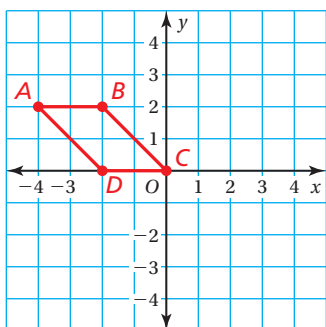


The coordinates of the reflected image are $P'(0, 0)$, $Q'(2, 0)$, $R'(1, 3)$, $S'(4, 3)$, and $T'(0, 7)$.



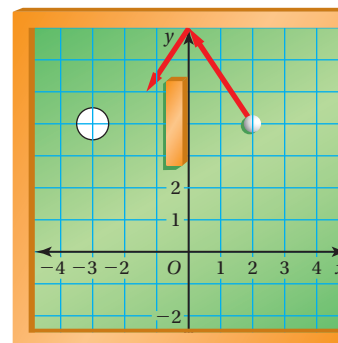
Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.



6. You design a logo using the figure shown at the left. You want both the x -axis and the y -axis to be lines of reflection. Describe how to use reflections to complete the design. Then draw the logo in the coordinate plane.

7. **DIG DEEPER!** You hit the golf ball along the path shown, so that its final location is a reflection in the y -axis of its starting location.
- Does the golf ball land in the hole? Explain.
 - Your friend tries the shot from the same starting location. He bounces the ball off the wall at the point $(-0.5, 7)$ so that its path is a reflection. Does the golf ball land in the hole?



2.2 Practice



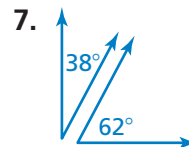
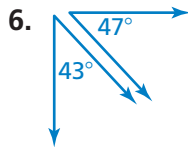
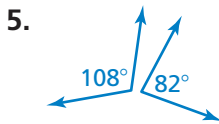
Go to BigIdeasMath.com to get HELP with solving the exercises.

► Review & Refresh

The vertices of a quadrilateral are $P(-1, -1)$, $Q(0, 4)$, $R(3, 1)$, and $S(1, -2)$. Draw the figure and its image after the translation.

1. 7 units down
2. 3 units left and 2 units up
3. $(x + 4, y - 1)$
4. $(x - 5, y - 6)$

Tell whether the angles are *complementary*, *supplementary*, or *neither*.



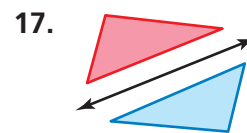
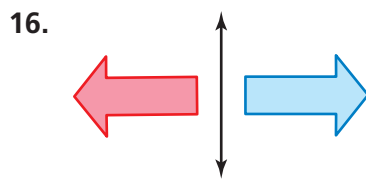
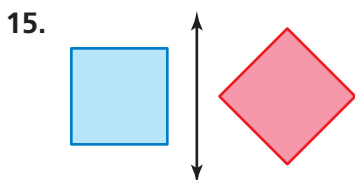
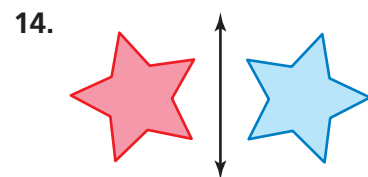
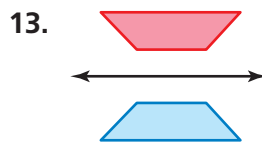
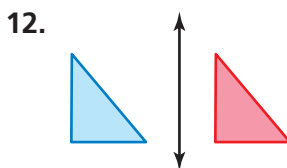
8. 36 is 75% of what number?
- A. 27 B. 48 C. 54 D. 63

► Concepts, Skills, & Problem Solving

DESCRIBING RELATIONSHIPS Describe the relationship between the given point and the point $A(5, 3)$ in terms of reflections. (See Exploration 1, p. 49.)

9. $B(5, -3)$ 10. $C(-5, -3)$ 11. $D(-5, 3)$

IDENTIFYING A REFLECTION Tell whether the blue figure is a reflection of the red figure.



REFLECTING FIGURES Draw the figure and its reflection in the x -axis. Identify the coordinates of the image.

18. $A(3, 2)$, $B(4, 4)$, $C(1, 3)$
19. $M(-2, 1)$, $N(0, 3)$, $P(2, 2)$
20. $H(2, -2)$, $J(4, -1)$, $K(6, -3)$, $L(5, -4)$
21. $D(-2, -5)$, $E(0, -1)$, $F(2, -1)$, $G(0, -5)$

REFLECTING FIGURES Draw the figure and its reflection in the y -axis. Identify the coordinates of the image.

22. $Q(-4, 2), R(-2, 4), S(-1, 1)$

23. $T(4, -2), U(4, 2), V(6, -2)$

24. $W(2, -1), X(5, -2), Y(5, -5), Z(2, -4)$

25. $J(2, 2), K(7, 4), L(9, -2), M(3, -1)$

26. **MP REASONING** Which letters look the same when reflected in the line?



MP STRUCTURE The coordinates of a point and its image after a reflection are given. Identify the line of reflection.

27. $(2, -2) \rightarrow (2, 2)$

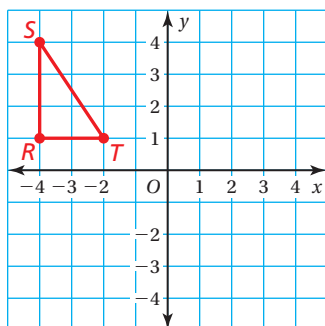
28. $(-4, 1) \rightarrow (4, 1)$

29. $(-2, -5) \rightarrow (4, -5)$

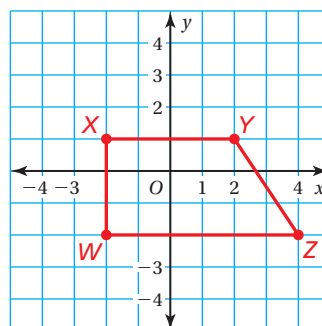
30. $(-3, -4) \rightarrow (-3, 0)$

TRANSFORMING FIGURES Find the coordinates of the figure after the transformations.

31. Translate the triangle 1 unit right and 5 units down. Then reflect the image in the y -axis.



32. Reflect the trapezoid in the x -axis. Then translate the image 2 units left and 3 units up.



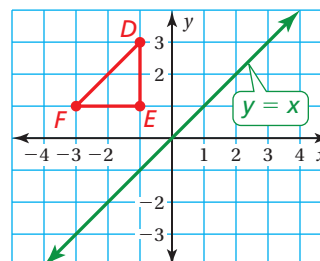
33. **MP REASONING** In Exercises 31 and 32, is the original figure identical to the final image? Explain.



34. **CRITICAL THINKING** Hold a mirror to the left side of the photo of the vehicle.

- What word do you see in the mirror?
- Why do you think it is written that way on the front of the vehicle?

35. **DIG DEEPER!** Reflect the triangle in the line $y = x$. How are the x - and y -coordinates of the image related to the x - and y -coordinates of the original triangle?



2.3 Rotations

Learning Target: Rotate figures in the coordinate plane.

- Success Criteria:**
- I can identify a rotation.
 - I can find the coordinates of a figure rotated about the origin.
 - I can use coordinates to rotate a figure about the origin.

EXPLORATION 1

Rotating Figures

Work with a partner.

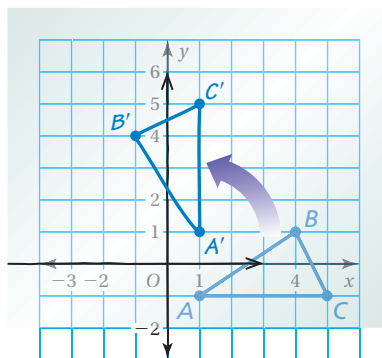
- a. For each figure below, draw the figure in the coordinate plane. Then copy the axes and the figure onto a piece of transparent paper. Turn the transparent paper and align the origin and the axes with the coordinate plane. For each pair of figures, describe the angle of rotation.

Math Practice

Explain the Meaning

What does it mean to rotate a figure about the origin?

- point
- line segment
- line
- triangle
- rectangle



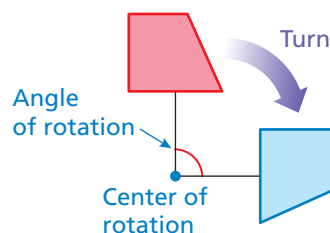
- b. When you rotate figures, what do you notice about sides, angles, and parallel lines?
- c. Describe the relationship between each point below and the point $A(3, 6)$ in terms of rotations.
- $B(-3, -6)$ $C(6, -3)$ $D(-6, 3)$
- d. What are the coordinates of a point $P(x, y)$ after a rotation 90° counterclockwise about the origin? 180° ? 270° ?

2.3 Lesson

Key Vocabulary

rotation, p. 56
 center of rotation, p. 56
 angle of rotation, p. 56

A **rotation**, or *turn*, is a transformation in which a figure is rotated about a point called the **center of rotation**. The number of degrees a figure rotates is the **angle of rotation**.

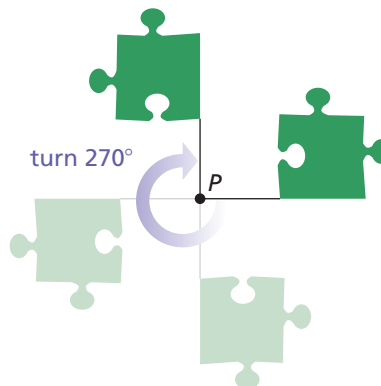


EXAMPLE 1 Identifying a Rotation

You must rotate the puzzle piece 270° clockwise about point P to fit it into a puzzle. Which piece fits in the puzzle as shown?

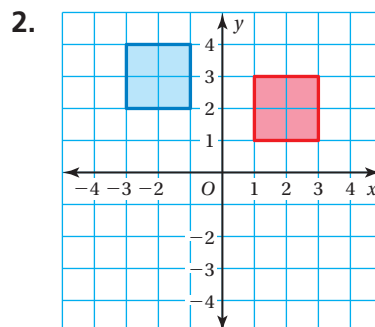
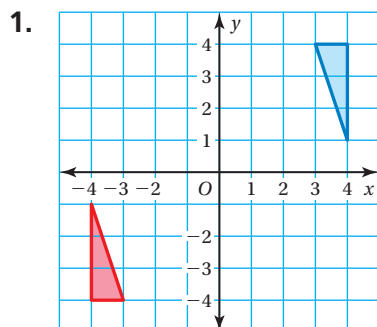


Rotate the puzzle piece 270° clockwise about point P .



So, the correct answer is **C**.

Try It Tell whether the blue figure is a rotation of the red figure about the origin. If so, give the angle and direction of rotation.



When rotating figures, it may help to sketch the rotation in several steps, as shown in Example 1.

You can use coordinate rules to find the coordinates of a point after a rotation of 90° , 180° , or 270° about the origin.

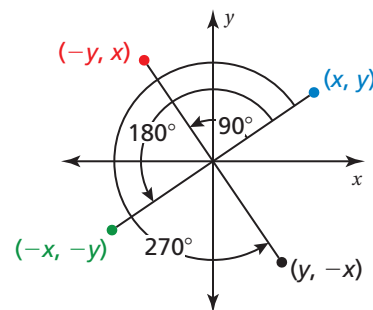
Key Idea

A counterclockwise rotation of n° is the same as a clockwise rotation of $(360 - n)^\circ$. Similarly, a clockwise rotation of n° is the same as a counterclockwise rotation of $(360 - n)^\circ$.

Rotations in the Coordinate Plane

When a point (x, y) is rotated counterclockwise about the origin, the following are true.

- For a rotation of 90° , $(x, y) \rightarrow (-y, x)$.
- For a rotation of 180° , $(x, y) \rightarrow (-x, -y)$.
- For a rotation of 270° , $(x, y) \rightarrow (y, -x)$.



In a rotation, the original figure and its image are identical.

EXAMPLE 2 Rotating a Figure

A 180° clockwise rotation and a 180° counterclockwise rotation have the same image. So, you do not need to specify direction when rotating a figure 180° .

The vertices of a trapezoid are $W(-4, 2)$, $X(-3, 4)$, $Y(-1, 4)$, and $Z(-1, 2)$. Rotate the trapezoid 180° about the origin. What are the coordinates of the image?

A point (x, y) rotated 180° about the origin results in an image with coordinates $(-x, -y)$.

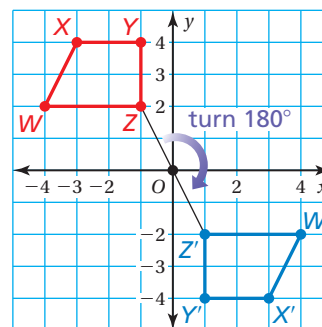
$$(x, y) \longrightarrow (-x, -y)$$

$$W(-4, 2) \longrightarrow W'(4, -2)$$

$$X(-3, 4) \longrightarrow X'(3, -4)$$

$$Y(-1, 4) \longrightarrow Y'(1, -4)$$

$$Z(-1, 2) \longrightarrow Z'(1, -2)$$



The coordinates of the image are $W'(4, -2)$, $X'(3, -4)$, $Y'(1, -4)$, and $Z'(1, -2)$.

Try It The vertices of a figure are given. Rotate the figure as described. Find the coordinates of the image.

3. $J(-4, -4)$, $K(-4, 2)$, $L(-1, 0)$, $M(-2, -3)$; 180° about the origin
4. $P(-3, 2)$, $Q(6, 1)$, $R(-1, -5)$; 90° counterclockwise about the origin
5. $A(5, 3)$, $B(4, -1)$, $C(1, -1)$; 90° clockwise about the origin

EXAMPLE 3

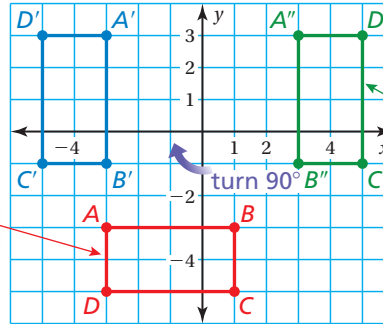
Using More Than One Transformation

The vertices of a rectangle are $A(-3, -3)$, $B(1, -3)$, $C(1, -5)$, and $D(-3, -5)$. Rotate the rectangle 90° clockwise about the origin, and then reflect it in the y -axis. What are the coordinates of the image?

Common Error

Be sure to pay attention to whether a rotation is clockwise or counterclockwise.

Draw $ABCD$ and rotate it 90° clockwise.



Reflect the rotated figure in the y -axis.

The coordinates of the image are $A''(3, 3)$, $B''(3, -1)$, $C''(5, -1)$, and $D''(5, 3)$.

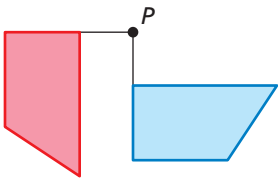
Try It

6. The vertices of a triangle are $P(-1, 2)$, $Q(-1, 0)$, and $R(2, 0)$. Rotate the triangle 180° about the origin, and then reflect it in the x -axis. What are the coordinates of the image?

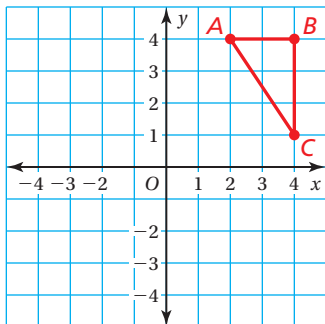


Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.



7. **IDENTIFYING A ROTATION** Tell whether the blue figure is a rotation of the red figure about point P . If so, give the angle and direction of rotation.
8. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.



What are the coordinates of the image after a 90° clockwise rotation about the origin?

What are the coordinates of the image after a 270° clockwise rotation about the origin?

What are the coordinates of the image after turning the figure 90° to the right about the origin?

What are the coordinates of the image after a 270° counterclockwise rotation about the origin?

EXAMPLE 4

Modeling Real Life



A carousel is represented in a coordinate plane with the center of the carousel at the origin. You and three friends sit at $A(-4, -4)$, $B(-3, 0)$, $C(-1, -2)$, and $D(-2, -3)$. At the end of the ride, your positions have rotated 270° clockwise about the center of the carousel. What are your locations at the end of the ride?

A rotation of 270° clockwise about the origin is the same as a rotation of 90° counterclockwise about the origin. Use coordinate rules to find the locations after a rotation of 90° counterclockwise about the origin.

A point (x, y) rotated 90° counterclockwise about the origin results in an image with coordinates $(-y, x)$.

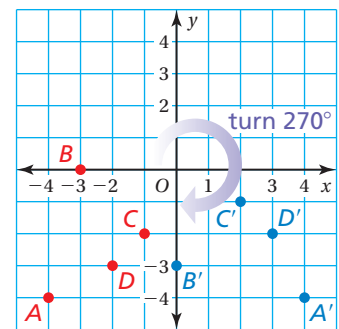
$$(x, y) \longrightarrow (-y, x)$$

$$A(-4, -4) \longrightarrow A'(4, -4)$$

$$B(-3, 0) \longrightarrow B'(0, -3)$$

$$C(-1, -2) \longrightarrow C'(2, -1)$$

$$D(-2, -3) \longrightarrow D'(3, -2)$$



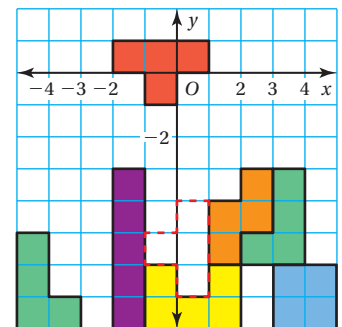
Your locations at the end of the ride are $A'(4, -4)$, $B'(0, -3)$, $C'(2, -1)$, and $D'(3, -2)$.



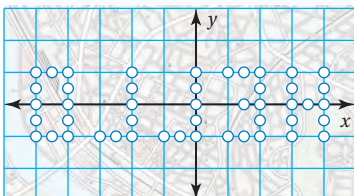
Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

9. You move the red game piece to the indicated location using a rotation about the origin, followed by a translation. What are the coordinates of the vertices of the game piece after the rotation? Justify your answer.



10. **DIG DEEPER!** Skytyping is a technique that airplanes use to write messages in the sky. The coordinate plane shows a message typed in the sky over a city, where the positive y -axis represents north. What does the message say? How can you transform the message so that it is read from north to south?



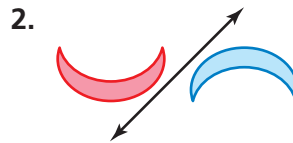
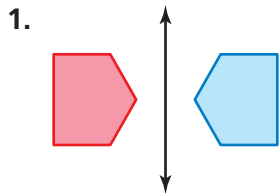
2.3 Practice



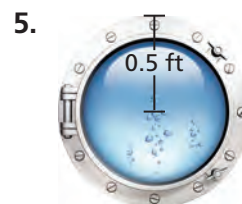
Go to BigIdeasMath.com to get HELP with solving the exercises.

► Review & Refresh

Tell whether the blue figure is a reflection of the red figure.



Find the circumference of the object. Use 3.14 or $\frac{22}{7}$ for π .



► Concepts, Skills, & Problem Solving

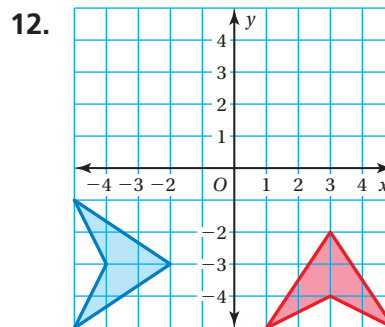
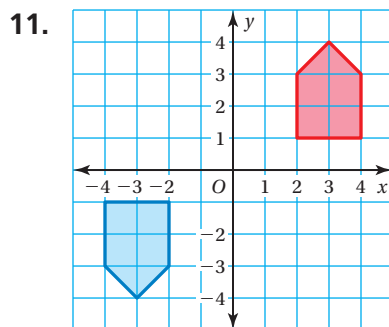
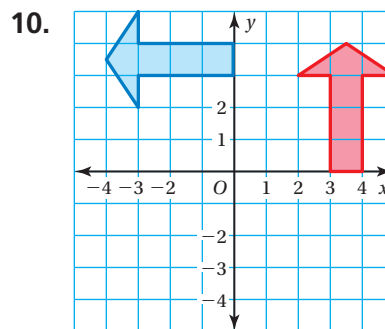
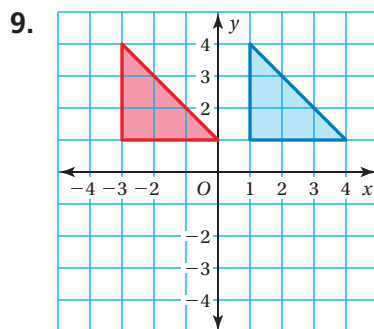
DESCRIBING RELATIONSHIPS Describe the relationship between the given point and the point $A(2, 7)$ in terms of rotations. (See Exploration 1, p. 55.)

6. $B(7, -2)$

7. $C(-7, 2)$

8. $D(-2, -7)$

IDENTIFYING A ROTATION Tell whether the blue figure is a rotation of the red figure about the origin. If so, give the angle and direction of rotation.



ROTATING A FIGURE The vertices of a figure are given. Rotate the figure as described. Find the coordinates of the image.

13. $A(2, -2), B(4, -1), C(4, -3), D(2, -4)$
 90° counterclockwise about the origin

14. $F(1, 2), G(3, 5), H(3, 2)$
 180° about the origin

15. $J(-4, 1), K(-2, 1), L(-4, -3)$
 90° clockwise about the origin

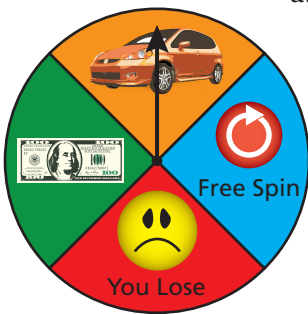
16. $P(-3, 4), Q(-1, 4), R(-2, 1), S(-4, 1)$
 270° clockwise about the origin

17. $W(-6, -2), X(-2, -2), Y(-2, -6), Z(-5, -6)$
 270° counterclockwise about the origin

18. $A(1, -1), B(5, -6), C(1, -6)$
 90° counterclockwise about the origin

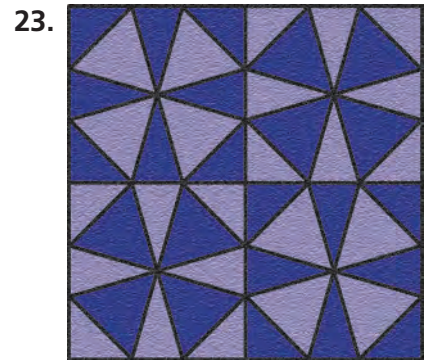
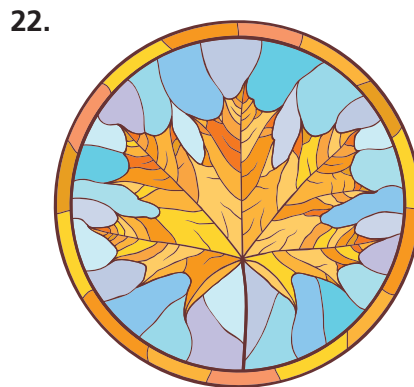
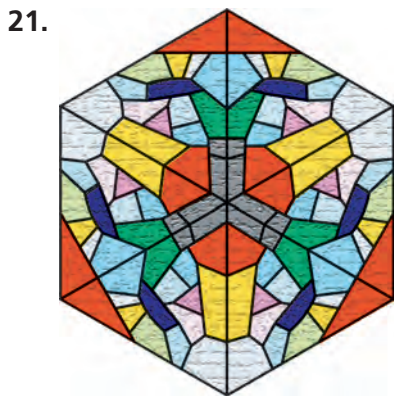
19. **YOU BE THE TEACHER** The vertices of a triangle are $A(4, 4), B(1, -2)$, and $C(-3, 0)$. Your friend finds the coordinates of the image after a rotation 90° clockwise about the origin. Is your friend correct? Explain your reasoning.

$A(4, 4) \longrightarrow A'(-4, 4)$
 $B(1, -2) \longrightarrow B'(2, 1)$
 $C(-3, 0) \longrightarrow C'(0, -3)$



20. **MP PROBLEM SOLVING** A game show contestant spins the prize wheel shown. The arrow remains in a fixed position while the wheel rotates. The wheel stops spinning, resulting in an image that is a rotation 270° clockwise about the center of the wheel. What is the result?

MP PATTERNS A figure has *rotational symmetry* if a rotation of 180° or less produces an image that fits exactly on the original figure. Determine whether the figure has rotational symmetry. Explain your reasoning.

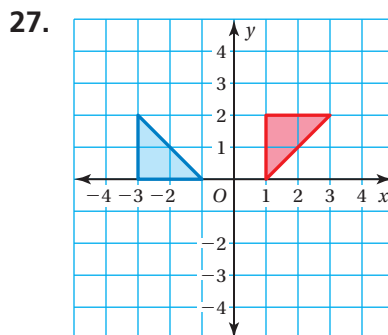
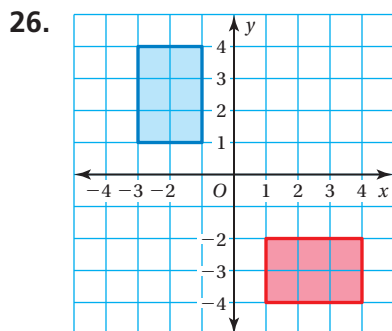


USING MORE THAN ONE TRANSFORMATION The vertices of a figure are given. Find the coordinates of the image after the transformations given.

24. $R(-7, -5), S(-1, -2), T(-1, -5)$
 Rotate 90° counterclockwise about the origin. Then translate 3 units left and 8 units up.

25. $J(-4, 4), K(-3, 4), L(-1, 1), M(-4, 1)$
 Reflect in the x -axis, and then rotate 180° about the origin.

CRITICAL THINKING Describe two different sequences of transformations in which the blue figure is the image of the red figure.



28. **MP REASONING** A trapezoid has vertices $A(-6, -2)$, $B(-3, -2)$, $C(-1, -4)$, and $D(-6, -4)$.

- Rotate the trapezoid 180° about the origin. What are the coordinates of the image?
- Describe a way to obtain the same image without using rotations.

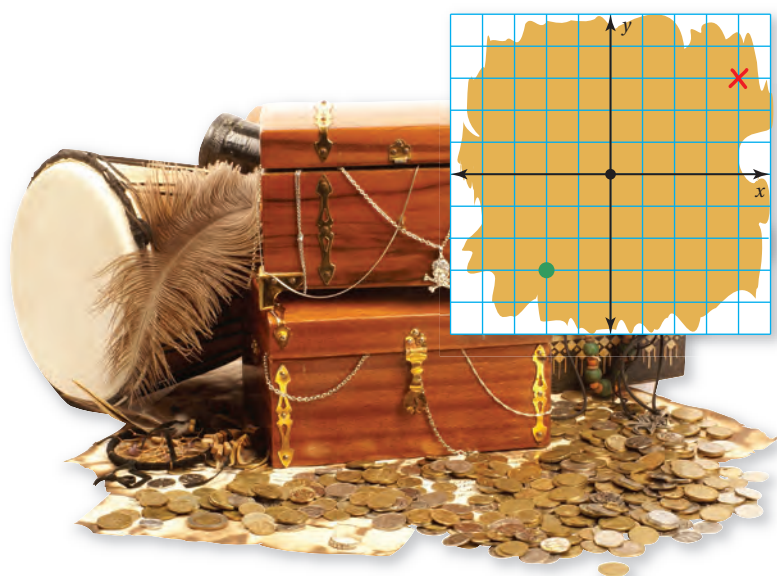
ROTATING A FIGURE The vertices of a figure are given. Rotate the figure as described. Find the coordinates of the image.

29. $D(2, 1)$, $E(2, -2)$, $F(-1, 4)$
 90° counterclockwise about vertex D

30. $L(-4, -3)$, $M(-1, -1)$, $N(2, -2)$
 180° about vertex M

31. $W(-5, 0)$, $X(-1, 4)$, $Y(3, -1)$, $Z(0, -4)$
 270° counterclockwise about vertex W

32. $D(-3, -4)$, $E(-5, 2)$, $F(1, -1)$, $G(3, -7)$
 270° clockwise about vertex E



33. **MP LOGIC** You want to find the treasure located on the map at \times . You are located at \bullet . The following transformations will lead you to the treasure, but they are not in the correct order. Find the correct order. Use each transformation exactly once.

- Rotate 180° about the origin.
- Reflect in the y -axis.
- Rotate 90° counterclockwise about the origin.
- Translate 1 unit right and 1 unit up.

34. **DIG DEEPER!** You rotate a triangle 90° counterclockwise about the origin. Then you translate its image 1 unit left and 2 units down. The vertices of the final image are $(-5, 0)$, $(-2, 2)$, and $(-2, -1)$. What are the vertices of the original triangle?

2.4 Congruent Figures

Learning Target: Understand the concept of congruent figures.

- Success Criteria:**
- I can identify congruent figures.
 - I can describe a sequence of rigid motions between two congruent figures.

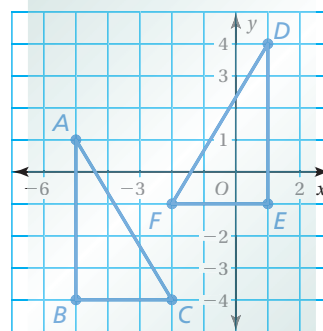
EXPLORATION 1

Transforming Figures

Work with a partner.

- a. For each pair of figures whose vertices are given below, draw the figures in a coordinate plane. Then copy one of the figures onto a piece of transparent paper. Use transformations to try to obtain one of the figures from the other figure.

- $A(-5, 1)$, $B(-5, -4)$, $C(-2, -4)$ and $D(1, 4)$, $E(1, -1)$, $F(-2, -1)$
- $G(1, 2)$, $H(2, -6)$, $J(5, 0)$ and $L(-1, -2)$, $M(-2, 6)$, $N(-5, 0)$
- $P(0, 0)$, $Q(2, 2)$, $R(4, -2)$ and $X(0, 0)$, $Y(3, 3)$, $Z(6, -3)$
- $A(0, 4)$, $B(3, 8)$, $C(6, 4)$, $D(3, 0)$ and $F(-4, -3)$, $G(-8, 0)$, $H(-4, 3)$, $J(0, 0)$
- $P(-2, 1)$, $Q(-1, -2)$, $R(1, -2)$, $S(1, 1)$ and $W(7, 1)$, $X(5, -2)$, $Y(3, -2)$, $Z(3, 1)$



Math Practice

Interpret Results

When you can use translations, reflections, and rotations to obtain one figure from another, what do you know about the side lengths and angle measures of the figures?

- b. Which pairs of figures in part (a) are identical? Explain your reasoning.
- c. Figure A and Figure B are identical. Do you think there must be a sequence of transformations that obtains Figure A from Figure B? Explain your reasoning.

2.4 Lesson

Key Vocabulary

rigid motion, p. 64
 congruent figures,
 p. 64
 congruent angles,
 p. 64
 congruent sides,
 p. 64

Reading

The symbol \triangle means *triangle*. The symbol \cong means *is congruent to*. In the Key Idea,

$$\triangle ABC \cong \triangle DEF.$$

In diagrams, matching arcs indicate congruent angles.

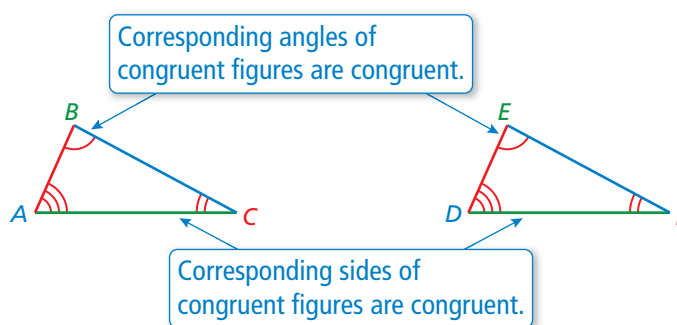
A **rigid motion** is a transformation that preserves length and angle measure. Translations, reflections, and rotations are rigid motions.

Key Idea

Congruent Figures

Two figures are **congruent figures** when one can be obtained from the other by a sequence of rigid motions. Congruent figures have the same size and the same shape. Angles with the same measure are called **congruent angles**. Sides with the same measure are **congruent sides**.

The triangles below are congruent.



Sides

$$\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}, \overline{AC} \cong \overline{DF}$$

Angles

$$\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$$

EXAMPLE 1 Identifying Congruent Figures

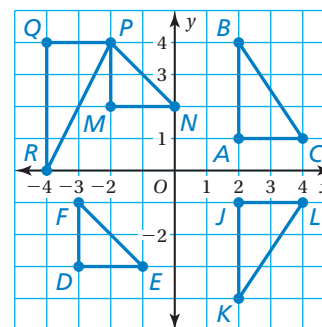
Common Error

When writing a congruence statement, make sure to list the vertices of the figures in the correct order.

Identify any congruent figures in the coordinate plane.

$\triangle DEF$ is a translation 1 unit left and 5 units down of $\triangle MNP$. So, $\triangle DEF$ and $\triangle MNP$ are congruent.

$\triangle ABC$ is a reflection in the x -axis of $\triangle JKL$. So, $\triangle ABC$ and $\triangle JKL$ are congruent.

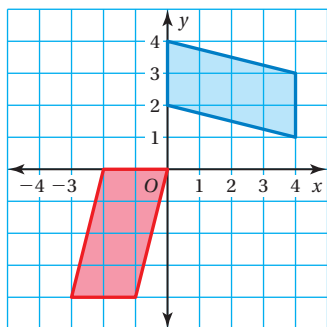


Try It

1. A triangle has vertices $X(0, 4)$, $Y(4, 4)$, and $Z(4, 2)$. Is $\triangle XYZ$ congruent to any of the triangles in Example 1? Explain.

EXAMPLE 2

Describing a Sequence of Rigid Motions

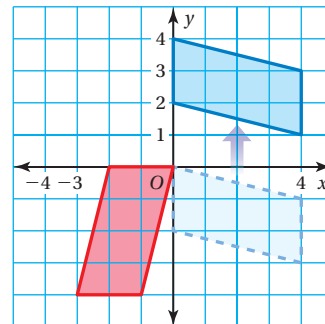


The red figure is congruent to the blue figure. Describe a sequence of rigid motions between the figures.

The orientations of the figures are different. You can rotate the red figure 90° to match the orientation of the blue figure.

After rotating the red figure, you can translate its image to the blue figure.

So, one possible sequence of rigid motions is to rotate the red figure 90° counterclockwise about the origin and then translate the image 4 units up.



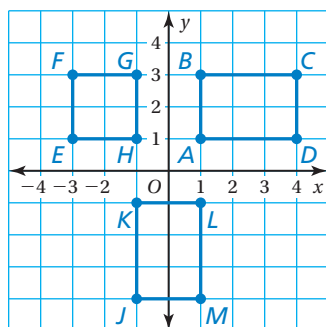
Try It

- Describe a different sequence of rigid motions between the figures.



Self-Assessment for Concepts & Skills

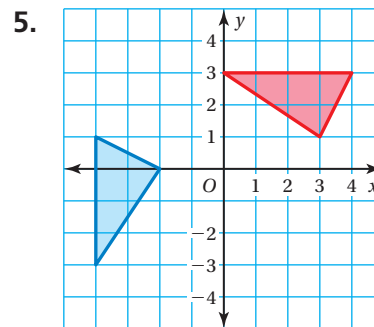
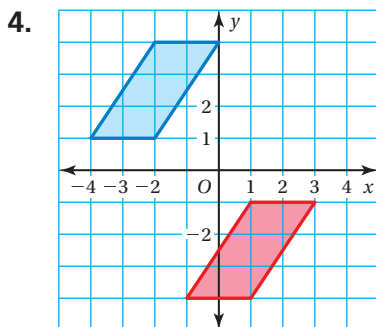
Solve each exercise. Then rate your understanding of the success criteria in your journal.



- IDENTIFYING CONGRUENT FIGURES** Use the coordinate plane shown.

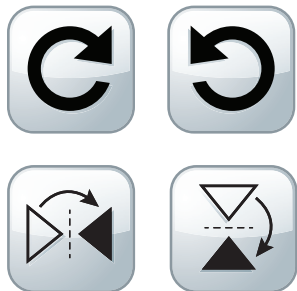
- Identify any congruent figures.
- A rectangle has vertices $W(-4, -1)$, $X(-4, 2)$, $Y(-1, 2)$, and $Z(-1, -1)$. Is Rectangle $WXYZ$ congruent to any of the rectangles in the coordinate plane? Explain.

RIGID MOTIONS The red figure is congruent to the blue figure. Describe a sequence of rigid motions between the figures.

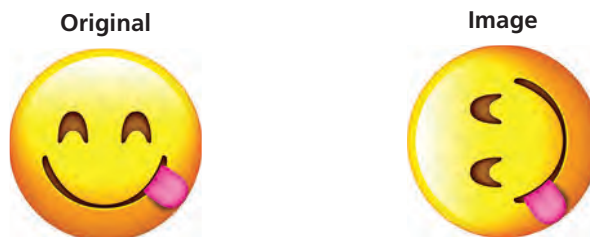


EXAMPLE 3

Modeling Real Life



You can use the buttons shown at the left to transform objects in a computer program. You can rotate objects 90° in either direction and reflect objects in a horizontal or vertical line. How can you transform the emoji as shown below?



When you rotate the emoji 90° counterclockwise, the tongue is in the wrong place. Reflect the emoji in a horizontal line to move the tongue to the correct location.

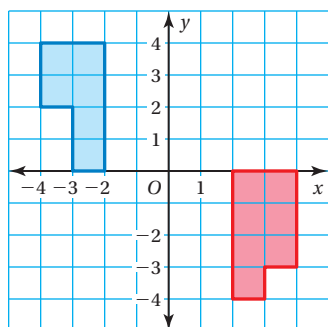


▶ To transform the emoji as shown, you can use a 90° counterclockwise rotation followed by a reflection in a horizontal line.



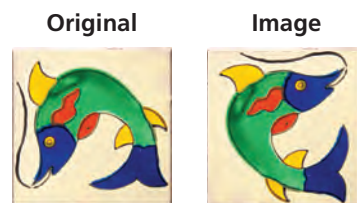
Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.



6. In the coordinate plane at the left, each grid line represents 50 feet. Each figure represents a pasture.
 - a. Are the figures congruent? Use rigid motions to justify your answer.
 - b. How many feet of fencing do you need to enclose each pasture?

7. A home decorator uses a computer to design a floor tile. How can the decorator transform the tile as shown?



2.4 Practice



Go to BigIdeasMath.com to get HELP with solving the exercises.

► Review & Refresh

The vertices of a figure are given. Rotate the figure as described. Find the coordinates of the image.

- $A(1, 3), B(2, 5), C(3, 5), D(2, 3)$
90° counterclockwise about the origin
- $F(-2, 1), G(-1, 3), H(3, 1)$
180° about the origin

Factor the expression using the greatest common factor.

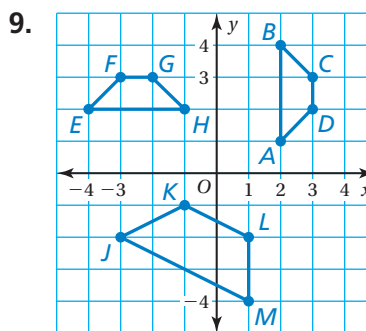
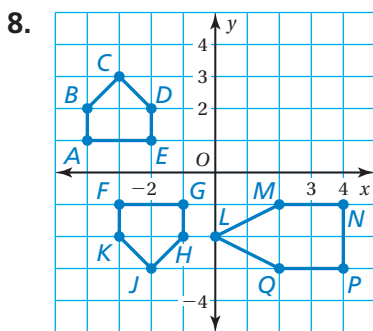
- $4n - 32$
- $3w + 66$
- $2y - 18$

► Concepts, Skills, & Problem Solving

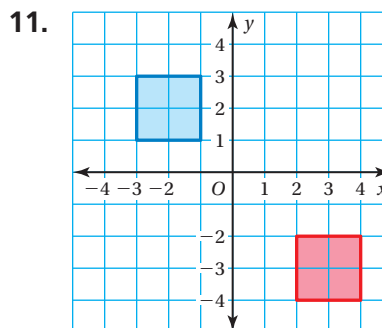
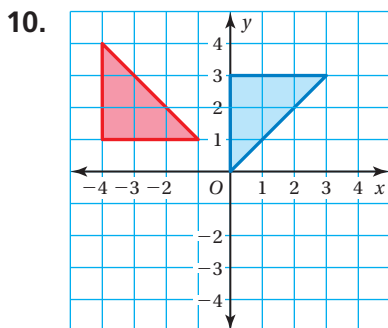
TRANSFORMING FIGURES The vertices of a pair of figures are given. Determine whether the figures are identical. (See Exploration 1, p. 63.)

- $G(0, 0), H(3, 2), J(1, -2)$ and $L(-1, 0), M(2, 2), N(0, -3)$
- $A(-2, -1), B(-2, 2), C(-1, 1), D(-1, -2)$ and $F(-2, 0), G(-1, 1), H(2, 1), J(1, 0)$

IDENTIFYING CONGRUENT FIGURES Identify any congruent figures in the coordinate plane.

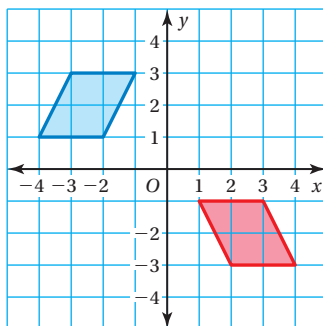


DESCRIBING A SEQUENCE OF RIGID MOTIONS The red figure is congruent to the blue figure. Describe a sequence of rigid motions between the figures.



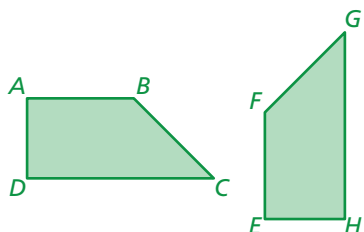
12. **YOU BE THE TEACHER** Your friend describes a sequence of rigid motions between the figures. Is your friend correct? Explain your reasoning.

Reflect the red figure in the x -axis, and then translate it left 5 units.

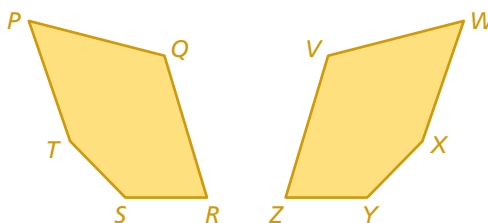


NAMING CORRESPONDING PARTS The figures are congruent. Name the corresponding angles and the corresponding sides.

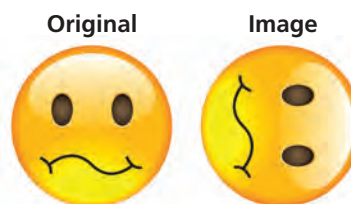
13.



14.



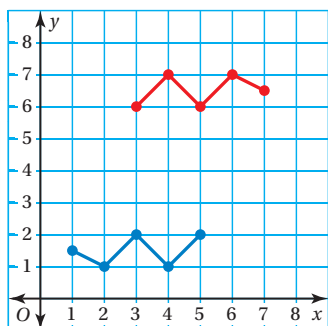
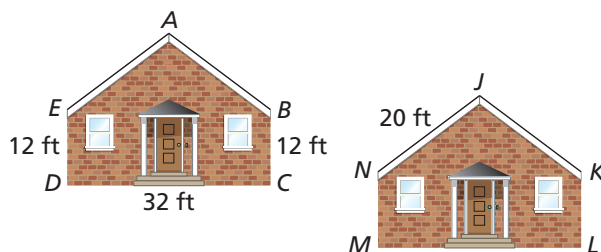
15. **MODELING REAL LIFE** You use a computer program to transform an emoji. How can you transform the emoji as shown?



16. **CRITICAL THINKING** Two figures are congruent. Are the areas of the two figures the same? the perimeters? Explain your reasoning.

17. **DIG DEEPER!** The houses are identical.

- What is the length of side LM ?
- Which angle of $JKLMN$ corresponds to $\angle D$?
- Side AB is congruent to side AE . What is the length of side AB ? What is the perimeter of $ABCDE$?



18. **MP REASONING** Two constellations are represented by the figures in the coordinate plane shown. Are the figures congruent? Justify your answer.

2.5 Dilations

Learning Target: Dilate figures in the coordinate plane.

- Success Criteria:**
- I can identify a dilation.
 - I can find the coordinates of a figure dilated with respect to the origin.
 - I can use coordinates to dilate a figure with respect to the origin.

The Meaning of a Word ► Dilate

When you have your eyes checked, the optometrist sometimes



dilates one or both of the pupils of your eyes.

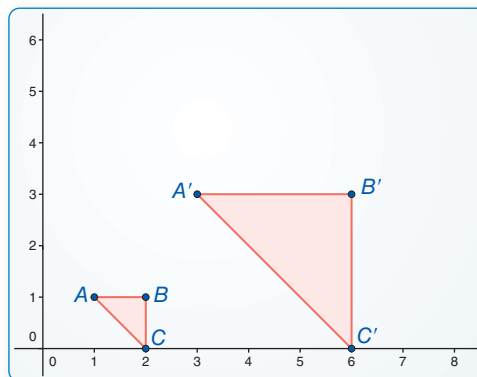


EXPLORATION 1

Dilating a Polygon

Work with a partner. Use geometry software.

- a. Draw a polygon in the coordinate plane. Then *dilate* the polygon with respect to the origin. Describe the scale factor of the image.



Available at BigIdeasMath.com.

Sample

Points

$$A(1, 1)$$

$$B(2, 1)$$

$$C(2, 0)$$

Segments

$$AB = 1$$

$$BC = 1$$

$$AC = 1.41$$

Angles

$$m\angle A = 45^\circ$$

$$m\angle B = 90^\circ$$

$$m\angle C = 45^\circ$$

- b. Compare the image and the original polygon in part (a). What do you notice about the sides? the angles?
- c. Describe the relationship between each point below and the point $A(x, y)$ in terms of dilations.

$$B(3x, 3y)$$

$$C(5x, 5y)$$

$$D(0.5x, 0.5y)$$

- d. What are the coordinates of a point $P(x, y)$ after a dilation with respect to the origin by a scale factor of k ?

Math Practice

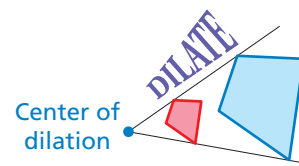
Consider Similar Problems

How does your previous work with scale drawings help you understand the concept of dilations?

2.5 Lesson

A scale drawing is an example of a dilation.

A **dilation** is a transformation in which a figure is made larger or smaller with respect to a point called the **center of dilation**. In a dilation, the angles of the image and the original figure are congruent.



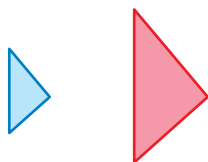
EXAMPLE 1 Identifying a Dilation

Key Vocabulary

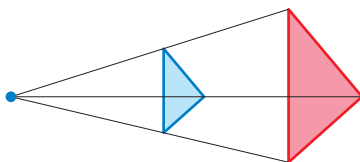
dilation, p. 70
center of dilation, p. 70
scale factor, p. 70

Tell whether the blue figure is a dilation of the red figure.

a.

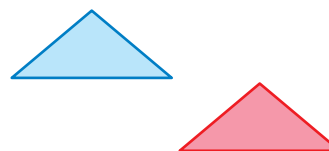


Lines connecting corresponding vertices meet at a point.



So, the blue figure is a dilation of the red figure.

b.



The figures have the same size and shape. The red figure *slides* to form the blue figure.

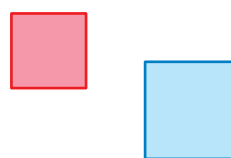
So, the blue figure is *not* a dilation of the red figure. It is a translation.

Try It Tell whether the blue figure is a dilation of the red figure.

1.



2.



In a dilation, the value of the ratio of the side lengths of the image to the corresponding side lengths of the original figure is the **scale factor** of the dilation.

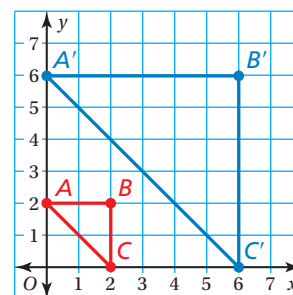
Key Idea

Dilations in the Coordinate Plane

Words To dilate a figure with respect to the origin, multiply the coordinates of each vertex by the scale factor k .

Algebra $(x, y) \rightarrow (kx, ky)$

- When $k > 1$, the dilation is an enlargement.
- When $k > 0$ and $k < 1$, the dilation is a reduction.



In this course, when the center of dilation is not specified, it is the origin.

EXAMPLE 2 Dilating a Figure

The vertices of a triangle are $A(1, 3)$, $B(2, 3)$, and $C(2, 1)$. Draw the image after a dilation with a scale factor of 3. Identify the type of dilation.

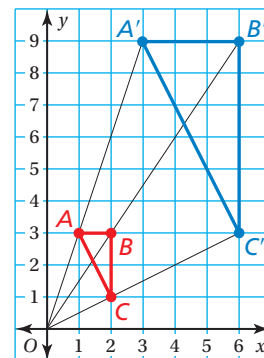
Multiply each x - and y -coordinate by the scale factor 3.

$$(x, y) \longrightarrow (3x, 3y)$$

$$A(1, 3) \longrightarrow A'(3 \cdot 1, 3 \cdot 3) \longrightarrow A'(3, 9)$$

$$B(2, 3) \longrightarrow B'(3 \cdot 2, 3 \cdot 3) \longrightarrow B'(6, 9)$$

$$C(2, 1) \longrightarrow C'(3 \cdot 2, 3 \cdot 1) \longrightarrow C'(6, 3)$$



You can check your answer by drawing a line from the origin through each vertex of the original figure. The vertices of the image should lie on these lines.

The image is shown at the right. The dilation is an *enlargement* because the scale factor is greater than 1.

Try It

3. **WHAT IF?** Triangle ABC is dilated by a scale factor of 2. What are the coordinates of the image?

EXAMPLE 3 Dilating a Figure

The vertices of a rectangle are $W(-4, -6)$, $X(-4, 8)$, $Y(4, 8)$, and $Z(4, -6)$. Draw the image after a dilation with a scale factor of 0.5. Identify the type of dilation.

Multiply each x - and y -coordinate by the scale factor 0.5.

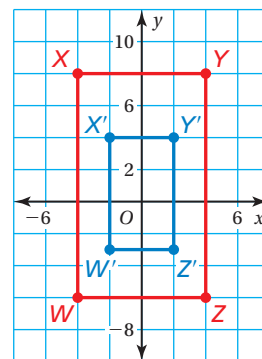
$$(x, y) \longrightarrow (0.5x, 0.5y)$$

$$W(-4, -6) \longrightarrow W'(0.5 \cdot (-4), 0.5 \cdot (-6)) \longrightarrow W'(-2, -3)$$

$$X(-4, 8) \longrightarrow X'(0.5 \cdot (-4), 0.5 \cdot 8) \longrightarrow X'(-2, 4)$$

$$Y(4, 8) \longrightarrow Y'(0.5 \cdot 4, 0.5 \cdot 8) \longrightarrow Y'(2, 4)$$

$$Z(4, -6) \longrightarrow Z'(0.5 \cdot 4, 0.5 \cdot (-6)) \longrightarrow Z'(2, -3)$$



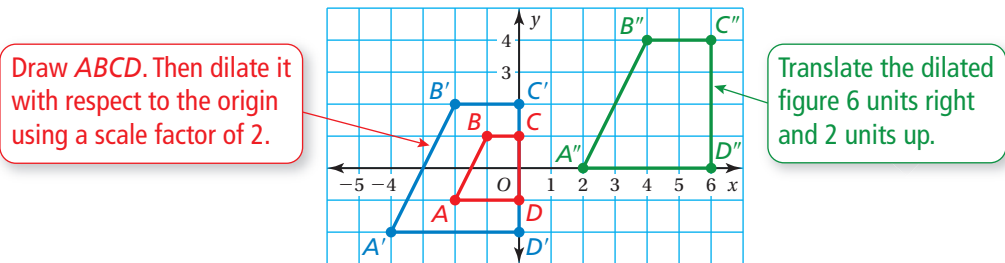
The image is shown at the right. The dilation is a *reduction* because the scale factor is greater than 0 and less than 1.

Try It

4. **WHAT IF?** Rectangle $WXYZ$ is dilated by a scale factor of $\frac{1}{4}$. What are the coordinates of the image?

EXAMPLE 4 Using More than One Transformation

The vertices of a trapezoid are $A(-2, -1)$, $B(-1, 1)$, $C(0, 1)$, and $D(0, -1)$. Dilate the trapezoid using a scale factor of 2. Then translate it 6 units right and 2 units up. What are the coordinates of the image?



► The coordinates of the image are $A''(2, 0)$, $B''(4, 4)$, $C''(6, 4)$, and $D''(6, 0)$.

Try It

5. **WHAT IF?** Trapezoid $ABCD$ is dilated using a scale factor of 3, and then rotated 180° about the origin. What are the coordinates of the image?

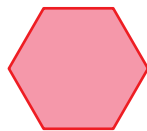
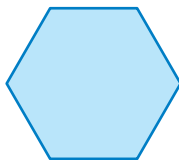


Self-Assessment for Concepts & Skills

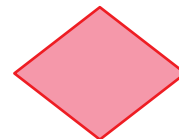
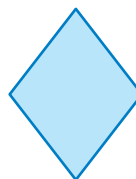
Solve each exercise. Then rate your understanding of the success criteria in your journal.

IDENTIFYING A DILATION Tell whether the blue figure is a dilation of the red figure.

6.



7.



8. **DILATING A FIGURE** The vertices of a rectangle are $J(4, 8)$, $K(12, 8)$, $L(12, 4)$, and $M(4, 4)$. Draw the image after a dilation with a scale factor of $\frac{1}{4}$. Identify the type of dilation.

9. **VOCABULARY** How is a dilation different from other transformations?

EXAMPLE 5

Modeling Real Life



A wildlife refuge is mapped on a coordinate plane, where each grid line represents 1 mile. The refuge has vertices $J(0, 0)$, $K(1, 3)$, and $L(4, 0)$. An expansion of the refuge can be represented by a dilation with a scale factor of 1.5. How much does the area of the wildlife refuge increase?

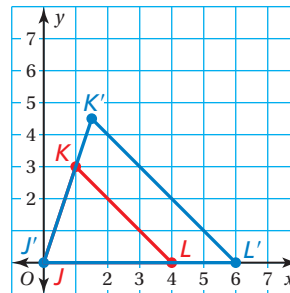
Multiply each x - and y -coordinate by the scale factor 1.5. Then find the area of each figure.

$$(x, y) \longrightarrow (1.5x, 1.5y)$$

$$J(0, 0) \longrightarrow J'(1.5 \cdot 0, 1.5 \cdot 0) \longrightarrow J'(0, 0)$$

$$K(1, 3) \longrightarrow K'(1.5 \cdot 1, 1.5 \cdot 3) \longrightarrow K'(1.5, 4.5)$$

$$L(4, 0) \longrightarrow L'(1.5 \cdot 4, 1.5 \cdot 0) \longrightarrow L'(6, 0)$$



The original figure is a triangle with a base of 4 miles and a height of 3 miles. The image has a base of 6 miles and a height of 4.5 miles. Use the formula for the area of a triangle to find the areas of the original figure and the image.

Original Figure

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}(4)(3)$$

$$= 6$$

Write the formula.

Substitute for b and h .

Simplify.

Image

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}(6)(4.5)$$

$$= 13.5$$

So, the area of the wildlife refuge increases $13.5 - 6 = 7.5$ square miles.



Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.



- A photograph is dilated to fit in a frame, so that its area after the dilation is 9 times greater than the area of the original photograph. What is the scale factor of the dilation? Explain.
- DIG DEEPER!** The location of a water treatment plant is mapped using a coordinate plane, where each unit represents 1 foot. The plant has vertices $(0, 0)$, $(0, 180)$, $(240, 180)$, and $(240, 0)$. You dilate the figure with a scale factor of $\frac{1}{3}$. What are the coordinates of the image? What do you need to change so that the image accurately represents the location of the plant? Explain your reasoning.

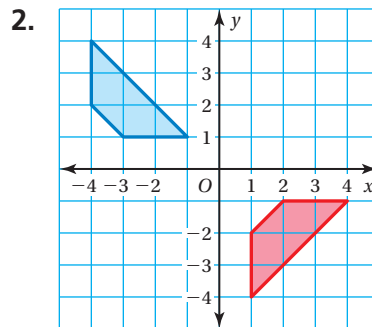
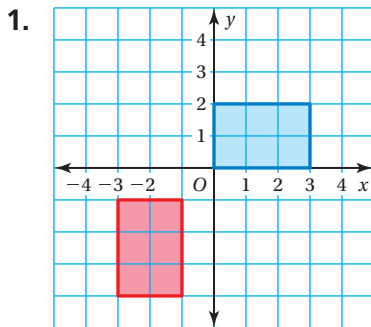
2.5 Practice



Go to BigIdeasMath.com to get HELP with solving the exercises.

► Review & Refresh

The red figure is congruent to the blue figure. Describe a sequence of rigid motions between the figures.



Tell whether the ratios form a proportion.

3. $3 : 5$ and $15 : 20$

4. 2 to 3 and 12 to 18

5. $7 : 28$ and $12 : 48$

► Concepts, Skills, & Problem Solving

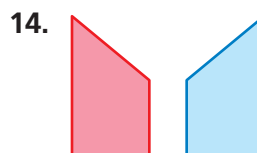
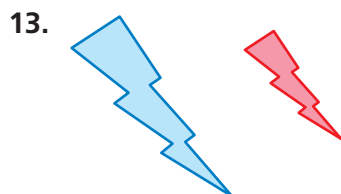
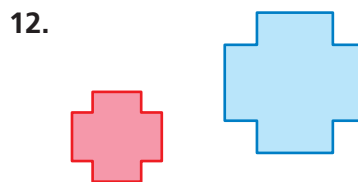
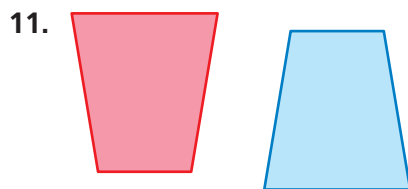
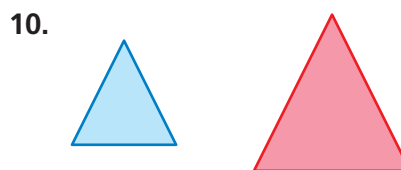
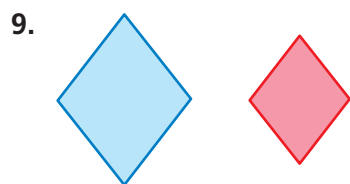
DESCRIBING RELATIONSHIPS Describe the relationship between the given point and the point $A(8, 12)$ in terms of dilations. (See Exploration 1, p. 69.)

6. $B(16, 24)$

7. $C(2, 3)$

8. $D(6, 9)$

IDENTIFYING A DILATION Tell whether the blue figure is a dilation of the red figure.



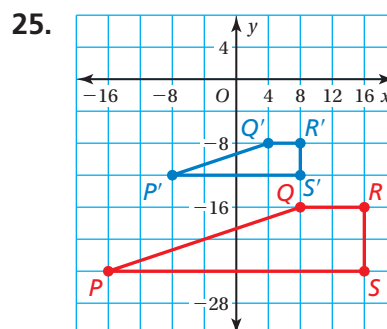
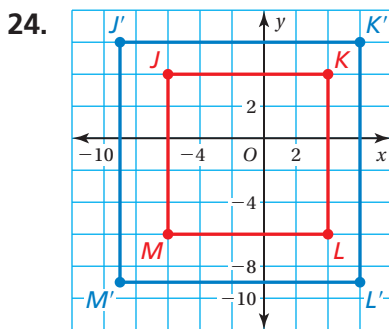
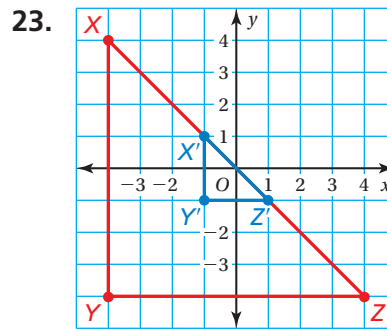
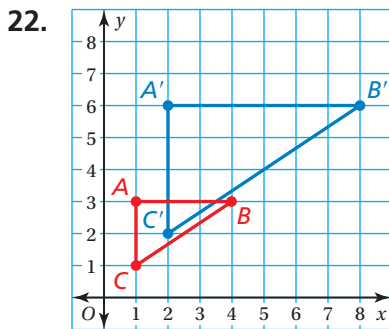
DILATING A FIGURE The vertices of a figure are given. Draw the figure and its image after a dilation with the given scale factor. Identify the type of dilation.

15. $A(1, 1), B(1, 4), C(3, 1); k = 4$ 16. $D(0, 2), E(6, 2), F(6, 4); k = 0.5$
 17. $G(-2, -2), H(-2, 6), J(2, 6); k = 0.25$ 18. $M(2, 3), N(5, 3), P(5, 1); k = 3$
 19. $Q(-3, 0), R(-3, 6), T(4, 6), U(4, 0); k = \frac{1}{3}$
 20. $V(-2, -2), W(-2, 3), X(5, 3), Y(5, -2); k = 5$

21. **YOU BE THE TEACHER** Your friend finds the coordinates of the image of $\triangle ABC$ after a dilation with a scale factor of 2. Is your friend correct? Explain your reasoning.

$A(2, 5) \rightarrow A'(2 \cdot 2, 2 \cdot 5) \rightarrow A'(4, 10)$
 $B(2, 0) \rightarrow B'(2 \cdot 2, 2 \cdot 0) \rightarrow B'(4, 0)$
 $C(4, 0) \rightarrow C'(2 \cdot 4, 2 \cdot 0) \rightarrow C'(8, 0)$

FINDING A SCALE FACTOR The blue figure is a dilation of the red figure. Identify the type of dilation and find the scale factor.

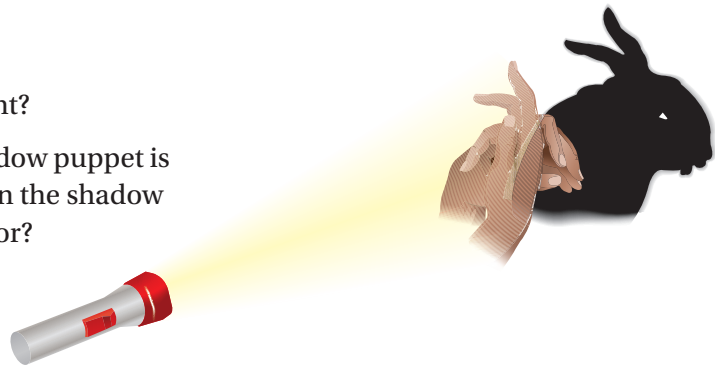


USING MORE THAN ONE TRANSFORMATION The vertices of a figure are given. Find the coordinates of the image after the transformations given.

26. $A(-5, 3), B(-2, 3), C(-2, 1), D(-5, 1)$
 Reflect in the y -axis. Then dilate using a scale factor of 2.
 27. $F(-9, -9), G(-3, -6), H(-3, -9)$
 Dilate using a scale factor of $\frac{2}{3}$. Then translate 6 units up.
 28. $J(1, 1), K(3, 4), L(5, 1)$
 Rotate 90° clockwise about the origin. Then dilate using a scale factor of 3.

29. **MP LOGIC** You can use a flashlight and a shadow puppet (your hands) to project shadows on the wall.

- Identify the type of dilation.
- What does the flashlight represent?
- The length of the ears on the shadow puppet is 3 inches. The length of the ears on the shadow is 4 inches. What is the scale factor?
- Describe what happens as the shadow puppet moves closer to the flashlight. How does this affect the scale factor?

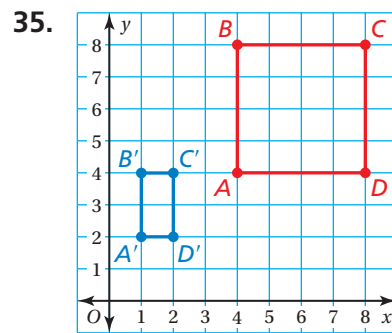
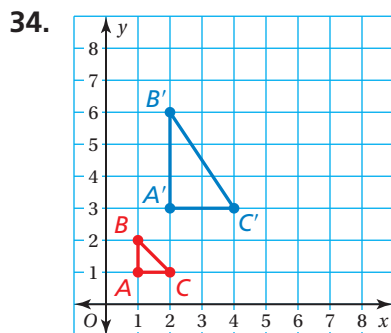


30. **MP REASONING** A triangle is dilated using a scale factor of 3. The image is then dilated using a scale factor of $\frac{1}{2}$. What scale factor can you use to dilate the original triangle to obtain the final image? Explain.

CRITICAL THINKING The coordinate notation shows how the coordinates of a figure are related to the coordinates of its image after transformations. What are the transformations? Are the figure and its image congruent? Explain.

31. $(x, y) \rightarrow (2x + 4, 2y - 3)$ 32. $(x, y) \rightarrow (-x - 1, y - 2)$ 33. $(x, y) \rightarrow \left(\frac{1}{3}x, -\frac{1}{3}y\right)$

MP STRUCTURE The blue figure is a transformation of the red figure. Use coordinate notation to describe the transformation. Explain your reasoning.



36. **MP NUMBER SENSE** You dilate a figure using a scale factor of 2, and then translate it 3 units right. Your friend translates the same figure 3 units right and then dilates it using a scale factor of 2. Are the images congruent? Explain.
37. **MP PROBLEM SOLVING** The vertices of a trapezoid are $A(-2, 3)$, $B(2, 3)$, $C(5, -2)$, and $D(-2, -2)$. Dilate the trapezoid with respect to vertex A using a scale factor of 2. What are the coordinates of the image? Explain the method you used.
38. **DIG DEEPER!** A figure is dilated using a scale factor of -1 . How can you obtain the image without using a dilation? Explain your reasoning.

2.6 Similar Figures

Learning Target: Understand the concept of similar figures.

- Success Criteria:**
- I can identify similar figures.
 - I can describe a similarity transformation between two similar figures.

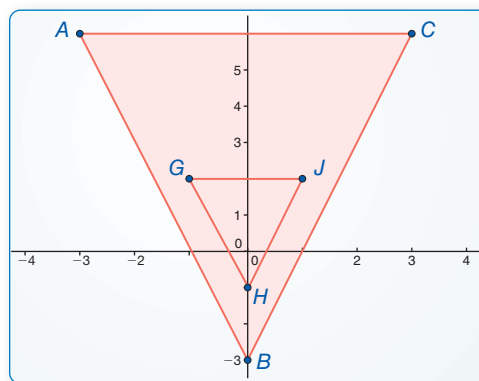
EXPLORATION 1

Transforming Figures

Work with a partner. Use geometry software.

- a. For each pair of figures whose vertices are given below, draw the figures in a coordinate plane. Use dilations and rigid motions to try to obtain one of the figures from the other figure.

- $A(-3, 6), B(0, -3), C(3, 6)$ and $G(-1, 2), H(0, -1), J(1, 2)$
- $D(0, 0), E(3, 0), F(3, 3)$ and $L(0, 0), M(0, 6), N(-6, 6)$
- $P(1, 0), Q(4, 2), R(7, 0)$ and $X(-1, 0), Y(-4, 6), Z(-7, 0)$
- $A(-3, 2), B(-1, 2), C(-1, -1), D(-3, -1)$ and $F(6, 4), G(2, 4), H(2, -2), J(6, -2)$
- $P(-2, 2), Q(-1, -1), R(1, -1), S(2, 2)$ and $W(2, 8), X(3, 3), Y(7, 3), Z(8, 8)$



Available at BigIdeasMath.com.

Math Practice

Interpret Results

When you need a dilation to obtain one figure from another, what does it tell you about the side lengths and angle measures of the figures?

- b. Is a scale drawing represented by any of the pairs of figures in part (a)? Explain your reasoning.
- c. Figure A is a scale drawing of Figure B. Do you think there must be a sequence of transformations that obtains Figure A from Figure B? Explain your reasoning.

2.6 Lesson

Key Vocabulary

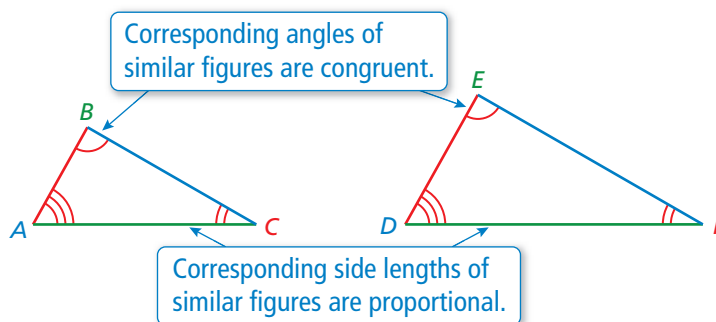
similarity transformation, p. 78
similar figures, p. 78

Dilations do not preserve length, so dilations are not rigid motions. A **similarity transformation** is a dilation or a sequence of dilations and rigid motions.

Key Idea

Similar Figures

Two figures are **similar figures** when one can be obtained from the other by a similarity transformation. Similar figures have the same shape but not necessarily the same size. The triangles below are similar.



Side Lengths

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

Angles

$$\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$$

Reading

The symbol \sim means *is similar to*.

In the Key Idea,

$$\triangle ABC \sim \triangle DEF.$$

EXAMPLE 1 Identifying Similar Figures

Determine whether $\triangle ABC$ and $\triangle JKL$ are similar.

Compare the coordinates of the vertices.

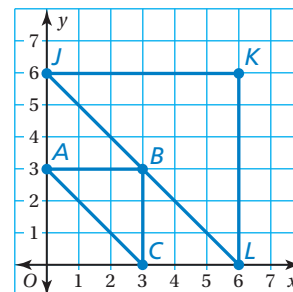
$$A(0, 3) \longrightarrow A'(2 \cdot 0, 2 \cdot 3) \longrightarrow J(0, 6)$$

$$B(3, 3) \longrightarrow B'(2 \cdot 3, 2 \cdot 3) \longrightarrow K(6, 6)$$

$$C(3, 0) \longrightarrow C'(2 \cdot 3, 2 \cdot 0) \longrightarrow L(6, 0)$$

$\triangle JKL$ is a dilation of $\triangle ABC$ using a scale factor of 2.

► So, $\triangle ABC$ and $\triangle JKL$ are similar.



Common Error

When writing a similarity statement, make sure to list the vertices of the figures in the correct order.

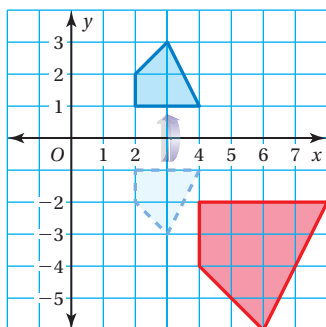
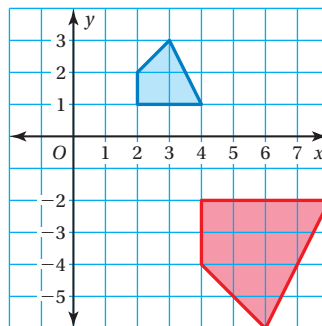
Try It

1. A triangle has vertices $D(0, 4)$, $E(5, 4)$, and $F(5, 0)$. Is $\triangle DEF$ similar to $\triangle ABC$ and $\triangle JKL$ in Example 1? Explain.

EXAMPLE 2

Describing a Similarity Transformation

The red figure is similar to the blue figure. Describe a similarity transformation between the figures.



By comparing corresponding side lengths, you can see that the blue figure is one-half the size of the red figure. So, begin by dilating the red figure with respect to the origin using a scale factor of $\frac{1}{2}$.

After dilating the red figure, you need to reflect the figure in the x -axis.

▶ So, one possible similarity transformation is to dilate the red figure with respect to the origin using a scale factor of $\frac{1}{2}$ and then reflect the image in the x -axis.

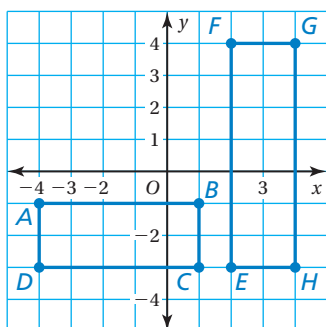
Try It

- Can you reflect the red figure first, and then perform the dilation to obtain the blue figure? Explain.

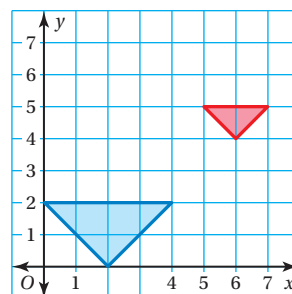


Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

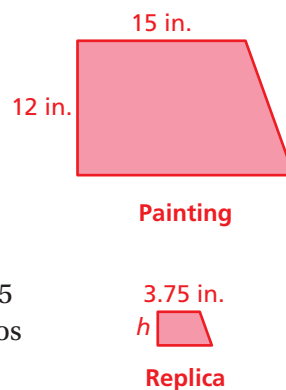


- IDENTIFYING SIMILAR FIGURES** In the coordinate plane at the left, determine whether Rectangle $ABCD$ is similar to Rectangle $EFGH$. Explain your reasoning.
- SIMILARITY TRANSFORMATION** The red triangle is similar to the blue triangle. Describe a similarity transformation between the figures.



EXAMPLE 3**Modeling Real Life**

An artist draws a replica of a painting that is on a remaining piece of the Berlin Wall. The painting includes a red trapezoid. The shorter base of the similar trapezoid in the replica is 3.75 inches. What is the height h of the trapezoid in the replica?



Because the trapezoids are similar, corresponding side lengths are proportional. So, the ratios $3.75 : 15$ and $h : 12$ are equivalent. Use the values of the ratios to write and solve a proportion to find h .

$$\frac{3.75}{15} = \frac{h}{12}$$

Write a proportion.

$$12 \cdot \frac{3.75}{15} = 12 \cdot \frac{h}{12}$$

Multiplication Property of Equality

$$3 = h$$

Simplify.

▶ So, the height of the trapezoid in the replica is 3 inches.

Another Method The replica is a scale drawing of the painting with a scale factor of $\frac{3.75}{15} = \frac{1}{4}$. So, the height of the trapezoid in the replica is $\frac{1}{4}$ the height of the trapezoid in the painting, $\frac{1}{4}(12) = 3$ inches. ✓

**Self-Assessment for Problem Solving**

Solve each exercise. Then rate your understanding of the success criteria in your journal.



- A medical supplier sells gauze in large and small rectangular sheets. A large sheet has a length of 9 inches and an area of 45 square inches. A small sheet has a length of 4 inches and a width of 3 inches. Are the sheets similar? Justify your answer.
- The sail on a souvenir boat is similar in shape to the sail on a sailboat. The sail on the sailboat is in the shape of a right triangle with a base of 9 feet and a height of 24 feet. The height of the souvenir's sail is 3 inches. What is the base of the souvenir's sail?
- DIG DEEPER!** A coordinate plane is used to represent a cheerleading formation. The vertices of the formation are $A(-4, 4)$, $B(0, 8)$, $C(4, 4)$, and $D(0, 6)$. A choreographer creates a new formation similar to the original formation. Three vertices of the new formation are $J(-2, -2)$, $K(0, -4)$, and $L(2, -2)$. What is the location of the fourth vertex? Explain.

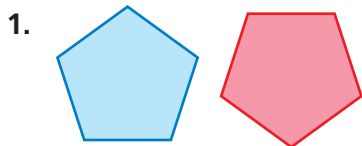
2.6 Practice



Go to BigIdeasMath.com to get HELP with solving the exercises.

► Review & Refresh

Tell whether the blue figure is a dilation of the red figure.



3. You solve the equation $S = \ell w + 2wh$ for w . Which equation is correct?

A. $w = \frac{S - \ell}{2h}$

B. $w = \frac{S - 2h}{\ell}$

C. $w = \frac{S}{\ell + 2h}$

D. $w = S - \ell - 2h$

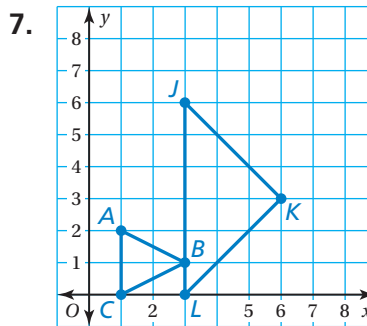
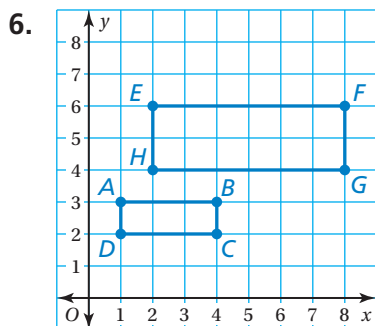
► Concepts, Skills, & Problem Solving

TRANSFORMING FIGURES The vertices of a pair of figures are given. Determine whether a scale drawing is represented by the pair of figures. (See Exploration 1, p. 77.)

4. $A(-8, -2), B(-4, 2), C(-4, -2)$ and $G(2, -1), H(4, -1), J(2, -3)$

5. $A(0, 3), B(3, 4), C(5, 3), D(3, 2)$ and $F(-4, 4), G(-1, 5), H(5, 3), J(3, 2)$

IDENTIFYING SIMILAR FIGURES Determine whether the figures are similar. Explain your reasoning.



IDENTIFYING SIMILAR FIGURES Draw the figures with the given vertices in a coordinate plane. Which figures are similar? Explain your reasoning.

8. Rectangle A: $(0, 0), (4, 0), (4, 2), (0, 2)$

Rectangle B: $(0, 0), (-6, 0), (-6, 3), (0, 3)$

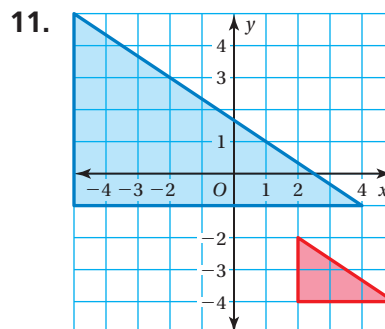
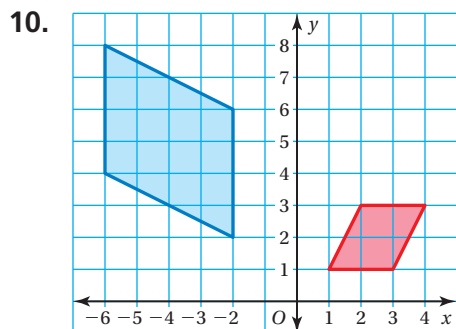
Rectangle C: $(0, 0), (4, 0), (4, 2), (0, 2)$

9. Figure A: $(-4, 2), (-2, 2), (-2, 0), (-4, 0)$

Figure B: $(1, 4), (4, 4), (4, 1), (1, 1)$

Figure C: $(2, -1), (5, -1), (5, -3), (2, -3)$

DESCRIBING A SIMILARITY TRANSFORMATION The red figure is similar to the blue figure. Describe a similarity transformation between the figures.



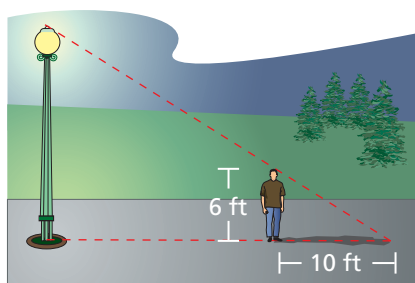
12. **MODELING REAL LIFE** A barrier in the shape of a rectangle is used to retain oil spills. On a blueprint, a similar barrier is 9 inches long and 2 inches wide. The width of the actual barrier is 1.2 miles. What is the length of the actual barrier?

13. **MP LOGIC** Are the following figures *always*, *sometimes*, or *never* similar? Explain.
 a. two triangles b. two squares c. two rectangles

14. **CRITICAL THINKING** Can you draw two quadrilaterals each having two 130° angles and two 50° angles that are *not* similar? Justify your answer.

15. **MP REASONING** The sign is rectangular.

- You increase each side length by 20%. Is the new sign similar to the original? Explain your reasoning.
- You increase each side length by 6 inches. Is the new sign similar to the original? Explain your reasoning.

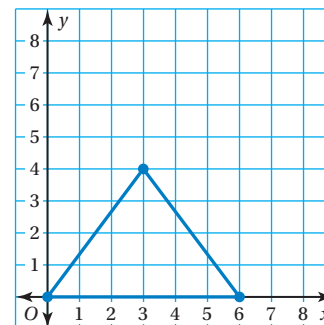


16. **DIG DEEPER!** A person standing 20 feet from a streetlight casts a shadow as shown. How many times taller is the streetlight than the person? Assume the triangles are similar.

17. **GEOMETRY** Use a ruler to draw two different isosceles triangles similar to the one shown. Measure the heights of each triangle.

- Are the ratios of the corresponding heights equivalent to the ratios of the corresponding side lengths?
- Do you think this is true for all similar triangles? Explain.

18. **CRITICAL THINKING** Given $\triangle ABC \sim \triangle DEF$ and $\triangle DEF \sim \triangle JKL$, is $\triangle ABC \sim \triangle JKL$? Justify your answer.



2.7 Perimeters and Areas of Similar Figures

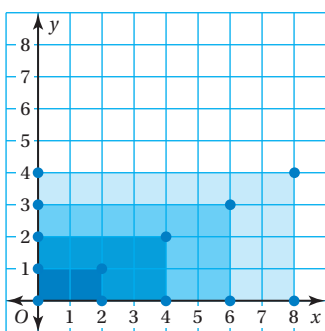
Learning Target: Find perimeters and areas of similar figures.

- Success Criteria:**
- I can use corresponding side lengths to compare perimeters of similar figures.
 - I can use corresponding side lengths to compare areas of similar figures.
 - I can use similar figures to solve real-life problems involving perimeter and area.

EXPLORATION 1

Comparing Similar Figures

Work with a partner. Draw a rectangle in the coordinate plane.



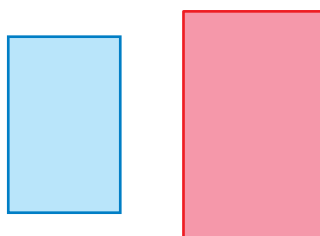
- a. Dilate your rectangle using each indicated scale factor k . Then complete the table for the perimeter P of each rectangle. Describe the pattern.

Original Side Lengths	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$
$P =$ 					

- b. Compare the ratios of the perimeters to the ratios of the corresponding side lengths. What do you notice?
- c. Repeat part (a) to complete the table for the area A of each rectangle. Describe the pattern.

Original Side Lengths	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$
$A =$ 					

- d. Compare the ratios of the areas to the ratios of the corresponding side lengths. What do you notice?
- e. The rectangles shown are similar. You know the perimeter and the area of the red rectangle and a pair of corresponding side lengths. How can you find the perimeter of the blue rectangle? the area of the blue rectangle?



Math Practice

Look for Patterns

How can you use the pattern in part (c) to find the area of the rectangle after a dilation using any scale factor?

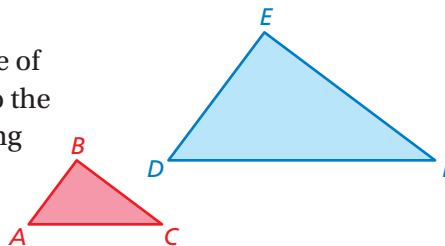
2.7 Lesson

Key Idea

Perimeters of Similar Figures

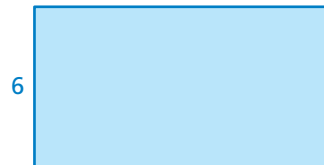
When two figures are similar, the value of the ratio of their perimeters is equal to the value of the ratio of their corresponding side lengths.

$$\frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle DEF} = \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$



EXAMPLE 1 Comparing Perimeters of Similar Figures

Find the value of the ratio (red to blue) of the perimeters of the similar rectangles.



$$\frac{\text{Perimeter of red rectangle}}{\text{Perimeter of blue rectangle}} = \frac{4}{6} = \frac{2}{3}$$

▶ The value of the ratio of the perimeters is $\frac{2}{3}$.

Try It

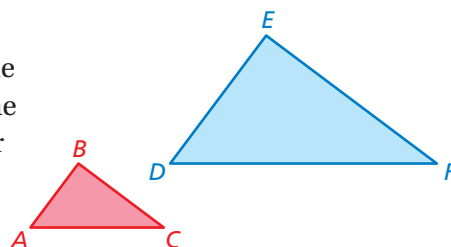
- The height of Figure A is 9 feet. The height of a similar Figure B is 15 feet. What is the value of the ratio of the perimeter of A to the perimeter of B?

Key Idea

Areas of Similar Figures

When two figures are similar, the value of the ratio of their areas is equal to the square of the value of the ratio of their corresponding side lengths.

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$$

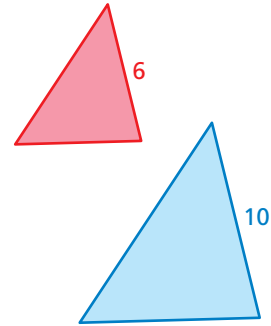


You can think of the red rectangle as a scale drawing of the blue rectangle, where the ratio of the side lengths is the scale, and the value of the ratio is the scale factor.

EXAMPLE 2**Comparing Areas of Similar Figures**

Find the value of the ratio (red to blue) of the areas of the similar triangles.

$$\begin{aligned} \frac{\text{Area of red triangle}}{\text{Area of blue triangle}} &= \left(\frac{6}{10}\right)^2 \\ &= \left(\frac{3}{5}\right)^2 \\ &= \frac{9}{25} \end{aligned}$$



► The value of the ratio of the areas is $\frac{9}{25}$.

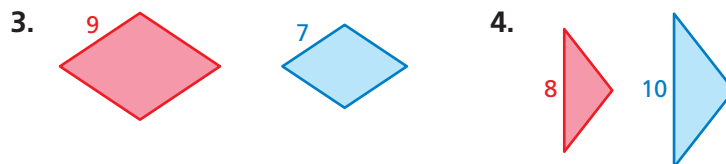
Try It

- The base of Triangle P is 8 meters. The base of a similar Triangle Q is 7 meters. What is the value of the ratio of the area of P to the area of Q?

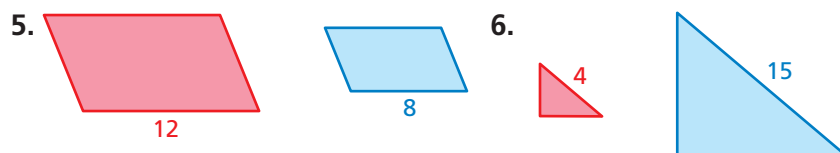
**Self-Assessment for Concepts & Skills**

Solve each exercise. Then rate your understanding of the success criteria in your journal.

COMPARING PERIMETERS OF SIMILAR FIGURES Find the value of the ratio (red to blue) of the perimeters of the similar figures.



COMPARING AREAS OF SIMILAR FIGURES Find the value of the ratio (red to blue) of the areas of the similar figures.

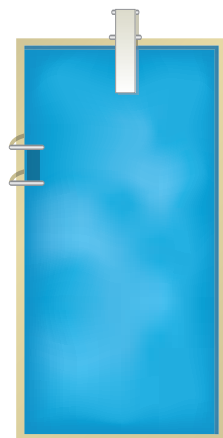


EXAMPLE 3

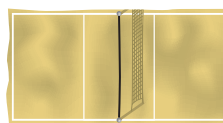
Modeling Real Life

A swimming pool is similar in shape to a volleyball court. Find the perimeter P and the area A of the pool.

The rectangular pool and the court are similar. So, use the ratio of corresponding side lengths to write and solve proportions to find the perimeter and the area of the pool.



18 yd



10 yd

Area = 200 yd²
Perimeter = 60 yd

Perimeter

$$\frac{\text{Perimeter of court}}{\text{Perimeter of pool}} = \frac{\text{Width of court}}{\text{Width of pool}}$$

$$\frac{60}{P} = \frac{10}{18}$$

$$1080 = 10P$$

$$108 = P$$

Area

$$\frac{\text{Area of court}}{\text{Area of pool}} = \left(\frac{\text{Width of court}}{\text{Width of pool}} \right)^2$$

$$\frac{200}{A} = \left(\frac{10}{18} \right)^2$$

$$\frac{200}{A} = \frac{100}{324}$$

$$64,800 = 100A$$

$$648 = A$$

So, the perimeter of the pool is 108 yards, and the area is 648 square yards.



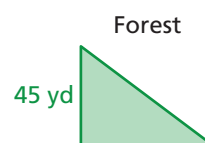
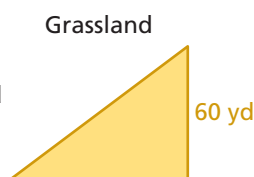
Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

7. Two similar triangular regions are prepared for development.



Grassland Perimeter = 240 yd
Grassland Area = 2400 yd²



- It costs \$6 per foot to install fencing. How much does it cost to surround the forest with a fence?
 - The cost to prepare 1 square yard of grassland is \$15 and the cost to prepare 1 square yard of forest is \$25. Which region costs more to prepare? Justify your answer.
8. **DIG DEEPER!** You buy a new television with a screen similar in shape to your old television screen, but with an area four times greater. The size of a television screen is often described using the distance between opposite corners of the screen. Your old television has a 30-inch screen. What is the size of your new television screen? Explain.

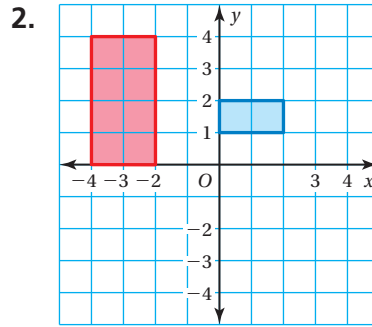
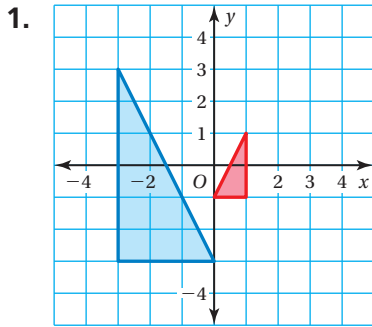
2.7 Practice



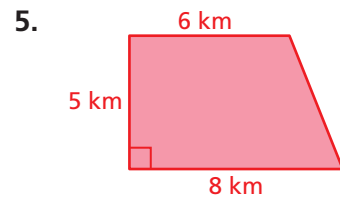
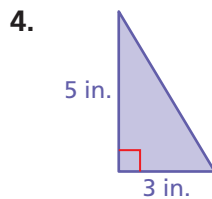
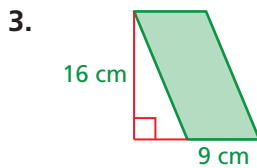
Go to BigIdeasMath.com to get HELP with solving the exercises.

► Review & Refresh

The red figure is similar to the blue figure. Describe a similarity transformation between the figures.



Find the area of the figure.

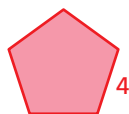
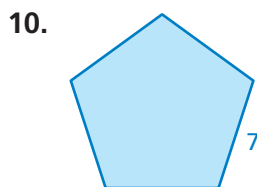
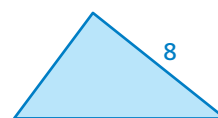


► Concepts, Skills, & Problem Solving

COMPARING SIMILAR FIGURES Dilate the figure using the indicated scale factor k . What is the value of the ratio (new to original) of the perimeters? the areas? (See Exploration 1, p. 83.)

- a triangle with vertices $(0, 0)$, $(0, 2)$, and $(2, 0)$; $k = 3$
- a square with vertices $(0, 0)$, $(0, 4)$, $(4, 4)$, and $(4, 0)$; $k = 0.5$

PERIMETERS AND AREAS OF SIMILAR FIGURES Find the values of the ratios (red to blue) of the perimeters and areas of the similar figures.

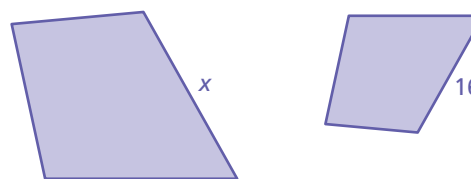


USING SIMILAR FIGURES The figures are similar. Find x .

12. The ratio of the perimeters is 7 : 10.



13. The ratio of the perimeters is 8 : 5.



14. **COMPARING AREAS** The playing surfaces of two foosball tables are similar. The ratio of the corresponding side lengths is 10 : 7. What is the ratio of the areas?

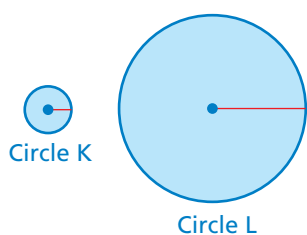
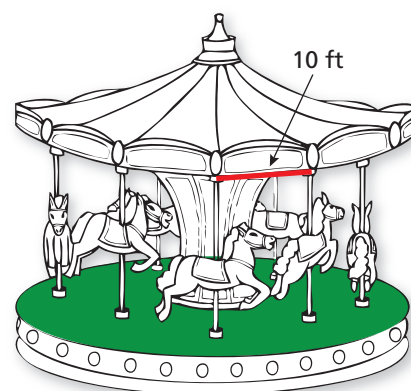


15. **CRITICAL THINKING** The ratio of the side length of Square A to the side length of Square B is 4 : 9. The side length of Square A is 12 yards. What is the perimeter of Square B?

16. **MODELING REAL LIFE** The cost of the piece of fabric shown is \$1.31. What would you expect to pay for a similar piece of fabric that is 18 inches by 42 inches?

17. **MP PROBLEM SOLVING** A scale model of a merry-go-round and the actual merry-go-round are similar.

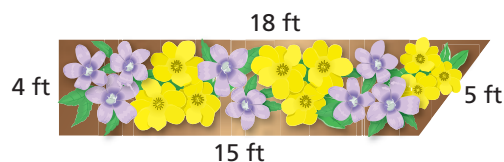
- How many times greater is the base area of the actual merry-go-round than the base area of the scale model? Explain.
- What is the base area of the actual merry-go-round in square feet?



18. **MP STRUCTURE** The circumference of Circle K is π . The circumference of Circle L is 4π . What is the value of the ratio of their circumferences? of their radii? of their areas?

19. **GEOMETRY** A triangle with an area of 10 square meters has a base of 4 meters. A similar triangle has an area of 90 square meters. What is the height of the larger triangle?

20. **MP PROBLEM SOLVING** You need two bottles of fertilizer to treat the flower garden shown. How many bottles do you need to treat a similar garden with a perimeter of 105 feet?



21. **MP REPEATED REASONING** Three square mirrors are used for a light reflection experiment. The ratio of the side length of Mirror A to the side length of Mirror B is 5 : 6. The ratio of the area of Mirror B to the area of Mirror C is 16 : 25. The perimeter of Mirror C is 280 centimeters. What is the area of Mirror A? Justify your answer.

2

Connecting Concepts

Using the Problem-Solving Plan

1. A scale drawing of a helipad uses a scale of 1 ft : 20 ft. The scale drawing has an area of 6.25 square feet. What is the area of the actual helipad?

Understand the problem.

You know the scale of the drawing and the area of the helipad in the drawing. You are asked to find the area of the actual helipad.

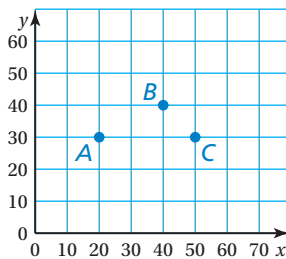


Make a plan.

A scale drawing is similar to the actual object. So, use the scale 1 ft : 20 ft and the ratio $6.25 \text{ ft}^2 : A \text{ ft}^2$ to write and solve a proportion that represents the area A of the actual helipad.

Solve and check.

Use the plan to solve the problem. Then check your solution.



2. The locations of three cargo ships are shown in the coordinate plane. Each ship travels at the same speed in the same direction. After 1 hour, the x - and y -coordinates of Ship A increase 80%. Use a translation to describe the change in the locations of the ships. Then find the new coordinates of each ship.



3. All circles are similar. A circle with a radius of 2 inches is dilated, resulting in a circle with a circumference of 22π inches. What is the scale factor? Justify your answer.

Performance Task

Master Puppeteer

At the beginning of this chapter, you watched a STEAM Video called "Shadow Puppets." You are now ready to complete the performance task related to this video, available at BigIdeasMath.com. Be sure to use the problem-solving plan as you work through the performance task.





► Review Vocabulary

Write the definition and give an example of each vocabulary term.

transformation, p. 44

image, p. 44

translation, p. 44

reflection, p. 50

line of reflection, p. 50

rotation, p. 56

center of rotation, p. 56

angle of rotation, p. 56

rigid motion, p. 64

congruent figures, p. 64

congruent angles, p. 64

congruent sides, p. 64

dilation, p. 70

center of dilation, p. 70

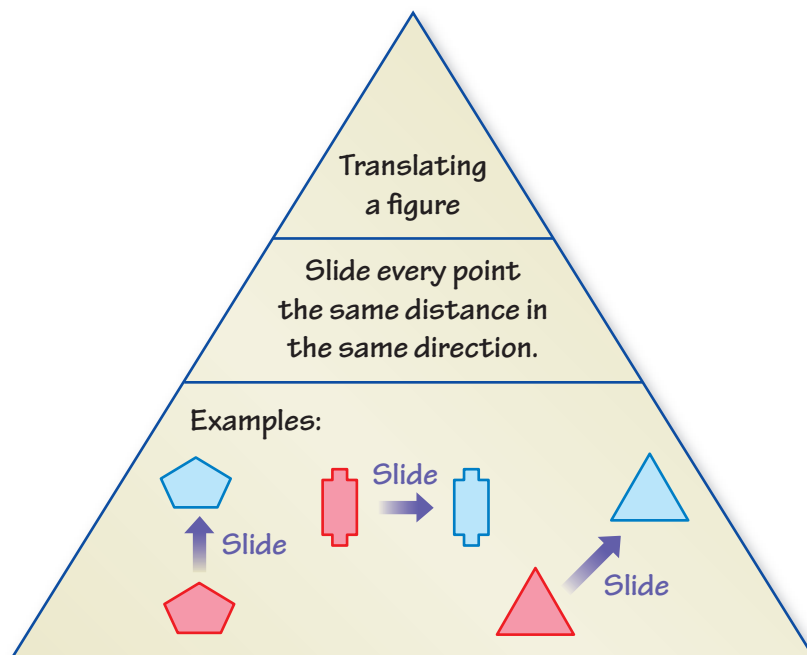
scale factor, p. 70

similarity transformation, p. 78

similar figures, p. 78

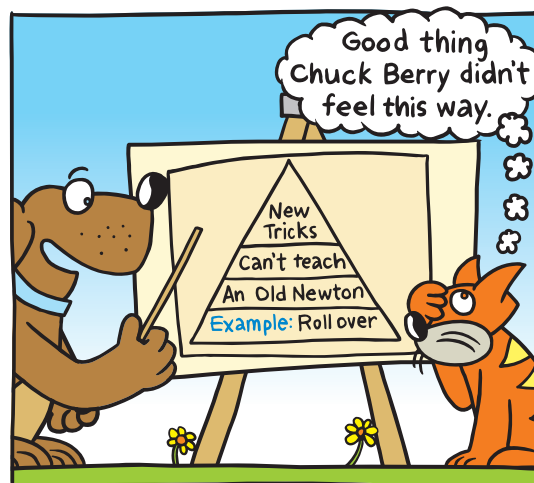
► Graphic Organizers

You can use a **Summary Triangle** to explain a concept. Here is an example of a Summary Triangle for *translating a figure*.



Choose and complete a graphic organizer to help you study the concept.

1. reflecting a figure
2. rotating a figure
3. congruent figures
4. dilating a figure
5. similar figures
6. perimeters of similar figures
7. areas of similar figures



"I hope my owner sees my **Summary Triangle**. I just can't seem to learn 'roll over.'"

▶ Chapter Self-Assessment

As you complete the exercises, use the scale below to rate your understanding of the success criteria in your journal.

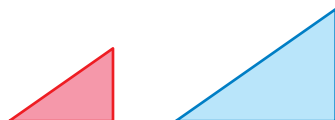


2.1 Translations (pp. 43–48)

Learning Target: Translate figures in the coordinate plane.

Tell whether the blue figure is a translation of the red figure.

1.



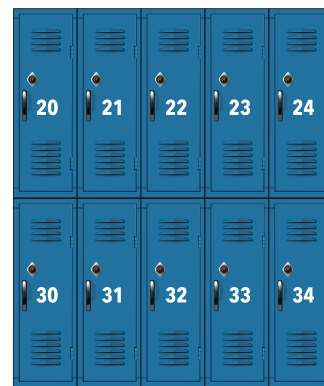
2.



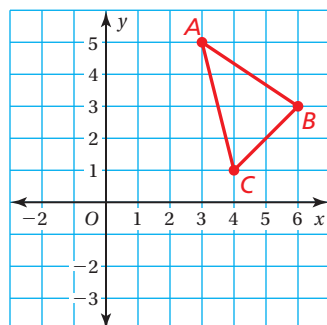
3. The vertices of a quadrilateral are $W(1, 2)$, $X(1, 4)$, $Y(4, 4)$, and $Z(4, 2)$. Draw the figure and its image after a translation 3 units left and 2 units down.

4. The vertices of a triangle are $A(-1, -2)$, $B(-2, 2)$, and $C(-3, 0)$. Draw the figure and its image after a translation 5 units right and 1 unit up.

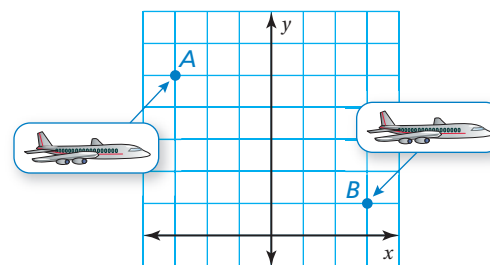
5. Your locker number is 20 and your friend's locker number is 33. Describe the location of your friend's locker relative to the location of your locker.



6. Translate the triangle 4 units left and 1 unit down. What are the coordinates of the image?



7. Describe a translation of the airplane from point A to point B.

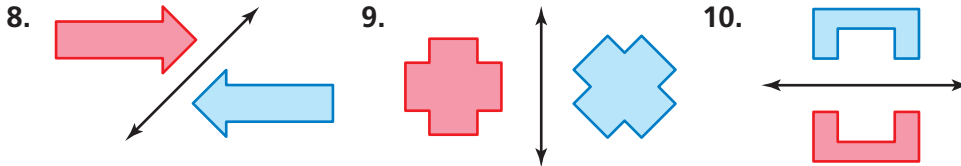




2.2 Reflections (pp. 49–54)

Learning Target: Reflect figures in the coordinate plane.

Tell whether the blue figure is a reflection of the red figure.



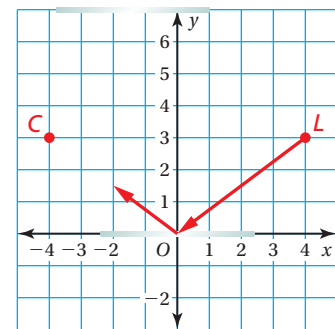
Draw the figure and its reflection in (a) the x -axis and (b) the y -axis. Identify the coordinates of the image.

11. $A(2, 0), B(1, 5), C(4, 3)$ 12. $D(-5, -5), E(-5, 0), F(-2, -2), G(-2, -5)$
13. The vertices of a rectangle are $E(-1, 1), F(-1, 3), G(-5, 3), H(-5, 1)$. Find the coordinates of the figure after reflecting in the x -axis, and then translating 3 units right.

The coordinates of a point and its image after a reflection are given. Identify the line of reflection.

14. $(-1, -3) \rightarrow (1, -3)$ 15. $(2, 1) \rightarrow (2, -1)$

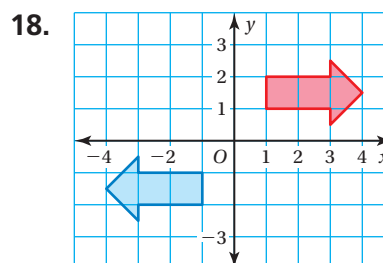
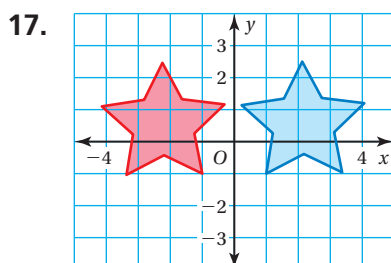
16. You perform an experiment involving angles of refraction with a laser pen. You point a laser pen from point L at a mirror along the red path and the image is a reflection in the y -axis.
- Does the light reach a cat at point C ? Explain.
 - You bounce the light off the top mirror so its path is a reflection. What line of reflection is needed for the light to reach the cat?



2.3 Rotations (pp. 55–62)

Learning Target: Rotate figures in the coordinate plane.

Tell whether the blue figure is a rotation of the red figure about the origin. If so, give the angle and the direction of rotation.



The vertices of a triangle are $A(-4, 2)$, $B(-2, 2)$, and $C(-3, 4)$. Rotate the triangle as described. Find the coordinates of the image.

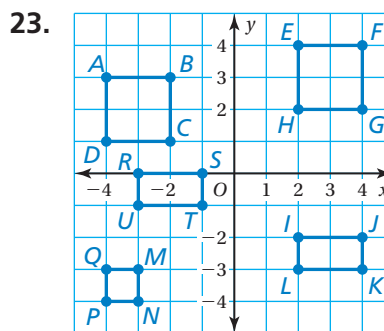
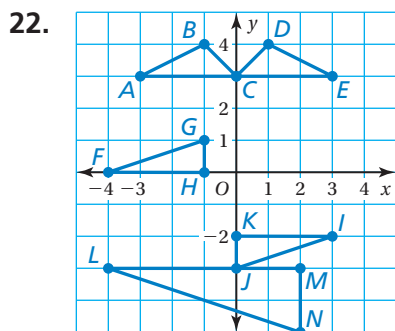
19. 180° about the origin
20. 270° clockwise about the origin
21. A bicycle wheel is represented in a coordinate plane with the center of the wheel at the origin. Reflectors are placed on the bicycle wheel at points $(7, 4)$ and $(-5, -6)$. After a bike ride, the reflectors have rotated 90° counterclockwise about the origin. What are the locations of the reflectors at the end of the bike ride?



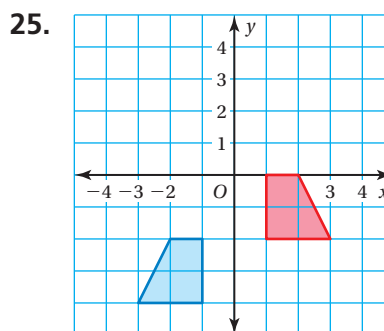
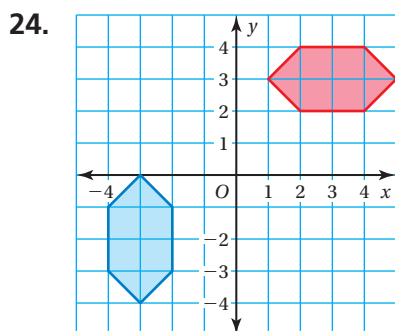
2.4 Congruent Figures (pp. 63–68)

Learning Target: Understand the concept of congruent figures.

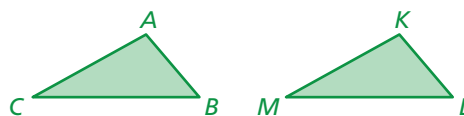
Identify any congruent figures in the coordinate plane.



The red figure is congruent to the blue figure. Describe a sequence of rigid motions between the figures.

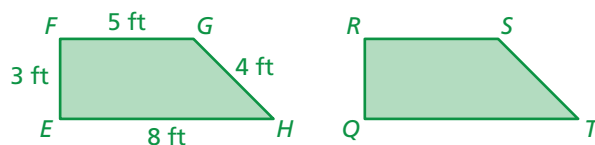


26. The figures are congruent. Name the corresponding angles and the corresponding sides.



27. Trapezoids $EFGH$ and $QRST$ are congruent.

- What is the length of side QR ?
- Which angle in $QRST$ corresponds to $\angle H$?
- What is the perimeter of $QRST$?





2.5 Dilations (pp. 69–76)

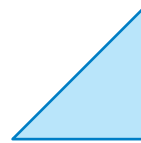
Learning Target: Dilate figures in the coordinate plane.

Tell whether the blue figure is a dilation of the red figure.

28.



29.



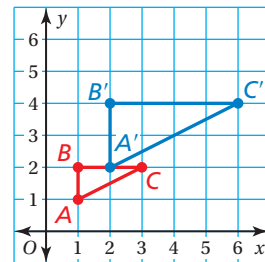
The vertices of a figure are given. Draw the figure and its image after a dilation with the given scale factor. Identify the type of dilation.

30. $P(-3, -2), Q(-3, 0), R(0, 0); k = 4$

31. $B(3, 3), C(3, 6), D(6, 6), E(6, 3); k = \frac{1}{3}$

32. The blue figure is a dilation of the red figure. Identify the type of dilation and find the scale factor.

33. The vertices of a rectangle are $Q(-6, 2), R(6, 2), S(6, -4),$ and $T(-6, -4)$. Dilate the rectangle with respect to the origin using a scale factor of $\frac{3}{2}$. Then translate it 5 units right and 1 unit down. What are the coordinates of the image?



2.6 Similar Figures (pp. 77–82)

Learning Target: Understand the concept of similar figures.

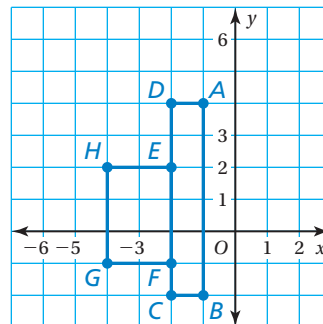
34. Determine whether the two figures are similar. Explain your reasoning.

35. Draw figures with the given vertices in a coordinate plane. Which figures are similar? Explain your reasoning.

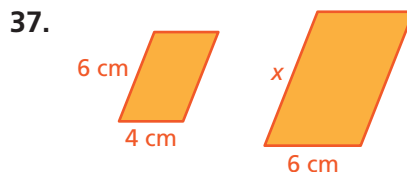
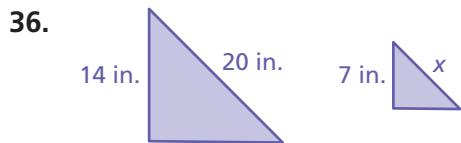
Triangle A: $(-4, 4), (-2, 4), (-2, 0)$

Triangle B: $(-2, 2), (-1, 2), (-1, 0)$

Triangle C: $(6, 6), (3, 6), (3, 0)$



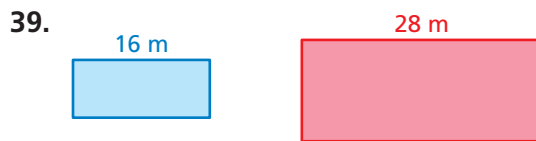
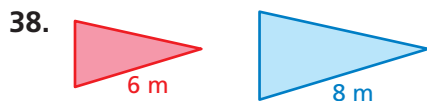
The figures are similar. Find x .



2.7 Perimeters and Areas of Similar Figures (pp. 83–88)

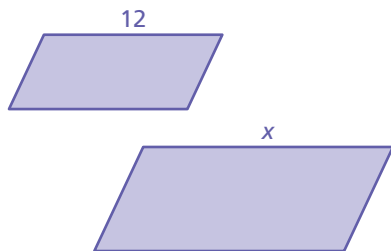
Learning Target: Find perimeters and areas of similar figures.

Find the values of the ratios (red to blue) of the perimeters and areas of the similar figures.

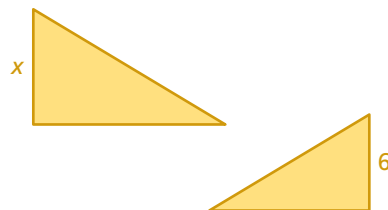


The figures are similar. Find x .

40. The ratio of the perimeters is 5 : 7.



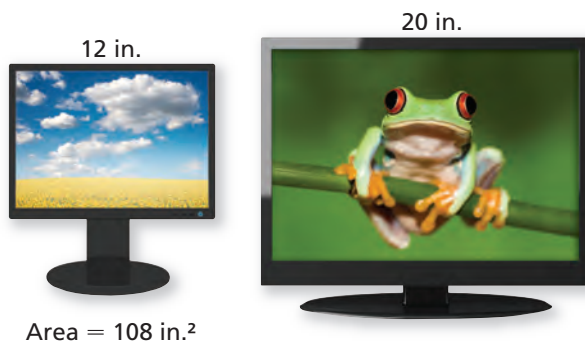
41. The ratio of the perimeters is 6 : 5.



42. Two photos are similar. The ratio of the corresponding side lengths is 3 : 4. What is the ratio of the areas?

43. The ratio of side lengths of Square A to Square B is 2 : 3. The perimeter of Square A is 16 inches. What is the area of Square B?

44. The TV screen is similar to the computer screen. What is the area of the TV screen?

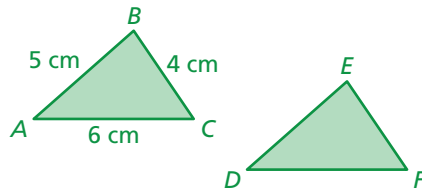


2

Practice Test

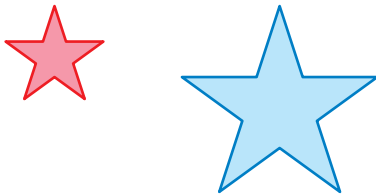
Triangles ABC and DEF are congruent.

- Which angle of DEF corresponds to $\angle C$?
- What is the perimeter of DEF ?

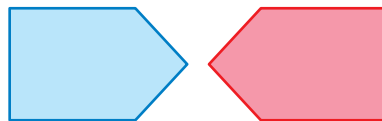


Tell whether the blue figure is a *translation*, *reflection*, *rotation*, or *dilation* of the red figure.

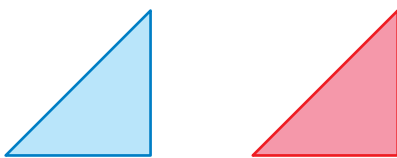
3.



4.



5.

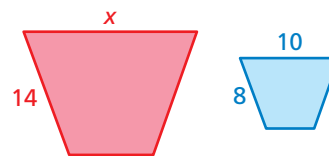


6.

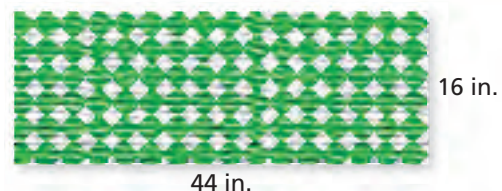


The vertices of a triangle are $A(2, 5)$, $B(1, 2)$, and $C(3, 1)$. Find the coordinates of the image after the transformations given.

- Reflect in the y -axis.
- Rotate 90° clockwise about the origin.
- Reflect in the x -axis, and then rotate 90° counterclockwise about the origin.
- Dilate with respect to the origin using a scale factor of 2. Then translate 2 units left and 1 unit up.
- In a coordinate plane, draw Rectangle A: $(-4, 4)$, $(0, 4)$, $(0, 2)$, $(-4, 2)$; Rectangle B: $(-2, 2)$, $(0, 2)$, $(0, 1)$, $(-2, 1)$; and Rectangle C: $(-6, 6)$, $(0, 6)$, $(0, 3)$, $(-6, 3)$. Which figures are similar? Explain your reasoning.
- Translate a point (x, y) 3 units left and 5 units up. Then translate the image 5 units right and 2 units up. What are the coordinates of the point after the translations?
- The two figures are similar. (a) Find the value of x . (b) Find the values of the ratios (red to blue) of the perimeters and of the areas.



- A wide-screen television measures 36 inches by 54 inches. A movie theater screen measures 42 feet by 63 feet. Are the screens similar? Explain.
- You want to use the rectangular piece of fabric shown to make a pair of curtains for your window. Name the types of congruent shapes you can make with one straight cut. Draw an example of each type.



2

Cumulative Practice



- A clockwise rotation of 90° is equivalent to a counterclockwise rotation of how many degrees?
- The formula $K = C + 273.15$ converts temperatures from degrees Celsius C to Kelvin K . Which of the following formulas is *not* correct?

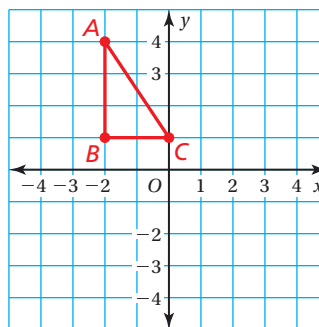
 - $K - C = 273.15$
 - $C = K - 273.15$
 - $C - K = -273.15$
 - $C = K + 273.15$
- You want to solve the equation $-3(x + 2) = 12x$. What should you do first?

 - Subtract 2 from each side.
 - Add 3 to each side.
 - Multiply each side by -3 .
 - Divide each side by -3 .
- Which value of x makes the equation $\frac{3}{4}x = 12$ true?

 - 9
 - $11\frac{1}{4}$
 - 16
 - 48

- A triangle is graphed in the coordinate plane. What are the coordinates of the image after a translation 3 units right and 2 units down?

- $A'(1, 4), B'(1, 1), C'(3, 1)$
- $A'(1, 2), B'(1, -1), C'(3, -1)$
- $A'(-2, 2), B'(-2, -1), C'(0, -1)$
- $A'(0, 1), B'(0, -2), C'(2, -2)$



6. Your friend solved the equation in the box shown. What should your friend do to correct the error that he made?

- A. Add $\frac{2}{5}$ to each side to get $-\frac{x}{3} = -\frac{1}{15}$.
- B. Multiply each side by -3 to get $x + \frac{2}{5} = \frac{7}{5}$.
- C. Multiply each side by -3 to get $x = 2\frac{3}{5}$.
- D. Subtract $\frac{2}{5}$ from each side to get $-\frac{x}{3} = -\frac{5}{10}$.

$$-\frac{x}{3} + \frac{2}{5} = -\frac{7}{15}$$

$$-\frac{x}{3} + \frac{2}{5} - \frac{2}{5} = -\frac{7}{15} - \frac{2}{5}$$

$$-\frac{x}{3} = -\frac{13}{15}$$

$$3 \cdot \left(-\frac{x}{3}\right) = 3 \cdot \left(-\frac{13}{15}\right)$$

$$x = -2\frac{3}{5}$$

7. Your teacher dilates the rectangle using a scale factor of $\frac{1}{2}$.



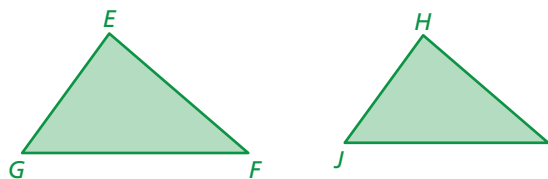
What is the area of the dilated rectangle in square inches?

8. Your cousin earns \$9.25 an hour at work. Last week she earned \$222.00. How many hours did she work last week?

- F. $\frac{1}{24}$ hour
- G. 22 hours
- H. 24 hours
- I. 212.75 hours

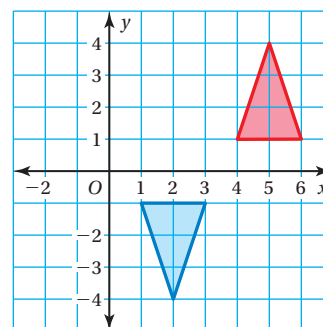
9. Triangle EFG is a dilation of Triangle HIJ .

Which proportion is *not* true for Triangle EFG and Triangle HIJ ?



- A. $\frac{EF}{FG} = \frac{HI}{IJ}$
- B. $\frac{EG}{HI} = \frac{FG}{IJ}$
- C. $\frac{GE}{EF} = \frac{JH}{HI}$
- D. $\frac{EF}{HI} = \frac{GE}{JH}$

10. The red figure is congruent to the blue figure. Which of the following is a sequence of rigid motions between the figures?

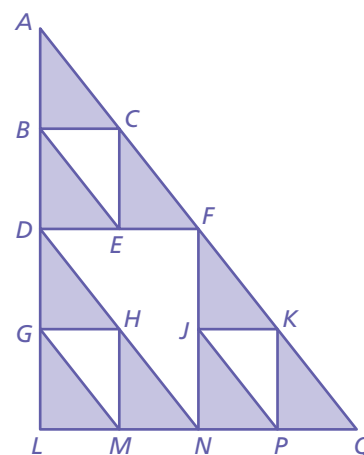


- F. Reflect the red triangle in the x -axis, and then translate it 3 units left.
- G. Reflect the red triangle in the x -axis, and then translate it 3 units right.
- H. Reflect the red triangle in the y -axis, and then translate it 3 units left.
- I. Rotate the red triangle 90° clockwise about the origin.

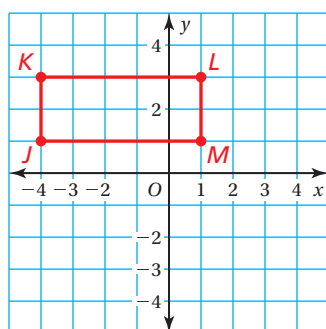
11. Several transformations are used to create the pattern.

Think
Solve
Explain

- Part A* Describe the transformation of Triangle GLM to Triangle DGH .
- Part B* Describe the transformation of Triangle ALQ to Triangle GLM .
- Part C* Triangle DFN is a dilation of Triangle GHM . Find the scale factor.



12. A rectangle is graphed in the coordinate plane.



What are the coordinates of the image after a reflection in the y -axis?

- A. $J'(4, 1), K'(4, 3), L'(-1, 3), M'(-1, 1)$
- B. $J'(-4, -1), K'(-4, -3), L'(1, -3), M'(1, -1)$
- C. $J'(1, 4), K'(3, 4), L'(3, -1), M'(1, -1)$
- D. $J'(-4, 1), K'(-4, 3), L'(1, 3), M'(1, 1)$